EXERCISES 1.1 AND 1.2

1. During a local campaign, eight Republican and five Democratic candidates are nominated for president of the school board.

a) If the president is to be one of these candidates, how many possibilities are there for the eventual winner?

b) How many possibilities exist for a pair of candidates (one from each party) to oppose each other for the eventual election?

c) Which counting principle is used in part (a)? in part (b)?

2. Answer part (c) of Example 1.6.

3. Buick automobiles come in four models, 12 colors, three engine sizes, and two transmission types. (a) How many distinct Buicks can be manufactured? (b) If one of the available colors is blue, how many different blue Buicks can be manufactured?

4. The board of directors of a pharmaceutical corporation has 10 members. An upcoming stockholders' meeting is scheduled to approve a new slate of company officers (chosen from the 10 board members).

a) How many different slates consisting of a president, vice president, secretary, and treasurer can the board present to the stockholders for their approval?

b) Three members of the board of directors are physicians. How many slates from part (a) have (i) a physician nominated for the presidency? (ii) exactly one physician appearing on the slate? (iii) at least one physician appearing on the slate?

5. While on a Saturday shopping spree Jennifer and Tiffany witnessed two men driving away from the front of a jewelry shop, just before a burglar alarm started to sound. Although everything happened rather quickly, when the two young ladies were questioned they were able to give the police the following information about the license plate (which consisted of two letters followed by four digits) on the get-away car. Tiffany was sure that the second letter on the plate was either an O or a Q and the last digit was either a 3 or an 8. Jennifer told the investigator that the first letter on the plate was either a C or a G and that the first digit was definitely a 7. How many different license plates will the police have to check out?

6. To raise money for a new municipal pool, the chamber of commerce in a certain city sponsors a race. Each participant pays a $5 entrance fee and has a chance to win one of the different-sized trophies that are to be awarded to the first eight runners who finish.

a) If 30 people enter the race, in how many ways will it be possible to award the trophies?

b) If Roberta and Candice are two participants in the race, in how many ways can the trophies be awarded with these two runners among the top three?

7. A certain "Burger Joint" advertises that a customer can have his or her hamburger with or without any or all of the following: catsup, mustard, mayonnaise, lettuce, tomato, onion, pickle, cheese, or mushrooms. How many different kinds of hamburger orders are possible?

8. Matthew works as a computer operator at a small university. One evening he finds that 12 computer programs have been submitted earlier that day for batch processing. In how many ways can Matthew order the processing of these programs if (a) there are no restrictions? (b) he considers four of the programs higher in priority than the other eight and wants to process those four first? (c) he first separates the programs into four of top priority, five of lesser priority, and three of least priority, and he wishes to process the 12 programs in such a way that the top-priority programs are processed first and the three programs of least priority are processed last?

9. Patter's Pastry Parlor offers eight different kinds of pastry and six different kinds of muffins. In addition to bakery items one can purchase small, medium, or large containers of the following beverages: coffee (black, with cream, with sugar, or with cream and sugar), tea (plain, with cream, with sugar, with cream and sugar, with lemon, or with lemon and sugar), hot cocoa, and orange juice. When Carol comes to Patter's, in how many ways can she order

a) one bakery item and one medium-sized beverage for herself?

b) one bakery item and one container of coffee for herself and one muffin and one container of tea for her boss, Ms. Didio?

c) one piece of pastry and one container of tea for herself, one muffin and a container of orange juice for Ms. Didio, and one bakery item and one container of coffee for each of her two assistants, Mr. Talbot and Mrs. Gillis?

10. Pamela has 15 different books. In how many ways can she place her books on two shelves so that there is at least one book on each shelf? (Consider the books in each arrangement to be stacked one next to the other, with the first book on each shelf at the left of the shelf.)

