Randomness in Biology [2014 Aug Term]

Homework 6: Assigned Oct 13. Due Oct 20.

1. The most general one-step master equation in two variables is:

$$\frac{dp_{i,j}}{dt} = -(a_{i,j} + b_{i,j} + c_{i,j} + d_{i,j})p_{i,j} + a_{i-1,j}p_{i-1,j} + b_{i+1,j}p_{i+1,j} + c_{i,j-1}p_{i,j-1} + d_{i,j+1}p_{i,j+1}$$

a. Write down the deterministic ODE this would correspond to. I.e. an equation of the form

$$\frac{d}{dt}i = f(i,j), \quad \frac{d}{dt}j = g(i,j)$$

where *f* and *g* are written in terms of the *a,b,c,d* and *i, j* are considered continuous.

b. Suppose now that the coefficients all have the following linear (or affine) form:

$$x_{i,j} = x_0 + x_1 i + x_2 j$$

where this applies for $x = a, b, c, d$.

Derive an ODE for the means $< i >, < j >$. You should find it is identical to the deterministic case. This argument is easily extended to multiple variables.