Optimisation and Statistical Data Analysis
Exercise Set 12 (Kalman filter)

Problem 1

A sequence of position measurements of a mobile robot is

t_k (s)	1	2	3	4	5	6	7	8	9	10
y_k (m)	-0.083	0.028	0.285	0.780	0.757	1.076	1.173	1.409	1.521	1.773

The observation model is

$$y_k \,|\, u_k \sim \mathrm{norm}(u_k, 0.1)$$

where u_k is the position.

(a) Assume an integrated-random-walk motion model (slide 8) with $q = 0.001$. Use a Kalman filter with

$$m_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad P_0 = I$$

to find a 95% confidence interval for the position at time $t = 10$.

(b) Find a 95% confidence interval for the position at times $t = 11$ and $t = 12$.

(c) Repeat (a) using a steady state Kalman filter.

(d) Repeat (a–b) using a random walk model

$$u_k \,|\, u_{k-1} \sim \mathrm{norm}(u_{k-1}, 0.05)$$

and initial state $u_0 \sim \mathrm{norm}(0, 1)$.

Answers   1. (a) 1.7612 ± 0.1458,   (b) 1.9460 ± 0.2181, 2.1308 ± 0.3129,   (c) 1.7649 ± 0.1458,   (d) 1.7331 ± 0.1811, 1.7331 ± 0.4742, 1.7331 ± 0.6457