Discrete Mathematics QUIZ-1 INF. 114/10/13

1. Determine the truth value of each of the following implications. (12/115)

   (a) If $3 + 4 = 12$, then $3 + 2 = 6$.

   (b) If $3 + 3 = 6$, then $3 + 4 = 9$.

   (c) If Thomas Jefferson was the third president of the United States, then $2 + 3 = 5$.

   (d) If $x < 3$ then $x = 5$, $x \in \mathbb{Z}$.

2. (a) How many arrangements are there of all the letters in SOCIOLOGICAL (12/115)

   (b) In how many of the arrangements in part (a) are A and G adjacent?

   (c) In how many of the arrangements in part (a) are all the vowels adjacent?

3. Consider the following program segment, where i, j, k and m are integer variables. (14/115)

   (a)
```
for i := 1 to 20 do
  for j := 1 to i do
    for k := 1 to j do
      for m := 1 to k do
        print "*"
```
   (b)
```
for i := 1 to 20 do
  for j := 1 to i-1 do
    for k := 1 to j-1 do
      for m := 1 to k-1 do
        print "*"
```
   How many times is the print statement executed in this program segment of (a) and (b)?

4. (a) In how many ways can a particle move in the xy-plane from the point (2,3) to the point (6,9) if the moves that are allowed are the form: (13/115)

   (R): $(x, y) \to (x + 1, y)$; (U): $(x, y) \to (x, y+1)$; (D): $(x, y) \to (x+1, y+1)$?

   (b) How many of the paths in part (a) do not use the path from (3,4) to (3, 5) to (4,5) to (4,6)?

5. Determine the number of integer solutions for $x_1 + x_2 + x_3 + x_4 + x_5 < 40$, where

   (a) $x_j \geq 0,\ 1 \leq j \leq 5$ (b) $x_j \geq -3,\ 1 \leq j \leq 5$. (15/115)

6. Find the value of increment and sum after the given program segment is executed. (14/115)
```
increment := 0;    sum := 0;
for i := 1 to 13 do
  for j := 1 to i do
    for k=1 to j do
      increment := increment +1
      sum := sum + increment
```

7. Simplify the following compound statements. (18/115)

   (a) $(p \to q) \wedge (\neg q \wedge (r \vee \neg q))$

   b) $(p \vee q \vee r) \wedge (p \vee t \vee \neg q) \wedge (p \vee \neg t \vee r)$

   (c) $\neg[(p \vee q) \wedge (\neg p \vee q)] \vee [(\neg p \wedge q) \vee (p \wedge q)]$.

8. Define the connective "Nand" or "Not… and…" by $(p \uparrow q) \Leftrightarrow \neg(p \wedge q)$, for any statement p, q, represent the following using only this connectives in a manner of least cost. (17/115)

   (a) $p \wedge q$ (b) $p \leftrightarrow q$ (c) $p \oplus q$

Discrete Mathematics Mid-term. INF 114/10/28

1. Let p(x), q(x), and r(x) be the following open statements. p(x): $x^2 - 7x + 10 = 0$, r(x): $x < 0$, q(x): $x^2 - 2x - 3 = 0$. Determine the truth or falsity of the following statements, where the universe contains (a) all of the positive integers and (b) only the integers 2 and 5. (12/123)

   (I) $\forall x\, [p(x) \rightarrow \neg r(x)]$ T (II) $\forall x\, [q(x) \rightarrow r(x)]$ F (III) $\exists x\, [q(x) \rightarrow r(x)]$ T

2. (a) Find the coefficient of term v^2w^4xz in the expansion of $(3v+2w+x+y+z)^8$. (12/123) 120160

   (b) How many distinct terms are there in the complete expansion of $(3v+2w+x+y+z)^8$? 415

   (c) What is the sum of all coefficients in the complete expansion of $(3v+2w+x+y+z)^8$? 8^8

3. Simply the following statements by using the laws of logic. (12/123)

   (a) $p \vee q \vee (\neg p \wedge \neg q \wedge r)$. $p \vee r \vee q$

   (b) $[[[(p \wedge q) \wedge r] \vee [(p \wedge q) \wedge \neg r]] \vee \neg q] \rightarrow q$ q

4. Find all $x, y \in \mathbb{Z}$ such that $95 \cdot (2463 - 4 \cdot 512) - 77 \cdot 512$ (16/123)

   (a) $2463x + 512y = 1$

   (b) $1560x + 910y = 1430$ $7x + (-4b)$

5. Let a, b be integers so that $5a + 6b$ is a multiple of 19. Prove that 19 divides $7a + 16b$. (12/123)

   $19 \mid 5a+6b$

6. (a) With n a positive integer, evaluate the sum (16/123)

$$\binom{n}{0} + 2\binom{n}{1} + 2^2\binom{n}{2} + \cdots + 2^k\binom{n}{k} + \cdots + 2^n\binom{n}{n}$$

   $5a+6b$ $4a+b$

   (b) Determine x if $\sum_{i=0}^{75}\binom{75}{i}15^i = x^{150}$. $19 \mid 19a+19b$ $\frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3}$

   $7a$ $16b$

   $12a+3b$

7. A wheel of fortune has the numbers from 1 to 25 painted on it in a random manner. Show that regardless of how the numbers are suited, there are three consecutive numbers (on the wheel) whose sum is at least 39. (13/123)

8. (a) How many triangles are determined by the vertices of a regular polygon of 10 sides? 120 $\binom{10}{3}$

   (b) As in part (a), if no side of the polygon is to be a side of any triangle? (14/123) 50

9. The different ways we can write the number 4 as a sum of positive integers, where the order of the summands is considered relevant. These representations are called the compositions of 4 and is listed as follows: 4, 3+1, 1+3, 2+2, 2+1+1, 1+2+1, 1+1+2, 1+1+1+1. Show that for each positive integer m, there are 2^{m-1} compositions. (16/123)