11. Three small towns, designated by A, B, and C, are interconnected by a system of two-way roads, as shown in Fig. 1.4.

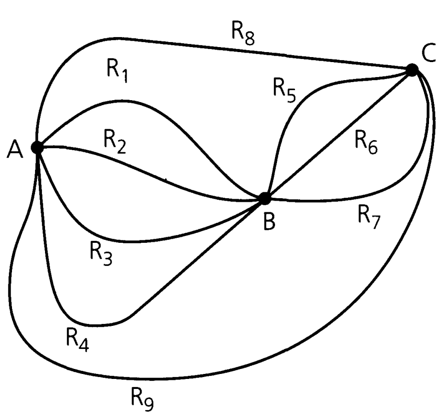

Figure 1.4

a) In how many ways can Linda travel from town A to town C?

b) How many different round trips can Linda travel from town A to town C and back to town A?

c) How many of the round trips in part (b) are such that the return trip (from town C to town A) is at least partially different from the route Linda takes from town A to town C? (For example, if Linda travels from town A to town C along roads R_1 and R_6, then on her return she might take roads R_6 and R_3, or roads R_7 and R_2, or road R_9, among other possibilities, but she does *not* travel on roads R_6 *and* R_1.)

12. List all the permutations for the letters a, c, t.

13. a) How many permutations are there for the eight letters a, c, f, g, i, t, w, x?

b) Consider the permutations in part (a). How many start with the letter t? How many start with the letter t and end with the letter c?

14. Evaluate each of the following.

a) $P(7, 2)$ **b)** $P(8, 4)$ **c)** $P(10, 7)$ **d)** $P(12, 3)$

15. In how many ways can the symbols a, b, c, d, e, e, e, e, e be arranged so that no e is adjacent to another e?

16. An alphabet of 40 symbols is used for transmitting messages in a communication system. How many distinct messages (lists of symbols) of 25 symbols can the transmitter generate if symbols can be repeated in the message? How many if 10 of the 40 symbols can appear only as the first and/or last symbols of the message, the other 30 symbols can appear anywhere, and repetitions of all symbols are allowed?

17. In the Internet each network interface of a computer is assigned one, or more, Internet addresses. The nature of these Internet addresses is dependent on network size. For the Internet Standard regarding reserved network numbers (STD 2), each address is a 32-bit string which falls into one of the following three classes: (1) A class A address, used for the largest networks, begins with a 0 which is then followed by a seven-bit *network number*, and then a 24-bit *local address*. However, one is restricted from using the network numbers of all 0's or all 1's and the local addresses of all 0's or all 1's. (2) The class B address is meant for an intermediate-sized network. This address starts with the two-bit string 10, which is followed by a 14-bit network number and then a 16-bit local address. But the local addresses of all 0's or all 1's are not permitted. (3) Class C addresses are used for the smallest networks. These addresses consist of the three-bit string 110, followed by a 21-bit network number, and then an eight-bit local address. Once again the local addresses of all 0's or all 1's are excluded. How many different addresses of each class are available on the Internet, for this Internet Standard?

18. Morgan is considering the purchase of a low-end computer system. After some careful investigating, she finds that there are seven basic systems (each consisting of a monitor, CPU, keyboard, and mouse) that meet her requirements. Furthermore, she also plans to buy one of four modems, one of three CD ROM drives, and one of six printers. (Here each peripheral device of a given type, such as the modem, is compatible with all seven basic systems.) In how many ways can Morgan configure her low-end computer system?

19. A computer science professor has seven different programming books on a bookshelf. Three of the books deal with C++, the other four with Java. In how many ways can the professor arrange these books on the shelf (a) if there are no restrictions? (b) if the languages should alternate? (c) if all the C++ books must be next to each other? (d) if all the C++ books must be next to each other and all the Java books must be next to each other?

20. Over the Internet, data are transmitted in structured blocks of bits called *datagrams*.

a) In how many ways can the letters in DATAGRAM be arranged?

b) For the arrangements of part (a), how many have all three A's together?

21. a) How many arrangements are there of all the letters in SOCIOLOGICAL?

b) In how many of the arrangements in part (a) are A and G adjacent?

c) In how many of the arrangements in part (a) are all the vowels adjacent?

22. How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000?

23. Twelve clay targets (identical in shape) are arranged in four hanging columns, as shown in Fig. 1.5. There are four red targets in the first column, three white ones in the second column, two green targets in the third column, and three blue ones in the fourth column. To join her college drill team, Deborah must break all 12 of these targets (using her pistol and only 12 bullets) and in so doing must always break the existing target at the bottom of a column. Under these conditions, in how many different orders can Deborah shoot down (and break) the 12 targets?

