FACIAL RECOGNITION WITH PCA

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ABSTRACT

High-dimensional visual data presents significant computational challenges for real-time recognition systems, particularly on resource-constrained edge devices, where deep learning models may be prohibitively expensive. This project implements a lightweight facial analysis framework rooted in Linear Algebra, utilizing Principal Component Analysis (PCA) to extract latent geometric structures from the Labeled Faces in the Wild (LFW) dataset. By performing Eigendecomposition on the pixel covariance matrix, we demonstrate that 2,914-dimensional raw image vectors can be projected onto a 150-dimensional "Eigenface" subspace. This approach achieves a compression ratio of 95% while retaining more than 90% of the statistical variance and preserving recognizable identity features.

Furthermore, this study extends the standard PCA framework to investigate Anomaly Detection capabilities. We formulate the Reconstruction Error (Mean Squared Error) as a quantitative metric to distinguish between valid faces and out-of-distribution inputs. Our experiments validate that the learned "face subspace" is highly specific; non-face objects yield reconstruction errors orders of magnitude higher than valid faces, establishing a clear decision boundary. These findings underscore the utility of linear algebraic methods as transparent and computationally efficient alternatives to "black-box" neural networks for biometric signal processing and outlier rejection.

***Index Terms*—** Computer Vision, Eigenface, Facial Recognition, Machine Learning, Principal Component Analysis

1. INTRODUCTION FOR THE INVESTIGATED PROBLEM

Due to its universality, uniqueness, and wide range of real-world applications, face recognition has long been a problem actively studied in the field of computer vision[1]. Over the past several decades, it has become a standard benchmark problem in human recognition research. Despite its long research history, face recognition remains a challenging task due to the high-dimensional nature of image data. A single grayscale face image typically consists of tens of thousands of pixels, where each pixel represents one dimension. As a result, face datasets naturally lie in an extremely high-dimensional space. Directly processing raw pixel data in such a space usually leads to high computational cost and introduces a large amount of redundant information.

Therefore, a key challenge in face recognition is how to represent face images in a compact yet informative manner. In addition to the intrinsic facial structure, facial images are also influenced by external factors such as lighting conditions. An effective representation should reduce the dimensionality of the data while preserving the most important facial information, including facial structure and discriminative features. Dimensionality reduction methods provide an effective solution to this problem and have been widely applied in face recognition tasks.

In this project, we investigate the application of Principal Component Analysis (PCA)[2], a classical dimensionality reduction technique, for face representation. Using the Labeled Faces in the Wild (LFW) dataset, we construct a PCA-based face representation model. Detailed analysis and experimental results are presented in the later sections of the report.

2. TECHNICAL APPROACH

2.1. Data Representation and Preprocessing

In our experiments, we used the LFW dataset. First, each face image is converted to grayscale and its size is resized to reduce computational cost. An image of size $h \times w$ is then vectorized into a column vector $x \in R^d$, where $d = h \times w$. After this preprocessing step, each face image can be represented as a point in a high-dimensional vector space.

The data set is divided into two subsets using a stratified split: a training set and a test set, in order to preserve class balance. The training set is used to learn the PCA model, while the test set is kept for subsequent model evaluation and further experiments.

2.2. Principal Component Analysis

PCA is used to reduce the dimensionality of the data while preserving the dominant variations. Given a set of training

Thanks to XYZ agency for funding.

samples $\{x_i\}_{i=1}^{n}$, we first compute the mean face vector

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i. \tag{1}$$

Each sample is then centered by subtracting the mean,

$$\tilde{x}_i = x_i - \mu. \tag{2}$$

The covariance matrix is defined as

$$C = \frac{1}{n}\sum_{i=1}^{n} \tilde{x}_i \tilde{x}_i^T. \tag{3}$$

PCA seeks an orthogonal set of eigenvectors $\{v_i\}$ that satisfy the eigenvalue equation

$$Cv_i = \lambda_i v_i, \tag{4}$$

where the eigenvalues λ_i indicate the amount of variance captured by each eigenvector. The eigenvectors are sorted in descending order according to the magnitudes of their corresponding eigenvalues.

