Lecture 2: From MDP Planning to RL Basics

CS234: RL

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Recap: Value Iteration (VI)

1. Initialize $V_0(s_i)$=0 for all states s_i,
2. Set k=1
3. Loop until [finite horizon, convergence]
   - For each state s,

$$V_{k+1}(s) = \max_a \left[r(s,a) + \gamma \sum_{s' \in S} p(s'|a,s) V_k(s') \right]$$

4. Extract Policy

Bellman backup

V_k is optimal value if horizon=k

1. Initialize $V_0(s_i)=0$ for all states s_i,
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3. Loop until [finite horizon, convergence]
   - For each state s,

$$V_{k+1}(s) = \max_a \left[r(s,a) + \gamma \sum_{s' \in S} p(s'|a,s) V_k(s') \right]$$

4. Extract Policy

Bellman backup

Value vs Policy Iteration

- Value iteration:
  - Compute optimal value if horizon=k
    - Note this can be used to compute optimal policy if horizon = k
  - Increment k
- Policy iteration:
  - Compute infinite horizon value of a policy
  - Use to select another (better) policy
  - Closely related to a very popular method in RL: policy gradient

Policy Iteration (PI)

1. i=0; Initialize $\pi_0(s)$ randomly for all states s
2. Converged = 0;
3. While i == 0 or $|\pi_i - \pi_{i-1}| > 0$
   - i=i+1 V^{π}
   - Policy evaluation
   - Policy improvement

Policy Evaluation

1. Use minor variant of value iteration

$$V_{k+1}(s) = \cancel{\max_a} \left[r(s,a) + \gamma \sum_{s' \in S} p(s'|s,a) V_k(s') \right]$$

Policy Evaluation

1. Use minor variant of value iteration

$$V_{k+1}(s) = \max_a \left[r(s,a) + \gamma \sum_{s' \in S} p(s'|s,a) V_k(s') \right]$$

$$V^{\pi}_{k+1}(s) = r(s,\pi(s)) + \gamma \sum_{s' \in S} p(s'|s,\pi(s)) V^{\pi}_k(s')$$

$\rightarrow$ restricts action to be one chosen by policy

Policy Evaluation

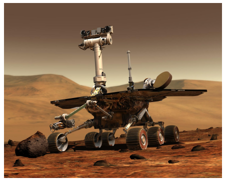
1. Use minor variant of value iteration

$$V_{k+1}(s) = \max_a \left[r(s,a) + \gamma \sum_{s' \in S} p(s'|s,a) V_k(s') \right]$$

$$V^{\pi}_{k+1}(s) = r(s,\pi(s)) + \gamma \sum_{s' \in S} p(s'|s,\pi(s)) V^{\pi}_k(s')$$

2. Analytic solution (for discrete set of states)
   - Set of linear equations (no max!)
   - Can write as matrices and solve directly for V

Policy Evaluation: Example

S1	S2	S3	S4	S5	S6	S7
Okay Field Site +1						Fantastic Field Site +10

- Deterministic actions of TryLeft or TryRight
- Reward: +1 in state S1, +10 in state S7, 0 otherwise
- Let $\pi_0(s)$=TryLeft for all states (e.g. always go left)
- Assume ϒ=0. What is the value of this policy in each s?

$$V_{k+1}^{\pi}(s) = r(s, \pi(s)) + \gamma \sum_{s' \in S} p(s'|s, \pi(s)) V_k^{\pi}(s')$$

Policy Improvement

- Have $V^{\pi}(s)$ for all s (from policy evaluation step!)
- Want to try to find a better (higher value) policy
- Idea:
  - Find the state-action Q value of doing an action followed by following π forever, for each state
  - Then take argmax of Qs

Policy Improvement

- Compute Q value of different 1st action and then following π_i

$$Q^{\pi_i}(s,a) = r(s,a) + \gamma \sum_{s' \in S} p(s'|s,a) V^{\pi_i}(s')$$

- Use to extract a new policy

$$\pi_{i+1}(s) = \arg\max_a Q^{\pi_i}(s,a)$$

Delving Deeper Into Improvement

$$Q^{\pi_i}(s,a) = r(s,a) + \gamma \sum_{s' \in S} p(s'|s,a) V^{\pi_i}(s')$$

$$\max_a Q^{\pi_i}(s,a) \geq V^{\pi_i}(s)$$

$$\pi_{i+1}(s) = \arg\max_a Q^{\pi_i}(s,a)$$

- So if take $\pi_{i+1}(s)$ then followed π_i forever,
  - expected sum of rewards would be at least as good as if we had always followed π_i
- But new proposed policy is to always follow π_{i+1} …

