Homework 1

Calculus 2016

September 8, 2016

Problem 1

a. If $0 < a < b$, and c is any real number, rewrite the sets $\{x : a < |x-c| < b\}$ in terms of intervals. Do the same for $a = 0$, and also give a clear description in words. In this case, it will be something like "all the numbers between [something] and [something else] except for [these one(s)]

b. Given $a < b$ real numbers, describe the intervals (a, b) using the absolute value function. That is, write $(a, b) = \{x : ...\}$ where "..." is some condition using absolute value.

c. Similarly, express $\{a, b\}$, i.e. the set containing precisely the two (different) numbers a and b (e.g. $\{-14.7, e + \pi\}$ or $\{1776, 1947\}$) using absolute values. [1]

Problem 2

1. Show that for any x, y, $||x| - |y|| \leq |x + y|$. (This is a useful counterpart to the triangle inequality and requires a very similar analysis.)

2. Write the set $\{x : |x^2 - 2x - 3| > x\}$ as a union of intervals (i.e. figure out explicitly for which x this statement is true)

[1] Hint: choose any a and b you like, and work it out first to see how it goes for the general case. But remember that doing it for a specific example is not enough–it is supposed to be true for $\{any\}$ a, b, c (satisfying the relevant conditions), so you need to do something that works for all possible choices. In math, we don't show a statement true by making observations, unless it's possible to directly observe every single case; we have to make an argument that shows that the statement is true $\{always\}$. When there are infinitely many possible choices of a,b,c as above, we can't directly check each case by hand; we need to have a general argument.