A general purpose Hyper – Heuristic based on Ant colony optimization

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1 Introduction

In the plan of the CHESC2001 competition [1], an hyper heuristic has been designed based on the Ant Colony Optimization method (from now ACO).

According to the competition goals, the algorithm was developed with the intention to produce an independent model of the problem domain and the particular instance to be solved. This means that the hyper-heuristic is abstracted from the features of each heuristic previously known; therefore, the work is based exclusively on the behavior observed during the problem resolution. These types of hyper-heuristics are known as heuristic selection and online learning [2].

The model presented in this document was implemented for Hyflex framework [3] provided for the participatants in the competition.

2 ACO Metaheuristic

A general model is presented for a combinatorial optimization problem:

$$P = (S, \Omega, f) \tag{1}$$

Where:

- S is a search space defined by a set of decision variables $X_k \in D_k$, $k = 1,2,...n$.
- Ω is a set of constraints over the assignment of decision variables.
- f is the objective function to be minimized. $f : S \rightarrow \Re_{+}^{0}$

A feasible solution for P denoted by $s \in S$, is defined as a complete valuation for the decision variables X_k who satisfies all of the constraints in Ω. A feasible solution $s^* \in S$ is called a global minimum for P when $f(s^*) \leq f(s) \ \forall s \in S$ is carried out .

The ACO's algorithm consists in a method of construction of solutions where a set of m ants properly combine solution components c_k^h, defined as the assignment of a value $v_k^h \in D_k$ for the decision variable X_k, generating - in this manner - an ant's path.

In order to obtain high quality combinations, the exploration is guided by a pheromone model t_{kh} related to each solution component c_k^h, which its value is defined by the ants in every construction step and represents - in sum - the quality of the solutions generated where the component was used.

For each construction step, the ants select the next component for their partial solution s_p through a function of probability $P(c_k^h \mid s_p)$. Besides the pheromone, the function is also determined by the value of the heuristic information related to the component, denoted by η_{kh}. The degree of influence of both values depends on the problem where the algorithm is applied and, in general, it is established in the formula by the positive constants α y β, for t and η respectively.

The general algorithm is presented in a pseudo-code below:

```
procedure ACO
  Inicialization
  while termination condition not met
    Construct Ant Solutions
    Apply Local Search (optional)
    Pheromone update
  end while
end procedure
```

A detailed description for this process can be consulted in [4].

3 ACO Hyper-Heuristic

The hyper-heuristic model for the minimization problem is an extension of the original model, as defined below:

$$P' = (P, S', \Omega') \tag{2}$$

Whre $P = (S, \Omega, f)$ is an instance of the minimization problem.

Here $S' = \{X'_k \in D'_k \mid k = 1,2,...n\}$, such that each domain of values D_k is a set of perturbation heuristics[1] c_k^h available to be applied in the construction step k to the instance solution $s_{k-1} \in S$ obtained in the previous step.

Formalizing, there is a feasible solution for P' called $s' \in S'$, which is a vectorial function defined as:

[1] Perturbation heuristics are those who work from a complete initial solution, unlike the constructive heuristics that start from an initial empty solution. [2].

$$s'(s_0): S \rightarrow S^n \mid s_0 \rightarrow \left\langle c_1^h(s_0) = s_1 \; , c_2^h(s_1) = s_2 \; , \ldots , c_n^h(s_{n-1}) = s_n \right\rangle \quad (3)$$

Ω' is the set of constraints over the use of heuristics. For this, the ACO application is considered an unrestricted model, therefore, in every step construction it will be possible to use any of the heuristics provided, independent of its particular features.

From the above, the paths generated by a set of ants are built on a grid of dimensions $H \times n$, with H equal to the number of heuristics available and n equal to the length established for the path.

