ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Lecture with Computer Exercises:
Modelling and Simulating Social Systems with MATLAB

Project Report

Zombie Outbreak: The Effect of Inter-State Cooperation on the Survival of Humanity

by

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Zurich
December 2012

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Contents

1 Abstract

This part needs updating once the rest of the report will be finished
Investigation of the application of the SIR model to a zombie outbreak has already been studied, raising the fear of dark days for humanity. However, we would like to deepen this investigation to a multi-state system to see how interactions between subpopulation may brighten the future of the human race. Moreover, we are interested in seeing to what extent the different paradigms of international politics, Realpolitik, Liberalism and Neoconservatism as defined by Daniel W. Drezner in Theories of International Politics and zombies may lead to different outcomes.

2 Individual contributions

M.G.M. and B.I.M.W. formulated the question in mathematical terms and discussed the implementation in MATLAB. M.G.M. wrote the code. M.G.M. and B.I.M.W. analysed and discussed the results and B.I.M.W. wrote the report.

3 Acknowledgments

We wish to thank Karsten Donnay and Stefano Balietti for their support in our work, fruitful discussions and open-mindedness to accept such a project. We also would like to thank the Chair of Sociology for the computational support provided with simulation time on the ETH cluster Brutus.

4 Introduction and Motivations

4.1 Zombie Outbreak

While models of international relationship have already been studied under different pressure components, the effect of a zombie outbreak on international collaboration and equilibrium is a question that has been underestimated and was never addressed to the best of our knowledge. It is remarkable that the effect of such an intense event has not been looked at, although the fear of zombies and the threat they represent is vivid for many of us as reflected by the importance of the zombie culture. Zombies, compared to other unnatural creatures such as vampires or aliens, have the very peculiar property not to be a minority inside the human civilization but rather to be in a way a part of it, just not quite as it was, i.e. zombified. Accordingly, zombies cause a much more deep-rooted fear as they not only threaten our lives but also the our very sense of identity as it questions our notion of what humanity is. The psychological effect of such a non-standard threat as well as the repercussion on the behaviour of large popluation systems such as states should be far from trivial. Accordingly, we decided to simulate an inter-state collaboration models in order to see the outcome on a large scale of such an extreme event. While some people might question the validity of such a study (no zombie has been observed so far), we think that the applicablity of such a reasoning could extend to a more probable large-scale epidemological disaster or simply, give a line of reasoning to cope with what former U.S. Secretery of Defense Donald Rumsfeld referred as the "unknown unknowns" of international security. Zombies might not be real, but the threat and stress they could impose on current world politics is.

4.2 Epidemological Studies

Epidemiological models have been well studies to see the evolution dynamics and spread of a disease in a population REFERENCE. They are based on a mass action model where interaction between susceptibles and infected together with an infection constant define the rate of infection. In the simplest case of such models, infected people can move to the immunized category also following a mass action law. This models have been shown to work in numerous cases and when corrections are added REFERENCE, they can very well represent the evolution and spread of an infectious disease in a population. However, an epidemiological model of communicating population has not been tested to the best of our knowledge.

4.3 Zombies: a Definition

The origin of the word "zombie" as well as its initial meaning is quite remote from modern pop culture description. The word itself is said to have originated in the voodoo language in the Caribbean [1]. The original description is that of a ritual where the wizard of a tribe would start controlling another person. The fact that this person doesn't have a soul anymore and that it is no under the control of another human being, i.e. it has no longer a free-will, makes it a zombie [1]. Some studies suggest that those "zombified" members of the tribe where actually administrated a cocktail of two natural drugs, one being a neurotoxin and the other a hallucinogenic drug REFERENCE. In fact, the neurotoxin damages the brain and turns the person into a vegetative state.

Zombies in popular culture differ a lot from this original etymological definition. The canon of the zombie literature have had numerous description and hypothesis on how they may emerge in a human population, as well as what might characterize them. Since Romero's "night of the living dead" REFERENCE, where zombies were said to have raised from some pseudo-magical event, the depiction and origin of zombies has considerably evolved. Recent zombie stories usually describe the genesis of flesh-heating monsters in an epidemiological sense, typically some sort of virus. Recent examples in the zombie culture are numerous and include (non-exhaustively) "Resident Evil", "28 Days Later". For simplicity, we will treat the emergence and zombification events as linked to an infectious disease of some sort, allowing the treatment of the problem with a modified epidemiological model. The big difference with a standard infectious disease where people usually get immunized, being transformed into a zombie is a on way process. The only way out is death and therefore, humans (susceptibles) can only decrease in our system.

4.4 Popular Believes in the Event of the Zombie Outbreak

It is interesting to notice that most zombie canon predict a very bleak outcome concerning the fate of humanity. Indeed, most movies/film describe an almost total disappearance of humans and the few survivors are rarely in a position that seem to be about to brighten up. While some might argue that the disappearance of humanity might not be such regretful event and might actually benefit our planet REFERENCE, we decided to see if the usual outcome and fate of the human race in case of a zombie outbreak might differ from those classical scenarios, and if so, under which set of particular conditions.

Mathematical modeling of a zombie outbreak in a single population has previously been simulated [2] but showed very little hope for humans in the case of such an unlikely event. The primarily reason for the annihilation of humans in all

of the presented scenarios lies in their models. In contrast to "classical" epidemiological models where infected people can recover and although having changed population statues (going from susceptible to removed in a immunized sense of the term), they do not actually die. This is very different for the zombie scenario, where now "removed" is no longer synonym of "immunized", but is actually a very nice way of putting "dead". Accordingly, under the considerations of the model presented, humans can only day and this eventually happens in every case (some set of parameters can give reprieve the inevitable fate).

4.5 Can a Different Treatment of the Problem Lead to a Different Outcome?

We rationalized that a more state-based description of the world population might actually help in brightening the outcome of a zombie outbreak. In fact, the world in divided into states and nations that apply their own laws and restrictions in terms of immigration. If immigration applies to humans, it might as well apply to zombies. In such a case, one might envision that in a given state under the threat of a zombie epidemiological disaster, flux of incoming susceptibles to help them kill the zombies or on the other hand the emigration of the survivors to a non-infected state might lead to various outcome. For example, it is imaginable to see the emergence of a zombie-only state where all the remaining survivor would have found shelter in another state. Or even better, that the help of susceptible from another state might help eradicate the new-coming zombie threat.

For those reason we decided to simulate a model of interacting sub-populations, each under an epidemiological-like treatment. The inner-state epidemiological model describes the emergences of zombies from a spreading disease point of view, while the populations fluxes between states would represent immigration/emigration of the populations under concern (humans and/or zombies).

