ASE-4046 Exercise 5 Solutions

Problem 1

The equation of the plane and of the cylinder are $g_1(x) = 0$ and $g_2(x) = 0$ where

```
g1=@(x) x(1)-x(2)+x(3)-1;
g2=@(x) x(1)^2+x(2)^2-1;
```

The candidate solution is the point

```
sqrt29=sqrt(sym(29));    % symbolic variable
x=[2/sqrt29 -5/sqrt29 1-7/sqrt29]';
```

Verify that the point is feasible, i.e. verify that $g_1(x) = 0$ and $g_2(x) = 0$:

```
[ g1(x) g2(x)]
```

ans = $\begin{pmatrix} 0 & 0 \end{pmatrix}$

The Lagrange multiplier equation is

$$\nabla f(x) + \lambda_1 \nabla g_1(x) + \lambda_2 \nabla g_2(x) = 0$$

where

$$f = x_1 + 2x_2 + 3x_3, \quad \nabla f = [1\ 2\ 3]^T, \quad \nabla g_1 = [1\ -1\ 1]^T, \quad \nabla g_2 = [2x_1\ 2x_2\ 0]^T$$

The three components of the Lagrange multiplier equation are

```
syms lambda_1 lambda_2
e1=1+lambda_1+2*x(1)*lambda_2==0
```

e1 =

$$\lambda_1 + \frac{4\sqrt{29}\,\lambda_2}{29} + 1 = 0$$

```
e2=2-lambda_1+2*x(2)*lambda_2==0
```

e2 =

$$2 - \frac{10\sqrt{29}\,\lambda_2}{29} - \lambda_1 = 0$$

```
e3=3+lambda_1==0
```

e3 = $\lambda_1 + 3 = 0$

This is a set of three linear equations in the two unknowns λ_1, λ_2.

Verify that the equation set has a solution:

```
s=solve([e1,e2,e3],[lambda_1,lambda_2]) % solve the equations
```

```
s =
    lambda_1: [1×1 sym]
    lambda_2: [1×1 sym]
```

```
[s.lambda_1 s.lambda_2]  % show the solution
```

```
ans =
```

$$\begin{pmatrix} -3 & \frac{\sqrt{29}}{2} \end{pmatrix}$$

Problem 2

The design variables are the coordinates of the sphere's centre (denote them $x_{1:3}$) and the sphere's radius (denote it x_4).

The cost function to be minimised is the sphere radius, which is coded in the function `f` at the end of this script.

A point with coordinates $z_{1:3}$ is inside the sphere if and only if

$$(x_1 - z_1)^2 + (x_2 - z_2)^2 + (x_3 - z_3)^2 \leq x_4^2$$

Each of the given points corresponds to a nonlinear inequality constraint. There are no nonlinear equality constraints. The nonlinear constraints are coded in two-variable function `con2(x,z)` at the end of this script. The given data is the list of point coordinates

```
z=[ 0.3423   -0.4614   -0.7857   -0.8973    0.4951
    1.1052    1.3299   -1.6986   -1.5359    0.2815
   -1.2265   -1.7246    0.8027    0.2764   -0.1611 ];
```

This anonymous function of one input argument encapsulates this data and is used by the solver.

```
con=@(x) con2(x,z);
```

The initial guess is a sphere with centre at the origin and with radius 3:

```
x0=[0 0 0 3]';
```

There are no linear inequality constraints, no linear equality constraints, and no upper-bound constraints:

```
A=[ ]; b=[ ]; Aeq=[ ]; beq=[ ]; ub=[];
```

The only lower-bound constraint is that the radius x_4 must be ≥ 0:

```
lb=[-inf,-inf,-inf,0];   % radius can't be negative
```

Solve the problem and display the solution:

```
[xopt]=fmincon(@f,x0,A,b,Aeq,beq,lb,ub,con);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the optimality tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.

<stopping criteria details>
```

The sphere's centre is located at

```
centre=xopt(1:3)
```

```
centre = 3×1
   -0.6235
   -0.1843
   -0.4609
```

The sphere's radius is

```
radius=xopt(4)
```

```
radius = 1.9789
```

From the values of the constraint function, we see that the second and third points are on the surface (the corresponding constraint function values are zero); the other points are inside the sphere:

```
con(xopt)
```

```
ans = 1×5
   -0.7342   -0.0000   -0.0000   -1.4707   -2.3578
```

Local functions

The cost function for Problem 2 is the radius of the sphere, which is the design variable x_4.

```
function fval=f(x)
fval=x(4);
end
```

The constraint function evaluates the right hand side of the inequality constraint

$$(x_1 - z_1)^2 + (x_2 - z_2)^2 + (x_3 - z_3)^2 - x_4^2 \leq 0$$

This version of the constraint function has two input arguments. In the calling script, the data array is defined and then an anonymous one-input function is created for use with the `fmincon` solver.

```
function [c,ceq]=con2(x,z)
c=(x(1)-z(1,:)).^2+(x(2)-z(2,:)).^2+(x(3)-z(3,:)).^2-x(4)^2;
ceq=[ ]; % no equality constraints
end
```