Monotonic Improvement in Policy

- For any two value functions V1 and V2, let V1 >= V2 $\rightarrow$ for all states s, V1(s) >= V2(s)
- Proposition: $V^{\pi'}$ >= V^{π} with strict inequality if π is suboptimal (where π' is the new policy we get from doing policy improvement)

Proof

$$V^{\pi}(s) \le \max_a Q^{\pi}(s,a)$$

$$= \sum_{s' \in S} p(s' \mid s, \pi'(s)) \left[R(s, \pi'(s), s') + \gamma V^{\pi}(s') \right]$$

$$\le \sum_{s' \in S} p(s' \mid s, \pi'(s)) \left[R(s, \pi'(s), s') + \gamma \max_{a'} Q^{\pi}(s', a') \right]$$

$$= \sum_{s' \in S} p(s' \mid s, \pi'(s)) \begin{bmatrix} R(s, \pi'(s), s') + \\ \gamma \sum_{s' \in S} p(s'' \mid s', \pi'(s'))(R(s', \pi'(s'), s'' + \gamma V^{\pi}(s'')) \end{bmatrix}$$

$$\ldots \le V^{\pi'}(s)$$

If Policy Doesn't Change ($\pi_{i+1}(s) = \pi_i(s)$ for all s) Can It Ever Change Again in More Iterations?

- Recall policy improvement step

$$Q^{\pi_i}(s,a) = r(s,a) + \gamma \sum_{s' \in S} p(s'|s,a) V^{\pi_i}(s')$$

$$\pi_{i+1}(s) = \arg\max_a Q^{\pi_i}(s,a)$$

Policy Iteration (PI)

1. i=0; Initialize $\pi_0(s)$ randomly for all states s
2. Converged = 0;
3. While i == 0 or $|\pi_i - \pi_{i-1}| > 0$
   - i=i+1
   - Policy evaluation: Compute V^π
   - Policy improvement:

$$Q^{\pi_i}(s,a) = r(s,a) + \gamma \sum_{s' \in S} p(s'|s,a) V^{\pi_i}(s')$$

$$\pi_{i+1}(s) = \arg\max_a Q^{\pi_i}(s,a)$$

Policy Iteration Can Take At Most |A|^|S| Iterations (Size of # Policies)

1. i=0; Initialize $\pi_0(s)$ randomly for all states s
2. Converged = 0;
3. While i == 0 or $|\pi_i - \pi_{i-1}| > 0$
   - i=i+1
   - Policy evaluation: Compute V^π
   - Policy improvement:

