Optimisation and Statistical Data Analysis

Exercise Set 12 Solutions

Set the size of figures.

```
figure; set(gcf,'position',[0 0 500 300])
```

Problem 1

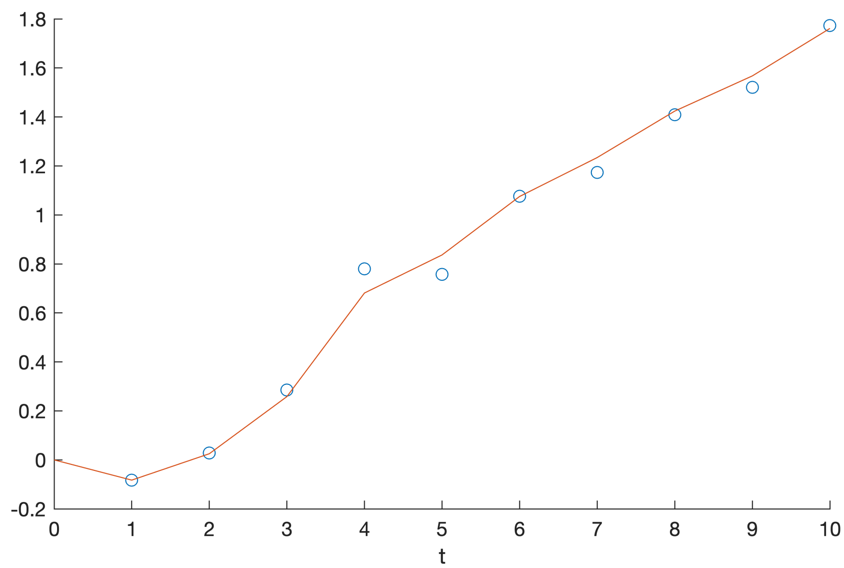
part (a)

Enter the data.

```
t=1:10;
y=[-0.083 0.028 0.285 0.780 0.757 1.076 1.173 1.409 1.521 1.773];
```

Kalman filter with integrated random walk model. The state has two components, position and velocity.

```
dt=1; nt=length(t);
q=0.001; A=[1 dt;0 1]; Q=diag([0,q*dt]);  H=[1 0]; R=0.1^2;
m0=[0;0]; P0=eye(2);
Ma=[ ]; m=m0; P=P0;
for k=1:nt
    m=A*m; P=A*P*A'+Q;
    S=H*P*H'+R; K=P*H'/S; P=P-K*S*K'; m=m+K*(y(k)-H*m);
    Ma(k)=m(1); % position estimate
end
plot(t,y,'o',[0 t],[m0(1) Ma])
xlabel('t'), box off
```



The 95% confidence interval of position (the first state component) is $m_{10|10}(1) \pm 1.96\sqrt{P_{10|10}(1,1)}$.

```
m(1)+[-1,1]*1.96*sqrt(P(1,1))
```

```
ans = 1×2
    1.6154    1.9070
```

Part (b)

The distribution of $x_{11} \mid y_{1:10}$ is found by applying a prediction step.

```
m=A*m; P=A*P*A'+Q;
m(1)+[-1,1]*1.96*sqrt(P(1,1))
```

```
ans = 1×2
    1.7279    2.1641
```

At $t = 11$ there is no measurement, so we omit the update step and make another prediction step to find the distribution of $x_{12} \mid y_{1:10}$.

```
m=A*m; P=A*P*A'+Q;
m(1)+[-1,1]*1.96*sqrt(P(1,1))
```

```
ans = 1×2
    1.8179    2.4437
```

Part (c)

The Kalman gain computed after 10 time steps is

```
K
```

```
K = 2×1
    0.5533
    0.2116
```

We can use this as an approximation of the steady-state gain in the steady-state filter.

Alternatively, we can use the solution of the Riccati equation.

