

9. Tucker, Alan. *Applied Combinatorics*, 4th ed. New York: Wiley, 2002.
 10. Whitworth, W. A. *Choice and Chance*. Reprint of the 1901 edition. New York: Hafner, 1965.

SUPPLEMENTARY EXERCISES

1. In the manufacture of a certain type of automobile, four kinds of major defects and seven kinds of minor defects can occur. For those situations in which defects do occur, in how many ways can there be twice as many minor defects as there are major ones?

2. A machine has nine different dials, each with five settings labeled 0, 1, 2, 3, and 4.

a) In how many ways can all the dials on the machine be set?

b) If the nine dials are arranged in a line at the top of the machine, how many of the machine settings have no two adjacent dials with the same setting?

3. Twelve points are placed on the circumference of a circle and all the chords connecting these points are drawn. What is the largest number of points of intersection for these chords?

4. A choir director must select six hymns for a Sunday church service. She has three hymn books, each containing 25 hymns (there are 75 different hymns in all). In how many ways can she select the hymns if she wishes to select (a) two hymns from each book? (b) at least one hymn from each book?

5. How many ways are there to place 25 different flags on 10 numbered flagpoles if the order of the flags on a flagpole is (a) not relevant? (b) relevant? (c) relevant and every flagpole flies at least one flag?

6. A penny is tossed 60 times yielding 45 heads and 15 tails. In how many ways could this have happened so that there were no consecutive tails?

7. There are 12 men at a dance. (a) In how many ways can eight of them be selected to form a cleanup crew? (b) How many ways are there to pair off eight women at the dance with eight of these 12 men?

8. In how many ways can the letters in WONDERING be arranged with exactly two consecutive vowels?

9. Dustin has a set of 180 distinct blocks. Each of these blocks is made of either wood or plastic and comes in one of three sizes (small, medium, large), five colors (red, white, blue, yellow, green), and six shapes (triangular, square, rectangular, hexagonal, octagonal, circular). How many of the blocks in this set differ from

a) the *small red wooden square* block in exactly one way? (For example, the *small red plastic square* block is one such block.)

b) the *large blue plastic hexagonal* block in exactly two ways? (For example, the *small red plastic hexagonal* block is one such block.)

10. Mr. and Mrs. Richardson want to name their new daughter so that her initials (first, middle, and last) will be in alphabetical order with no repeated initial. How many such triples of initials can occur under these circumstances?

11. In how many ways can the 11 identical horses on a carousel be painted so that three are brown, three are white, and five are black?

12. In how many ways can a teacher distribute 12 different science books among 16 students if (a) no student gets more than one book? (b) the oldest student gets two books but no other student gets more than one book?

13. Four numbers are selected from the following list of numbers: $-5, -4, -3, -2, -1, 1, 2, 3, 4$. (a) In how many ways can the selections be made so that the product of the four numbers is positive and (i) the numbers are distinct? (ii) each number may be selected as many as four times? (iii) each number may be selected at most three times? (b) Answer part (a) with the product of the four numbers negative.

14. Waterbury Hall, a university residence hall for men, is operated under the supervision of Mr. Kelly. The residence has three floors, each of which is divided into four sections. This coming fall Mr. Kelly will have 12 resident assistants (one for each of the 12 sections). Among these 12 assistants are the four senior assistants — Mr. DiRocco, Mr. Fairbanks, Mr. Hyland, and Mr. Thornhill. (The other eight assistants will be new this fall and are designated as junior assistants.) In how many ways can Mr. Kelly assign his 12 assistants if

a) there are no restrictions?

b) Mr. DiRocco and Mr. Fairbanks must both be assigned to the first floor?

c) Mr. Hyland and Mr. Thornhill must be assigned to different floors?

15. a) How many of the 9000 four-digit integers 1000, 1001, 1002, ..., 9998, 9999 have four distinct digits that are either increasing (as in 1347 and 6789) or decreasing (as in 6421 and 8653)?

b) How many of the 9000 four-digit integers 1000, 1001, 1002, ..., 9998, 9999 have four digits that are either non-decreasing (as in 1347, 1226, and 7778) or nonincreasing (as in 6421, 6622, and 9888)?

16. a) Find the coefficient of x^2yz^2 in the expansion of $[(x/2) + y - 3z]^5$.

- b) How many distinct terms are there in the complete expansion of

$$\left(\frac{x}{2} + y - 3z\right)^5 ?$$

- c) What is the sum of all coefficients in the complete expansion?