$$Q^{\pi_i}(s,a) = r(s,a) + \gamma \sum_{s' \in S} p(s'|s,a) V^{\pi_i}(s')$$

$$\pi_{i+1}(s) = \arg\max_a Q^{\pi_i}(s,a)$$

* For finite state and action spaces

Policy Iteration

Fewer Iterations

More expensive per iteration

Value Iteration

More iterations

Cheaper per iteration

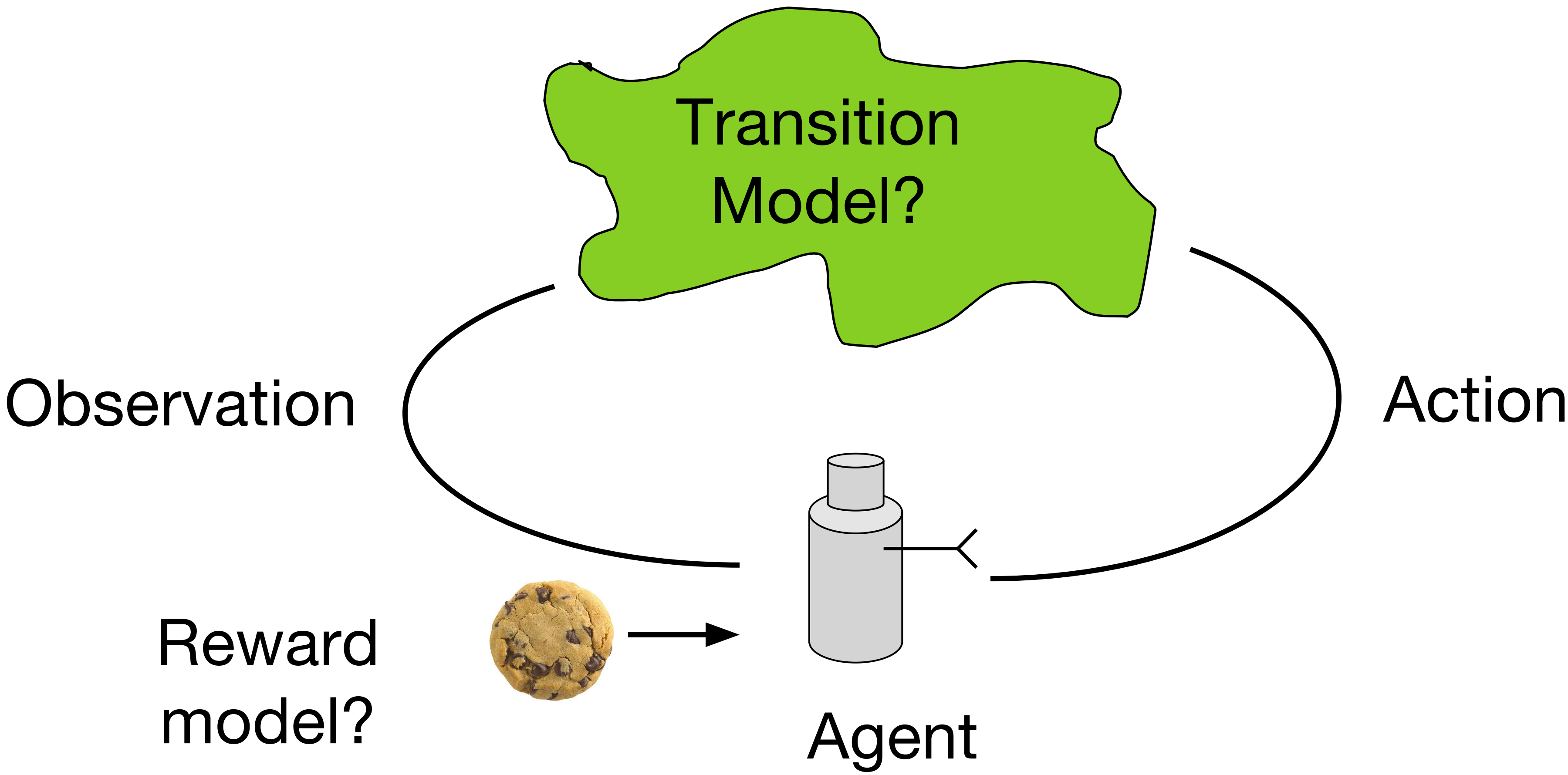
MDPs: What You Should Know

- Definition
- How to define for a problem
- MDP Planning: Value iteration and policy iteration
  - How to implement
  - Convergence guarantees
  - Computational complexity

Reasoning Under Uncertainty

Learn model of outcomes	Multi-armed bandits	**Reinforcement Learning**
Given model of stochastic outcomes	Decision theory	Markov Decision Processes
	Actions Don’t Change State of the World	**Actions Change State of the World**

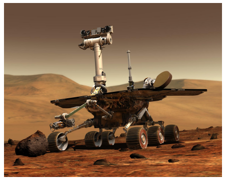
Reinforcement Learning



Goal: Maximize expected sum of future rewards

MDP Planning vs Reinforcement Learning

- No world models (or simulators)
- Have to learn how world works by trying things out

S1	S2	S3	S4	S5	S6	S7
Okay Field Site +1						Fantastic Field Site +10

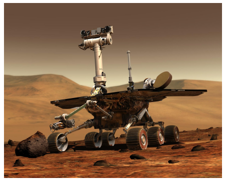
Policy Evaluation While Learning

- Before figuring out how should act
- 1st figure out how good a particular policy is (passive RL)

Passive RL

1. Estimate a model (and use to do policy evaluation)
2. Q-learning

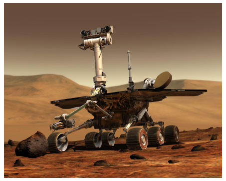
Learn a Model

S1	S2	S3	S4	S5	S6	S7
Okay Field Site +1						Fantastic Field Site +10

- Start in state S3, take TryLeft, go to S2
- In state S2, take TryLeft, go to S2
- In state S2, take TryLeft, go to S1
- What's an estimate of p(s'=S2| S=S2, a=TryLeft)?

Use Maximum Likelihood Estimate

E.g. Count & Normalize

S1	S2	S3	S4	S5	S6	S7
Okay Field Site +1						Fantastic Field Site +10

- Start in state S3, take TryLeft, go to S2
- In state S2, take TryLeft, go to S2
- In state S2, take TryLeft, go to S1
- What's an estimate of p(s'=S2| S=S2, a=TryLeft)?
  - 1/2

Model-Based Passive Reinforcement Learning

- Follow policy π
- Estimate MDP model parameters from data
    - If finite set of states and actions: count & average
- Use estimated MDP to do policy evaluation of π

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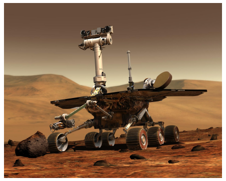
- Does this give us dynamics model parameter estimates for all actions?
- How good is the model parameter estimates?
- What about the resulting policy value estimate?

