Solution to HW3

Dilawar Singh

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1. (4 points) Refer to homework statement.

   (a) (2 points) **Solution:** For every y, we can find at least one x smaller than y such that $f(x)$ is smaller than $f(y)$.

   negation For any y, there is no x smaller than y such that $f(x)$ is smaller than $f(y)$. $\forall x$, $\nexists y$ such that if $x < y$ then $f(x) < f(y)$. Or, $\forall x$, $\exists y$ such that if $x < y$ then $f(x) \geq f(y)$.

   Now, rewriting the previous statement,

   For every x, there does not exists any y such that ....

   Using natural language, there could be multiple ways to write same thing . It is not easy to verify if all of those statements are equivalent. Mathematical symbols give a less error-prone way to operate on logical statements.

   (b) (2 points) **Solution:**

   There is at least one place where f is 0. Or, There is at least one x such that $f(x) = 0$.

   Negation There is no place where f is zero. Or, $\forall x \; f(x) \neq 0$.

2. (4 points) See the statement on google group.

   (a) (2 points) **Solution:**

   negation f is NOT increasing. $x < y \implies f(x) \geq f(y)$

   (b) (2 points) **Solution:**

Negation f DOES NOT have a local maxima at a i.e. in the δ neighbourhood of a, f does not have this shape $\vee$
$\forall\delta > 0, \exists h \in (a-\delta, a+\delta)$, such that $f(a) \geq f(a+h)$.

3. (12 points) The quadratic approximation goes like the following.
$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x)$.
And the solutions are plotted below. Note that this is not the replacement for algebraic analysis.

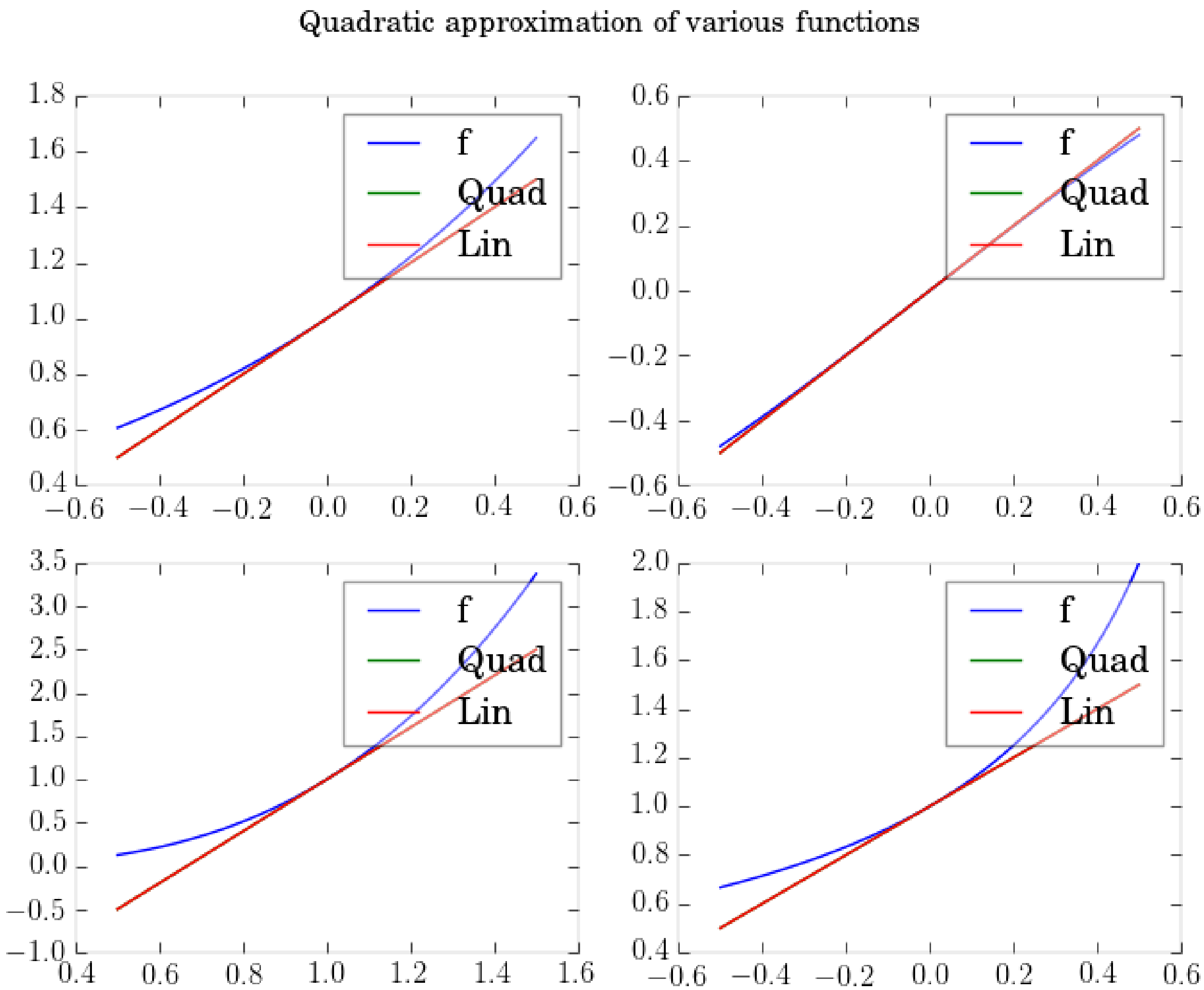

Figure 1: f is e^x, $sinx$, x^3, and $\frac{1}{1-x}$

4. (12 points) (a) (4 points) **Solution:** For f to be even, we must have $f(x) = f(-x)$. Therefore, for all x such that $ax^3+bx^2+cx+d = -ax^3+bx^3-cx+d$, f is even. That is $ax^3+cx = 0 \implies a = 0, c = 0$. See David's explanation on group. Similarly, for f to be odd, $-ax^3+bx^2-cx+d = -ax^3-bx^2-cx-d$. That is