17. a) In how many ways can 10 people, denoted A, B, ..., I, J, be seated about the rectangular table shown in Fig. 1.11, where Figs. 1.11(a) and 1.11(b) are considered the same but are considered different from Fig. 1.11(c)?

- b) In how many of the arrangements of part (a) are A and B seated on longer sides of the table across from each other?

18. a) Determine the number of nonnegative integer solutions to the pair of equations

$$x_1 + x_2 + x_3 = 6, \quad x_1 + x_2 + \dots + x_5 = 15, \\ x_i \geq 0, \quad 1 \leq i \leq 5.$$

- b) Answer part (a) with the pair of equations replaced by the pair of inequalities

$$x_1 + x_2 + x_3 \leq 6, \quad x_1 + x_2 + \dots + x_5 \leq 15, \\ x_i \geq 0, \quad 1 \leq i \leq 5.$$

19. For any given set in a tennis tournament, opponent A can beat opponent B in seven different ways. (At 6–6 they play a tie breaker.) The first opponent to win three sets wins the tournament. (a) In how many ways can scores be recorded with A winning in five sets? (b) In how many ways can scores be recorded with the tournament requiring at least four sets?

20. Given n distinct objects, determine in how many ways r of these objects can be arranged in a circle, where arrangements are considered the same if one can be obtained from the other by rotation.

21. For every positive integer n , show that

$$\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \dots = \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots$$

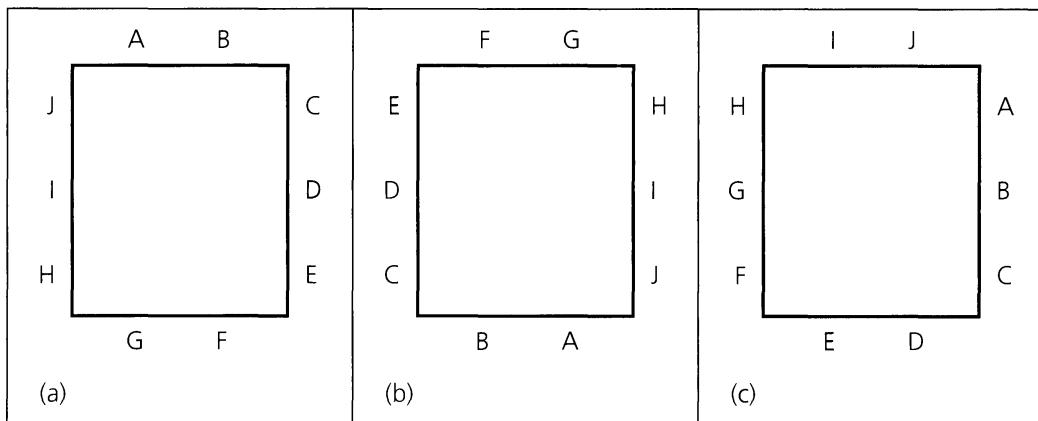


Figure 1.11

22. a) In how many ways can the letters in UNUSUAL be arranged?

- b) For the arrangements in part (a), how many have all three U's together?

- c) How many of the arrangements in part (a) have no consecutive U's?

23. Francesca has 20 different books but the shelf in her dormitory residence will hold only 12 of them.

- a) In how many ways can Francesca line up 12 of these books on her bookshelf?

- b) How many of the arrangements in part (a) include Francesca's three books on tennis?

24. Determine the value of the integer variable *counter* after execution of the following program segment. (Here i , j , k , l , m , and n are integer variables. The variables r , s , and t are also integer variables; their values — where $r \geq 1$, $s \geq 5$, and $t \geq 7$ — have been set prior to this segment.)

```
counter := 10
for i := 1 to 12 do
  for j := 1 to r do
    counter := counter + 2
  for k := 5 to s do
    for l := 3 to k do
      counter := counter + 4
    for m := 3 to 12 do
      counter := counter + 6
    for n := t downto 7 do
      counter := counter + 8
```

25. a) Find the number of ways to write 17 as a sum of 1's and 2's if order is relevant.

- b) Answer part (a) for 18 in place of 17.

- c) Generalize the results in parts (a) and (b) for n odd and for n even.

26. a) In how many ways can 17 be written as a sum of 2's and 3's if the order of the summands is (i) not relevant? (ii) relevant?

b) Answer part (a) for 18 in place of 17.

27. a) If n and r are positive integers with $n \geq r$, how many solutions are there to

$$x_1 + x_2 + \cdots + x_r = n,$$

where each x_i is a positive integer, for $1 \leq i \leq r$?

- b) In how many ways can a positive integer n be written as a sum of r positive integer summands ($1 \leq r \leq n$) if the order of the summands is relevant?

