Optimisation and Statistical Data Analysis

Exercise Set 9 Solutions

Problem 1

Part a

A 95% confidence interval for $\theta \mid \text{stats}_{2011}$ is

```
a=1; b=1;          % uniform prior
s=30546;           % boys
n=s+29415;         % girls
a=s+a; b=n-s+b;    % update
CI95=betainv([0.025,0.975],a,b)  % 95 percent confidence interval
```

```
CI95 = 1×2
    0.5054    0.5134
```

Part b

The posterior probability that $\theta \le 0.5$ is

```
betacdf(0.5,a,b)
```

```
ans = 1.9291e-06
```

and so the posterior probability that $\theta > 0.5$ is

```
format long; 1-ans
```

```
ans = 
   0.999998070929810
```

Revert to default display format.

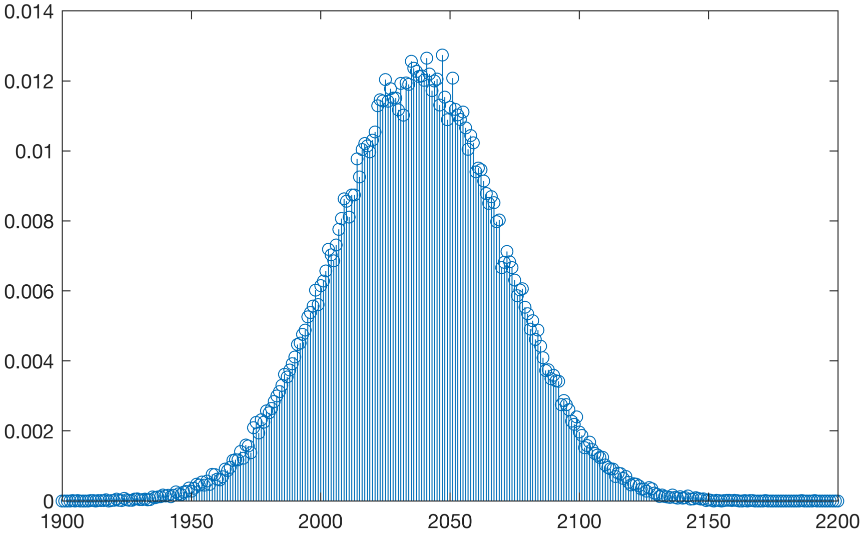
```
format
```

Part c

Modify the code from slide 13. Notice the "off-by-1" indexing of `p_pred` in the last line.

```
rng default; % for replicability
Nsamp=100000;
theta=betarnd(a,b,Nsamp,1); % samples of theta
n_pred=4000;
s_pred=binornd(n_pred,theta);     % samples of s
p_pred=histcounts(s_pred,-0.5:n_pred+0.5)/Nsamp;
```

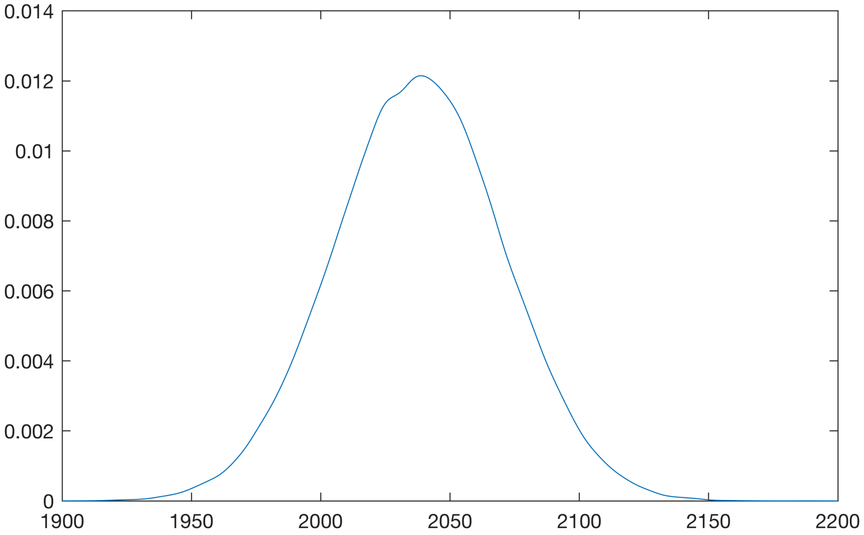
```
figure; set(gcf,'position',[0 0 500 300])
stem(1900:2200,p_pred(1901:2201))
```



The "noise" is due to the approximation and could be reduced by increasing `Nsamp`, the number of Monte Carlo samples.

