Solution to HW2

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1. (20 points) Refer to homework statement.

Solution:

Except for the last subplot, these plots shows how each function will transform given functions in homework. These two functions are in first subplot. They are called `fun1` and `fun2`.

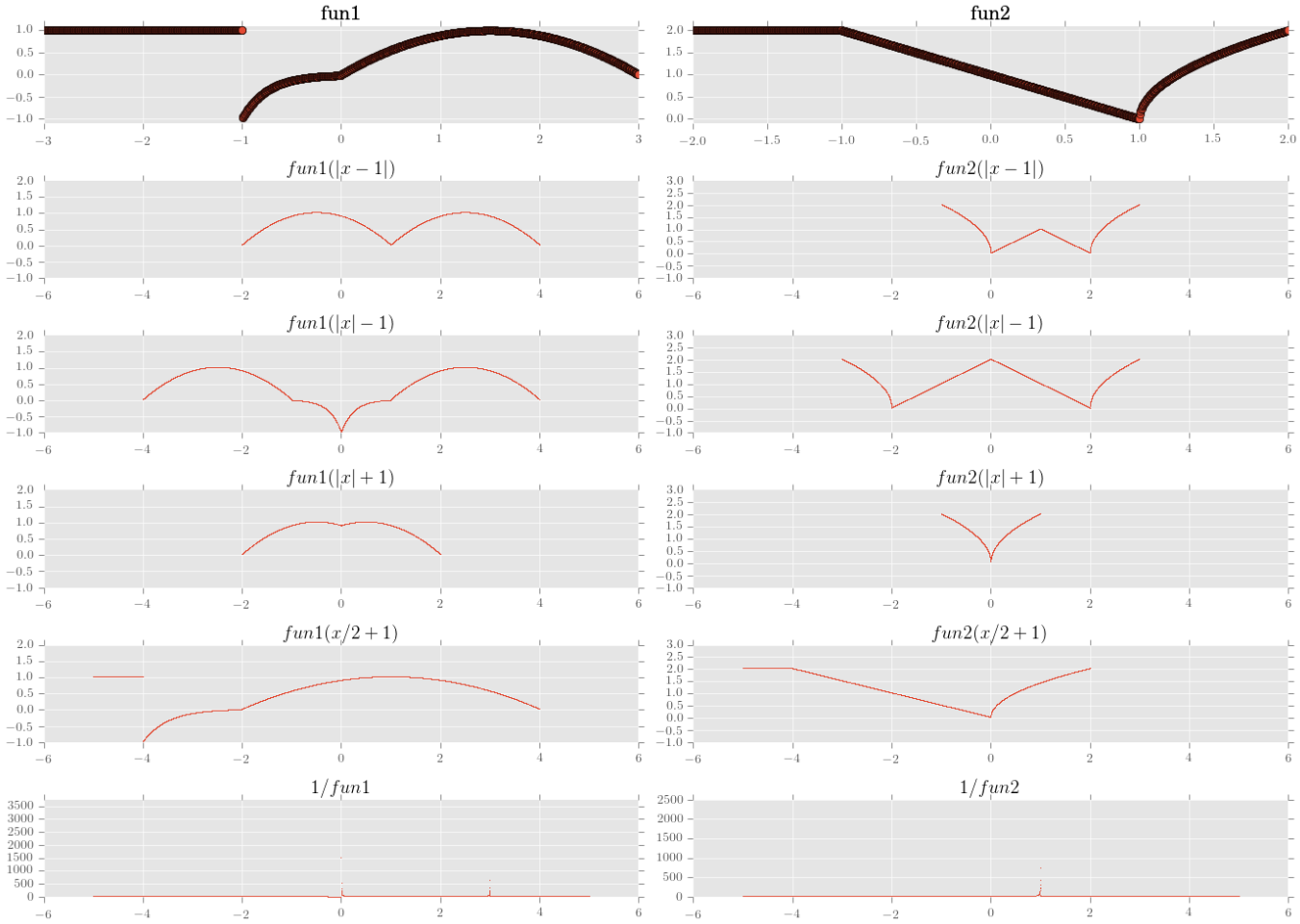

Figure 1: Solution to problem 1

2. (12 points) Refer to homework statement.

Solution:

See figure 2.