28. a) In how many ways can one travel in the xy -plane from $(1, 2)$ to $(5, 9)$ if each move is one of the following types:

(R): $(x, y) \rightarrow (x + 1, y)$; (U): $(x, y) \rightarrow (x, y + 1)$?

- b) Answer part (a) if a third (diagonal) move

(D): $(x, y) \rightarrow (x + 1, y + 1)$

is also possible.

29. a) In how many ways can a particle move in the xy -plane from the origin to the point $(7, 4)$ if the moves that are allowed are of the form:

(R): $(x, y) \rightarrow (x + 1, y)$; (U): $(x, y) \rightarrow (x, y + 1)$?

- b) How many of the paths in part (a) do not use the path from $(2, 2)$ to $(3, 2)$ to $(4, 2)$ to $(4, 3)$ shown in Fig. 1.12?

- c) Answer parts (a) and (b) if a third type of move

(D): $(x, y) \rightarrow (x + 1, y + 1)$

is also allowed.

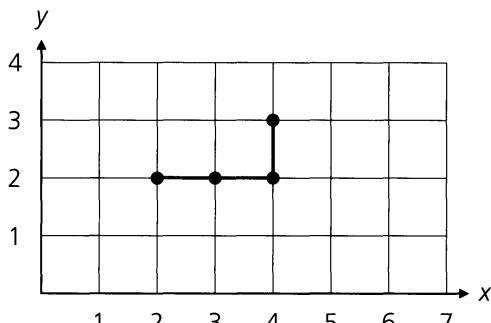


Figure 1.12

30. Due to their outstanding academic records, Donna and Katalin are the finalists for the outstanding physics student (in their college graduating class). A committee of 14 faculty mem-

bers will each select one of the candidates to be the winner and place his or her choice (checked off on a ballot) into the ballot box. Suppose that Katalin receives nine votes and Donna receives five. In how many ways can the ballots be selected, one at a time, from the ballot box so that there are always more votes in favor of Katalin? [This is a special case of a general problem called, appropriately, *the ballot problem*. This problem was solved by Joseph Louis François Bertrand (1822–1900).]

31. Consider the 8×5 grid shown in Fig. 1.13. How many different rectangles (with integer-coordinate corners) does this grid contain? [For example, there is a rectangle (square) with corners $(1, 1), (2, 1), (2, 2), (1, 2)$, a second rectangle with corners $(3, 2), (4, 2), (4, 4), (3, 4)$, and a third with corners $(5, 0), (7, 0), (7, 3), (5, 3)$.]

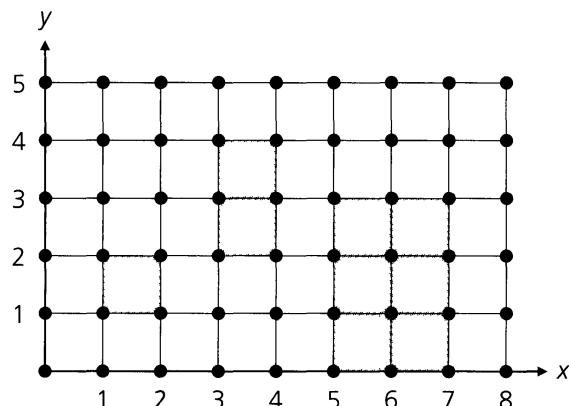


Figure 1.13

32. As head of quality control, Silvia examined 15 motors, one at a time, and found six defective (D) motors and nine in good (G) working condition. If she listed each finding (of D or G) after examining each individual motor, in how many ways could Silvia's list start with a run of three G's and have six runs in total?

33. In order to graduate on schedule, Hunter must take (and pass) four mathematics electives during his final six quarters. If he may select these electives from a list of 12 (that are offered every quarter) and he does not want to take more than one of these electives in any given quarter, in how many ways can he select and schedule these four electives?

34. In how many ways can a family of four (mother, father, and two children) be seated at a round table, with eight other people, so that the parents are seated next to each other and there is one child on a side of each parent? (Two seatings are considered the same if one can be rotated to look like the other.)

