Optimisation and Statistical Data Analysis
Exercise Set 10 (Gaussian mean)

Problem 1

Ten rock specimens were weighed on two different scales. The readings (in g) were

specimen	1	2	3	4	5	6	7	8	9	10
scale A	11.23	14.36	8.33	10.50	23.42	9.15	14.47	6.47	12.40	19.38
scale B	11.27	14.41	8.35	10.52	23.41	9.17	13.52	6.46	12.45	19.35

Find the median and 95% confidence interval of the scales' systematic difference $\mu_A - \mu_B$. How many specimens would need to be weighed in order to make the interval 10 times smaller? What's the probability that $\mu_A > \mu_B$?

Problem 2

Cuckoo eggs found in the nests of dunnocks had the following lengths (in mm)

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22.0 23.9 20.9 23.8 25.0 24.0 21.7 23.8 22.8 23.1
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(a) Plot the normal QQ plot of the data. What does the plot tell you?

(b) Plot the posterior density function for the "typical length" and find its median and 95% confidence interval.

(c) Plot the posterior density function for the length of the cuckoo egg found in the eleventh dunnock nest. On the same axes, plot a density histogram of the observations.

Extra problems

(a) You have a model-A clock whose error standard deviation is $10\,\mu$s. For the same money, you could either

(1) buy two more model-A clocks, or

(2) buy one model-B clock whose error standard deviation is $5\,\mu$s

Explain why option (2) gives the smaller error when all measurements are combined. What if the price of a model-B clock were 5 times the price of a model-A clock?

(b) Show that in problem 1, the probability that scale A will give a higher reading than scale B when weighing the eleventh specimen is 0.5954.

(c) Continuing problem 2, cuckoo eggs found in reed warbler nests had lengths

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23.2 22.0 22.2 21.2 21.6 21.9 22.0 22.9 22.8
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Show that we can be 96% sure that cuckoos lay smaller eggs in reed warbler nests than in dunnock nests.

Answers 1. 0.08, $(-0.1395, 0.2995)$, 100, 0.7845 2. (b) 23.10, $(22.20, 24.00)$