Moreover, we are interested in seeing to what extent the different paradigms of international politics, Realpolitik, Liberalism and Neoconservatism as defined by Daniel W. Drezner in Theories of International Politics and zombies may lead to different outcomes. Expected Results

As describe in Drezner's book, we except different equilibrium outcomes depending on the paradigm under consideration. He postulates the possibile appearance of zombie states under Realpolitik and Liberalism paradigms while Neoconservatism would not allow such an outcome.

5 Description of the Model

In order to simulate a multi-state system under epidemiological evolution we needed to define a clear mathematical framework treating both the population fluxes within states and among states. Such a treatment was necessary to represent the two-fold problem of a zombie outbreak at an international leve comprised of well-defined states:

i. Intra-state fluxes have a fixed physical definition and are invariable among states. They represent the true epidemiological part of our model

ii. Inter-state fluxes do not have a similar physical meaning and will depend greatly on the paradigm of international politics under consideration (*Realpolitik*, Liberal, Neoconservatorism). They do not represent epidemological variation *per se* but modelisation of immigration/emigration fluxes.

With such an approach, we would be able to separate the population evolutions at the international and domestic level. They idea would be see if variations of the international fluxes could influence the general (world) outcome in terms of survival of our species. Moreover, we were interested in the modalities of such a survival. Would the zombies be eradicated? Would some states disappear? Is the emergence of a zombie state possible? Of course, such a system is very complicated and in order to formulate a mathematical treatment of the model we made a few basic assumptions. Namely:

i. Zombification is an infectiously transmitted upon contact between a susceptible and a zombie.

ii. Zombification occurs instantaneously, and therefore no latent phase needs to be modeled.

iii. The outbreak occurs over a short amount of time, therefore both natural death rate and birth rate can be neglected.

iv. Only three homogenous population types are considered, susceptibles (S), zombies (Z) and removed (R).

We decided to simulate the microstate level under a modification of the classical SIR model REFERENCE called the SZR model (S for susceptible, Z for Zombies, R for removed) [2]. Contrary to the original SZR model developed by the authors, we made two modification in order to accommodate our interpretation of zombification. As we could not find an appropriate treatment of an epidemiological model with distinct subpopulations that would fit a zombie outbreak, we had to develop macrostate population transfer equation that would represent the simulated system. Since removed are effectively dead humans or head-shotted zombies, they will not transfer between states and accordingly, only the transfer of S and R needs to be considered.

5.1 Microstate Description

Each state (microstate) has three distinct populations, the susceptible (S), the zombies (Z) and the removed (R). This defines our SZR model. A scheme for the microstate fluxes is described in figure 1.

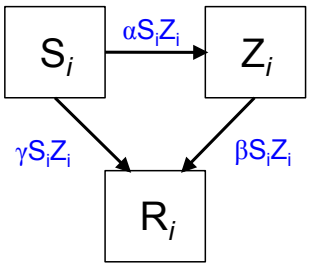

Figure 1: Description of population fluxes at the microstate level.

Contrary to the original SZR model by Munz *et al.* [2], we do not consider deads rising from their graves to become zombies. Indeed, we consider a very classical approach to the zombie type as described in modern culture such as removal of the head effectively kills a zombie. Moreover, we consider that non-natural death of susceptibles directly to removed is a mass-action based transfer of both the zombie and the susceptible population multiplied by a constant γ. The rational behind formulating the transfer as such comes from the consideration that those death are zombie-related, such as forced escapes, crowd panic effect, etc... In addition, susceptibles can become zombies through the same mass-action equation, where the frequency rate of infection is α. Taken together, those considerations give equation (1) for the variation of the susceptible population. The zombie population has an incoming flux from the susceptible with parameter α as previously described. They can also get killed through encounter with susceptible with a frequency rate β. Taken together, these two equations describe the flux of zombie at the microstate level (2). From those equation logically comes the equation(3), describing the flux of removed.

$$\Delta S_i^{micro} = -\alpha S_i Z_i - \gamma S_i Z_i = -(\alpha + \gamma) S_i Z_i \tag{1}$$

$$\Delta Z_i^{micro} = +\alpha S_i Z_i - \beta S_i Z_i = (\alpha - \beta) S_i Z_i \tag{2}$$

$$\Delta R_i^{micro} = +\beta S_i Z_i + \gamma S_i Z_i = (\beta + \gamma) S_i Z_i \tag{3}$$

It is interesting to already note that under such a model, which considers an outbreak occurring over a short amount of time, the susceptible population can

only decrease (two outgoing fluxes, no incoming flux) and the removed population increase (two incoming fluxes, no outgoing) no matter what. The zombie population evolution can however be positive or negative, depending on the ratio between α and β. Another interesting fact from this model is the identical mass-action description of the fluxes ($S_i \cdot Z_i$), meaning that the outcome will be dependent on the different ratio between the parameters α, β, γ.

5.2 Macrostate Description

Whilst description of the microstate was fairly obvious from literature precedents, a simple description of population fluxes at the macro-level proved to be challenging. Indeed, it had to describe the complexity of population immigration/emigration in a the complex international context of a large-scale epidemiological disaster. It also had to capture and describe the behavior of a so far unknown actor on the international scene, namely zombies. The baseline migration of susceptible (in the absence of a zombie outbreak) was supposed to be negligible in comparison to population exodus from zombie fear. We initially wanted to implement an other mass-action based transfer of susceptibles based on S to Z ratios between the states (susceptibles would not migrate to a state where humans would be overwhelmed). This idea might not reasonable since it implies knowledge of the infection statues of the state to be migrated in, which is unlikely to be the case in panic migration scheme. Moreover, this model proved to be very hard to implement for convergences criteria due to the high inter-dependance of the multiple sub-populations. It was therefore abandoned to the profit of a new, rougher model described by equation (4).

$$\Delta S_i^{macro} = \sum_{j \neq i} \left(\nu \langle \Delta S_j \rangle - \nu \langle \Delta S_i \rangle \right) \tag{4}$$

In this interpretation, susceptible emigrate symmetrically to all the other state irrespective of their infection statues (ignorance model). Emigration pressure is generated by calculating the mean of the last ten variation of susceptible in the state. This idea would be that if large death/zombification of susceptible over the recent updates occurred, the remaining population would be more inclined to move away from the "disaster" zone (panic model). Besides, the use of a sliding window to calculate this mean introduces a latent effect of the emigration, consistent with large crowd displacement in a moment of panic.

