Let $p(x)$, $q(x)$, and $r(x)$ be the following open statements.

$$\begin{aligned} &p(x) : x^2 - 7x + 10 = 0 \\ &r(x) : x < 0 \\ &q(x) : x^2 - 2x - 3 = 0 \end{aligned}$$

Determine the truth or falsity of the following statements, where the universe contains
(a) all of the positive integers
(b) only the integers 2 and 5

Ia

$$\forall x\ [p(x) \rightarrow \neg r(x)]$$

3

(a)

$$\begin{aligned} p \vee q \vee (\neg p \wedge \neg q \wedge r) &\iff p \vee q \vee [(\neg p \wedge \neg q) \wedge r] && \text{(associativity)} \\ &\iff [p \vee q \vee (\neg p \wedge \neg q)] \wedge [p \vee q \vee r] && \text{(distributivity)} \\ &\iff [(p \vee q \vee \neg p) \wedge (p \vee q \vee \neg q)] \wedge [p \vee q \vee r] && \text{(distributivity)} \\ &\iff [\top \wedge \top] \wedge [p \vee q \vee r] && \text{(law of excluded midd} \\ &\iff \top \wedge [p \vee q \vee r] && \text{(idempotent law)} \\ &\iff p \vee q \vee r && \text{(identity law)} \end{aligned}$$

(b)

$$\begin{aligned} [[[(p \wedge q) \wedge r] \vee [(p \wedge q) \wedge \neg r]] \vee \neg q] \rightarrow q &\iff [[(p \wedge q) \wedge (r \vee \neg r)] \vee \neg q] \rightarrow q && \text{(distributivit} \\ &\iff [[(p \wedge q) \wedge \top] \vee \neg q] \rightarrow q && \text{(law of exclu} \\ &\iff [(p \wedge q) \vee \neg q] \rightarrow q && \text{(identity law} \\ &\iff \neg[(p \wedge q) \vee \neg q] \vee q && \text{(p} \rightarrow \text{q def.)} \\ &\iff [\neg(p \wedge q) \wedge \neg\neg q] \vee q && \text{(De Morgan'} \\ &\iff [(\neg p \vee \neg q) \wedge q] \vee q && \text{(De Morgan'} \\ &\iff [(\neg p \wedge q) \vee (\neg q \wedge q)] \vee q && \text{(distributivit} \\ &\iff [(\neg p \wedge q) \vee \bot] \vee q && \text{(law of contr} \\ &\iff (\neg p \wedge q) \vee q && \text{(identity law} \\ &\iff q \wedge (\neg p \vee q) && \text{(distributivit} \\ &\iff q \wedge (q \vee \neg p) && \text{(commutativ} \\ &\iff q && \text{(absorption l} \end{aligned}$$

4

(a)

First, find the $\gcd(2463, 512)$ by short division

$$
\begin{aligned}
2463 &= 4 \cdot 512 + 415 \\
512 &= 1 \cdot 415 + 97 \\
415 &= 4 \cdot 97 + 27 \\
97 &= 3 \cdot 27 + 16 \\
27 &= 1 \cdot 16 + 11 \\
16 &= 1 \cdot 11 + 5 \\
11 &= 2 \cdot 5 + 1 \\
5 &= 5 \cdot 1 + 0
\end{aligned}
$$

So $\gcd(2463, 512) = 1$, and the equation already has 1 on the right side, so no simplification needed.

By the Extended Euclidean Algorithm, working backwards:

$$
\begin{aligned}
1 &= 11 - 2 \cdot 5 \\
&= 11 - 2 \cdot (16 - 1 \cdot 11) \\
&= -2 \cdot 16 + 3 \cdot 11 \\
&= -2 \cdot 16 + 3 \cdot (27 - 1 \cdot 16) \\
&= 3 \cdot 27 - 5 \cdot 16 \\
&= 3 \cdot 27 - 5 \cdot (97 - 3 \cdot 27) \\
&= -5 \cdot 97 + 18 \cdot 27 \\
&= -5 \cdot 97 + 18 \cdot (415 - 4 \cdot 97) \\
&= 18 \cdot 415 - 77 \cdot 97 \\
&= 18 \cdot 415 - 77 \cdot (512 - 1 \cdot 415) \\
&= -77 \cdot 512 + 95 \cdot 415 \\
&= -77 \cdot 512 + 95 \cdot (2463 - 4 \cdot 512) \\
&= 95 \cdot 2463 - 457 \cdot 512
\end{aligned}
$$

