

Assignment - Lecture 06

Dilawar Singh

1. Exorcism Maxwell's Demon

“Concerning Demons: 1. Who gave them this name? Thomson” –
In a letter from Maxwell to Tait

Unless demon does some work to create the heat-gradient in given system, the second law of thermodynamics – a heat engine with efficiency more than 1 – is violated. The demon is exorcised by showing that this is not the case and Demon has to do some work to create such a temperature gradient; even when closing and opening of gate does not involve any work.

Given that Demon is governed by laws of thermodynamics itself, an information theoretic approach is widely believed to exorcise Maxwell's Demon. The thesis was first proposed by Szilard where he asserts,

We show that it is a sort of a memory faculty, manifested by a system where measurements occur, that might cause a permanent decrease of entropy and thus a violation of the Second Law of Thermodynamics, were it not for the fact that measurements themselves are necessarily accompanied by a production of entropy.

Moreover, Szilard project which establishes that there is a cost to information acquisition is meant to protect the 2nd law not from the fluctuation phenomenon, but only from the intelligent accumulation of fluctuations.

Assume that we have 6 buckets in which we are to put 20 balls. In first case, we put 20 balls in 3 buckets; other 3 buckets are empty. The entropy of this system is $A \times \ln \frac{20!}{n_1!n_2!n_3!}$. If Demon partitions the balls into two sets of 10 balls each without doing any work, then the entropy of new system is $A \times \ln \frac{10!}{n_1!n_2!n_3!} + A \times \frac{10!}{n_1!n_2!n_3!}$, where n_1, n_2 , and n_3 are values from a distribution but with the constraint that $n_1 + n_2 + n_3 = 10$ or 20 depending on the context.

Then exorcising the Demon is as simple as showing that entropy in the second case is lower than the first case. Therefore Demon must have done some work to lower the entropy the system.

2. Plot $\frac{N_2}{N_1}$ for a two state system as a function of temperature (2 - 5000 K) for $\delta E = 0.05, 0.5, 1$, and 5 kcal/mole.

If the number of molecules in state i is N_i , and number of molecules in the ground-most state are N_0 ; and if $\Delta E = E_i - E_0$ then we have the following relationship:

$$N_i = N_0 \exp \frac{-\Delta E}{k_B T}$$

This equation is reduced to the following once we put the values of constants in dimensions given in assignment.

$$\frac{N_i}{N_0} = \exp \frac{-\Delta E}{1.987 \times 10^{-3} T}$$

Now we can plot the function. Plot is shown in figure .

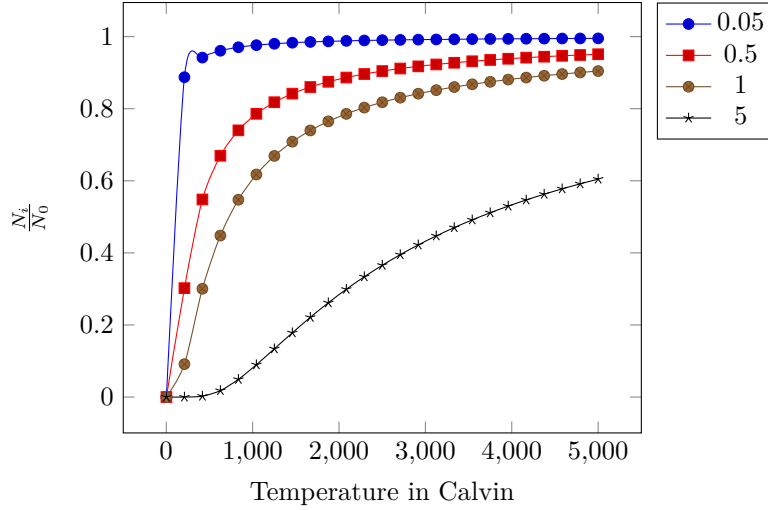


Figure 1: T and distribution for various ΔE values.

3. Plot of $\frac{N_i}{N_0}$ vs ΔE

This plot is in figure .

4. Sun Vs Human

The mass of Sun is estimated to be $1.989e30$ Kg and total power of Sun is $3.95e26$ Watt. Therefore, power of Sun per unit mass is $1.986e-4$ Watt per Kg. In 2008, it was estimated that we use 142.3 kWh per capita i.e. 39.53 W per capita. The average weight of human is 62 Kg. Therefore energy consumption per unit mass by a human is approximately 0.64 W per Kg.

The ratio of power produced by sun per unit mass to the ratio of power consumed by human per unit mass is 0.003255.

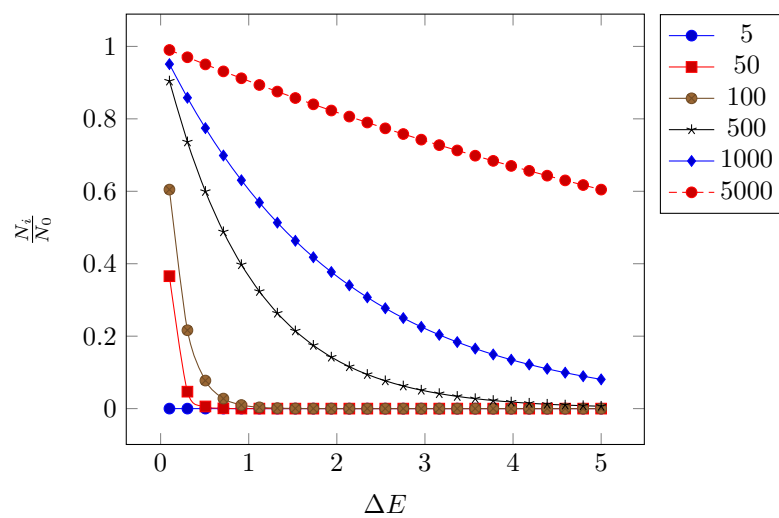


Figure 2: ΔE and distribution for various temperatures