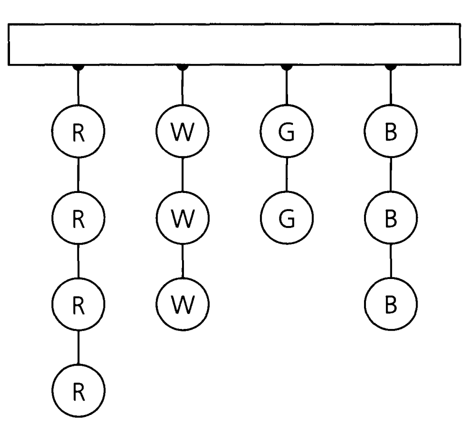

Figure 1.5

24. Show that for all integers $n, r \geq 0$, if $n + 1 > r$, then
$$P(n+1, r) = \left(\frac{n+1}{n+1-r}\right) P(n, r).$$

25. Find the value(s) of n in each of the following: (a) $P(n, 2) = 90$, (b) $P(n, 3) = 3P(n, 2)$, and (c) $2P(n, 2) + 50 = P(2n, 2)$.

26. How many different paths in the xy-plane are there from (0, 0) to (7, 7) if a path proceeds one step at a time by going either one space to the right (R) or one space upward (U)? How many such paths are there from (2, 7) to (9, 14)? Can any general statement be made that incorporates these two results?

27. a) How many distinct paths are there from $(-1, 2, 0)$ to $(1, 3, 7)$ in Euclidean three-space if each move is one of the following types?
$$\text{(H)}: (x, y, z) \to (x+1, y, z);$$
$$\text{(V)}: (x, y, z) \to (x, y+1, z);$$
$$\text{(A)}: (x, y, z) \to (x, y, z+1)$$

b) How many such paths are there from (1, 0, 5) to (8, 1, 7)?

c) Generalize the results in parts (a) and (b).

28. a) Determine the value of the integer variable *counter* after execution of the following program segment. (Here i, j, and k are integer variables.)

```
counter := 0
for i := 1 to 12 do
  counter := counter + 1
for j := 5 to 10 do
  counter := counter + 2
for k := 15 downto 8 do
  counter := counter + 3
```

b) Which counting principle is at play in part (a)?

29. Consider the following program segment where i, j, and k are integer variables.

```
for i := 1 to 12 do
  for j := 5 to 10 do
    for k := 15 downto 8 do
      print (i - j)*k
```

a) How many times is the **print** statement executed?

b) Which counting principle is used in part (a)?

30. A sequence of letters of the form abcba, where the expression is unchanged upon reversing order, is an example of a *palindrome* (of five letters). (a) If a letter may appear more than twice, how many palindromes of five letters are there? of six letters? (b) Repeat part (a) under the condition that no letter appears more than twice.

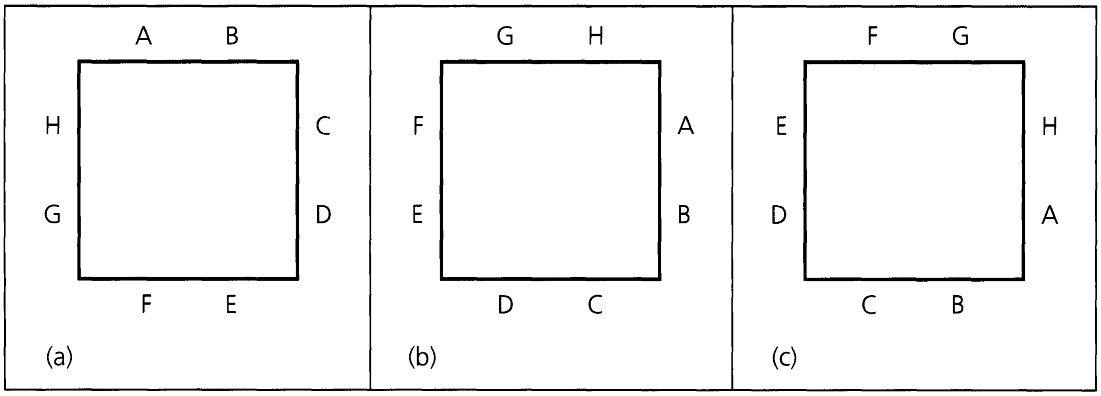

Figure 1.6

31. Determine the number of six-digit integers (no leading zeros) in which (a) no digit may be repeated; (b) digits may be repeated. Answer parts (a) and (b) with the extra condition that the six-digit integer is (i) even; (ii) divisible by 5; (iii) divisible by 4.

