Week 3

Problem 1

$$\mu = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$C = \begin{bmatrix} 2 & 0.2 \\ 0.2 & 0.5 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -0.8 \\ -0.8 & 1.5 \end{bmatrix}$$

$$C^{-1} = \frac{1}{7.5 - 0.64} \begin{bmatrix} 1.5 & 0.8 \\ 0.8 & 5 \end{bmatrix} = \begin{bmatrix} 0.2187 & 0.1166 \\ 0.1166 & 0.7289 \end{bmatrix}$$

$$\text{weight}; \quad w = C^{-1}\mu = \begin{bmatrix} 0.55 \\ 0.96 \end{bmatrix}$$

Problem 2 :

$$\text{class } 0: \ N(w^T\mu_0, w^T C_0 w) = N(\underset{m_0}{0.4082}, \underset{\sigma_0^2}{1.2897})$$

$$\text{class } 1: \ N(\mu^T\mu_1, w^T C_1 w) = N(\underset{m_1}{2.478}, \underset{\sigma_1^2}{0.7803})$$

$$\frac{p(w^T x|0)}{p(w^T x|1)} = \frac{\frac{1}{\sqrt{2\pi\sigma_0^2}}}{\frac{1}{\sqrt{2\pi\sigma_1^2}}} \frac{e^{-(y-m_0)^2/2\sigma_0^2}}{e^{-(y-m_1)^2/2\sigma_1^2}} > 1$$

$$= \frac{\sigma_1}{\sigma_0} e^{-\frac{(y-m_0)^2}{2\sigma_0^2} + \frac{(y-m_1)^2}{2\sigma_1^2}} > 1$$

$$= \log\left(\frac{\sigma_1}{\sigma_0}\right) + \log\left(e^{-\frac{(y-m_0)^2}{2\sigma_0^2} + \frac{(y-m_1)^2}{2\sigma_1^2}}\right) > 0$$

$$= \log(\sigma_1/\sigma_0) - \frac{(y-m_0)^2}{2\sigma_0^2} + \frac{(y-m_1)^2}{2\sigma_1^2} > 0$$

$$\Rightarrow (y-m_0)^2 \cdot 2\sigma_1^2 - (y-m_1)^2\, 2\sigma_0^2 < 4\log\left(\frac{\sigma_1}{\sigma_0}\right)\sigma_0^2\sigma_1^2$$

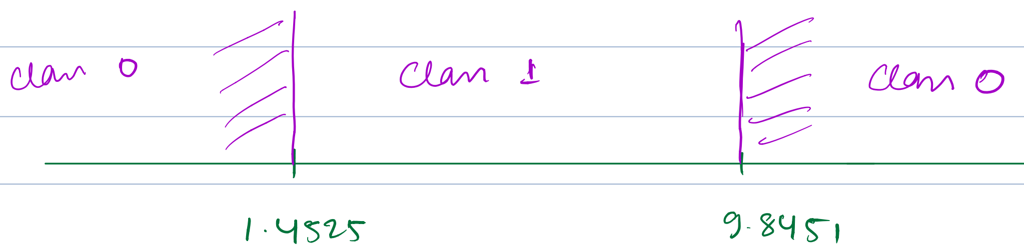
$$\Rightarrow (\sigma_1^2-\sigma_0^2)y^2 - 2ym_0\sigma_1^2 + \sigma_1^2 m_0^2 + 2ym_1\sigma_0^2 - \sigma_0^2 m_1^2 < 2\log\left(\frac{\sigma_1}{\sigma_0}\right)\sigma_0^2\sigma_1^2$$

$$\Rightarrow (\sigma_1^2-\sigma_0^2)y^2 + (2m_1\sigma_0^2 - 2m_0\sigma_1^2)y < 2\log\left(\frac{\sigma_1}{\sigma_0}\right)\sigma_0^2\sigma_1^2 - \sigma_1^2 m_0^2 + \sigma_0^2 m_1^2$$