$bx^2 + d = 0 \implies b = 0, d = 0.$

(b) (12 points) **Solution:**

(c) (4 points) **Solution:**
$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$$
\begin{aligned}
f'(-x) &= \lim_{h \to 0} \frac{f(-x+h)) - f(-x)}{h} && (1)\\
&= \lim_{h \to 0} \frac{f(-(x-h)) - f(x)}{h} && (2)\\
&= \lim_{h \to 0} \frac{f(x-h) - f(x)}{h} && (3)\\
&= \lim_{h \to 0} -\frac{f(x) - f(x-h)}{h} && (4)\\
&= \lim_{h \to 0} \frac{f(x) - f(x-h)}{-h} && (5)\\
&= f'(x) && (6)
\end{aligned}
$$

Or, $f'(-x) = f'(-x)\frac{d(-x)}{dx}$ (using chain rule). This gives us $f'(-x) = -f'(-x)$ therefore $f'(-x)$ is a even function.
Similarly (i.e. I am too lazy to do any work), one can show that g' is odd.

5. (20 points) See the statement.

Solution:

Following figure shows the solution:

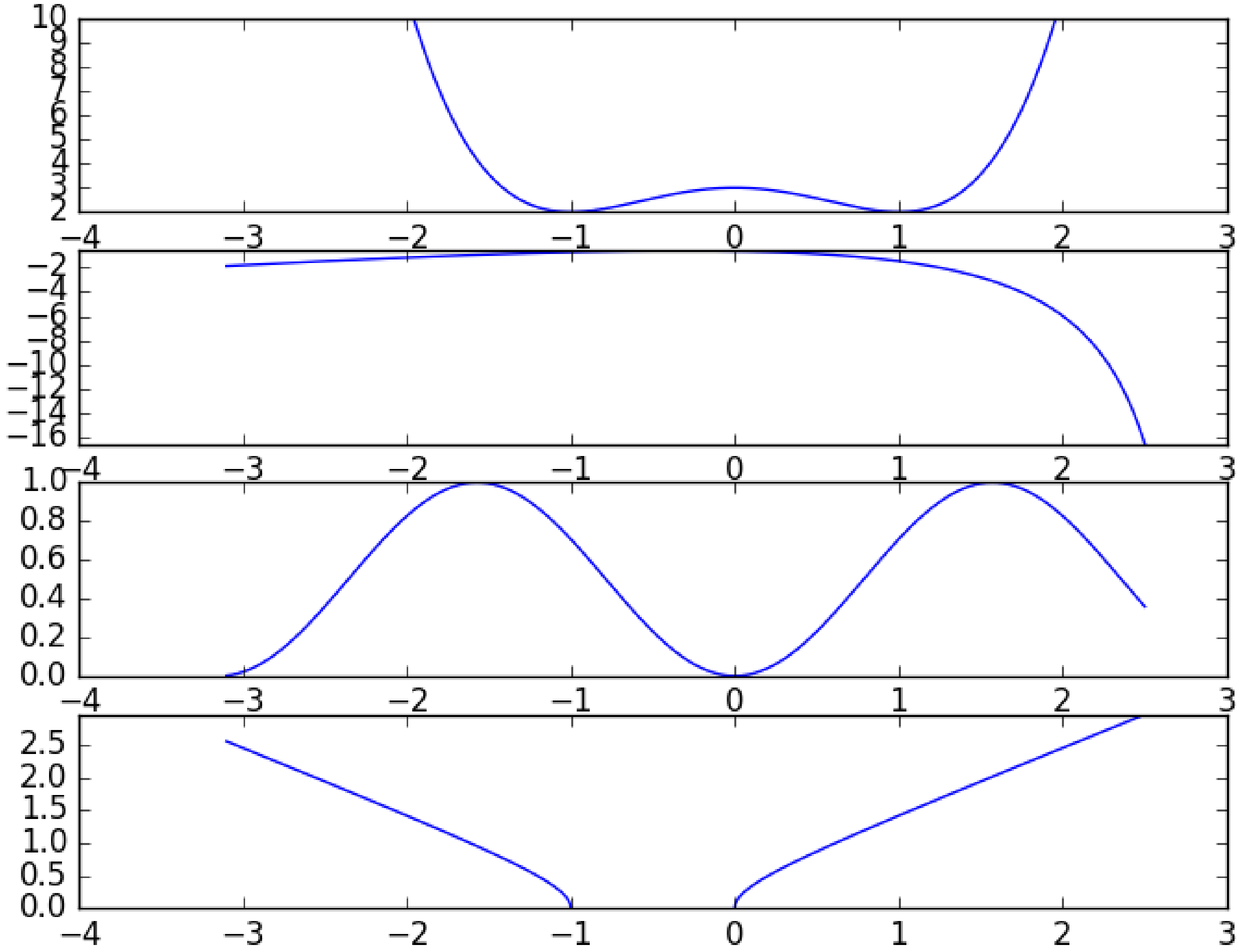

Figure 2: Solution to question 5. Functions are $x^4 - 2x^2 + 3$, $\frac{x^2+2}{x-3}$, $sin(x)^2$, and $\sqrt{x^2+x}$