32. a) Provide a combinatorial argument to show that if n and k are positive integers with $n = 3k$, then $n!/(3!)^k$ is an integer.

b) Generalize the result of part (a).

33. a) In how many possible ways could a student answer a 10-question true-false test?

b) In how many ways can the student answer the test in part (a) if it is possible to leave a question unanswered in order to avoid an extra penalty for a wrong answer?

34. How many distinct four-digit integers can one make from the digits 1, 3, 3, 7, 7, and 8?

35. a) In how many ways can seven people be arranged about a circular table?

b) If two of the people insist on sitting next to each other, how many arrangements are possible?

36. a) In how many ways can eight people, denoted A, B, . . . , H be seated about the square table shown in Fig. 1.6, where Figs. 1.6(a) and 1.6(b) are considered the same but are distinct from Fig. 1.6(c)?

b) If two of the eight people, say A and B, do not get along well, how many different seatings are possible with A and B not sitting next to each other?

37. Sixteen people are to be seated at two circular tables, one of which seats 10 while the other seats six. How many different seating arrangements are possible?

38. A committee of 15 — nine women and six men — is to be seated at a circular table (with 15 seats). In how many ways can the seats be assigned so that no two men are seated next to each other?

39. Write a computer program (or develop an algorithm) to determine whether there is a three-digit integer abc $(= 100a + 10b + c)$ where $abc = a! + b! + c!$.

Solutions

Chapter 1
Fundamental Principles of Counting

Sections 1.1 and 1.2—p. 11

1. a) 13 **b)** 40 **c)** The rule of sum in part (a); the rule of product in part (b)
3. a) 288 **b)** 24
5. $2 \times 2 \times 1 \times 10 \times 10 \times 2 = 800$ different license plates
7. 2^9 **9. a)** $(14)(12) = 168$ **b)** $(14)(12)(6)(18) = 18{,}144$ **c)** 73,156,608
11. a) $12 + 2 = 14$ **b)** $14 \times 14 = 196$ **c)** 182
13. a) $P(8, 8) = 8!$ **b)** $7!$ $6!$ **15.** $4! = 24$
17. Class A: $(2^7 - 2)(2^{24} - 2) = 2{,}113{,}928{,}964$
Class B: $2^{14}(2^{16} - 2) = 1{,}073{,}709{,}056$
Class C: $2^{12}(2^8 - 2) = 1{,}040{,}384$
19. a) $7! = 5040$ **b)** $(4!)(3!) = 144$ **c)** $(5!)(3!) = 720$ **d)** 288
21. a) $12!/(3!\,2!\,2!\,2!)$ **b)** $2[11!/(3!\,2!\,2!\,2!)]$ **c)** $[7!/(2!\,2!)][6!/(3!\,2!)]$
23. $12!/(4!\,3!\,2!\,3!) = 277{,}200$ **25. a)** $n = 10$ **b)** $n = 5$ **c)** $n = 5$
27. a) $(10!)/(2!\,7!) = 360$ **b)** 360
c) Let x, y, and z be any real numbers and let m, n, and p be any nonnegative integers. The number of paths from (x, y, z) to $(x + m, y + n, z + p)$, as described in part (a), is $(m + n + p)!/(m!\,n!\,p!)$.
29. a) 576 **b)** The rule of product
31. a) $9 \times 9 \times 8 \times 7 \times 6 \times 5 = 136{,}080$ **b)** 9×10^5
(i) (a) 68,880 (b) 450,000
(ii) (a) 28,560 (b) 180,000
(iii) (a) 33,600 (b) 225,000
33. a) 2^{10} **b)** 3^{10} **35. a)** $6!$ **b)** $2(5!) = 240$
37. $\binom{16}{10}9!\,5! = 348{,}713{,}164{,}800$