Description of the zombie migration was more problematic owing to the lack of an unambiguous zombie behavior in the literature. Two descriptions of behavourial types for zombies is usually found. It usually either is in the model of a random-walker, *i.e.* its pattern of motion is completely independent of its surrounding, or it is based on the flesh-craving version, *i.e.* zombies will go where flesh is present. Our initial try was to study a flesh-craving type of zombie. How-

ever, the implementation failed for the same reason as for the susceptible, namely the need for a mass-action description generating a too great dependance between all the sub-population and an impossibility to converge the simulation. We then decided to revise our description of the type of zombie consider in oder to simplify the inter-dependance. We describe a pseudo-flesh-craving zombie where emigration pressure to another state would be dictated by the ratio Z over S, in other terms, the little availability of "local" food would push the zombies to look for some new place. In this description, the knowledge of the other state's statues is unknown to the zombies, which is probably of fair representation of reality since zombies are not intelligent species and cannot get information through news channel etc... Equation (5) gives the mathematical description of this flux.

$$\Delta Z_i^{macro} = \sum_{j \neq i} \left(\eta Z_j \tanh\left(\frac{Z_j}{S_j}\right) - \eta Z_i \tanh\left(\frac{Z_i}{S_i}\right) \right) \tag{5}$$

Note the use of tanh for the ratio in order to avoid infinite emigration of zombies when all the susceptible of a state have either been turned into zombies or been killed, which would obviously be unrealistic. Since tanh can only take values between 0 and 1, we multiplied the equation by the actual zombie population, making the emigration proportional to the current zombie population. Figure 2 describes the relationship between two states, i and k.

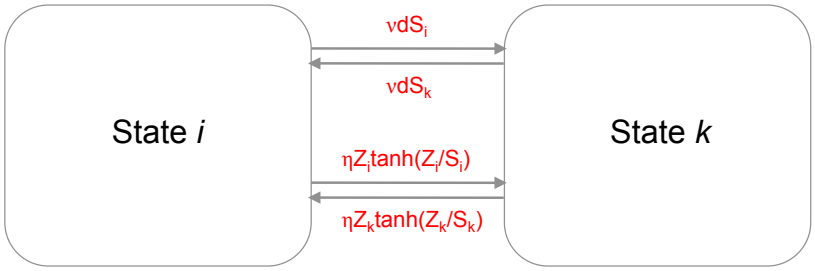

Figure 2: Description of population fluxes at the macrostate level.

5.3 Model with Three States

We decided to simulate the evolution of a zombie population (sparkled in a single state) in a system of three inter-connected state and see how the choice of parameters ν and η (international cooperation) might affect the outcome at the world population level and the effect on the different states. Figure 3 gives a representation of model we implemented. We initially wanted to make those macrostate parameters time-evolving and under a game-theoretical treatment, unfortunately, time-limitation made it impossible to implement (see 8.2 for details).

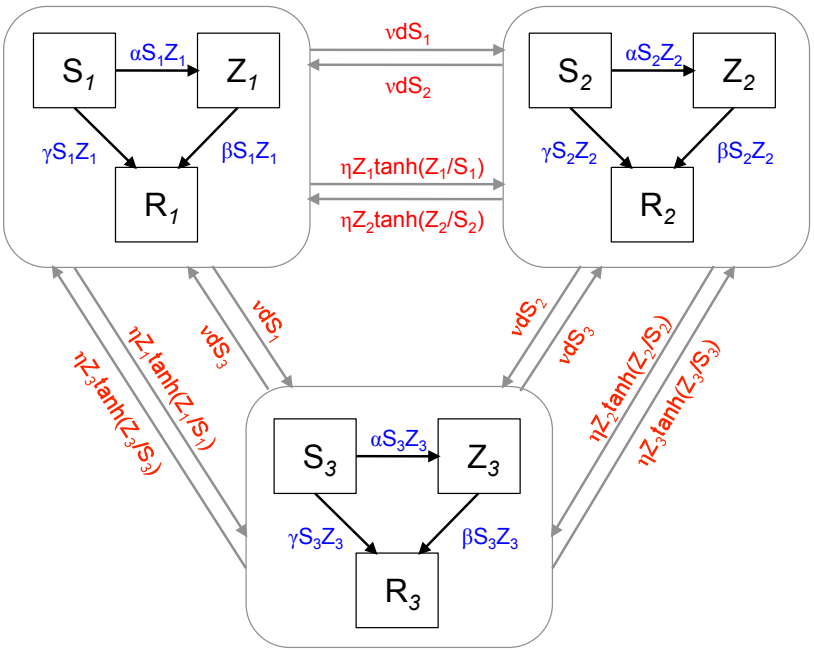

Figure 3: Schematic of our 3-state model including all variables. The parts in blue represent the fluxes at the microstate level while the parts in orange represent the macrostate population fluxes. This schematic represent our simulated system where the interaction of three states (microstates) is modeled

Note that due to the high inter-dependance of all the parameters, it is hard to deconvolute the effect to each parameter on the outcome of the individual simulation. Since quantitative data about parameters of zombification (α), zombie-killing (β) and death related to a zombie outbreak (γ) are absent from the literature it will be necessary to determine those (by parameter sweeping) before going on to vary the macrostate parameters (ν and η) in order to find the interesting regimes of our model located at the phase transitions.

6 Implementation

The implementation in MatLab works on two levels: one taking care of the simulation of the epidemiological model, the other one taking care of the sweeping of the epidemiological parameters. We will discuss the key parts of the implementation as well as some of the interesting details that had to ensure the coherence of the model.

6.1 The Outbreak Function

The outbreak function is the wrapping function of the epidemiological model. It first evaluates the different parameters, initialize the different matrices used throughout the simulation, calls the update function of our model and is finally responsible for the validation of the new data generated after each update cycle. The validation of the data allows to test if it worth continuing the simulation or if it should be shut down. This allows to save computational time. We implemented three exit policies in the simulation loop. First of all, we implemented two obvious safeguards when either the susceptible or the zombie population reaches zero. Continuing the simulation in either case would not make sense since the system already reached its final state. The third one is triggered when the different population evolution becomes too slow, indicating a relative steady state. Even though it might not be a true dynamical equilibrium, very little population variation over a very long time would not be of any significance within the framework of our model (e.g. a single zombie killing thousands of remaining humans over the course of a very long simulation simply because the infection to death rate as a particular value). In order to avoid unphysical convergences, we define the following control:

$$\langle|\Delta S|\rangle < x \;\&\&\; \langle|\Delta Z|\rangle < x \tag{6}$$

Where x is the threshold value. The mean of the absolute variation of susceptibles and zombies on the last 100 steps is calculated using a sliding window and compared to a threshold $x = 0.1$. If the values are smaller than the threshold, we assume a steady state or "quasi-equilibrium" and exit the simulation. Furthermore, a maximum number of step can be defined (default: 10^8 steps) as last exit condition (see 10.1 for details about the code).