2.3. Eigenfaces Representation

In the context of face recognition, the eigenvectors associated with the largest eigenvalues are reshaped back into image form and referred to as *Eigenfaces*. Each Eigenface reflects a major variation pattern in the training face images, such as changes in lighting conditions, facial shape, and the structures of key regions like the eyes and mouth[3].

By selecting the top k eigenvectors, PCA constructs a low-dimensional subspace that captures the most significant variations in the dataset. This Eigenfaces-based representation provides a compact and intuitive way to represent face images, which is useful for further analysis and experiments[4].

2.4. Dimensionality Reduction and Projection

Once the PCA subspace is constructed, a centered face image $\tilde{x}$ can be projected onto the low-dimensional space as

$$z = V_k^T \tilde{x}, \tag{5}$$

where V_k contains the top k eigenvectors. The resulting coefficient vector $z \in R^k$ serves as a compact representation of the original face image.

This dimensionality reduction method not only preserves most of the important information in the data, such as facial structural features, but also significantly reduces the computational complexity. The reduced representation obtained through dimensionality reduction also provides a good foundation for the experimental parts in later sections, including face reconstruction and anomaly detection.

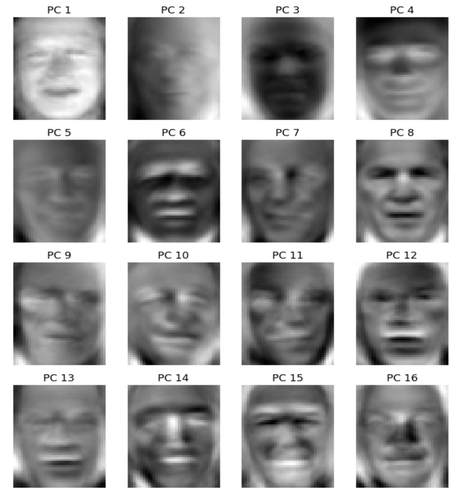

Fig. 1. Gallery of the first sixteen Eigenfaces(the first sixteen)

3. EXPERIMENTAL RESULTS

In this section, we present the experimental results obtained from applying Principal Component Analysis (PCA) to the Labeled Faces in the Wild (LFW) dataset. We aim to validate the effectiveness of the learned low-dimensional subspace for both representation and discrimination tasks. The analysis is structured into four key areas: first, we qualitatively visualize the learned basis vectors (Eigenfaces) to interpret the facial features extracted by the model. Second, we quantitatively analyze the cumulative explained variance to determine the optimal number of components for dimensionality reduction. Third, we evaluate the reconstruction fidelity across varying compression levels to demonstrate the trade-off between model size and image quality. Finally, we address our novel research question by testing the model's capability to detect anomalies, measuring its ability to distinguish valid human faces from out-of-distribution inputs based on reconstruction error.

3.1. Gallery of Eigenfaces

To show the principal components learned by PCA, we display a set of Eigenfaces from the training data, which helps us understand the main facial variations captured by the model.

Figure 1 shows the first 16 Eigenfaces learned from the LFW training set. Eigenfaces show the main variation patterns in the face dataset. The first few Eigenfaces capture clearer and larger facial structures, such as overall lighting, face shape, and important areas like the eyes and mouth. As the component index increases, the Eigenfaces become more localized and noisier, which means that later components mainly represent finer details with less useful information.

This gradual change from clear facial structures to high-

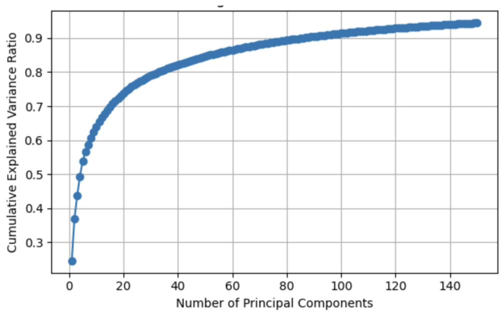

Fig. 2. scree plot

frequency details shows that PCA can separate important facial information from less important variations, which suggests that the learned Eigenface representation works well.