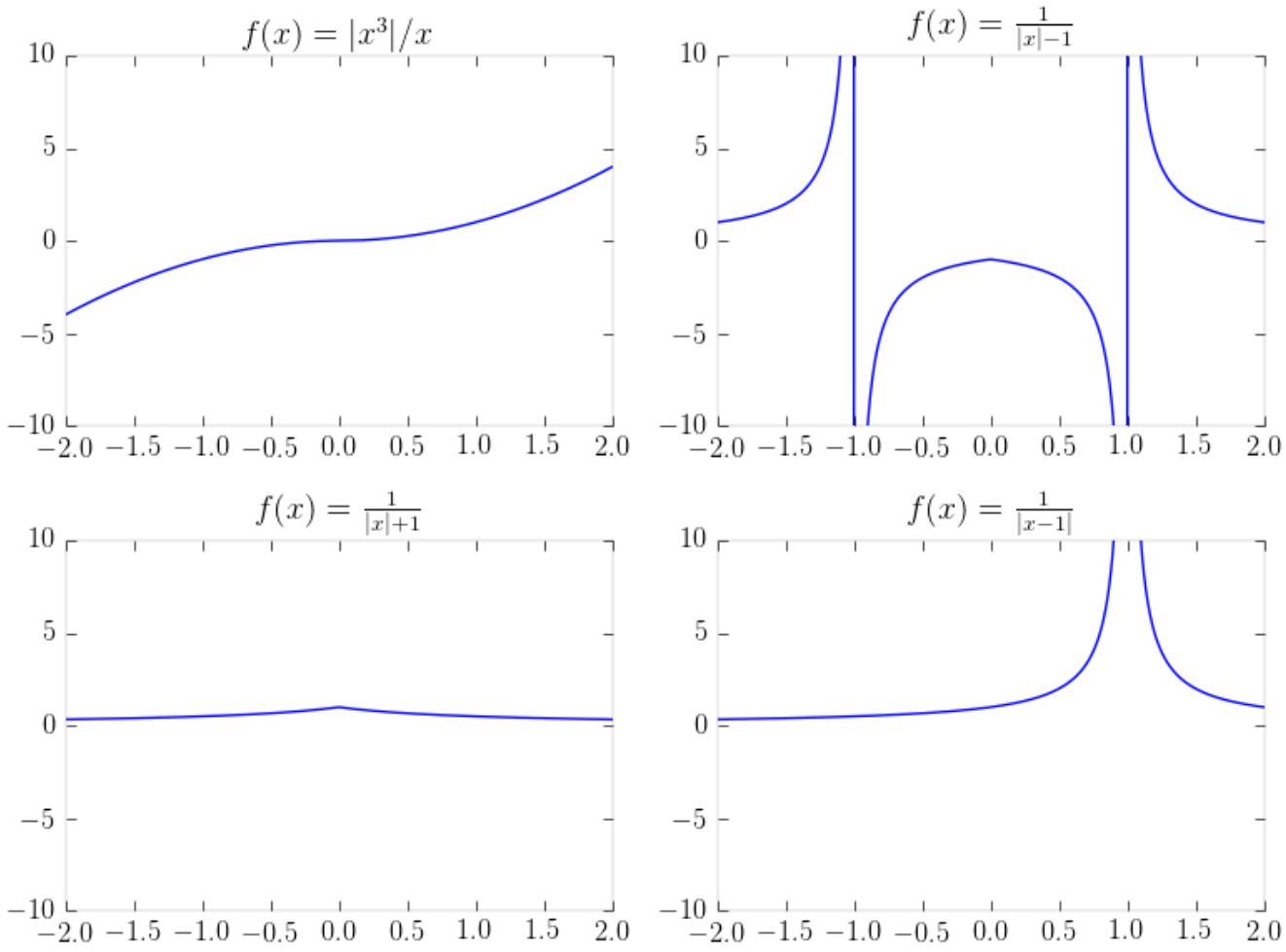

Figure 2: Solution 2

3. (5 points) Show that if $f(x) = \sqrt[3]{x}$, then for any $a \in \mathbb{R} \setminus \{0\}$ $f'(a) = 1/3a^{-2/3}$.

Solution:

We start with $f(a+h) = f(a) + hf'(a) + R(h)$ and $R(h)/h \to 0$ when $h \to 0$. We can write the following:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \tag{1}$$

This leads to,

$$f(\sqrt[3]{x}) = \lim_{h\to 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} \tag{2}$$
$$= \lim_{h\to 0} \frac{(x+h)^{1/3} - x^{1/3}}{h} \tag{3}$$
$$= \lim_{h\to 0} \frac{x^{1/3}\left(\left(1+\frac{h}{x}\right)^{1/3} - 1\right)}{h} \qquad = \lim_{h\to 0} \frac{x^{1/3}\left(1+\frac{h}{3x} + \ldots - 1\right)}{h} \tag{4}$$
$$= x^{1/3}\left(\frac{h}{3xh}\right) \tag{5}$$
$$= x^{-2/3}\left(\frac{1}{3}\right) \tag{6}$$

4. (5 points) Show that if $g(x) = \frac{1}{f(x)}$, then for any $a \in \mathbb{R}$ such that $f(a) \neq 0, g'(a) = -\frac{f'(a)}{f(a)^2}$.

Solution:

$$g'(a) = \lim_{h\to 0} \frac{g(a+h) - g(a)}{h} \tag{7}$$
$$= \lim_{h\to 0} \frac{\frac{1}{f(a+h)} - \frac{1}{f(a)}}{h} \tag{8}$$
$$= \lim_{h\to 0} \frac{f(a) - f(a+h)}{hf(a)f(a+h)} \tag{9}$$
$$= \lim_{\mathbf{h}\to\mathbf{0}} -\frac{\mathbf{f(a+h)} - \mathbf{f(h)}}{\mathbf{h}} \frac{1}{f(a)f(a+h)} \tag{10}$$
$$= -f'(a)\frac{1}{f(a)f(a)} \tag{11}$$

5. (5 points) Write the approximation for $\sqrt{99.8}$ and $\sqrt[3]{5.01^2 + 2}$.

Solution:

6. (8 points) Differentiate each of the following ....

Solution:

7. (8 points) Inverse function theorem

(a) (4 points) **Solution:**

$$f(x) = \sqrt{1+x^3} \tag{12}$$

$$\implies f^{-1}(x) = \frac{1}{\sqrt{1+x^3}} \tag{13}$$

$$(f^{-1})'(x) = -\frac{1}{2}(1+x^3)^{-3/2} \tag{14}$$

$$\tag{15}$$

(b) (4 points) **Solution:**

$$(f^{-1}(x))' = -f^{-2}(x)f'(x) \tag{16}$$

$$= -(1+x^3)^{-1}\frac{1}{2}(1+x^3)^{-1/2} \tag{17}$$

$$= -\frac{1}{2}(1+x^3)^{-3/2} \tag{18}$$