21. $\binom{n}{3} - \binom{n}{3} - n - n(n-4)$, $n \geq 4$
 23. a) $\binom{12}{9}$ b) $\binom{12}{9}(2^3)$ c) $\binom{12}{9}(2^9)(-3)^3$
 25. a) $\binom{4}{1,1,2} = 12$ b) 12 c) $\binom{4}{1,1,2}(2)(-1)(-1)^2 = -24$
 d) -216 e) $\binom{8}{3,2,1,2}(2^3)(-1)^2(3)(-2)^2 = 161,280$
 27. a) 2^3 b) 2^{10} c) 3^{10} d) 4^5 e) 4^{10}
 29. $n \binom{m+n}{m} = n \frac{(m+n)!}{m! n!} = \frac{(m+n)!}{m!(n-1)!} = (m+1) \frac{(m+n)!}{(m+1)(m!)(n-1)!}$
 $= (m+1) \frac{(m+n)!}{(m+1)!(n-1)!} = (m+1) \binom{m+n}{m+1}$

31. Consider the expansions of (a) $[(1+x) - x]^n$; (b) $[(2+x) - (x+1)]^n$; and (c) $[(2+x) - x]^n$.

33. a) $a_3 - a_0$ b) $a_n - a_0$ c) $\frac{1}{102} - \frac{1}{2} = \frac{-25}{51}$

Section 1.4–p. 34

1. a) $\binom{14}{10}$ b) $\binom{9}{5}$ c) $\binom{12}{8}$ 3. $\binom{23}{20}$ 5. a) 2^5 b) 2^n
 7. a) $\binom{35}{32}$ b) $\binom{31}{28}$ c) $\binom{11}{8}$ d) 1 e) $\binom{43}{40}$ f) $\binom{31}{28} - \binom{6}{3}$
 9. $n = 7$ 11. a) $\binom{14}{5}$ b) $\binom{11}{5} + 3\binom{10}{4} + 3\binom{9}{3} + \binom{8}{2}$
 13. a) $\binom{7}{4}$ b) $\sum_{i=0}^3 \binom{9-2i}{7-2i}$ 15. $\binom{23}{20}(24!)$ 17. a) $\binom{16}{12}$ b) 5^{12}
 19. $\binom{23}{4}$ 21. $24,310 = \sum_{i=1}^n i$ [for $n = \binom{12}{3}$]
 23. a) Place one of the m identical objects into each of the n distinct containers. This leaves $m - n$ identical objects to be placed into the n distinct containers, resulting in
 $\binom{n + (m - n) - 1}{m - n} = \binom{m - 1}{m - n} = \binom{m - 1}{n - 1}$ distributions.
 25. a) 2^9 b) 2^4
 27. a) $\binom{2+3-1}{3} = 4$ b) 10 c) 48 d) $\binom{3+4-1}{4}(\binom{2+3-1}{3}) + \binom{3+2-1}{2}(\binom{2+5-1}{5}) = 96$
 e) 180 f) 420

Section 1.5–p. 40

1. $\binom{2n}{n} - \binom{2n}{n-1} = \frac{(2n)!}{n! n!} - \frac{(2n)!}{(n-1)!(n+1)!} = \frac{(2n)!(n+1)}{(n+1)! n!} - \frac{(2n)! n}{n!(n+1)!} =$
 $\frac{(2n)![n(n+1)-n]}{(n+1)! n!} = \frac{1}{(n+1)} \frac{(2n)!}{n! n!} = \left(\frac{1}{n+1}\right) \binom{2n}{n}$
 3. a) 5 ($= b_3$); 14 ($= b_4$)
 b) For $n \geq 0$ there are b_n ($= \frac{1}{(n+1)} \binom{2n}{n}$) such paths from $(0, 0)$ to (n, n) .
 c) For $n \geq 0$ the first move is U and the last is R.
 5. Using the results in the third column of Table 1.10 we have:

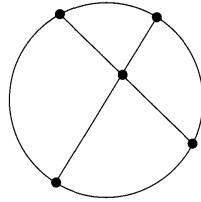
111000	110010	101010
1 2 3	1 2 5	1 3 5
4 5 6	3 4 6	2 4 6

7. There are b_5 ($= 42$) ways.
 9. (i) When $n = 4$ there are 14 ($= b_4$) such diagrams.
 (ii) For each $n \geq 0$, there are b_n different drawings of n semicircles on and above a horizontal line, with no two semicircles intersecting. Consider, for instance, the diagram in part (f) of Fig. 1.10. Going from left to right, write 1 the first time you encounter a semicircle and write 0 the second time that semicircle is encountered. Here we get the list 110100. The list 110010 corresponds with the drawing in part (g). This correspondence shows that the number of such drawings for n semicircles is the same as the number of lists of n 1's and n 0's where, as the list is read from left to right, the number of 0's never exceeds the number of 1's.

