

Solution to HW3

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1. (4 points) Refer to homework statement.

(a) (2 points) **Solution:** For every y , we can find at least one x smaller than y such that $f(x)$ is smaller than $f(y)$.

negation For any y , there is no x smaller than y such that $f(x)$ is smaller than $f(y)$. $\forall x, \nexists y$ such that if $x < y$ then $f(x) < f(y)$. Or, $\forall x, \exists y$ such that if $x < y$ then $f(x) \geq f(y)$.

Now, rewriting the previous statement,

For every x , there does not exist any y such that

Using natural language, there could be multiple ways to write same thing . It is not easy to verify if all of those statements are equivalent. Mathematical symbols give a less error-prone way to operate on logical statements.

(b) (2 points) **Solution:**

There is at least one place where f is 0. Or, There is at least one x such that $f(x) = 0$.

Negation There is no place where f is zero. Or, $\forall x f(x) \neq 0$.

2. (4 points) See the statement on google group.

(a) (2 points) **Solution:**

negation f is NOT increasing. $x < y \implies f(x) \geq f(y)$

(b) (2 points) **Solution:**

Negation f DOES NOT have a local maxima at a i.e. in the δ neighbourhood of a , f does not have this shape \vee
 $\forall \delta > 0, \exists h \in (a - \delta, a + \delta)$, such that $f(a) \geq f(a + h)$.

3. (12 points) The quadratic approximation goes like the following.

$$f(x + h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x).$$

And the solutions are plotted below. Note that this is not the replacement for algebraic analysis.

Quadratic approximation of various functions

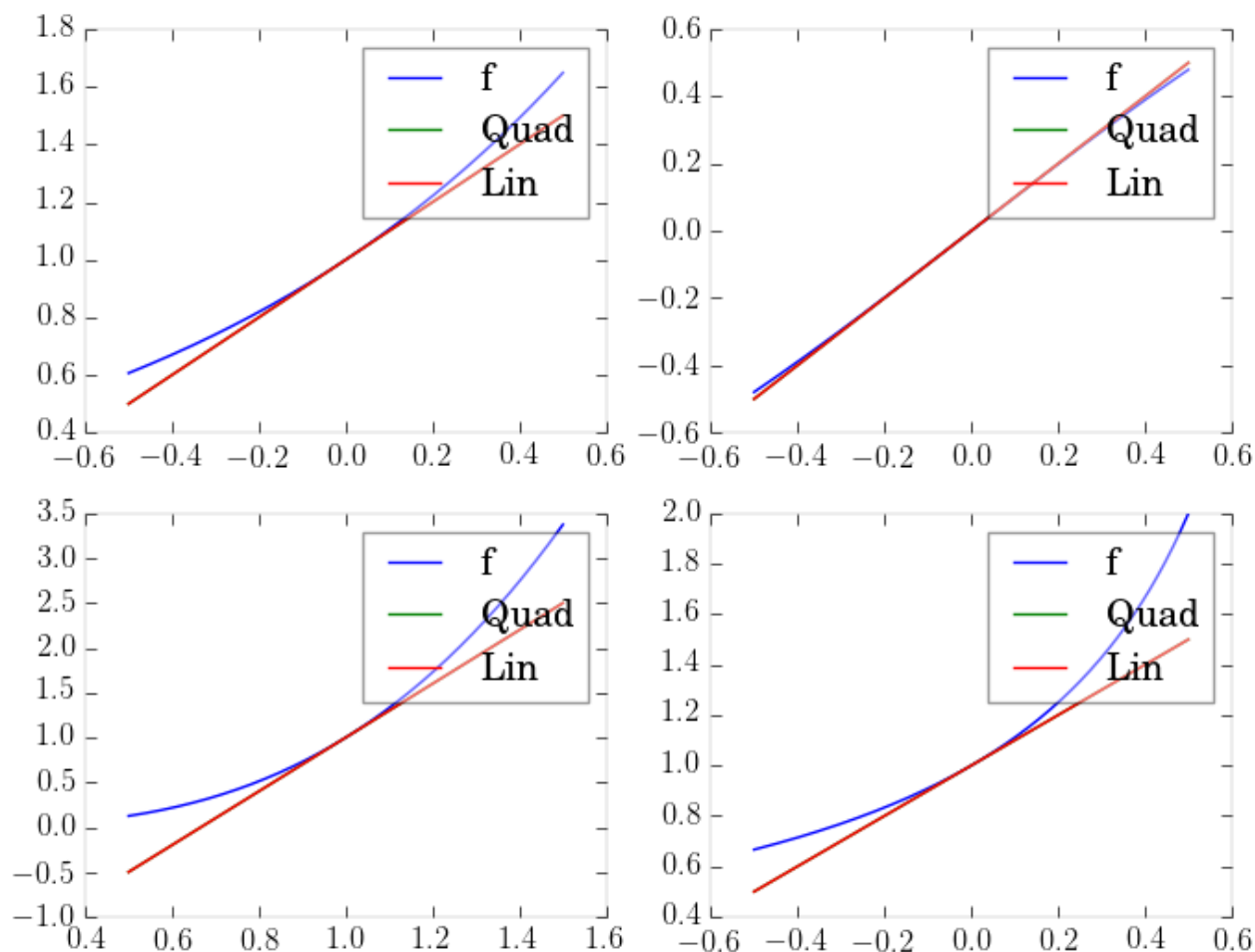


Figure 1: f is e^x , $\sin x$, x^3 , and $\frac{1}{1-x}$

4. (12 points) (a) (4 points) **Solution:** For f to be even, we must have $f(x) = f(-x)$. Therefore, for all x such that $ax^3 + bx^2 + cx + d = -ax^3 + bx^3 - cx + d$, f is even. That is $ax^3 + cx = 0 \implies x = 0, \pm\sqrt{\frac{-c}{a}}$. Similarly, for f to be odd,

$$-ax^3 + bx^2 - cx + d = -ax^3 - bx^2 - cx - d. \text{ That is } bx^2 + d = 0 \implies x = \pm \sqrt{\frac{-d}{b}}.$$

