ASE-4046 Exercise 1 Solutions

Problem 1

a) fl(23.1454545 + 0.232976)

= fl(2.315×10^{1} + 2.330×10^{-1}) [machine numbers, t=3 digits after decimal point

= fl($(2.315 + 0.02330) \times 10^{1}$) [equalise exponents

= fl(2.33830×10^{1}) [add mantissas

= 2.338×10^{1} [machine number

b) fl(23.1454545 / 0.232976)

= fl(2.315×10^{1} / 2.330×10^{-1}) [machine numbers

= fl($0.99356223 \cdots \times 10^{2}$) [divide mantissas, subtract exponents

= 9.936×10^{1} [machine number

Problem 2

Part (a)

The terms $a = e^{x}$ and $b = e^{-x}$ are nearly equal at $x = 10^{-5}$, so the difference $a - b$ is inaccurate in floating point arithmetic.

```
x=1e-5;
format long  % show lots of digits
a=exp(x)
```

```
a =
   1.000010000050000
```

```
b=exp(-x)
```

```
b =
   0.999990000050000
```

```
a-b
```

```
ans =
     2.000000000024205e-05
```

A Taylor polynomial approximation of the function can be found using the symbolic toolbox.

```
syms X;
f=exp(X)-exp(-X);
```

```
P = taylor(f,X,'order',4)
```

P =

$$\frac{X^3}{3} + 2\,X$$

Evaluate the Taylor polynomial.

```
p=sym2poly(P);     % extract polynomial coefficients
polyval(p,x)       % evaluate the polynomial
```

```
ans =
     2.000000000033334e-05
```

Retaining more terms of the Taylor series gives the same answer. This indicates that for this x the Taylor polynomial of degree 3 is sufficiently accurate.

```
P6 = taylor(f,X,'order',6);
polyval(sym2poly(P6),x)
```

```
ans =
     2.000000000033334e-05
```

An alternative solution is to rewrite the expression using a mathematical identity.

```
2*sinh(x)
```

```
ans =
     2.000000000033334e-05
```

Part (b)

The square root terms are nearly equal at $x = 10^{-2}$, so the expression is inaccurate in floating point arithmetic.

```
x=1e-2;
format long  % show lots of digits
a=sqrt(1+x^2);
b=sqrt(1-x^2);
1/(a-b)
```

```
ans =
     9.999999987511092e+03
```

Apply some algebraic manipulations to find a mathematically equivalent formula without the subtraction of nearly equal values.

$$\frac{1}{\sqrt{1+x^2}-\sqrt{1-x^2}}\times 1=\frac{1}{\sqrt{1+x^2}-\sqrt{1-x^2}}\times\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}=$$

$$\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{(1+x^2)-(1-x^2)}=\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{2x^2}$$

Evaluate the expression.

```
(sqrt(1+x^2)+sqrt(1-x^2))/(2*x^2)
```

```
ans =
     9.999999987499998e+03
```

An alternative solution is to find a Taylor polynomial of the denominator.

```
syms X;
P=taylor(sqrt(1+X^2)-sqrt(1-X^2),X,'order',7)
```

P =

$$\frac{X^6}{8}+X^2$$

Evaluate the expression with the Taylor polynomial in the denominator.

```
1/polyval(sym2poly(P),x)
```

```
ans =
     9.999999987499998e+03
```

Problem 3

The unit roundoff is $\mu=\frac{1}{2}2^{-t}=\frac{1}{2}2^{-63}=2^{-64}\approx 5.421\times 10^{-20}$

realmin $=2^{L}=2^{(-2^{14})}=2^{-16384}\approx 8.405\times 10^{-4933}$

realmax$\approx 2^{U+1}=2^{(2^{14}+1)}=2^{16385}\approx 2.379\times 10^{4932}$

Problem 4

The given code computes the zeros of x^2-10^9x+1 as

```
a=1; b=-1e9; c=1; % coefficients
d=sqrt(b^2-4*a*c) % discriminant
```

```
d = 1.0000e+09
```

```
r1=-(b-d)/(2*a)  % one root
```

```
r1 = 1.0000e+09
```

```
r2=-(b+d)/(2*a) % the other root
```

```
r2 = 0
```

The value of `r2` should be positive, not 0. The error is caused by cancellation in the computation of the term b+d, which is a subtraction of nearly equal quantities. Computing the roots with the formula from slide 21 gives accurate results

```
r1 = -(b+sign(b)*d)/(2*a)
```

```
r1 = 1.0000e+09
```

```
r2 = c/(a*r1)
```

```
r2 = 1.0000e-09
```

The Matlab function for computing polynomial roots also gives accurate results.

```
r=roots([a b c]);
r(1), r(2)
```

```
ans = 1.0000e+09
ans = 1.0000e-09
```