Therefore:

$$2463 \cdot 95 + 512 \cdot (-457) = 1$$

By the linear equation:

$$y = \frac{-2463}{512} x + \frac{1}{512}$$

The slope:

$$\frac{y - y_0}{x - x_0} = \frac{-2463}{512}$$

So the answer:

$$\boxed{x = 95 + 512t, \quad y = -457 - 2463t, \quad \text{for } t \in \mathbb{Z}}$$

(b)

First, find the $\gcd(1560, 910, 1430)$ by short division

$$\begin{aligned}\gcd(1560, 910, 1430) &= \gcd((1560, 910), 1430)\\ &= \gcd(130, 1430)\\ &= 130\end{aligned}$$

so simplify it

$$12x + 7y = 11$$

by the Extended Euclidean Algorithm,

$$\begin{aligned}12 &= 1 \cdot 7 + 5\\ 7 &= 1 \cdot 5 + 2\\ 5 &= 2 \cdot 2 + 1\\ 2 &= 2 \cdot 2 + 0\end{aligned}$$

Inverse:

$$\begin{aligned}1 &= 5 - 2 \cdot 2\\ &= 5 - 2 \cdot (7 - 1 \cdot 5)\\ &= -2 \cdot 7 + 3 \cdot 5\\ &= -2 \cdot 7 + 3 \cdot (12 - 1 \cdot 7)\\ &= 3 \cdot 12 - 5 \cdot 7\end{aligned}$$

multiply:

$$\begin{aligned}12 \cdot 3 + 7 \cdot (-5) &= 1\\ 12 \cdot 33 + 7 \cdot (-55) &= 11\end{aligned}$$

by the linear equation

$$y = \frac{-12}{7}x + \frac{11}{7}$$

the slope

$$\frac{y - y_0}{x - x_0} = \frac{-12}{7}$$

So the answer

$$\boxed{x = 33 + 7t, \quad y = -55 - 12t, \quad \text{for } t \in \mathbb{Z}}$$

5

Photo

3

(a)

$$\begin{aligned} p \vee q \vee (\neg p \wedge \neg q \wedge r) &\iff p \vee q \vee [(\neg p \wedge \neg q) \wedge r] && \text{(associativity)} \\ &\iff [p \vee q \vee (\neg p \wedge \neg q)] \wedge [p \vee q \vee r] && \text{(distributivity} \\ &\iff [(p \vee q \vee \neg p) \wedge (p \vee q \vee \neg q)] \wedge [p \vee q \vee r] && \text{(distributivity} \\ &\iff [T \wedge T] \wedge [p \vee q \vee r] && \text{(law of exclud} \\ &\iff T \wedge [p \vee q \vee r] && \text{(idempotent l} \\ &\iff p \vee q \vee r && \text{(identity law)} \end{aligned}$$

(b)

$$\begin{aligned} [[(p \wedge q) \wedge r] \vee [(p \wedge q) \wedge \neg r]] \vee \neg q] \to q &\iff [[(p \wedge q) \wedge (r \vee \neg r)] \vee \neg q] \to q && \text{(distr} \\ &\iff [[(p \wedge q) \wedge T] \vee \neg q] \to q && \text{(law} \\ &\iff [(p \wedge q) \vee \neg q] \to q && \text{(iden} \\ &\iff \neg[(p \wedge q) \vee \neg q] \vee q && \text{(p} \to \\ &\iff [\neg(p \wedge q) \wedge \neg\neg q] \vee q && \text{(De M} \\ &\iff [(\neg p \vee \neg q) \wedge q] \vee q && \text{(De M} \\ &\iff [(\neg p \wedge q) \vee (\neg q \wedge q)] \vee q && \text{(distr} \\ &\iff [(\neg p \wedge q) \vee \bot] \vee q && \text{(law} \\ &\iff (\neg p \wedge q) \vee q && \text{(iden} \\ &\iff q \wedge (\neg p \vee q) && \text{(distr} \\ &\iff q \wedge (q \vee \neg p) && \text{(com} \\ &\iff q && \text{(abso} \end{aligned}$$