(b) (12 points) **Solution:**

(c) (4 points) **Solution:**

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(-x) = \lim_{h \rightarrow 0} \frac{f(-x+h) - f(-x)}{h} \quad (1)$$

$$= \lim_{h \rightarrow 0} \frac{f(-(x-h)) - f(x)}{h} \quad (2)$$

$$= \lim_{h \rightarrow 0} \frac{f(x-h) - f(x)}{h} \quad (3)$$

$$= \lim_{h \rightarrow 0} -\frac{f(x) - f(x-h)}{h} \quad (4)$$

$$= \lim_{h \rightarrow 0} \frac{f(x) - f(x-h)}{-h} \quad (5)$$

$$= f'(x) \quad (6)$$

Or, $f'(-x) = f'(-x) \frac{d(-x)}{dx}$ (using chain rule). This gives us $f'(-x) = -f'(-x)$ therefore $f'(-x)$ is a even function.

Similarly (i.e. I am too lazy to do any work), one can show that g' is odd.

5. (20 points) See the statement.

Solution:

Following figure shows the solution:

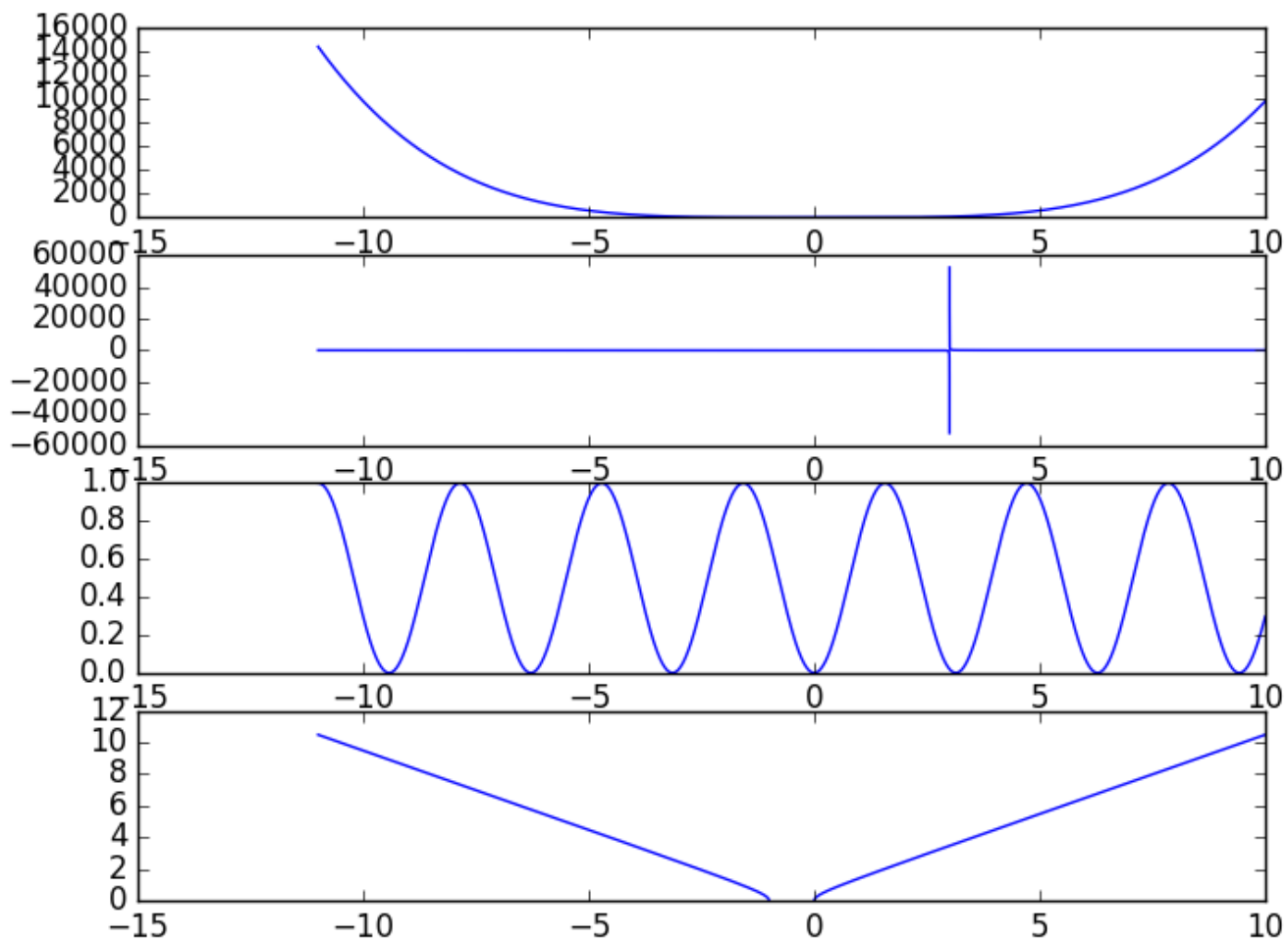


Figure 2: Solution to question 5. Functions are $x^4 - 2x^2 + 3$, $\frac{x^2+2}{x-3}$, $\sin(x)^2$, and $\sqrt{x^2 + x}$