Discrete Mathematics Quiz-1 INF 111/10/19

1. (a) Find the number of arrangements of the letters in HELLOWHELLOW ? (12/115)

   (b) How many of these arrangements have no consecutive L's ?

2. There are 4 balls, labeled 1, 2, 3, and 4 respectively, in a box. If the ball taken from the box with repetition allows, then find the probability that the number on the ball, taken for the second time, is greater than the number on the ball, taken for the first time. (12/115)

3. Simply the following statements by using the laws of logic. (15/115)

   (a) $(\neg p \vee q) \wedge (p \wedge (p \wedge q))$. (b) $(p \rightarrow q) \vee (\neg q \wedge (r \vee \neg q))$

   (c) $[[[(p \wedge q) \wedge r] \vee [(p \wedge q) \wedge \neg r]] \vee \neg q] \rightarrow p$.

4. Find the value of *counter* after the following program segment is executed. (15/115)

(a)
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counter := 0
  for i := 1 to n do
    for j := 1 to i do
      for k := 1 to j do
        counter := counter + 5
```

(b)
```
counter := 0
  for i := 1 to n do
    for j := 1 to i-1 do
      for k := 1 to j-1 do
        for l := 1 to k-1 do
          counter := counter + 5
```

5. (a) In how many ways can a particle move in the xy-plane from the point (2,3) to the point (6,9) if the moves that are allowed are the form: (12/115)

   (R): $(x, y) \rightarrow (x + 1, y)$; (U): $(x, y) \rightarrow (x, y+1)$; (D): $(x, y) \rightarrow (x+1, y+1)$?

   (b) How many of the paths in part (a) do not use the path from (3,4) to (3, 5) to (4,5) to (4,6)?

6. Let p and q be primitive statements. Determine each of the following is a tautology or NOT?

   (a) $p \rightarrow (p \wedge q)$ (b) $[p \wedge (p \rightarrow q)] \rightarrow q$ (c) $[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$ (10/115)

   (d) $[(p \wedge q) \vee \neg p] \rightarrow q$ (e) $(p \wedge q) \rightarrow p$.?

7. In how many ways can one parenthesize the product *abcdefg*? (10/115)

8. An island has two tribes of native. Any native from the first tribe always tells the truth, while any native from the other tribe always lies. You arrive at the island and ask a native if there is gold on the island. "There is gold on the island if and only if I always lie." Is there gold on the island? What is your conclusions and explains it in detail. (15/115)

9. For any positive integer n, show that (14/115)

$$\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \binom{n}{6} + \cdots = \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \binom{n}{7} + \cdots$$

Discrete Mathematics   Mid-term.   111/11/11   INF

1. Sixteen people are to be seated at two circular tables, one of which seats 10 while the other seats six. How many different seating arrangements are possible? (10/120)

5! × 9!

2. Let $p(x)$, $q(x)$, and $r(x)$ denote the following open statements. (10/120)

$p(x)$: $x^2 - 8x + 15 = 0$, $(x-3)(x-5)$ $q(x)$: x is odd, $r(x)$: $x > 0$.

For the universe of all integers, determine the truth or falsity of each of the following statements.

(a) $\forall x\, [p(x) \rightarrow q(x)]$, (b) $\forall x\, [q(x) \rightarrow p(x)]$, (c) $\forall x\, [\neg q(x) \rightarrow \neg\, p(x)]$,

(d) $\exists x\, [p(x) \rightarrow (q(x) \wedge r(x))]$, (e) $\forall x\, [(p(x) \vee q(x)) \rightarrow r(x)]$.

3. Simply the following statements by using the laws of logic. (12/120)

(a) $p \vee q \vee (\neg\, p \wedge \neg\, q \wedge r)$.

(b) $[[[(p \wedge q) \wedge r] \vee [(p \wedge q) \wedge \neg r]] \vee \neg q] \rightarrow q$

4. Find all $x, y \in \mathbb{Z}$ such that (16/120)

(a) $2463x + 512y = 1$

(b) $1560x + 910y = 1430$

5. In how many ways can we distribute nine identical white balls into four distinct containers so that

(a) no container is left empty? (13/120)

(a) $C^9_1 \times C^8_2 \times C^6_3 \times C^3_3$

(b) the fourth container has an odd number of balls in it?