Model-Based Passive Reinforcement Learning

- Follow policy π
- Estimate MDP model parameters from data
  - If finite set of states and actions: count & average
- Use estimated MDP to do policy evaluation of π

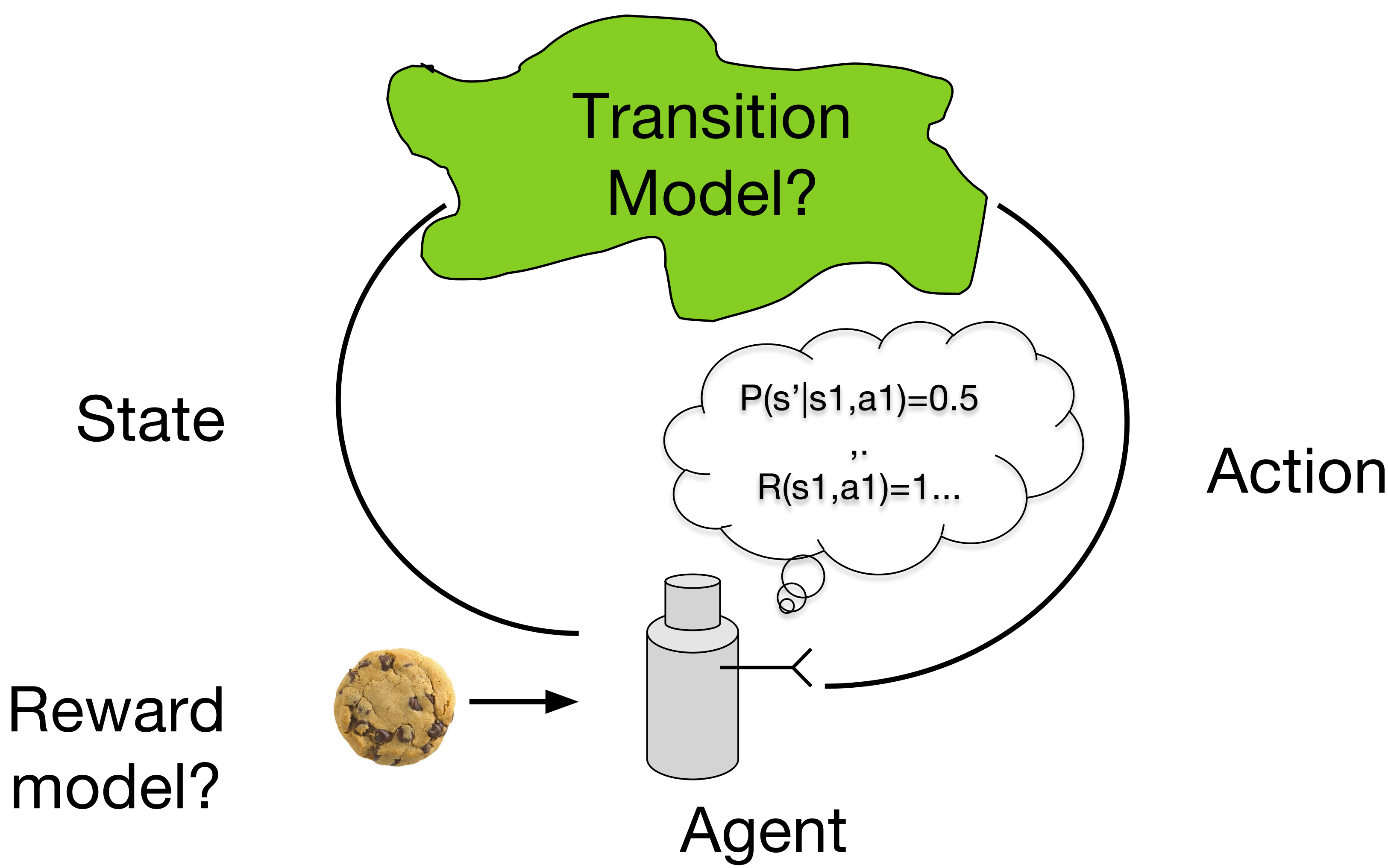
- Does this give us dynamics model parameter estimates for all actions?
  - No. But all ones need to estimate the value of the policy.
- How good is the model parameter estimates?
  - Depends on amount of data we have
- What about the resulting policy value estimate?
  - Depends on quality of model parameters

Good Estimate if Use 2 Data Points?

