

## Randomness in Biology [2014 Aug Term]

### Homework 5: Assigned Oct 1. Due Oct 8.

1. Radioactive decay is governed by the following Master equation

$$\frac{d}{dt} p_n = \gamma(-(n)p_n + (n+1)p_{n+1})$$

where  $n$  represents the number of radioactive nuclei in a lump of uranium, and  $p_n$  is the number of lumps with precisely  $n$  radioactive nuclei. We derived in class that the equation for the mean is:

$$\frac{d}{dt} \langle n \rangle = -\gamma \langle n \rangle$$

(a) Derive the equation obeyed by the second moment  $\langle n^2 \rangle$ .

(b) Starting with  $p_n = \delta(n - n_0)$ , i.e. only  $p_{n_0} = 1$ , all others are 0, solve for the first and second moments as functions of time.

(c) If we started with a large number of lumps of equal size, at what timepoint would the size of the lumps be most variable in absolute terms?