9 balls 4 boxes (1,2,3,3) (2,2,2,3) 9 = 1+2+3+3 = 2+2+2+3

(1,2,2,4) (2,3,1,3) = 1+2+2+4 = 2+3+1+3

(1,1,1,5) = 1+1+1+5

6. Let a, b be integers so that $5a + 6b$ is a multiple of 19. Prove that 19 divides $7a + 16b$. (12/120)

7. Consider the moves R: $(x,y) \rightarrow (x+1,y)$ and U: $(x,y) \rightarrow (x,y+1)$, travel in the xy-plane. In how many ways can one go (15/120)

(a) from (0, 0) to (6, 6) and may touch but not rise above the line $y = x$?

(b) from (3, 10) to (10, 17) and may touch but not rise above the line $y = x + 7$? (1, 8)

(c) from (0,0) to (9,4) and the number of U's may never exceed the number of R's along the path taken?

8. For any statement p, q, represent the following: (I) $p \wedge q$ (II) $p \oplus q$ by using only (18/120)

(a) NAND (b) NOR connectives.

9. Let $a, b, c \in \mathbb{Z}^+$, with $c = \mathrm{lcm}(a, b)$. If d is a common multiple of a and b, then show that $c \mid d$.

(14/120)

Discrete Mathematics Quiz-2 INF 111/12/21

1. Let $f: A \to B$ be a function with finite sets A, B then fills in the blanks bellow. (15/117)

(a) if f is one to one then (1) $|A| \le |B|$ or $|f(A)| = |A|$ (b) if f is not one to one then (2) $|f(A)| < |A|$

(c) if f is onto then (3) $B = f(A)$ or $|B| = |f(A)|$ or $|A| \ge |B|$ (d) if f is one to one and onto then (4) $|A| = |B|$

(e) $|f| =$ (5) $|A|$

2. Let $f, g: Z^+ \to Z^+$, where for all $x \in Z^+$, $f(x) = x + 1$ and $g(x) = \max\{1, x-1\}$, then (14/117)

(a) What is the range of f? $f(Z^+) = Z^+ - \{-\}$

(b) Is the function f one-to-one (y/n)? Is f an onto function (y/n)? yes (1-1), no (onto)

$\sum_{k=0}^{n} (-1)^k \binom{n}{n-k} (n-k)^m$

(c) What is the range of g? $g(Z^+) = Z^+$

(d) Is the function g one-to-one (y/n)? Is g an onto function (y/n)? no (1-1), yes onto

3. Find the number of positive divisors of $n = 2^{15} \times 3^9 \times 5^{10} \times 7^{11} \times 11^3 \times 13^6 \times 37^{10}$ such that

(a) divisible by 1,166,400,000 (12/117)

(b) perfect squares that are divisible by $2^4 \times 3^5 \times 5^2 \times 11^2 \times 37$

(c) perfect cubes that are multiple of $2^{10} \times 3^9 \times 5^2 \times 7^5 \times 11^2 \times 13^2 \times 37^2$.

4. If U = {1,2,3,4,5}, A = {1,2,3}, and B = {2,4,5}, determine the following: (15/117)

(a) the number of relations from A to B; (b) the number of binary relations on A; (c) the number of relations from A to B that contain (1,2) and (1,5); (d) the number of relations from A to B that contain exactly five ordered pairs; and (e) the number of binary relations on A that contain at least seven elements.

a) 512 b) 512 c) 128 A: d) 126^4 e) 46^{15}

5^{15} 3×5^9

5. Let A = $\{x, a, b, c, d, e, f\}$. (13/117)

(a) How many closed binary operations f on A satisfy $f(a, b) = c$? 7^{48}

(b) How many of the functions f in part (a) have x as an identity? 7^{35}

(c) How many of the functions f in part (a) have an identity? 5×7^{35}

(d) How many of the functions f in part (c) are commutative? 5×7^{20}

6. For each of the following functions f: R → R, determine whether or not f is invertible. If it is then find f^{-1} and if it is not then explain the reason. (14/117)

(a) $f = \{ (x,y) \mid ax + by = c, b \neq 0\}$ if $a = 0$ then f is not 1-1, f^{-1}: does not exist

if $a \neq 0$ then $f^{-1}(x) = \frac{1}{a}((-bx)$, $a \neq 0$, $b \neq 0$

(b) $f = \{ (x,y) \mid y = x^3\}$

(c) $f = \{ (x,y) \mid y = x^4 - x\}$

b) $f^{-1}(x) = \sqrt[3]{x}$

c) $\because f(0) = f(1) = 0 \Rightarrow f$ is not 1-1 $\Rightarrow f^{-1}$ does not exist

7. Let $f: A \to B$, $g: B \to C$ be functions then, (18/117)

(a) Prove that if both f and g are bijective then $g \circ f: A \to C$ is bijective ; and

(b) If $g \circ f: A \to C$ is one-to-one, then what are the necessary conditions do f and g be needed?

b) Necessary conditions: (1) $f: A \to B$ is one to one. (2) $g': f(A) \to C$ is one to one where $g'(b) = g(b)\ \forall b \in f(A)$