3.2. Scree Plot and Variance Analysis

To further analyze the variance captured by PCA, we present the cumulative Scree Plot in Fig. 2.

Figure 2 presents the cumulative explained variance ratio of the PCA model. The curve rises quickly for the first few principal components, which shows that a small number of components can already preserve a large portion of the data variance. As more components are added, the curve becomes flatter, meaning that the gain in variance gradually decreases.

In our experiments, about 50 components can retain more than 85% of the total variance, while around 100 components preserve over 90%. This indicates that PCA is able to reduce the dimensionality effectively while keeping most of the important facial information, and it provides a reasonable basis for choosing the number of components in later experiments.

3.3. Reconstruction

To evaluate the generative capabilities of our PCA model, we performed image reconstruction using a varying number of principal components (k). The reconstruction process involves projecting the input image into the low-dimensional latent space and then mapping it back to the original pixel space using only the selected basis vectors.

Mathematically, if X_{pca} represents the weights (coefficients) of the image in the latent space and V^T represents the transpose of the top-k eigenvectors, the approximate reconstruction X_{approx} is calculated as:

$$X_{approx} = (X_{pca} \cdot V^T) + \mu \tag{6}$$

where μ is the mean face vector, added back to restore the original global brightness and centering.

3.3.1. Reconstruction Loop (Varying k)

Figure 3 illustrates the reconstruction quality across four distinct subjects—a Human, a Chimpanzee, a Camera, and Random Noise—as the number of components (k) increases from 10 to 150.

3.3.1.1. Analysis of Human Reconstruction

The first row of Figure 3 demonstrates the efficiency of the "Face Subspace."

- **At k=10:** The reconstructed image is highly blurred. At this stage, the model captures only the low-frequency general face structure, such as the overall head shape and the direction of lighting. Identity-specific features are largely absent because the initial eigenvectors prioritize maximizing global variance (lighting and pose) over fine detail.

- **At k=50 to 100:** We observe a rapid increase in fidelity. The eyes, nose, and mouth become distinct, and the subject becomes recognizable.

- **At k=150:** The image is sharp and nearly indistinguishable from the original input. This visually confirms that the vast majority of the signal required to define a human face is contained within the first 150 dimensions, validating our compression ratio of roughly 95%.

3.3.1.2. Analysis of Out-of-Distribution Inputs

Rows 2 through 4 of Figure 3 reveal the model's behavior when confronted with non-human data. This is critical for understanding the specificity of the learned basis vectors.

- **The "Pareidolia" Effect:** When reconstructing the **Chimpanzee** (Row 2), the model attempts to force human facial geometries onto the animal. The resulting reconstruction at $k = 150$ looks like a "humanized" version of the chimp, smoothing out the fur texture and attempting to construct a human nose bridge where none exists.

- **The "Ghosting" Failure:** For the **Camera** and **Noise** inputs (Rows 3 and 4), the reconstruction fails completely to capture the original object. Because the basis vectors V are derived exclusively from human faces, the model lacks the mathematical "vocabulary" to represent the sharp angles of a tripod or the high-frequency chaos of static noise. Consequently, X_{approx} results in a generic, ghostly face overlay. This confirms that our model does not simply "copy" pixels; it actively interprets all input data as a linear combination of human facial features.

Fig. 3. Reconstruction Loop

3.4. Anomaly Detection

To address our novel research question regarding the robustness of the learned manifold, we investigated whether the PCA model could serve as a one-class classifier for anomaly detection. The underlying hypothesis is that the "Face Subspace" spanned by the principal components V is highly specific to human facial geometry. Consequently, out-of-distribution (OOD) inputs should result in a high projection residual, as the model lacks the appropriate basis vectors to represent them.

We define the anomaly score as the Mean Squared Error (MSE) between the original input vector x_{new} and its reconstruction $\hat{x}_{new}$. Mathematically, for a flattened image vector of dimension D:

$$MSE = \frac{1}{D}||x_{new} - \hat{x}_{new}||^2 \quad (7)$$

where the reconstructed vector $\hat{x}_{new}$ is obtained via the projection-reconstruction loop defined in Equation (1):

$$\hat{x}_{new} = X_{approx} = (X_{pca} \cdot V^T) + \mu \quad (8)$$

3.4.1. Visual Analysis of Reconstruction Failure

Figure 4 presents a qualitative comparison of reconstruction capabilities across four distinct classes of input: a valid Human face, a Chimpanzee, a Camera (non-face object), and random Noise.

