Homework 1

1. $X[n] = A\cos[n] + \omega[n] \; ; \; \omega[n] \sim N(0,\sigma^2)$

$$X[n] - A\cos[n] = \omega[n]$$

$$p(x|A) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left[-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1}\left(X[n] - A\cos[n]\right)^2\right]$$

$$\ln p(x|A) = -\frac{N}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{n=0}^{N-1}\left(x[n] - A\cos[n]\right)^2$$

$$\frac{\partial \ln p(x|A)}{\partial A} = -\frac{1}{\sigma^2}\sum_{n=0}^{N-1}\left(x[n] - A\cos[n]\right)\left(-\cos[n]\right) = 0$$

$$\Rightarrow \frac{1}{\sigma^2}\sum_{n=0}^{N-1}\left(x[n] - A\cos[n]\right)\left(\cos[n]\right) = 0$$

$$\Rightarrow \sum_{n=0}^{N-1} x[n]\cdot\cos[n] - A\sum_{n=0}^{N-1}\cos^2[n] = 0$$

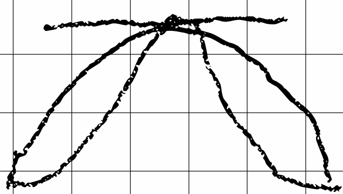
$$\Rightarrow \sum_{n=0}^{N-1} x[n]\cdot\cos[n] = A\sum_{n=0}^{N-1}\cos^2[n]$$

$\lambda x = \lambda$

$$\Rightarrow A = \frac{\sum_{n=0}^{N-1} x[n]\cos[n]}{\sum_{n=0}^{N-1}\cos^2[n]}$$

$\ln(e^{x+z}) = x+z$

$\frac{1}{2\sigma^2} e^{(x-\mu)^2}$



$\ln(a^b) = b\ln(a)$

$$P(x;\lambda)=\frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$\prod_{n=0}^{N-1}\frac{e^{-\lambda}\lambda^{x_n}}{x_n!}$$

$$\ln\left(\prod_{n=0}^{N-1}\frac{e^{-\lambda}\lambda^{x_n}}{x_n!}\right)=\sum_{n=0}^{N-1}\ln\left(\frac{e^{-\lambda}\lambda^{x_n}}{x_n!}\right)$$

$$=\sum\ln(e^{-\lambda}\lambda^{x_n})-\sum\ln(x_n!)=$$

$$=\sum\ln(e^{-\lambda})+\sum\ln(\lambda^{x_n})-\sum\ln(x_n!)=$$

$$=-\sum\lambda+\sum x_n\ln(\lambda)-\sum\ln(x_n!)$$

$$\frac{\partial p(x;\lambda)}{\partial\lambda}=\sum x_n\frac{1}{\lambda}-n=0$$

$$\cancel{}=\sum x_n\frac{1}{\lambda}=n \qquad \boxed{\frac{\sum x_n}{n}=\lambda}$$

$$\cos(2x)=-\sin(2x)\cdot 2$$