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4:30 PM

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3a, 3b

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Compositions of 4

4	3+1	2+1+1	1+1+1+1
	1+3	1+2+1	
	2+2	1+1+2	

we use ball and sticks to

OOOO	OOO+O	OO+O+O	O+O+O+O
	O+OOO	O+OO+O	
	OO+OO	O+O+OO	

$$\binom{3}{0} + \binom{3}{1} + \binom{3}{2} + \binom{3}{3} = \sum_{k=0}^{3} \binom{3}{k} = 2^3$$

So the composition of m

$$\binom{m-1}{0} + \binom{m-1}{1} + \binom{m-1}{2} + \binom{m-1}{m-1} = \sum_{k=0}^{m-1} \binom{m-1}{k} = 2^{m-1}$$

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9.

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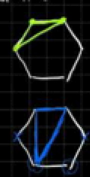
21.
vertex → vertices 頂點

a)

n=4 sides $\Rightarrow \binom{4}{3}$ 4个頂点取3个

n $\Rightarrow \binom{n}{3}$ #

b) EX. n=6



$\binom{6}{3} - 6 - 6(6-4)$

所以
$\binom{n}{3} - n - n(n-4)$, $n \geq 4$ #

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這是Exercise 1.3 21題

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n代入 10即是

8a
8b

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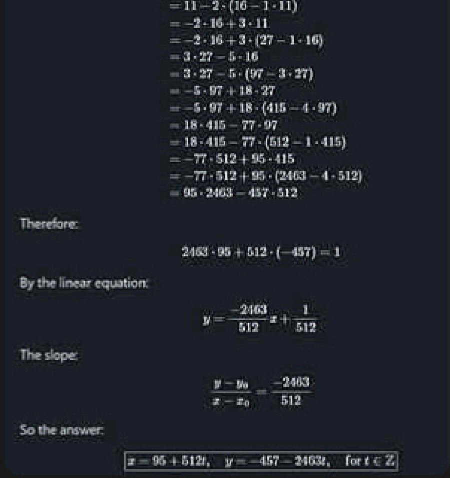
By the Extended Euclidean Algorithm, working backwards:

$1 = 11 - 2 \cdot 5$



$$
\begin{aligned}
&= 11 - 2 \cdot (16 - 1 \cdot 11) \\
&= -2 \cdot 16 + 3 \cdot 11 \\
&= -2 \cdot 16 + 3 \cdot (27 - 1 \cdot 16) \\
&= 3 \cdot 27 - 5 \cdot 16 \\
&= 3 \cdot 27 - 5 \cdot (97 - 3 \cdot 27) \\
&= -5 \cdot 97 + 18 \cdot 27 \\
&= -5 \cdot 97 + 18 \cdot (415 - 4 \cdot 97) \\
&= 18 \cdot 415 - 77 \cdot 97 \\
&= 18 \cdot 415 - 77 \cdot (512 - 1 \cdot 415) \\
&= -77 \cdot 512 + 95 \cdot 415 \\
&= -77 \cdot 512 + 95 \cdot (2463 - 4 \cdot 512) \\
&= 95 \cdot 2463 - 457 \cdot 512
\end{aligned}
$$

Therefore:

$$2463 \cdot 95 + 512 \cdot (-457) = 1$$

By the linear equation:

$$y = \frac{-2463}{512} x + \frac{1}{512}$$

The slope:

$$\frac{y - y_0}{x - x_0} = \frac{-2463}{512}$$

So the answer:

$$\boxed{x = 95 + 512t, \quad y = -457 - 2463t, \quad \text{for } t \in \mathbb{Z}}$$

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4b

Read 20
4:38 PM

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Photo

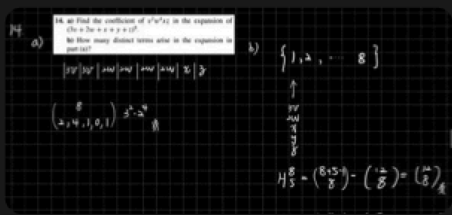
4a

Read 19
4:38 PM

Palm Tree

猜題專家

4:39 PM



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You unsent a message.