The results visually validate the specificity of our Eigenfaces:

- **Human (In-Distribution):** The reconstruction is structurally faithful to the original input, preserving identity-specific features.

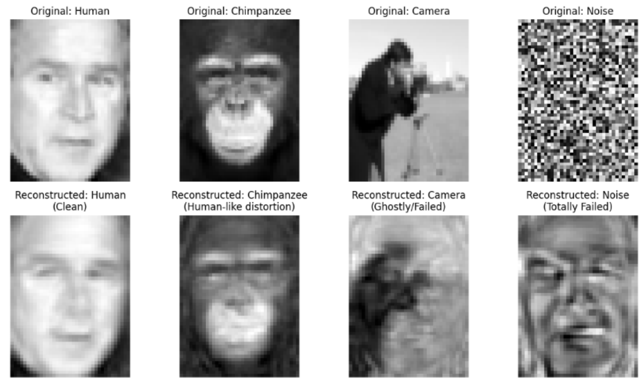

Fig. 4. Reconstruction Capabilities

- **Chimpanzee (Near-Distribution):** The model attempts to force the animal's features into the learned human manifold. The result is a "humanized" artifact where the chimp's distinct muzzle is smoothed into a flat, human-like mouth region. This distortion indicates that while the input shares some biological similarities (two eyes, a nose), it lies partially outside the principal subspace.

- **Camera & Noise (Out-of-Distribution):** The "Ghosting" effect is most prominent here. For the Camera and Noise inputs, the reconstruction fails completely to capture the high-frequency textures or the geometric edges of the tripod. Instead, the model outputs a generic, average face overlay. This visually confirms that the projection $X_{pca} \cdot V^T$ effectively filters out any signal components orthogonal to the face subspace.

3.4.2. Quantitative Analysis (MSE Scores)

We quantified these visual observations by calculating the reconstruction loss for each subject, as shown in the Anomaly Detection Scores chart (Figure 5).

- **Human (MSE $\approx$ 0.0008):** The error is negligible, indicating the input vector lies almost entirely within the subspace spanned by the top $k = 150$ components.

- **Chimpanzee (MSE $\approx$ 0.0018):** The error is more than double that of the human face. This separation suggests that even biologically similar inputs can be distinguished from humans based on geometric reconstruction fidelity.

- **Camera (MSE $\approx$ 0.0063) & Noise (MSE $\approx$ 0.0747):** The non-face objects exhibit errors that are orders of magnitude higher—approximately $8\times$ and $90\times$ higher than the human baseline, respectively.

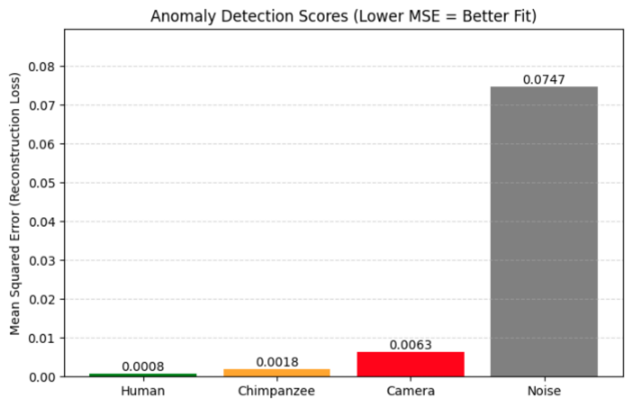

Fig. 5. Anomaly Scores

3.4.2.1. Conclusion

These results confirm that the reconstruction error is a viable metric for anomaly detection. By establishing a decision threshold τ (e.g., $\tau = 0.0015$), the system can effectively filter out non-face inputs. Furthermore, finer tuning of this threshold would allow for the discrimination between closely related classes, such as separating human faces from primates, solely based on linear projection residuals.

