

Linear Algebra (MT1004)

BS(AI), BS(CS), BS(CY), BS(DS), BS(SE)

Course Instructors

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Sessional-I Exam

Total Time (Hrs): 1

Total Marks: 45

Total Questions: 4

Date: September 22, 2025

Roll No

Section

Student Signature

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Attempt all the questions

[CLO #1: Apply elementary row operations to solve linear systems of equations.]
Question # 1 [17 Marks]

(a) Let the solution of $Ax = b$ be

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

The matrix B and vector d are obtained by performing the elementary row operation $R_2 \leftrightarrow R_2 + 2R_1$ on A and b respectively. What will be the solution of the equation $Bx = d$? Determine the order of A . (2)

(b) For any matrices A of size 3×4 and B of size 4×3 , what is the maximum possible number of pivot positions in A and in B ? If both A and B have 3 pivot positions, comment on the nature of the solution sets of the systems $Ax = b$ and $Bx = d$. (3)

(c) Consider the following matrix A and vector b :

$$A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & 4 & 0 \\ -2 & 4 & -6 & -2 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 2 \\ -10 \end{bmatrix}.$$

(i) Find the relation between the solution sets of $Ax = b$ and $Ax = 0$. (5)

(ii) Determine whether the columns of A span the whole of \mathbb{R}^3 . If not, find the equation of span of the columns of A . (3)

(iii) Find the basis and the dimension of the $Nul(A)$ and the $Col(A)$. (4)

[CLO #2: Evaluate mathematical expressions to compute quantities that deal with linear systems and eigenvalue problems.]
Question # 2 [7 Marks]

(a) Suppose we represent a dataset by a matrix $D_{100 \times 50}$ (100 samples, 50 features). If $\text{rank}(D) = 20$, what is the dimension of the column space, row space, null space, and left null space of D ? (2)

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- (b) Consider the following set of polynomials in \mathbb{P}_2 (the space of polynomials of degree 2 or less than 2)

$$S = \{1 + x, x + 2x^2, 2x\}.$$

Find a basis for the subspace of \mathbb{P}_2 spanned by S .

[CLO #2: Evaluate mathematical expressions to compute quantities that deal with linear systems and eigenvalue problems.]

[15 Marks]

Question # 3

- (a) Suppose $T : M_{2 \times 2} \rightarrow \mathbb{R}^5$ be a linear transformation.
- (i) If $\dim(\text{Range}(T)) = 3$, can we conclude that T is onto? What can you say about its nullity and rank? (2)
- (ii) If T is one-to-one, what must be the dimension of $\text{Range}(T)$? (1)
- (b) Let $T : \mathbb{P}_1 \rightarrow \mathbb{R}^3$ be a linear transformation defined by

$$T(\mathbf{p}(x)) = \begin{bmatrix} \mathbf{p}(-1) \\ \mathbf{p}(0) \\ \mathbf{p}(1) \end{bmatrix}, \quad \text{where } \mathbf{p}(x) = ax + b.$$

- (i) Find $T(\mathbf{p}(x))$ when $\mathbf{p}(x) = 2x + 1$. (2)
- (ii) Find the matrix of the transformation T . (4)
- (iii) Compute the kernel of T . Explain how this relates to one-to-one. (3)
- (iv) Compute the range (image) of T . Determine whether T is onto. (3)

[CLO #2: Evaluate mathematical expressions to compute quantities that deal with linear systems and eigenvalue problems.]

[6 Marks]

Question # 4

Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ be the basis of \mathbb{R}^2 , where

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{c}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{c}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

- (a) Find the change-of-coordinates matrix from \mathcal{B} to \mathcal{C} . (4)
- (b) Find the \mathcal{C} -coordinate vector for \mathbf{x} when $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$. (2)

Good Luck!