To obtain a smooth (but still approximate) plot use

```
ksdensity(s_pred,1900:2200)
```



The posterior predictive probability that the number of male births is > 2000 is approximately

```
sum(s_pred>2000)/Nsamp
```

```
ans = 0.8738
```

which is high, but much smaller than the answer in Part b. Here there is uncertainty (randomness) in observations **and** uncertainly about θ, so the distribution has more spread than in Part b, where there is only the uncertainty about θ.

Part d

According to

http://pxnet2.stat.fi/PXWeb/pxweb/fi/StatFin/StatFin__vrm__synt/statfin_synt_pxt_001.px/?rxid=4de473ce-429f-46f6-924a-4d464318b342

the number of boys born in Finland in 2012 was 30308 and the number of girls was 29185.

Update the parameters of the updated posterior distribution and compute its 95% confidence interval.

```
s=30308;          % boys
n=s+29185;        % girls
a=s+a; b=n-s+b;   % update
CI95=betainv([0.025,0.975],a,b)
```

```
CI95 = 1×2
    0.5066    0.5123
```

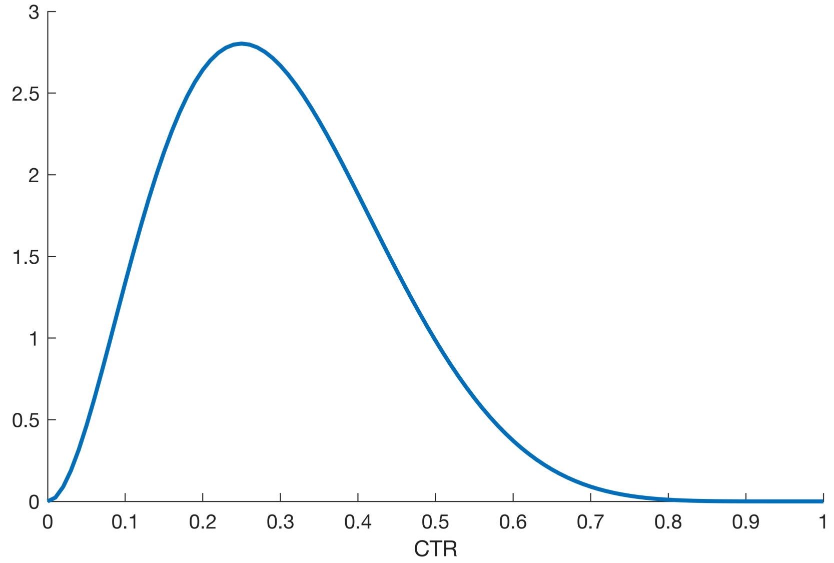
The interval is a bit narrower than in part (a) because there is more data.