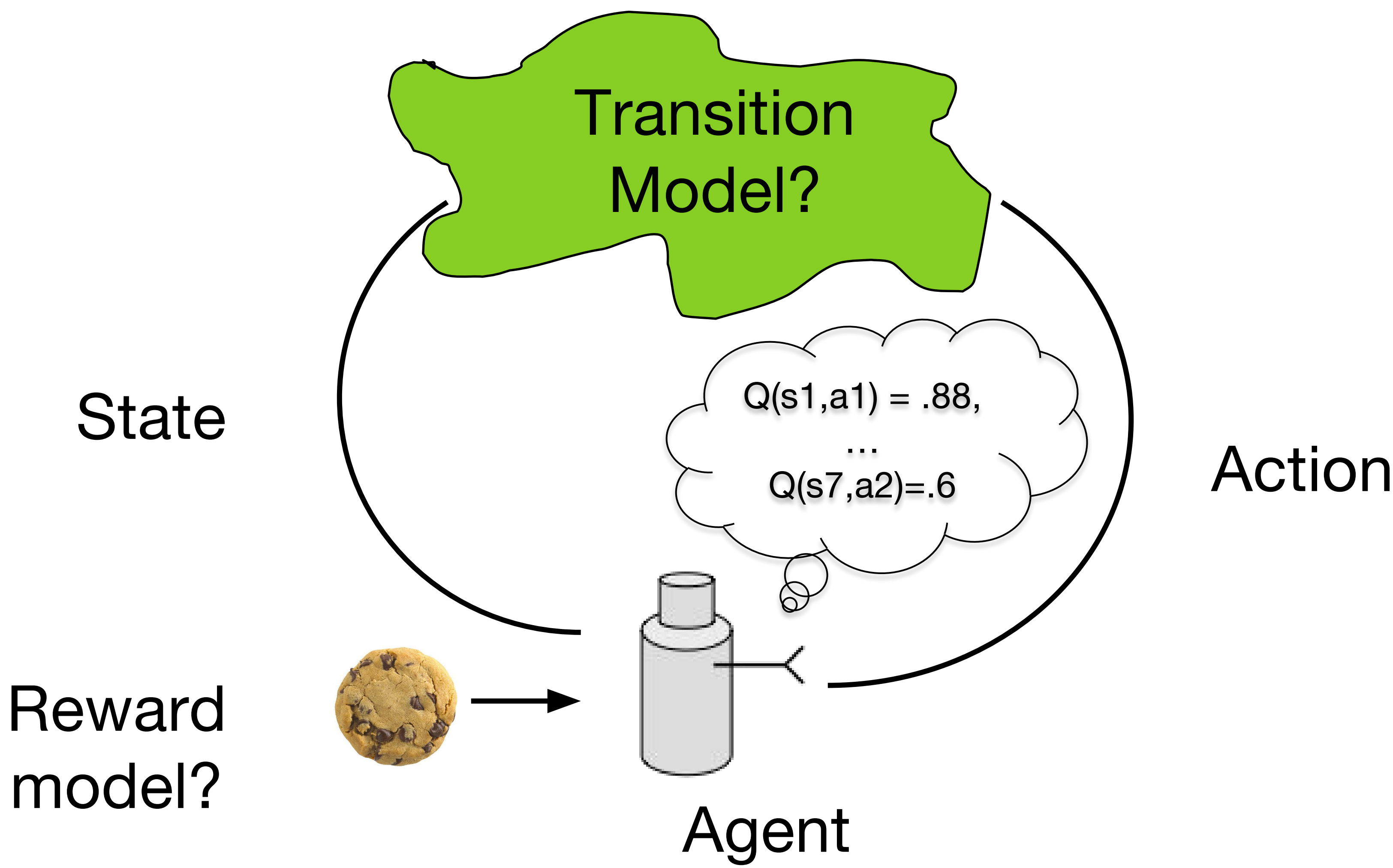
S1	S2	S3	S4	S5	S6	S7
Okay Field Site +1						Fantastic Field Site +10

- Start in state S3, take TryLeft, go to S2, r=0
- In state S2, take TryLeft, go to S2, r = 0
- In state S2, take TryLeft, go to S1,
- What's an estimate of p(s'=S2| S=S2, a=TryLeft)?
  - 1/2

Model-based Passive RL:
Agent has an estimated model in its head



Model-free Passive RL:
Only maintain estimate of Q



Q-values

- Recall that $Q^{\pi}(s,a)$ values are
  - expected discounted sum of rewards over H step horizon
  - if start with action a and follow π
- So how could we directly estimate this?