$$\Rightarrow -0.5094y^2 + 5.755\,y < 7.2845$$

$$\Rightarrow 0.5094y^2 - 5.755y + 7.2845 > 0$$

$$\Rightarrow (y-1.4525)(y-9.8451) > 0$$



If $1.4525 < T < 9.8451 \rightarrow$ Class 1

Else,

class 0

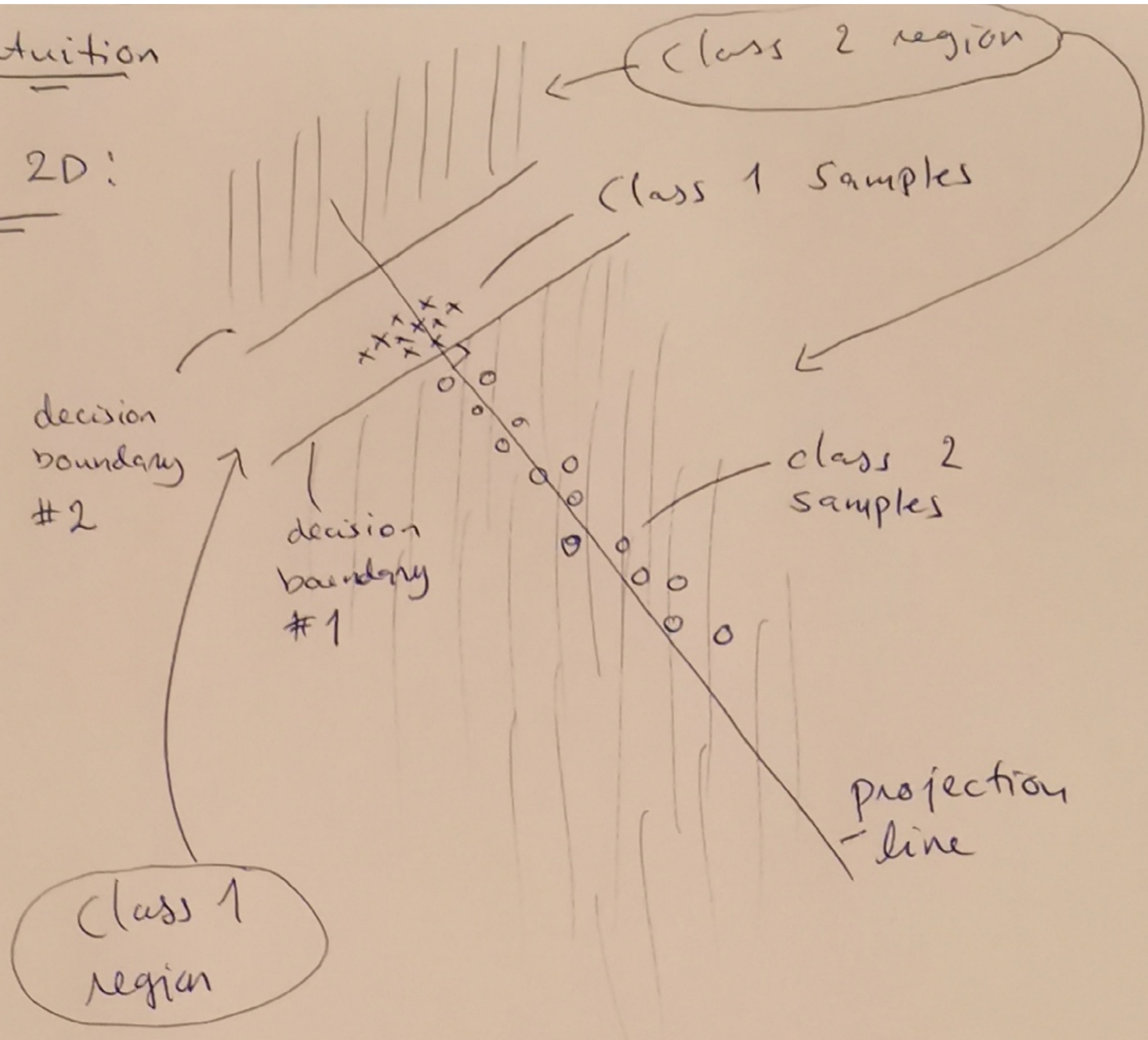
Since,

$$v = w^T x = \begin{bmatrix} 0.55 & 0.96 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 2.4781$$

Thus, $x \in$ class 1

Intuition

In 2D:



In 1D after projection: