Randomness in Biology [2014 Aug Term]

Homework 7: Assigned Oct 20. Due Oct 27.

Flipping a genetic switch

1. We have seen in class that the following differential equation describes a protein which activates its own transcription. This is equivalent to a double-well potential, where the two wells correspond to states of low and high gene expression.

$$\frac{dx}{dt} = \frac{v_0 + v_1 K_1 K_2 x^2}{1 + K_1 K_2 x^2} - \gamma x$$

Use the following parameters: $v_0 = 12.5, v_1 = 200, \gamma = 1, K_1 K_2 = 10^{-4}$ or $K_1 K_2 = 10^{-6}$.

For each value of $K_1 K_2$, estimate the steady-state distribution of gene expression levels using the following approaches:

a. A Langevin stochastic differential equation by adding an appropriate Wiener term.

b. A Gillespie simulation.

c. The approximate form $\mathcal{P}(x) = \frac{A}{f(x)+g(x)} \exp\left(2 \int \frac{f(x)-g(x)}{f(x)+g(x)} dx\right)$

d. Use the Langevin method to find the mean first passage time to transit between the two deterministic steady states for the case $K_1 K_2 = 10^{-4}$.

e. Bonus. Start with $x(t = 0) = x_{low}^{ss}$ and run the Langevin equation till you reach $x = x^{transition}$. When this occurs, store the path segment to the right of x_{low}^{ss}. Print out 10 paths of this kind. What do you see?