6.2 The Update Function

The update function takes care of the evolution of the different populations at each cycle. It applies the different equations of the models and uses safeguards to avoid unphysical results. It first calculates the the variation-to-be of each population of each state based on the current populations. It does this in a sequential manner, first the susceptibles and then the zombies. After the update, it checks

for the actual variations obtained. This is done in order to ensure that variations larger than the actual population size cannot occur. The first constraint is applied during the computation of the flux exiting each states. Due to the structure of the program, negatives population values can arise. Though these populations will be extremely small ($> -10^{-4}$), resulting in small negative exiting flux, they will lead to negative entering flux in the other states. These flux being considered as positive into the next control procedure, negative values have to be avoided.

The second constraint is applied after the computation of all flux. There is the possibility for the negative flux (death, contamination, emigration) to be bigger than the population plus the positive flux (immigration). In such case, we apply a simple algorithm to avoid the negative population:

1. the negative flux are normalized and multiplied by the population plus the positive flux,
2. the incoming flux of the two other states are corrected to fit the values,
3. the validation is applied to the two other states.

This algorithm is applied until all discrepancy is removed. Note that negative values up to 10^{-4} are allowed and that during the first step of the algorithm, in the case where the sum of the population and incoming flux is negative, this value is zeroed. This method allows a relatively fast convergence of the values while retaining physical significance of the model and its implementation. (see 10.2 for details about the code).

6.3 The Sweep Function

The sweep function acts as a wrapper function for the sweeping of each parameters between two values using defined steps. The results of each individual simulation is stored in a dedicated folder. It also assigns a id string to the sweep. This id allows to resume the sweep with the first non completed simulation, this feature was added due to the duration of the sweep with sufficient resolution. (see 10.3 for details about the code).

7 Simulation Results and Discussion

7.1 Population Time-Evolution under Different Regimes

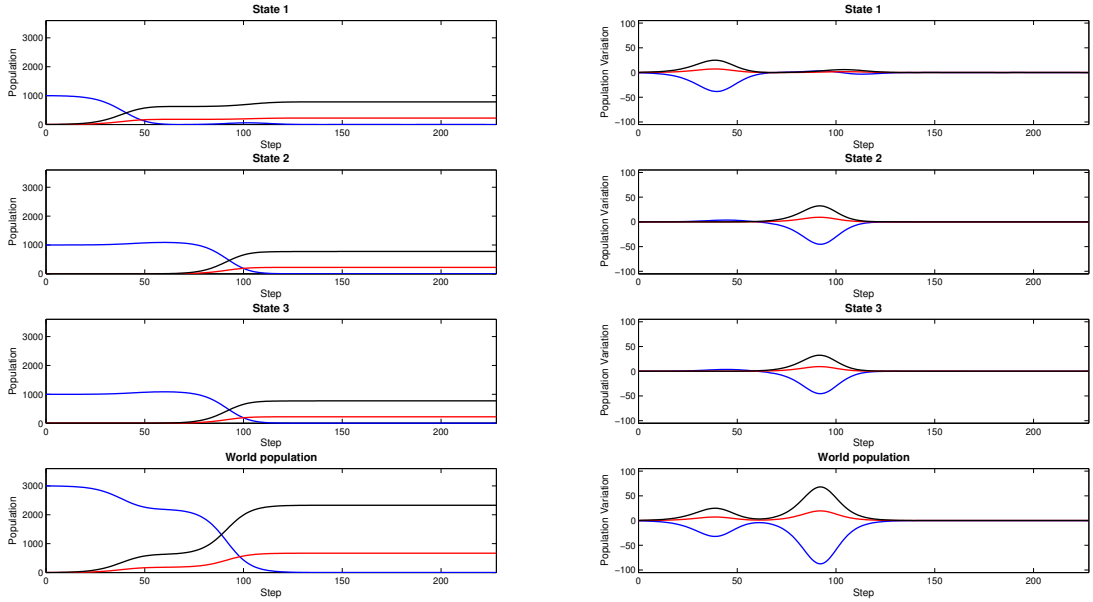

Figure 4: Doomsday scenario. Emergence of zombies in state 1 leads to full contamination of the world's population (blue = susceptibles, red = zombies, black = removed, $\alpha = 1.5 \cdot 10^{-4}, \beta = 5 \cdot 10^{-6}, \gamma = 5 \cdot 10^{-4}, \nu = 0.1, \eta = 1.5 \cdot 10^{-4}$).

7.2 Phase Transition at the Microstate

7.3 Phase Transition at the Macrostate Level

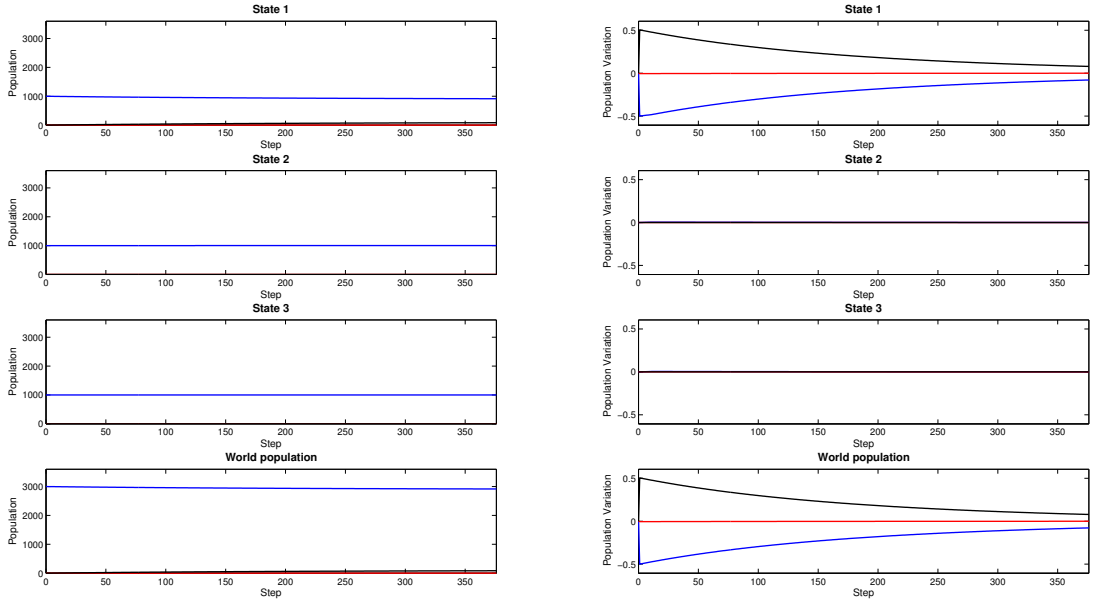

Figure 5: Survival of humanity by zombie eradication. As soon as zombies appear in state 1, they get killed until the outbreak is contained (blue = susceptibles, red = zombies, black = removed, $\alpha = 1.5 \cdot 10^{-8}, \beta = 5 \cdot 10^{-6}, \gamma = 5 \cdot 10^{-4}, \nu = 0.01, \eta = 1.5 \cdot 10^{-4}$).

