Homework 1 CS-712

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1 Problem 1

1.1 a : 4 - adjacent

As show in the **Table 1** for 4 - adjacent the answer is no, this is because $\forall i \in [0-3], S1[i][3] \neq S2[i][0]$. So the column 3 in S1 and the column 0 in s2 have no over laps with respect to $V = \{1\}$

1.2 b : 8 - adjacent

Ths is an adjacent connection show by the two underscored numbers in S1 and S2 ($S1[3][3] = S2[0][3]$ on the diagonal)

1.3 Union Question

$S1 \cup S2$ (as shown in the far right of Table 1) is both 4-adjacent, denoted by bold numbers and 8-adjacent denoted by underscored numbers.

Table 1: S1 || S2 ||| S1 ∪ S2

0	0	0	0	0	0	1	1	0	0	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}$
0	0	1	0	0	1	0	0	0	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}$	0
0	0	1	0	$\underline{1}$	1	0	0	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}$	0
0	1	1	$\underline{1}$	0	0	0	0	0	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}$

2 Problem 2

The images shown below are quite different, but their histograms are the same. Suppose that each image is blurred with a 3*3 averaging mask. Would the histograms of the blurred images still be equal? Please explain.

No, The left side blue effect will have pixes such as the following

0	0	1
0	0	1
0	0	1

0	1	1
0	1	1
0	1	1

how ever the right had side will get a blur filer of

1	0	0
0	1	1
0	1	1

which will yield a intensity that the left image won't have.

$\therefore$ the images will be different.

3 Problem 3

Consider the image segment shown

	3	1	2	1	(q)
	2	2	3	2	
	1	2	1	1	
(p)	1	0	1	2	

3.1 a : Let $V = \{0, 1\}$

	x	1	x	1	(q)
	$\underline{\mathbf{x}}$	$\underline{\mathbf{x}}$	$\underline{\mathbf{x}}$	$\underline{\mathbf{x}}$	
	1	x	1	1	
(p)	1	0	1	x	

There is no way for 4 -adjacent or 8 - adjacent to get from $p \rightarrow q$, as $row[2]$ has no members of the set as shown above in underscored $\underline{\text{x}}$

3.2 b : Let $V = \{1, 2\}$

$$
\begin{array}{ccccc}
 & \text{x} & \underline{\mathbf{1}} & \underline{\mathbf{2}} & \underline{\mathbf{1}} \quad \text{(q)} \\
 & \underline{\mathbf{2}} & \underline{\mathbf{2}} & \text{x} & \underline{\mathbf{2}} \\
 & \underline{\mathbf{1}} & \underline{\mathbf{2}} & \underline{\mathbf{1}} & \underline{\mathbf{1}} \\
\text{(p)} & \underline{\mathbf{1}} & \text{x} & 1 & 2
\end{array}
$$

A path exists for both **4-adjacent** and **8-adjacent** highlighted above.

4 Problem 4

Perform histogram equalization on the following histogram, where r is the intensity level and n is the number of pixels with the corresponding intensity level. You need to find out the histogram after the equalization.

r	n	$n \div (M \times N)$
0	400	0.09765625
1	700	0.170898438
2	800	0.1953125
3	900	0.219726563
4	500	0.122070313
5	400	0.09765625
6	196	0.047851563
7	200	0.048828125
$\sum$	4096	M = 64
$\sqrt{\sum}$ ::	64	N = 64

5 Problem 5

Give a 3*3 mask for performing unsharp masking in a single pass through an image. Assume that the average image is obtained using the filter below.

$$\frac{1}{9}\times \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

$f(x,y)$ = original image, $\bar{f}(x,y)$= blur image, $g_{mask} = f(x,y) - \bar{f}(x,y)$
$g(x,y) = f(x,y) + g_{mask} \Rightarrow g(x,y) = f(x,y) + f(x,y) - \bar{f}(x,y)$
$\therefore 2f(x,y) - \bar{f}(x,y)$

6 Problem 6

Textbook Problem 3.28 Show that subtracting the Laplacian from an image is proportional to unsharp masking. Use the definition for the Laplacian given below:

$\bigtriangledown^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$

$f(x,y) - \bigtriangledown^2 f(x,y) =$
$= f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)]$
$= 6f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) + 1f(x,y)]$
$= 5 \times [\frac{6}{5} \times f(x,y) -$
$\frac{1}{5} \times [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) + 1f(x,y)]]$

$= 5 \times [\frac{6}{5} f(x,y) - \bar{f}(x,y)]$
$\therefore f(x,y) - \bigtriangledown^2 f(x,y) \propto f(x,y) - \bar{f}(x,y)$

7 Problem 7

Textbook Problem 4.12 Consider a checkerboard image in which each square is 1mm * 1 mm. Assuming that the image extends infinitely in both coordinate directions, what is the minimum sampling rate (in samples/mm) required to avoid aliasing? (Hint; consider the image in the form of . . .

101010101 . . . .The period of this signal is 2mm.)

$P = 2mm; frequency = \frac{1}{period}$
$max_{frequency} = \frac{1}{p} \Rightarrow max_{frequency} = \frac{1}{2} = 0.5\frac{cycles}{mm}$
to avoid the aliasing $2 \times max_{frequency} = 2(0.5) = 1$

$\therefore$ each sample would need to exceed $\frac{1sample}{mm}$

8 Problem 8

Prove that the 1DFT satisfies

$$F(0) = \sum_{x=0}^{M-1} f_x$$

$$F(u) = \sum_{x=0}^{M-1} f_x \times e^{\frac{-j2\pi ux}{M}}$$

$e^{\frac{-j2\pi ux}{M}}$ where $u = 0 \Rightarrow e^0 = 1$

$$F(0) = \sum_{x=0}^{M-1} f_x \times 1 = \sum_{x=0}^{M-1} f_x \blacksquare$$