Optimisation and Statistical Data Analysis
Exercise Set 11 (Linear Regression)

Problem 1

A sequence of position measurements of a mobile robot is

t_k (s)	1	2	3	4	5	6	7	8	9	10
y_k (m)	−0.083	0.028	0.285	0.780	0.757	1.076	1.173	1.409	1.521	1.773

Assume the measurement model $y_k \,|\, \beta, \sigma^2 \sim \text{norm}(\beta_1 t_k + \beta_2, \sigma)$ and the default prior distribution, i.e. $p(\beta, \sigma^2) \propto \sigma^{-2}$.

(a) Find the posterior 95% confidence intervals for β_1 and β_2 given $y_{1:10}$.

(b) Plot the posterior mean and 95% confidence intervals for positions $\beta_1 t + \beta_2$ over the interval $0 \leq t \leq 12$.

(c) Plot the posterior mean and 95% confidence intervals for predicted position measurements $\tilde{y}$ over the interval $0 \leq t \leq 12$. Show the actual measurements on the same plot. Assess the model's goodness of fit.

(d) Assess goodness of fit using a QQ plot of the scaled residuals.

(e) Now assume that the standard deviation of measurement noise is $\sigma = 0.1$ and that the prior distribution is

$$\beta \sim \text{MVN}(\left[\begin{smallmatrix}0\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}100 & 0\\0 & 100\end{smallmatrix}\right]).$$

Plot the sequence of means of $\beta_1 \,|\, y_{1:k}, \sigma^2$ and $\beta_2 \,|\, y_{1:k}, \sigma^2$ for $k = 0, 1, 2, \ldots, 10$. Find the posterior 95% confidence intervals for β_1 and β_2 given $y_{1:10}$.

Problem 2

The following table shows the average temperature (in °F) and the number of automobile crashes resulting in fatalities in the USA in 2008.

month k	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
avg temp t_k	17	18	29	43	55	65	70	68	59	48	37	22
# crashes y_k	297	280	267	350	328	386	419	410	331	356	326	311

Using the model of Problem 1, find the posterior probability that $\beta_1 > 0$. Discuss whether this result justifies the news headline

> New research proves that driving in winter is safer than at any other time of the year

Answers

1. (a) 0.2077 ± 0.0261, -0.2705 ± 0.1621
(e) 0.2077 ± 0.0216, -0.2705 ± 0.1339.
2. 1.2×10^{-4}