Q-values

$$Q^{\pi_i}(s,a) = r(s,a) + \gamma \sum_{s' \in S} p(s'|s,a) V^{\pi_i}(s')$$

- Want to approximate the above with data
- Note if only following π, only get data for a=π(s)

Q-values

$$Q^{\pi_i}(s,a) = r(s,a) + \gamma \sum_{s' \in S} p(s'|s,a) V^{\pi_i}(s')$$

- Want to approximate the above with data
- Note if only following π, only get data for a=π(s)

- TD-learning
  - Approximate expectation with samples
  - Approximate future reward with estimate

Temporal Difference Learning

$$V^{\pi}(s) = r(s, \pi(s)) + \gamma \sum_{s' \in S} p(s'|s, \pi(s)) V^{\pi}(s')$$

- Maintain estimate of $V^{\pi}(s)$ for all states
  - Update $V^{\pi}(s)$ each time after each transition (s, a, s', r)
  - Likely outcomes s' will contribute updates more often
  - Approximating expectation over next state with samples
  - Running average

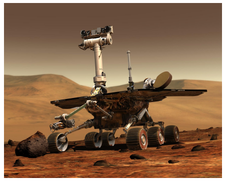
$$V_{samp}(s) = r + \gamma V^{\pi}(s')$$

Decrease learning rate over time (why?)

$$V^{\pi}(s) = (1 - \alpha) V^{\pi}(s) + \alpha V_{samp}(s)$$

$$V_{samp}(s) = r + \gamma V^{\pi}(s')$$

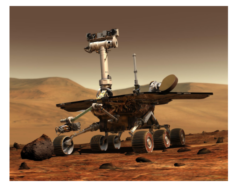
$$V^{\pi}(s) = (1-\alpha)V^{\pi}(s) + \alpha V_{samp}(s)$$

S1	S2	S3	S4	S5	S6	S7
Okay Field Site +1						Fantastic Field Site +10

- Policy: TryLeft in all states, use alpha = 0.5, γ=1
- Set V^{π}=[0 0 0 0 0 0 0],
- Start in state S3, take TryLeft, get r=0, go to S2
  - V_{samp}(S3) = 0 + 1 * 0 = 0
  - V^{π}(S3)=(1-0.5)*0 + .5*0 = 0 (no change!)

$$V_{samp}(s) = r + \gamma V^{\pi}(s')$$

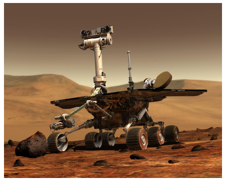
$$V^{\pi}(s) = (1-\alpha)V^{\pi}(s) + \alpha V_{samp}(s)$$

S1	S2	S3	S4	S5	S6	S7
Okay Field Site +1						Fantastic Field Site +10

- Policy: TryLeft in all states, use alpha = 0.5, γ=1
- Set V^{π}=[0 0 0 0 0 0 0],
- Start in state S3, take TryLeft, go to S2, get r=0
- V^{π}=[0 0 0 0 0 0 0]
- In state S2, take TryLeft, get r=0, go to S1
  - V_{samp}(S2) = 0 + 1 * 0 = 0
  - V^{π}(S2)=(1-0.5)*0 + .5*0 = 0 (no change!)

$$V_{samp}(s) = r + \gamma V^{\pi}(s')$$

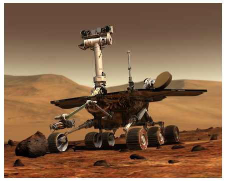
$$V^{\pi}(s) = (1-\alpha)V^{\pi}(s) + \alpha V_{samp}(s)$$

S1	S2	S3	S4	S5	S6	S7
Okay Field Site +1						Fantastic Field Site +10

- Policy: TryLeft in all states, use alpha = 0.5, γ=1
- Start in state S3, take TryLeft, go to S2, get r=0
- In state S2, take TryLeft, go to S1, get r=0
- V^{π}=[0 0 0 0 0 0 0]
- In state S1, take TryLeft, go to S1, get r=+1
  - $V_{samp}(S1) = 1 + 1 * 0 = 1$
  - $V^{\pi}(S1)=(1-0.5)*0 + .5*1 = 0.5$

$$V_{samp}(s) = r + \gamma V^{\pi}(s')$$

$$V^{\pi}(s) = (1-\alpha)V^{\pi}(s) + \alpha V_{samp}(s)$$

S1	S2	S3	S4	S5	S6	S7
Okay Field Site +1						Fantastic Field Site +10

- Policy: TryLeft in all states, use alpha = 0.5, γ=1
- Start in state S3, take TryLeft, go to S2, get r=0
- In state S2, take TryLeft, go to S1, get r=0
- V^{π}=[0 0 0 0 0 0 0]
- In state S1, take TryLeft, go to S1, get r=+1
- V^{π}=[0.5 0 0 0 0 0 0]

Problems with Passive Learning

- Want to make good decisions
- Initial policy may be poor -- don’t know what to pick
- And getting only experience for that policy

Can We Learn Optimal Values & Policy?

- Consider acting randomly in the world
- Can such experience allow the agent to learn the optimal values and policy?