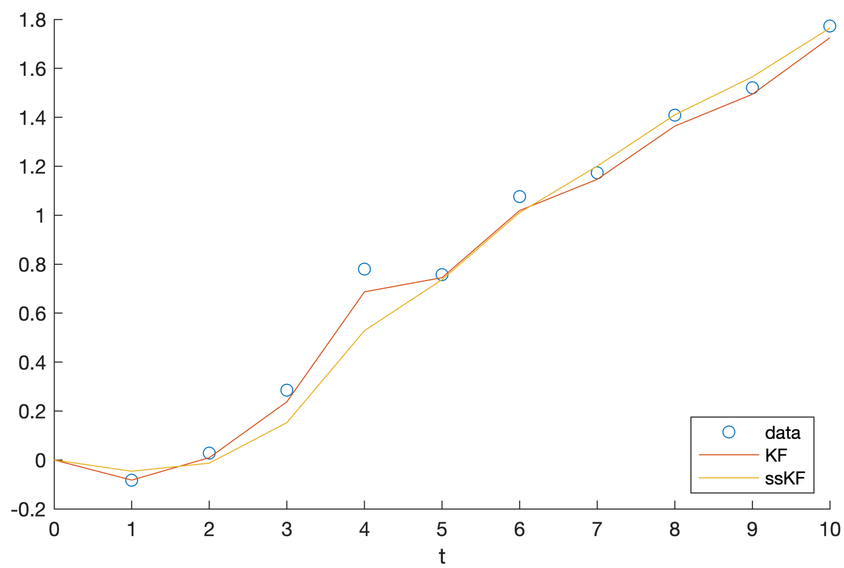
```
X=dare(A',H',Q,R); S=H*X*H'+R; K=X*H'/S, P=X-K*S*K';
```

```
K = 2×1
    0.5531
    0.2114
```

Steady state Kalman filter

```
Mss=[ ]; m=m0;
for k=1:nt
```

```
    m=A*m;
    m=m+K*(y(k)-H*m);
    Mss(k)=m(1); % position estimate
end
plot(t,y,'o',[0 t],[m0(1) M],[0 t],[m0(1) Mss])
xlabel('t'), box off, legend('data','KF','ssKF','location','southeast')
```



Confidence interval of position estimate.

```
m(1)+[-1,1]*1.96*sqrt(P(1,1))
```

```
ans = 1×2
    1.6191    1.9106
```

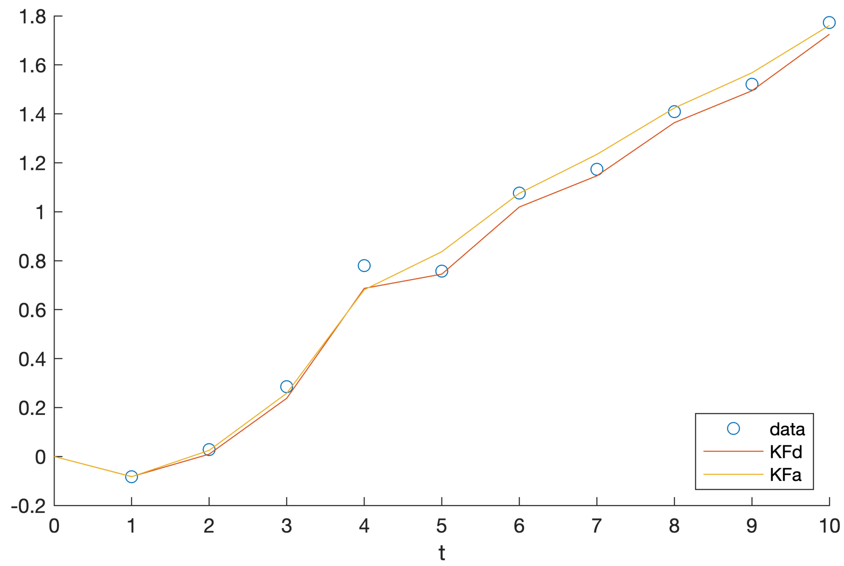
Part (d)

The random walk model has a single state. The model is

```
A=1; Q=0.2^2; H=1; m0=0; P0=1;
```

Kalman filter. The path estimated with the random walk model is not as smooth as the path estimated with the integrated random walk model.

```
Md=[ ]; m=m0; P=P0;
for k=1:nt
    m=A*m; P=A*P*A'+Q;
    S=H*P*H'+R; K=P*H'/S; P=P-K*S*K'; m=m+K*(y(k)-H*m);
    Md(k)=m(1); % position estimate
end
plot(t,y,'o',[0 t],[m0(1) Md],[0 t],[m0(1) Ma])
legend('data','KFd','KFa','location','southeast')
xlabel('t'), box off
```



Confidence interval for position at t=10

```
m(1)+[-1,1]*1.96*sqrt(P(1,1))
```

```
ans = 1×2
    1.5467    1.9035
```

Confidence intervals for position at t=11 and t=12. The predicted position stays constant after the last measurement (random walk has constant mean) but the interval width increases.

```
m=A*m; P=A*P*A'+Q;  % predict one step ahead
m(1)+[-1,1]*1.96*sqrt(P(1,1))
```

```
ans = 1×2
    1.2945    2.1558
```

```
m=A*m; P=A*P*A'+Q;  % predict next step ahead
m(1)+[-1,1]*1.96*sqrt(P(1,1))
```

```
ans = 1×2
    1.1428    2.3075
```