The figure 1 is an example for a generation of paths (one iteration in the algorithm), with $m = 2$ ants, length path $n = 3$ and $H = 3$. In this case the solutions obtained for P' are:

$s'_1(s_0) = \left\langle h_1(s_0) = s_{11} \; , h_2(s_{11}) = s_{12} \; , h_1(s_{12}) = s_{13} \right\rangle$ and

$s'_2(s_0) = \left\langle h_2(s_0) = s_{21} \; , h_2(s_{21}) = s_{22} \; , h_3(s_{22}) = s_{23} \right\rangle$

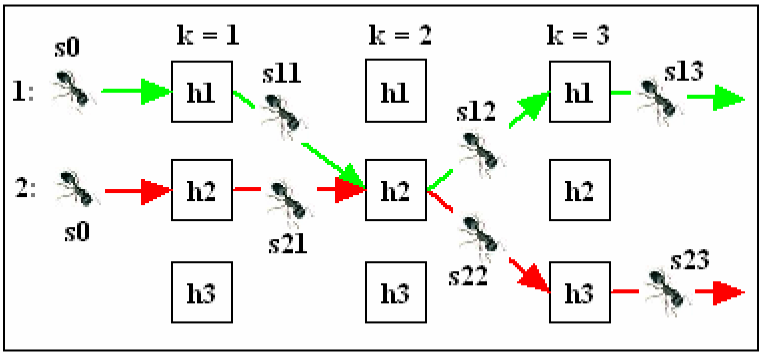

Fig. 1 Example of ant's path generation in the hyper-heuristic.

For this model, as well as the original, the selection probability function is given by the formula (4).

$$P(c_k^h \mid s'_p) = \frac{t_{kh}{}^{\alpha} \cdot \eta_{kh}{}^{\beta}}{\sum_{c_k^l \in N(s'_p)} t_{kl}{}^{\alpha} \cdot \eta_{kl}{}^{\beta}} \qquad (4)$$

Where $N(s'_p)$ is defined as the set of components which maintain the feasibility of s'_p. As $\Omega' = \phi$, $N(s'_p)$ represents the all heuristics available.

As mentioned above, α and β are the measure with the pheromone and heuristic information influences in the selection probability.

In the formula (5) the value calculation is appreciated for t and η. In synthesis, the pheromone will be built from the average measurement of improvements obtained by the historical solutions, where the component has involved (MC_{hk}). In other words, the average value of $f(s_0) - f(s_n)$ of the solutions generated by any ant j in any iteration i, as $c_k^h \in s'_{ij}$. If the component has never been used, the improvement is 0.

Finally, as the improvements can be negative, to prevent giving more probability components of worse solutions, full scale moves improvements averaging starting the worst result by 1.

At the same way, the heuristic information also corresponds to the historical improvement average (M_{kh}), but now from the specific improvements that the component had, and, not the complete solution s'_{ij}.

$$\begin{aligned}
t_{kh} &= MC_{kh} + 1 - MIN\{MC_{kl} \mid l = 1,2,\dots,H\} \\
\eta_{kh} &= M_{kh} + 1 - MIN\{M_{kl} \mid l = 1,2,\dots,H\} \\
MC_{kh} &= \begin{Bmatrix} (\sum_{c_k^h \in s'_{ij}} (f(s_0) - f(s_n)))/E & ,E > 0 \\ 0 & ,E = 0 \end{Bmatrix} \\
M_{kh} &= \begin{Bmatrix} (\sum_{c_k^h \in s'_{ij}} (f(s_{k-1}) - f(s_k)))/E & ,E > 0 \\ 0 & ,E = 0 \end{Bmatrix} \\
E &= \#\{s'_{ij} \mid c_k^h \in s'_{ij}\}, \forall i \forall j
\end{aligned} \tag{5}$$

For the first iteration of the algorithm, a random input solution is generated 0. From the second generation, the new input solutions will be the minimum established in the previous iteration, in any step of construction k.

In order to improve the exploration of solutions for P, a constraint is applied to the previous rule, when the chosen input for the next iteration is equals to the global minimum established in previous iterations. In this case, the second best solution of the generation is chosen as the next s_0.

4 References

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