Problem 2

Part a

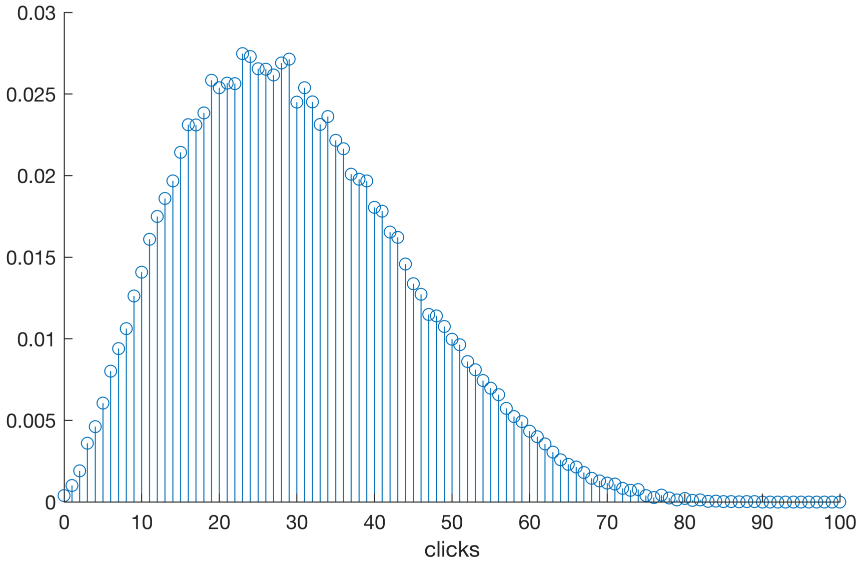
Plot the density function of the prior for CTR.

```
a=3; b=7;                    % parameters of beta prior
CTR=0:.01:1;                 % CTR values for plot
plot(CTR,betapdf(CTR,a,b),'linewidth',2)
box off, xlabel('CTR')
```



Plot the prior predictive density using Monte Carlo approximation as in slide 13.

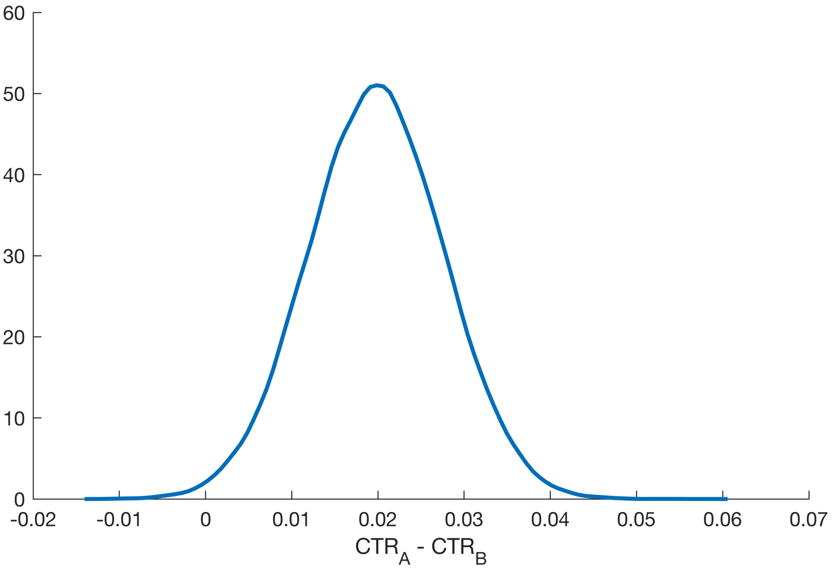
```
rng default, Nsamp=100000;
CTR_samp=betarnd(a,b,Nsamp,1);
n_pred=100;     % number of visits
s_pred=binornd(n_pred,CTR_samp);
p_pred=histcounts(s_pred,-0.5:n_pred+0.5)/Nsamp;
stem(0:n_pred,p_pred);
box off, xlabel('clicks')
```



Part b

Generate Monte Carlo samples from the posterior distribution of the CTR difference (as in slide 23) and plot the density.

```
s=[2654 2214]; n=[7706 6821];  % observations
rng default
Nsamp=100000;
a=s+a; b=n-s+b;
CTR_A=betarnd(a(1),b(1),Nsamp,1);
CTR_B=betarnd(a(2),b(2),Nsamp,1);
[p_diff,ctr]=ksdensity(CTR_A-CTR_B);
plot(ctr,p_diff,'linewidth',2)
box off, xlabel('CTR_{A} - CTR_{B}')
```



The probability that the difference is positive is

```
Prob=sum(CTR_A>CTR_B)/Nsamp
```

```
Prob = 0.9947
```

It is almost certain that the ad with image A has a higher click-through rate than the ad with image B.