8 Summary and Outlook

8.1 Regimes that Allowed the Survival of Humanity

8.2 Further Work: Inter-State fluxes under Game-Theoretical Treatment

For each state (microstate), we defined a SZR model that evaluates the evolution of the different populations under studies: susceptibles (S), zombies (Z) and removed (R). Epidemological-like (mass-action) transfer of populations between the states (at the macrostate level, *i.e.* international level) also occurs as defined above and models the refugees and zombie transfer across states. Those transfers are dependent of the parameter ν for susceptible transfer and parameter η for the zombies. Within the scope of this semester work, we decided to simulate the different paradigms of inter national politics by simply fixing values of both η and ν

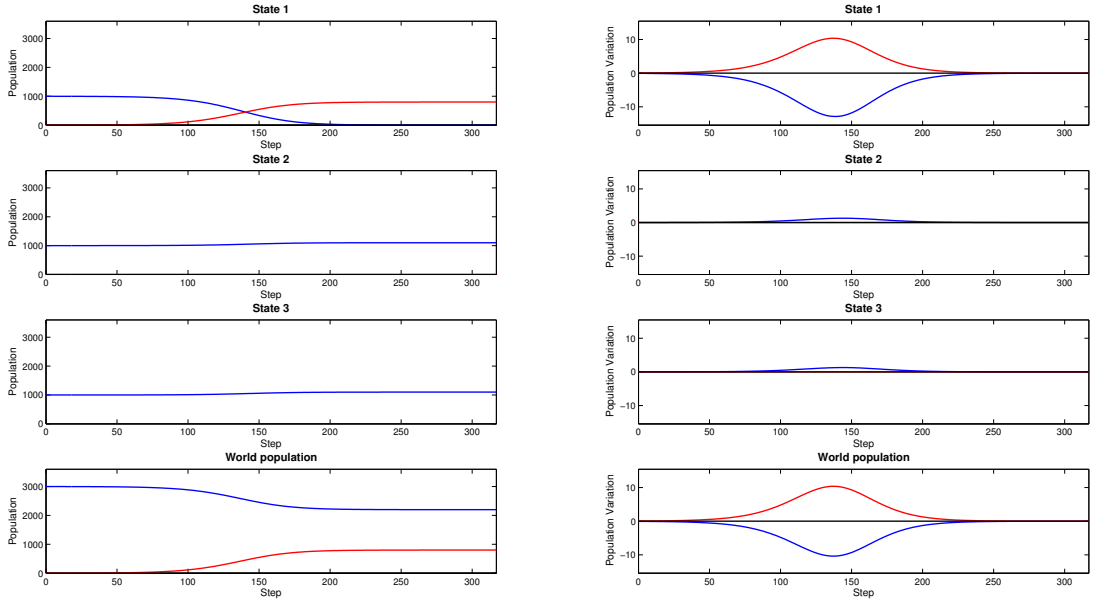

Figure 6: Survival of humanity and emergence of a zombie state. Emergence of zombies in state 1 leads to the exodus of the remaining population to states 2 and 3. State 1 then becomes fully infested of zombies and the remaining of the world's population is in state 2 and 3 (blue = susceptibles, red = zombies, black = removed, $\alpha = 5 \cdot 10^{-5}, \beta = 5 \cdot 10^{-10}, \gamma = 5 \cdot 10^{-10}, \nu = 0.1, \eta = 0$).

at the beginning of each simulation and observe the effect on the outcome. Both the relative and absolute values of those parameters are supposed to represent the paradigm under consideration (*e.g.* a very small value of ν would be representative of neoconservatism within the framework of our model since it drastically limits the possibility of immigration of refugees into a given state). Of course, such a treatment is very static and does not take into account the likely variation over time of international cooperation elements (change in immigration politics, military action on foreign soil, etc...). This change in foreign policies is even likely to be more pronounced for the case of a zombie outbreak since such an event would be unprecedented and decision-makers would have a hard time figuring out in such a short time what position to adopt.

Therefore, a nice way to make the macrostate parameters (ν, η) dynamic in order to represent such changes would be to make them functions of a game-theoretical treatment. In such a model, population fluxes would still be treated within the framework of the current model, with the only variation that the macrostate parameters would be time-evolving. Redefininig ν and η would be under a cost-hypothesis model as defined in game-theory and then the cost-hypothesis

themselves would become fixed parameters over the course of a simulation, defined in order to represent the different paradigms of international relationships. Accordingly, macrostate fluxes could vary over the course of the simulation, possibly having an effect on the outcome but those variation would be anisotropic and depend on foreign policies bias. The apparition of zombies in one state would start the game. Each state would then evolve on the domestic and international level. The domestic level will follow a standard SZR model, whereas the international level will introduce exchange in the population of suceptibles and zombies between the states. These exchanges will be influenced by the state decisions on foreign policies such as humanitarian or military actions determined by our game theory framework. The Game-Theoretical framework is defined as the possibility of undertaking military action of foreign soil (exporting S) or changing the refugee politics by modifying the mu parameter (allowing more S to come into one's state, and with a collateral cost of having more zombies crossing as well). Each action will be defined with a specific payoff, which in turn will depend on the international cooperation system under scrutiny. For simplicity, models should be treated homogenously, i.e. all states should adopt the same international politics paradigm. Finally, a "feedback" loop on the payoff depending on the success of a previously undertaken action (positive or negative affectation of the payoffs) could be introduced. This effect could be a modeling of the psychological effect of a successful or unsuccessful action on future action, for example the effectiveness of a military attack. This effect will be made as to converge after a certain time to model the wearing out of the psychological effect over time. The system will be implemented as a step-based update. This implies the ignorance of the actors (the states) of the action of the other actors. This rationalisation comes as the idea that the outbreak would occur over a short period of time, forcing for rapid decision-making and therefore not allow a reaction-based decision-making process.