Recall Model-Based Passive Reinforcement Learning

- **Follow policy π**
- Estimate MDP model params from observed transitions & rewards
  - If finite set of states and actions, count & avg counts
- Use estimated MDP to do policy evaluation of π

Recall Model-Based Passive Reinforcement Learning

- **Choose actions randomly**
- Estimate MDP model params from observed transitions & rewards
  - If finite set of states and actions, count & avg counts
- **Use estimated MDP to compute estimate of optimal value and policy**
- **Will policy converge to optimal value & policy**
  - **(In limit of infinite data)?**

Yes, if have reachability

- When acting randomly forever, still need to be able to visit each state and take each action many times
- Want all states to be reachable from any other state
- Quite mild assumption but doesn't always hold

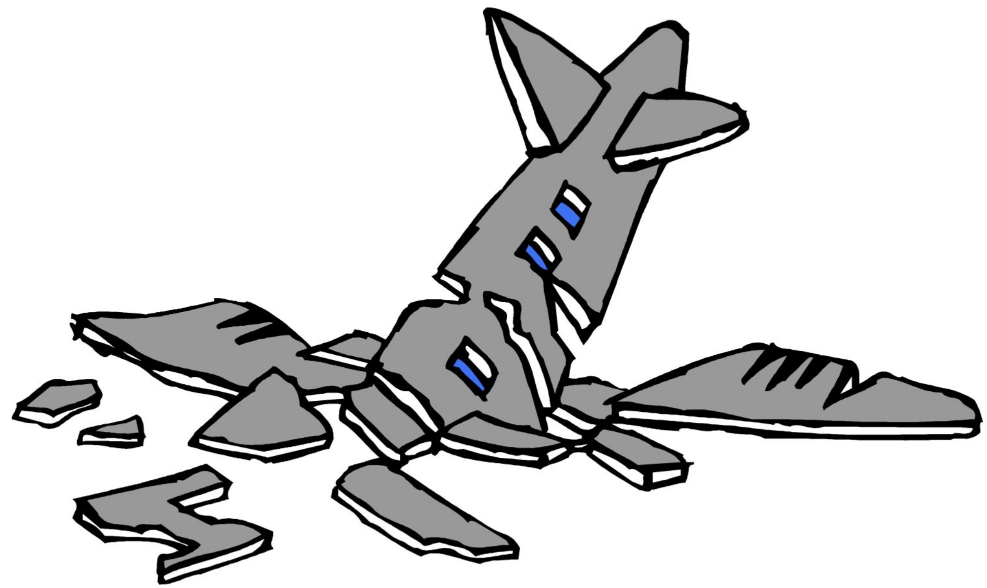


Image source: http://ancient-heritage.blogspot.com/2014/05/crash-course-on-flying-in-face-of-logic.html

Model-Free Learning w/Random Actions

- TD learning for policy evaluation:
  - As act in the world go through (s,a,r,s',a',r',...)
  - Update V^{π} estimates at each step
- Over time updates mimic Bellman updates
- Now do for Q values

Q-Learning

- Update Q(s,a) every time experience (s,a,s’,r)
  - Create new sample estimate

$$
\begin{aligned}
Q_{samp}(s,a) &= r + \gamma V(s') \\
&= r + \gamma \max_{a'} Q(s',a')
\end{aligned}
$$

  - Update estimate of Q(s,a)