8. (a) Let n = 88,200. In how many ways can one factor n as ab where $1 < a < n$, $1 < b < n$, and $\gcd(a, b) = 1$. (Note : Here order is not relevant.) (16/117)

(b) Answer part (a) for $n = 970{,}200$.

(c) Generalize the results in parts (a) and (b).

Discrete Mathematics Final INF 112/1/13

1. At the CH Company, John, the supervisor, has a secretary, Teresa, and four other administrative assistants. If there are seven accounts to be processed, in how many ways can Joan assign accounts so that each assistant works on at least one account and Teresa's work includes the most expensive account? (12/123)

6. 5 6. 4 (1,1,1,3) (1,1,2,2) (1,1,1,1,2)

2. Determine the constants b and c if $a_n = c_1 + c_2(7^n)$, $n \geq 0$, is the general solution of the relation $a_{n+2} + ba_{n+1} + ca_n = 0$, $n \geq 0$. (10/123)

$x^2+bx+c=0$ $(x-1)(x-7)=0$ $x^2-8x+7=0$ $b=-8$ $c=7$

3. Each of the following functions $f: N \times N \rightarrow N$, where $N = Z^+ \cup \{0\}$, is a binary operation on N. Determine in each case whether f has an identity or not. If it has, then find that identity. (15/123)

(a) $f(x,y) = x + y - xy$ (b) $f(x,y) = max\{x,y\}$

(c) $f(x,y) = x^y$ $x \neq 0$ $y \geq 0$ (d) $f(x,y) = x + y - 3$ $x+y-3 \geq 0$

4. Let A={a, b, c, d, e, f}, B ={w, x, y, z} and f be a binary operation on A then find the number of (13/123)

(a) relations from A to B; $2^{6\times 4} = 2^{24}$

(b) commutative functions f that has an identity;

(c) commutative functions f with $f(b, c) = a$ and identity e.

5. Let $\alpha \pm i\beta$ be the roots of the characteristic equation of the recurrence relation $c_n a_n + c_{n-1}a_{n-1} + c_{n-2}a_{n-2} = 0$ where $\alpha, \beta, c_n \neq 0, c_{n-1}$, and $c_{n-2} \neq 0$ are real. Prove that the general solution of the recurrence relation is : (14/123)

$$a_n = \rho^n (A\cos n\theta + B\sin n\theta), \text{ where } \rho = \sqrt{\alpha^2 + \beta^2},\ \tan\theta = \frac{\beta}{\alpha}, \text{ and } A, B \text{ are constant.}$$

6. Solve the following recurrence relations. (16/123)

(a) $a_{n+1} - a_n = 3n^2 - n,\ n \geq 0, a_0 = 3$

(b) $a_n + na_{n-1} = n!$ for $n \geq 1,\ a_0 = 1$.

a) $a_{n+1} = 3n^2 - n + a_n$ $\sum_{i=0}^{n} 3n^2 - n = f(n) =$ $a_1 = 3$ $a_{n+1} = f(n) + a_n$ $a_n = f(n) + a_0$ $a_1 = f(n) + a_0$ $a_2 = f(n) + a_1$ $a_3 = f(n) + a_2$ $3\sum_{i=0}^{n} n^2 - \sum_{i=0}^{n} n = 3\left(\frac{1}{6}(n+1)(2n+1)n\right) - \frac{(1+n)n}{2}$

b) $a_1 + 1 = 1$ $a_1 = 0$ $a_2 + 0 = 2$ $a_2 = 2$ $a_3 + 6 = 3!$

7. Let $D_n = \begin{vmatrix} 5 & 3 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 2 & 5 & 3 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 2 & 5 & 3 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 5 & 3 & \cdots & 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & \cdots & 2 & 5 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 2 & 5 & 3 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 2 & 5 \end{vmatrix}_{nxn}$ be a n x n determinant. Find and solve a recurrence relation for the value of D_n? (13/123)

$125-30-30 = 65$ $D_1 = 5$ $D_2 = 19$ $D_n = 5D_{n-1} + 19D_{n-2}$ $D_1 = 5D_0 +$ $D_3 = 5D_2 + 19D_1$ 95 $95 - 30 = 65$ $5D_3 - 6D_1$ $D_n = 5D_{n-1} - 6D_{n-2}$ $D_n - 5D_{n-1} + 6D_{n-2} = 0$ $x^2 - 5x + 6x = 0$ $(x-3)(x-2) = 0$ $x = 3$ or 2 $A(3)^n + B(2)^n = a_n$

8. Let n lines be drawn in the plane such that each line intersects every other line but no three lines are ever coincident. For $n \geq 0$, let a_n count the number of regions into which the plane is separated by the n lines. Find and solve a recurrence relation for a_n. (14/123)

9. Prove or disprove: Let f: A $\rightarrow$ B, g : B $\rightarrow$ C be functions then, (16/123)

(a) If $g \circ f$: A $\rightarrow$ C is one to one, then g is one to one.

(b) If $g \circ f$: A $\rightarrow$ C is onto, then g is onto.