Solution to Assignment 2

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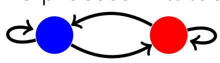
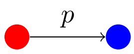
Let me start with solutions from assignment submission.

1 Drawing without replacement

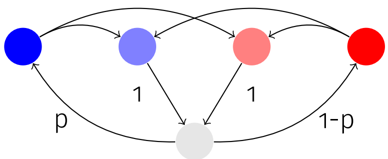
We have a jar with N balls, out of which b are blue and rest are red. I draw a ball at random but do not put it back. Clearly it is not a Markovian process since at each step the probability of drawing red (or blue) ball is changing. At time zero, probability of drawing a blue ball is $\frac{b}{N}$ which changes to either $\frac{b-1}{N-1}$ if drew blue ball or $\frac{b}{N-1}$ if I drew red ball now. I can define states in the process. Let say they are `BLUE_DRAWN` and `RED_DRAWN`. The transition graph of this process will look like the following.



Where arrow in graph shows the transition to head state given system is in tail state now. For example,  depicts the process of drawing blue ball given that red was drawn just before. The number on arrow p is the probability of this transition. I leave it upto you to write the transition matrix for this system using these two states and convince yourself that this is not Markovian.

A couple of attempts were made to turn it into a Markovian by introducing more states: a `NULL_BLUE`  and `NULL_RED`  states.

The state transition graph is following.



The state transition graph still does not have constant probabilities. In short, I could not turn this non-Markovian process to Markovian. However, if someone has come so far, I've given them full credit. If you know a way to convert this to Markovian process, let me know.

2 By *merging* states

A solution is given in submissions as following. Consider you are playing the game of "snake and ladder". You consider the position of player on the board coarsely. You note down the row (e.g. row 1, 2, etc.) rather than cell (2x1, 9x4, etc.). The claim is that this stochastic process sampled coarsely is not a Markovian process because *transition from row i to row j is dependant on previous transition $x \to i$* .

The idea is correct and has been awarded full marks. However 'snake and ladder' seen coarsely as row to row transition may still be Markovian. If merging some states of a Markovian process still leads to a Markovian process, such processes are callled *lumpable* [Kemeny and Snell, 1976; Peng 1996]. Lumping is computationally expensive process.

Simulation of 'snake and ladder' game suggest that coarse level description may still be Markovian for N games (where N > 4). The transition probabilities matrix does not change whether we describe the process at the level of cells or at the level of rows.

2.1 Simulation of game

I simulated a game of snake and ladder 10000 times and computed the transition probabilities; first by taking cell level view and second by taking coarse cell level view.

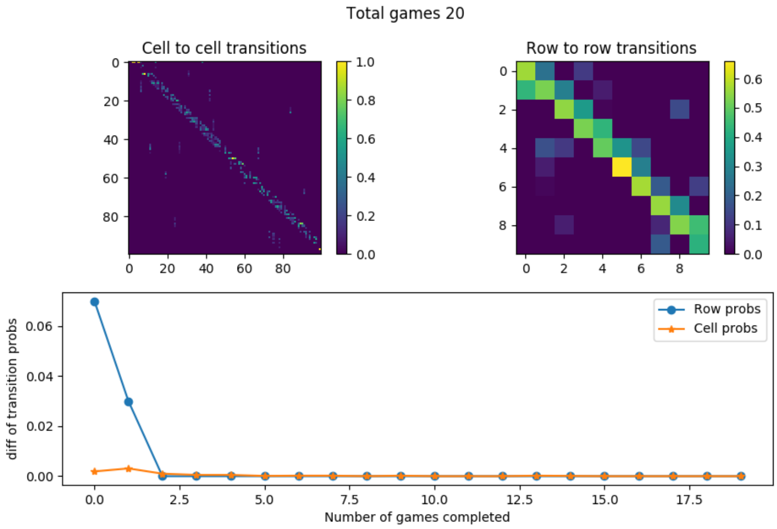
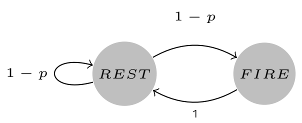


Figure 1: Subplot 1 shows the transition probabilities of cell to cell transitions after 20 games. Subplot 2 shows the transitions probabilities of row to row transitions after 20 games. Both of these do not change after roughly 3 games (shown in subplot 3)

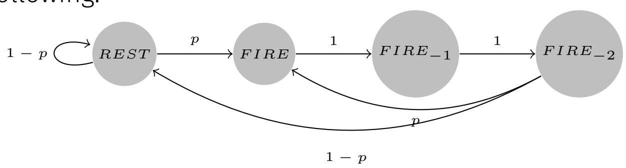
3 Non Markovian Neuron

Let assume that we are watching a neuron every 1 ms. We note down $\uparrow$ when the neuron fires and _ otherwise. We notice that probability of neuron firing is roughly p. We also know that if a neuron just has fired, then it can not fire for next 2 ms (2 steps) (neuron's **refactory period**). As long as we are describing this process using two states ↑ and _, it is not Markovian since probability of neuron firing is dependant on weather it has fired in previous two steps or not.



Above transition graph represents a Markovian process and can handle refactory period of 1 step (how?). Many of firing pattern can be approximated by such a system.

To make it Markovian when refactory period is longer than 1 step, we introduce two more state variables $FIRE-1$ and $FIRE-2$. The transition diagram is following:



Its easy to see that state $FIRE_2$ and state $REST$ can be merged together and process still be Markovian.