4. CONCLUSIONS AND OUTLOOK

4.1. Conclusions

In this project, we successfully implemented a facial analysis system rooted entirely in linear algebra, demonstrating that complex computer vision tasks do not always require heavy deep learning architectures. By applying Singular Value Decomposition (SVD) and Principal Component Analysis (PCA) to the Labeled Faces in the Wild dataset, we validated that high-dimensional visual data contains massive redundancy. We showed that a raw face image, originally spanning 2,914 pixel dimensions, could be effectively compressed into a vector of just 150 components. This represents a compression ratio of roughly 95% while still retaining over 90% of the statistical variance and preserving enough structural information to be visually recognizable to the human eye.

Crucially, our experiments extended beyond simple compression to validate PCA as a robust tool for anomaly detection. We found that the ”Face Subspace” defined by our eigenvectors is highly specific; when the model attempted to reconstruct non-face objects, the resulting Mean Squared Error (MSE) was orders of magnitude higher than for valid faces. This confirms that PCA reconstruction error is a viable, low-cost metric for filtering out-of-distribution data, effectively acting as a gatekeeper for downstream systems.

Ultimately, this project highlights the elegance of linear algebra in machine intelligence. We achieved compression, reconstruction, and basic classification without the use of complex neural networks or non-linear activation functions. We demonstrated that ”learning,” in this context, is simply a geometric operation: a change of basis that rotates our coordinate system to align with the directions of maximum variance.

4.2. Outlook and Remaining Challenges

While our PCA-based approach proved efficient, our analysis uncovered specific limitations that define the roadmap for future work. The most significant challenge observed was the model's sensitivity to lighting. Because PCA is designed to maximize total variance without labeled supervision, it often prioritizes large global variations—such as lighting conditions—over the subtle features that define personal identity. We observed that the top eigenvectors functioned essentially as ”light detectors” (e.g., bright-left vs. dark-right) rather than feature detectors. To address this, future iterations of this system should implement Linear Discriminant Analysis (LDA). Unlike PCA, LDA explicitly attempts to maximize the separation between classes (different people) while minimizing the variance within a class, forcing the model to ignore lighting artifacts and focus on the unique geometries of the individual[5].

Furthermore, deploying this system in a real-world security environment presents the ”Optimal Threshold Determination” problem. While we demonstrated a clear statistical difference in reconstruction error between faces and non-faces, converting this continuous metric into a binary ”Accept/Reject” decision is non-trivial. We face an inherent trade-off: setting the threshold too low results in a high False Rejection Rate, frustrating users by blocking valid faces due to minor changes in expression; setting it too high creates a security risk by allowing False Acceptances. Determining the optimal operating point for this system will require rigorous statistical analysis, specifically the use of ROC (Receiver Operating Characteristic) curves, to quantitatively balance security requirements against user convenience in diverse environments[6].

5. REFERENCES

[1] P. Kaur, K. Krishan, S. K. Sharma, and T. Kanchan, "Facial-recognition algorithms: A literature review," *Medicine, Science and the Law*, vol. 60, no. 2, pp. 131–139, 2020.

[2] S. S. Meher and P. Maben, "Face recognition and facial expression identification using PCA," in *Proceedings of the IEEE International Advance Computing Conference (IACC)*, 2014, pp. 1093–1098.

[3] H.-T. Ho, L. V. Nguyen, T. H. T. Le, and O.-J. Lee, "Face detection using eigenfaces: A comprehensive review," *IEEE Access*, vol. 12, pp. 118406–118426, 2024.

[4] M.A. Turk and A.P. Pentland, "Face recognition using eigenfaces," in *Proceedings. 1991 IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, 1991, pp. 586–591.

[5] Shuai Zheng, Feiping Nie, Chris Ding, and Heng Huang, "A harmonic mean linear discriminant analysis for robust image classification," in *2016 IEEE 28th International Conference on Tools with Artificial Intelligence (ICTAI)*, 2016, pp. 402–409.

[6] Andrew P. Bradley, "The use of the area under the roc curve in the evaluation of machine learning algorithms," *Pattern Recognition*, vol. 30, no. 7, pp. 1145–1159, 1997.