11. $\left(\frac{1}{7}\right) \binom{12}{6} (6!)(6!) = \left(\frac{1}{7}\right) (12!) = 68,428,800$

1. $\binom{4}{1} \binom{7}{2} + \binom{4}{2} \binom{7}{4} + \binom{4}{3} \binom{7}{6}$
 3. Select any four of these twelve points (on the circumference). As seen in the figure, these points determine a pair of chords that intersect. Consequently, the largest number of points of

intersection for all possible chords is $\binom{12}{4} = 495$.



5. a) 10^{25} b) $(10)(11)(12) \cdots (34) = 34!/9!$ c) $(25!)\binom{24}{9}$
 7. a) $C(12, 8)$ b) $P(12, 8)$ 9. a) 12 b) 49
 11. $(1/11)[11!/(5! 3! 3!)]$
 13. a) $\binom{5}{4} + \binom{5}{2}\binom{4}{2} + \binom{4}{4}$ (ii) $\binom{8}{4} + \binom{6}{2}\binom{5}{2} + \binom{7}{4}$ (iii) $\binom{8}{4} + \binom{6}{2}\binom{5}{2} + \binom{7}{4} - 9$
 b) $\binom{5}{1}\binom{4}{3} + \binom{5}{3}\binom{4}{1}$ (ii) and (iii) $\binom{5}{1}\binom{6}{3} + \binom{7}{3}\binom{4}{1}$
 15. a) $2\binom{9}{4} + \binom{9}{3} = 343$ b) $[2\binom{12}{4} - 9] + [\binom{12}{3} - 1] = 1200$
 17. a) $(5)(9!)$ b) $(3)(8!)$
 19. a) $\binom{4}{2}7^5$ b) $2[\binom{3}{2}7^4 + \binom{4}{2}7^5]$
 21. $0 = (1 + (-1))^n = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^n \binom{n}{n}$, so
 $\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots = \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \cdots$
 23. a) $P(20, 12) = 20!/8!$ b) $\binom{17}{9}(12!)$
 25. a) $\binom{9}{1} + \binom{10}{3} + \cdots + \binom{16}{15} + \binom{17}{17} = \sum_{k=0}^8 \binom{9+k}{1+2k}$ b) $\sum_{k=0}^9 \binom{9+k}{2k}$
 c) $n = 2k + 1, k \geq 0: \sum_{i=0}^k \binom{k+1+i}{1+2i}$
 $n = 2k, k \geq 1: \sum_{i=0}^k \binom{k+i}{2i}$
 27. a) $\binom{r+(n-r)-1}{n-r} = \binom{n-1}{n-r} = \binom{n-1}{r-1}$
 b) $\sum_{r=1}^n \binom{n-1}{r-1} = \binom{n-1}{0} + \binom{n-1}{1} + \cdots + \binom{n-1}{n-1} = 2^{n-1}$
 29. a) $11!/(7! 4!)$ b) $[11!/(7! 4!)] - [4!/(2! 2!)][4!/(3! 1!)]$
 c) $[11!/(7! 4!)] + [10!/(6! 3! 1!)] + [9!/(5! 2! 2!)] + [8!/(4! 1! 3!)] + [7!/(3! 4!)]$ [in part (a)]
 $\{[11!/(7! 4!)] + [10!/(6! 3! 1!)] + [9!/(5! 2! 2!)] + [8!/(4! 1! 3!)] + [7!/(3! 4!)]\}$
 $- \{[4!/(2! 2!)] + [3!/(1! 1! 1!)] + [2!/2!]\} \times \{[4!/(3! 1!)] + [3!/(2! 1!)]\}$ [in part (b)]
 31. $\binom{9}{2}\binom{6}{2} = 540$ 33. $\binom{6}{4}(12)(11)(10)(9) = 178,200$

Chapter 2

Fundamentals of Logic

Section 2.1 – p. 54

- The sentences in parts (a), (c), (d), and (f) are statements. The other two sentences are not.
- a) 0 b) 0 c) 1 d) 0
- a) If triangle ABC is equilateral, then it is isosceles.
 b) If triangle ABC is not isosceles, then it is not equilateral.
 d) Triangle ABC is isosceles, but it is not equilateral.
- a) If Darci practices her serve daily then she will have a good chance of winning the tennis tournament.
 b) If you do not fix my air conditioner, then I shall not pay the rent.
 c) If Mary is to be allowed on Larry's motorcycle, then she must wear her helmet.
- Statements (a), (e), (f), and (h) are tautologies.
- a) $2^5 = 32$ b) 2^n 13. $p: 0; r: 0; s: 0$
- a) $m = 3, n = 6$ b) $m = 3, n = 9$ c) $m = 18, n = 9$ d) $m = 4, n = 9$
 e) $m = 4, n = 9$
17. Dawn