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這是Exercise 1.4 14題

2a
2b

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2c

係數和 = 8^8
(1代入全都未知數)

Read 14
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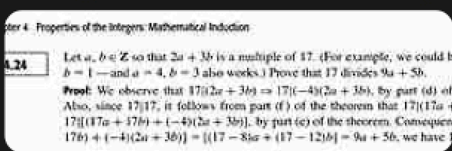

ter 4 Properties of the Integers: Mathematical Induction

4.24

Let $a, b \in \mathbb{Z}$ so that $2a + 3b$ is a multiple of 17. (For example, we could h
$b = 1$ — and $a = 4, b = 3$ also works.) Prove that 17 divides $9a + 5b$.

Proof: We observe that $17|(2a + 3b) \Rightarrow 17|(-4)(2a + 3b)$, by part (d) of
Also, since $17|17$, it follows from part (f) of the theorem that $17|(17a +$
$17|[(17a + 17b) + (-4)(2a + 3b)]$, by part (e) of the theorem. Consequen
$17b) + (-4)(2a + 3b)] = [(17 - 8)a + (17 - 12)b] = 9a + 5b$, we have

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5

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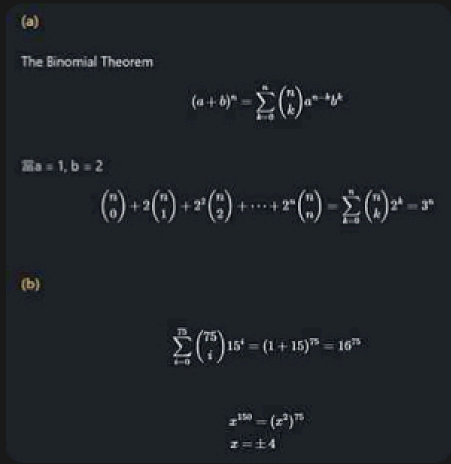
來自 Example 4.24



(a)

The Binomial Theorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

當a = 1, b = 2

$$\binom{n}{0} + 2\binom{n}{1} + 2^2\binom{n}{2} + \cdots + 2^n\binom{n}{n} = \sum_{k=0}^{n} \binom{n}{k} 2^k = 3^n$$

(b)

$$\sum_{i=0}^{75} \binom{75}{i} 15^i = (1+15)^{75} = 16^{75}$$

$$x^{150} = (x^2)^{75}$$
$$x = \pm 4$$

Read 11
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112 期中考古題

6a 6b

Read 10
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Example 4.12 的寫法是數學歸納法

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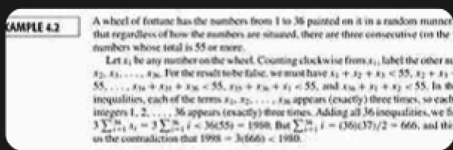

AMPLE 4.2

A wheel of fortune has the numbers from 1 to 36 painted on it in a random manner. that regardless of how the numbers are situated, there are three consecutive (on the numbers whose total is 55 or more.

Let x_1 be any number on the wheel. Counting clockwise from x_1, label the other $x_2, x_3, \ldots, x_{36}$. For the result to be false, we must have $x_1 + x_2 + x_3 < 55$, $x_2 + x_3 +$ $55, \ldots, x_{34} + x_{35} + x_{36} < 55$, $x_{35} + x_{36} + x_1 < 55$, and $x_{36} + x_1 + x_2 < 55$. In the inequalities, each of the terms $x_1, x_2, \ldots, x_{36}$ appears (exactly) three times, so each integers $1, 2, \ldots, 36$ appears (exactly) three times. Adding all 36 inequalities, we fi $3\sum_{i=1}^{36} x_i = 3\sum_{i=1}^{36} i < 36(55) = 1980$. But $\sum_{i=1}^{36} i = (36)(37)/2 = 666$, and thi us the contradiction that $1998 = 3(666) < 1980$.

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7.

(換數字)

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(抱歉傳的順序有點亂)

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