ASE-4046 Exercise Set 4 (Nonlinear least squares)

Problem 1

In exercise 3.1, you fitted an exponential model of the form $v \approx a_1 \, \mathrm{e}^{a_2 x}$ to the data

x	210.2	197	191.4	185.7	183.2	181.15
v	28.559	21.892	19.758	17.267	16.385	15.919

In this exercise you will fit the model by minimising the cost function

$$f(a) = \sum_i (v_i - a_1 \, \mathrm{e}^{a_2 x_i})^2$$

a. Do one iteration of the Gauss-Newton method, using the answer of exercise 3.1 as the initial estimate.

b. Find the minimiser of $f(a)$ using `lsqnonlin`.

c. What is the cost function that is minimised by the solution of exercise 3.1?

Problem 2

Satellite navigation systems such as GPS and Galileo use trilateration to compute the receiver's position. Each satellite transmits coded messages that tell its current position s and the transmission time. The difference between the transmission time and the reception time, multiplied by the speed of light, is called the *pseudo-range* and is denoted y. The satellite's atomic clock is very precise, but the receiver's clock is not. The receiver clock error v is one of the unknowns of the navigation problem. If u denotes the receiver's position, then the pseudo-range to the ith satellite is

$$y_i \approx \|u - s_i\| + v$$

The satnav trilateration problem consists of solving for $x = (u_1, u_2, u_3, v)$ by minimizing the sum of squared residuals $\sum_i r_i^2$, where $r_i = y_i - \|u - s_i\| - v$.

Given the data (in metres)

i	s_i(1)	s_i(2)	s_i(3)	y_i
1	7766188.44	-21960535.34	12522838.56	22228206.42
2	-25922679.66	-6629461.28	31864.37	24096139.11
3	-5743774.02	-25828319.92	1692757.72	21729070.63
4	-2786005.69	-15900725.8	21302003.49	21259581.09

use `lsqnonlin` to find the receiver position. Use TUT Sähkötalo

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u=[2.7950e+06; 1.2361e+06; 5.5797e+06]
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as the initial "guess".

Answers 1a. $v \approx 0.4004 \, \mathrm{e}^{0.0203x}$ 2. $(-2430745.10, -4702345.11, 3546568.71)$ (located in Riverside, California)