$$
Q(s,a) = (1-\alpha)Q(s,a) + \alpha Q_{samp}(s,a)
$$

Q-Learning Properties

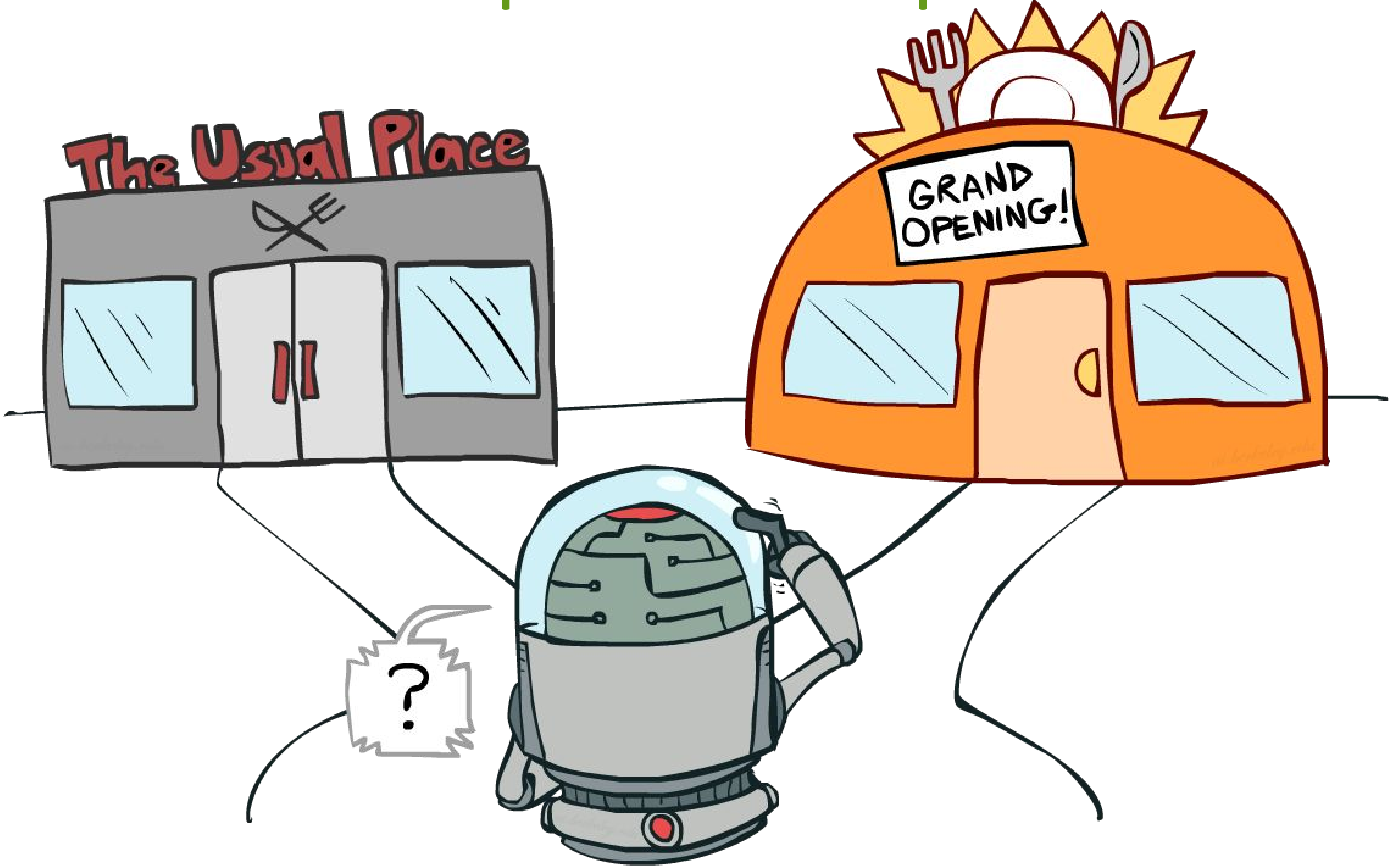
- If acting randomly*, Q-learning converges Q*
  - Optimal Q values
  - Finds optimal policy
- Off-policy learning
  - Can act in one way
  - But learning values of another π (the optimal one!)

*Again, under mild reachability assumptions

Towards Gathering High Reward

- Fortunately, acting randomly is sufficient, but not necessary, to learn the optimal values and policy
- Ultimately want to learn to get large reward

To Explore or Exploit?



Drawing by Ketrina Yim

Simple Approach: E-greedy

- With probability 1-e
  - Choose argmax_a Q(s,a)
- With probability e
  - Select random action

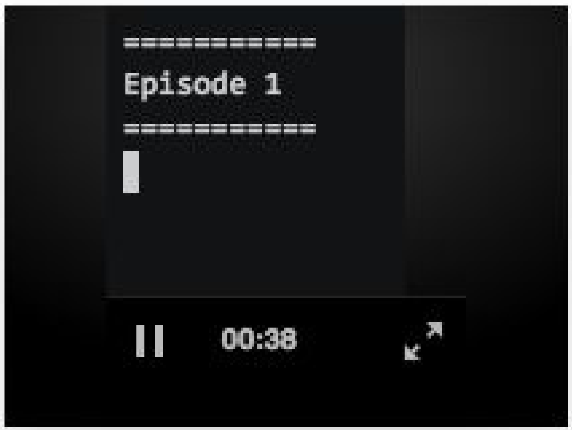
- Guaranteed to compute optimal policy
- But even after millions of steps still won't always be following argmax of Q(s,a))

Greedy in Limit of Infinite Exploration (GLIE)

- E-Greedy approach
- But decay epsilon over time
- Eventually will be following optimal policy almost all the time

- We'll talk more about exploration/exploitation later in the course

Homework 1 Will Be Released This Week



FrozenLake-v0

Find a safe path across a grid of ice and water tiles.

FrozenLake8x8-v0

- Review/practice basic MDP planning
- Get familiar with Open AI gym for basic RL

What You Should Know

- Define MDP, Bellman operator, contraction, model, Q-value, policy
- Contrast MDP planning and RL
- Be able to implement
  - Value iteration, policy iteration, Q-learning and model-based RL
- Contrast benefits and weaknesses of Q-learning and model-based RL
  - On homework!
  - Data efficiency, computational complexity, etc.