$$\textit{Microstate treatment} \Rightarrow \textit{SZR model}$$

$$\textit{Macrostate treatment} \Rightarrow \begin{cases} \Delta S_i^{macro} = \sum_{j \neq i} \left(\nu \langle \Delta S_j \rangle - \nu \langle \Delta S_i \rangle \right) \\ \Delta Z_i^{macro} = \sum_{j \neq i} \left(\eta Z_j \tanh\left(\frac{Z_j}{S_j}\right) - \eta Z_i \tanh\left(\frac{Z_i}{S_i}\right) \right) \end{cases}$$

$$\textit{where } \nu = f(GT) \ , \ \eta = f(GT)$$

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10 Appendix

10.1 outbreak.m

```
function dump = outbreak( varargin )

    %
    % OUTBREAK runs the simulation of a zombie oubreak in a 3 states
    %   system.
    %
    %   DUMP = OUTBREAK( ARGS )
    %       ARGS are name-value pairs. Possible arguments are :
    %           params: structure containing rate parameters (alpha, beta,
    %                   gamma, eta, nu) as arrays (3x1 for alpha, beta and
    %                   gamma or 3x2 for eta and nu).
    %           paramfile: path to a mat-file storing the parameter
    %                   structure.
    %           zombies: 3x1 array containing the number of initial zombie
    %                   per state (default: 0, 0, 0).
    %           population: 3x1 array containing the number of initial
    %                   susceptible population per state (default: 1000,
    %                   1000, 1000).
    %           steps: maximal number of steps for the simulation (default:
    %                   1e8).
    %           silent:  level of output for the simulation:
    %                           0 = all outputs
    %                           1 = all outputs excepts graphs
    %                           2 = no outputs
    %           dslength: size of the sliding window used for the
    %                   calculation of the mean of dS (used as weight
    %                   factor for the inter-state susceptible transfer).
    %
    %       DUMP is a structure containing the following fields:
    %           S: 4xn array containing the evolution of the susceptible
    %              population.
    %           dS: 4xn array containing the evolution of the susceptible
    %              population's variation.
    %           Z: 4xn array containing the evolution of the zombie
    %              population.
    %           dZ: 4xn array containing the evolution of the zombie
    %              population's variation.
    %           R: 4xn array containing the evolution of the removed
    %              population.
    %           dR: 4xn array containing the evolution of the removed
    %              population's variation.
    %           step: number of steps performed.
    %           alpha, beta, gamma, eta, nu: transfer rate parameters.
    %           time: simulation time.
    %


```

```
48      %% Variable initialization
49      silent = 0;
50      dslength = 10;
51      maxSteps = 1e8;
52      rates = struct( 'alpha', [ 0.00095 ; 0.00095 ; 0.00095 ], 'beta', ...
            [0.00025 ; 0.00025 ; 0.00025 ], 'gamma', [ 0.00005 ; 0.00005 ; ...
            0.00005 ], 'mu', [ 0.00000005 , 0.00000005 ; 0.00000005 , ...
            0.00000005 ; 0.00000005 , 0.00000005 ], 'nu', [ 0.00000005 , ...
            0.00000005 ; 0.00000005 , 0.00000005 ; 0.00000005 , 0.00000005 ], ...
            'eta', [ 0.00000005 , 0.00000005 ; 0.00000005 , 0.00000005 ; ...
            0.00000005 , 0.00000005 ] );
53      zombies = [ 0 ; 0 ; 0 ; 0 ];
54      populations = [ 1e3 ; 1e3 ; 1e3 ; 3e3 ];
55
56      % Structure storing populations and populations variation
57      states = struct( 'pop', zeros( 4, 3 ), 'dpop', zeros( 4, 3 ) );
58
59      % See exit condition at the end of the loop.
60      exitThreshold = 1e-1;
61
62      p = [ 'alpha' ; 'beta ' ; 'gamma' ; 'eta  ' ; 'mu   ' ; 'nu   ' ];
63      reverseStr = '';
64
65
66      %% Argument parsing
67      % All argument are optional.
68      for i = 1:2:nargin
69
70          % Argument are passed in name-value pairs. If i + 1 does not exist,
71          % then the value is absent and the program stops.
72          if( nargin < i + 1 )
73
74              error( [ 'Missing argument for parameter "' varargin{ i } ...
                    '".' ] );
75          end
76
77          switch varargin{ i }
78
79              % params excepts a structure containing the rates values for the
80              % epidemiologic model. Not all fields have to be present. If a
81              % field is missing, the default value is used.
82              case 'params'
83                  for j = 1:6
84
85                      if isfield( varargin{ i + 1 }, strtrim( p( j, : ) ) )
86
87                          rates.( strtrim( p( j, : ) ) ) = varargin{ i + 1 ...
                                }.( strtrim( p( j, : ) ) );
88                      end
89                  end
90
91              % paramfile excepts the path to a mat-file containing a
92              % parameter structure (refer to parms).
```

```
93              case 'paramfile'
94                  pfile = load( varargin{ i + 1 }, '-mat' );
95                  for j = 1:6
96
97                      if isfield( pfile{ i + 1 }, strtrim( p( j, : ) ) )
98
99                          rates.( strtrim( p( j ) ) ) = pfile{ i + 1 }.( ...
                                strtrim( p( j ) ) );
100                     end
101                 end
102
103             % zombies excepts an array containing the initial zombie
104             % populations in each state.
105             case 'zombies'
106                 temp = varargin{ i + 1 };
107                 for j = 1:3
108
109                     zombies( j ) = temp( j );
110                     zombies( 4 ) = zombies( 4 ) + temp( j );
111                 end
112
113             % zombies excepts an array containing the initial susceptible
114             % population in each state.
115             case 'population'
116                 temp = varargin{ i + 1 };
117                 populations( 4 ) = 0;
118                 for j = 1:3
119
120                     populations( j ) = temp( j );
121                     populations( 4 ) = populations( 4 ) + populations( j );
122                 end
123
124             % steps excepts the maximum number of steps allowed for the
125             % simulation.
126             case 'steps'
127                 maxSteps = varargin{ i + 1 };
128
129             % silent except a int defining the output type :
130             % 2 : No output
131             % 1 : All outputs but the graphs
132             % 0 : Normal output
133             case 'silent'
134                 silent = varargin{ i + 1 };
135
136             % dslength excepts the size of the sliding window used for the
137             % calculation of the mean of dS (used as weight factor for the
138             % inter-state susceptible transfer).
139             case 'dslength'
140                 dslength = varargin{ i + 1 };
141
142             otherwise
143                 error( [ 'Unknown parameter "', varargin{ i }, '".' ] );
144         end
```

```
145     end
146
147
148
149     %% Initialization of simulation
150
151     % Setting the initial populations.
152     states.pop( :, 1 ) = populations;
153     states.pop( :, 2 ) = zombies;
154
155     % Initialization of the sliding window for inter-state population
156     % transfer.
157     dshistory = zeros( 3, dslength );
158
159     % Setting up the initial dump structure. Large matrices are used to
160     % avoid the need of dynamic memory allocation to improve code
161     % efficency.
162     d = zeros( 4, max( 200, .0001 * maxSteps ) );
163     dump = struct( 'S', d, 'dS', d, 'Z', d, 'dZ', d, 'R', d, 'dR', d, ...
            'step', 1, 'alpha', mean( rates.alpha ), 'beta', mean( rates.beta ...
            ), 'gamma', mean( rates.gamma ), 'eta', mean( mean( rates.eta ) ...
            ), 'nu', mean( mean( rates.nu ) ), 'time', 0 );
164     dump.S( :, 1 ) = states.pop( 1:4, 1 );
165     dump.Z( :, 1 ) = states.pop( 1:4, 2 );
166     dump.R( :, 1 ) = states.pop( 1:4, 3 );
167
168     % Initialize timer.
169     tic;
170
171     %% Update loop.
172     for i = 1:maxSteps
173
174         % Output of step # and current population count.
175         if silent < 2
176
177             msg = sprintf('Processing step %d... Human population %d, ...
                    Zombie population %d\n', i, dump.S( 4, dump.step ), ...
                    dump.Z( 4, dump.step ) );
178             fprintf([ reverseStr, msg]);
179
180             reverseStr = repmat(sprintf('\b'), 1, length(msg));
181         end
182
183         % Update function (see update.m for details).
184         [ states, rates, dshistory ] = update( states, rates, dshistory );
185
186         % Dumping of the latest state of the simulation.
187         % Update of the step number.
188         dump.step = i + 1;
189
190         % Dumping of the various population and population variation.
191         dump.S( :, dump.step ) = states.pop( :, 1 );
192         dump.dS( :, dump.step ) = states.dpop( :, 1 );
```

```
193         dump.Z( :, dump.step ) = states.pop( :, 2 );
194         dump.dZ( :, dump.step ) = states.dpop( :, 2 );
195         dump.R( :, dump.step ) = states.pop( :, 3 );
196         dump.dR( :, dump.step ) = states.dpop( :, 3 );
197 
198         % Stops simulation if S is equal to 0.
199         if states.pop( 4, 1 ) == 0
200 
201             if silent < 2
202 
203                 disp( ' ' );
204                 disp( 'End of the human race.' );
205             end
206             break;
207         end
208 
209         % Stops simulation if Z is equal to 0.
210         if states.pop( 4, 2 ) == 0
211 
212             if silent < 2
213 
214                 disp( ' ' );
215                 disp( 'Humanity survived.' );
216             end
217             break;
218         end
219 
220         % Stops simulation if the average fluctuation of both dS and dZ
221         % over a sliding window is lower than the threshold (simulation
222         % reached equilibrium in the limit of our time frame).
223         if i > 100 && mean( abs( dump.dS( 4, ( i - 100 ):i ) ) ) < ...
                exitThreshold && mean( abs( dump.dZ( 4, ( i - 100 ):i ) ) ) < ...
                exitThreshold
224 
225 
226             if silent < 2
227 
228                 disp( ' ' );
229                 disp( 'Equilibrium reached.' );
230             end
231             break;
232         end
233     end
234 
235     % Store simulation time.
236     dump.time = toc;
237 
238     % Resizing of the dump matrices before plotting.
239     dump.S = dump.S( :, 1:dump.step );
240     dump.dS = dump.dS( :, 1:dump.step );
241     dump.Z = dump.Z( :, 1:dump.step );
242     dump.dZ = dump.dZ( :, 1:dump.step );
243     dump.R = dump.R( :, 1:dump.step );
```

```
244     dump.dR = dump.dR( :, 1:dump.step );
245 
246     if ¬silent
247 
248         % Plot the simulation history (see plotResults.m for details).
249         plotResults( dump );
250     end
251 end
```

10.2 update.m

```
1     function [ states, rates, dshistory ] = update( states, rates, ...
          dshistory )
2 
3     %
4     % UPDATE compute the population evolution for one step of the
5     %   epidemiological model.
6     %
7     % [ STATES, RATES, DSHISTORY, DUMP ] = UPDATE( STATES, RATES, ...
          DSHISTORY, DUMP )
8     %       STATES is the structure storing the populations and population
9     %           variations as defined in the OUTBREAK function.
10    %       RATES is the structure containing the epidemiological transfer
11    %           rates as defined in the OUTBREAK function.
12    %       DSHISTORY is the array containing the sliding window for
13    %           inter-state susceptible transfer.
14    %
15 
16    %% Initialization of parameters :
17 
18    s = states.pop( 1:3, 1 );
19    z = states.pop( 1:3, 2 );
20 
21    alpha = rates.alpha;
22    beta = rates.beta;
23    gamma = rates.gamma;
24    nu = rates.nu;
25    eta = rates.eta;
26 
27    % Permutation matrix used to set the input inter-state flux from the
28    % inter-state output flux to avoid redundant calculations. See flux
29    % update.
30    permutation = [ 0 0 1 ; 1 0 0 ; 0 1 0 ];
31 
32    % Matrix containing the coordinates of the input inter-state flux from
33    % each state (line) to the other two (block of two colons). See flux
34    % correction.
35    iFluxCoor = [ 2 2 3 1 ; 3 2 1 1 ; 1 2 2 1 ];
36 
```

```
37      %% Update of the susceptible population.
38
39      % Depletion of the susceptible population goes according to :
40      %
41      %   S  -> Z  = - alpha * s * z
42      %   S  -> R  = - gamma * s * z
43      %   S1 -> S2 = nu * dSmean
44      %   S1 -> S3 = nu * dSmean
45      %
46      % Immigration is the only source of susceptible population.
47      %
48      % dS is a 3x6 matrix containing for each state :
49      %
50      %   Row 1 : n( S  -> Z  )
51      %   Row 2 : n( S  -> R  )
52      %   Row 3 : n( S1 -> S2 ) | n( S2 -> S3 ) | n( S3 -> S1 )
53      %   Row 4 : n( S1 -> S3 ) | n( S2 -> S1 ) | n( S3 -> S2 )
54      %   Row 5 : n( S2 -> S1 ) | n( S3 -> S2 ) | n( S1 -> S3 )
55      %   Row 6 : n( S3 -> S1 ) | n( S1 -> S2 ) | n( S2 -> S3 )
56      %
57      % The sum of each line gives the population variation for the state.
58
59      % Mean variation of the susceptible in the sliding window.
60      dsmean = min( mean( dshistory, 2 ), 0 );
61
62      dS = zeros( 3, 6 );
63      dS( :, 1 ) = min( - alpha .* s .* z, 0 );
64      dS( :, 2 ) = min( - gamma .* s .* z, 0 );
65      dS( :, 3 ) = nu( :, 1 ) .* dsmean;
66      dS( :, 4 ) = nu( :, 2 ) .* dsmean;
67      dS( :, 5 ) = - permutation * permutation * dS( :, 4 );
68      dS( :, 6 ) = - permutation * dS( :, 3 );
69
70
71      % There is the possibility that any of the |-dSi| is larger than the
72      % actual population in the state (negative population fluctuation larger
73      % than the actual population). If so, a correction is applied to
74      % avoid negative population.
75      % As long as any state falls into this category:
76      while sum( sum( dS( :, 1:4 ), 2 ) + s + sum( dS( :, 5:6 ), 2 ) < ...
            -1e-6 )
77
78          for i = 1:3
79
80              % Correct the state that falls into this category:
81              if( sum( dS( i, 1:4 ), 2 ) + ( s( i ) + sum( dS( i, 5:6 ), 2 ...
                    ) ) < -1e-6 )
82
83                  % Population is considered equal to the sum of previous
84                  % population and the inter-state input transfer. The
85                  % negative contributions to the dS are reduced so that it
86                  % equals this population.
87                  dS( i, 1:4 ) = dS( i, 1:4 ) / abs( sum( dS( i, 1:4 ) ) ) ...
```

```
                    * max( sum( dS( i, 5:6 ) ) + s( i ), 0 );
88
89                  % The effect of the correction on the output flux is
90                  % applied to the other state input flux.
91                  dS( iFluxCoor( i, 1 ), 4 + iFluxCoor( i, 2 ) ) = - dS( i, ...
                        3 );
92                  dS( iFluxCoor( i, 3 ), 4 + iFluxCoor( i, 4 ) ) = - dS( i, ...
                        4 );
93              end
94          end
95          % Note that the output flux can only be lower than the original flux.
96          % Therefore, the input flux for the two other states can only
97          % be smaller than before the correction. Accordingly, the
98          % process has to be repeated until all states have a population
99          % differential such that it doesn't lead to a negative population.
100     end
101
102     %% Update of the zombie population.
103
104     % Depletion of the zombie populations goes according to :
105     %
106     %   S  -> Z  = alpha * s * z
107     %   Z  -> R  = - beta * s * z
108     %   Z1 -> Z2 = - eta * z * tanh( z / s )
109     %   Z1 -> Z3 = - eta * z * tanh( z / s )
110     %
111     % Conversion from susceptible and "immigration" are the two sources of
112     % zombies.
113     %
114     % dZ is a 3x6 matrix containing for each states :
115     %
116     %   Row 1 : n( S  -> Z  )
117     %   Row 2 : n( Z  -> R  )
118     %   Row 3 : n( Z1 -> Z2 ) | n( Z2 -> Z3 ) | n( Z3 -> Z1 )
119     %   Row 4 : n( Z1 -> Z3 ) | n( Z2 -> Z1 ) | n( Z3 -> Z2 )
120     %   Row 5 : n( Z2 -> Z1 ) | n( Z3 -> Z2 ) | n( Z1 -> Z3 )
121     %   Row 6 : n( Z3 -> Z1 ) | n( Z1 -> Z2 ) | n( Z2 -> Z3 )
122     %
123
124     dZ = zeros( 3, 6 );
125     dZ( :, 1 ) = max( - dS( :, 1 ), 0 );
126     dZ( :, 2 ) = min( - beta .* s .* z, 0 );
127     dZ( :, 3 ) = min( - eta( :, 1 ) .* z .* tanh( z ./ s ), 0 );
128     dZ( :, 4 ) = min( - eta( :, 2 ) .* z .* tanh( z ./ s ), 0 );
129     dZ( :, 5 ) = - permutation * permutation * dZ( :, 4 );
130     dZ( :, 6 ) = - permutation * dZ( :, 3 );
131
132
133     % The same control procedure as for the susceptible population is
134     % applied to avoid negative population of zombies. See susceptible
135     % correction for details on the procedure.
136     while sum( ( dZ( :, 1 ) + sum( dZ( :, 5:6 ), 2 ) + z ) + sum( dZ( :, ...
            2:4 ), 2 ) < - 1e-6 )
```

```
137
138         for i = 1:3
139
140             if( ( dZ( i, 1 ) + sum( dZ( i, 5:6 ), 2 ) + z( i ) ) + sum( ...
                    dZ( i, 2:4 ) ) < - 1e-6 )
141
142                 dZ( i, 2:4 ) = dZ( i, 2:4 ) * max( sum( dZ( i, 5:6 ) ) + ...
                        dZ( i, 1 ) + z( i ), 0 ) / abs( sum( dZ( i, 2:4 ) ) );
143
144                 dZ( iFluxCoor( i, 1 ), 4 + iFluxCoor( i, 2 ) ) = - dZ( i, ...
                        3 );
145                 dZ( iFluxCoor( i, 3 ), 4 + iFluxCoor( i, 4 ) ) = - dZ( i, ...
                        4 );
146             end
147         end
148     end
149
150     %% Update of the removed population
151
152     % Variation in the removed population is strictly positive and comes
153     % from both the susceptible and zombie population:
154     %
155     %   S  -> R  = alpha * s * z
156     %   Z  -> R  = beta * s * z
157     %
158     % The value obtained for dS and dZ are used:
159
160     dR = - [ dS( :, 2 ), dZ( :, 2 ) ];
161
162     %% Update each population
163
164     % Compilation of the populations variations
165     states.dpop( 1:3, : ) = [ sum( dS, 2 ), sum( dZ, 2 ), sum( dR, 2 ) ];
166
167     % Update of the sliding window matrix for the susceptible.
168     [ ~, dsSize ] = size( dshistory );
169     dshistory( :, 2:dsSize ) = dshistory( :, 1:( dsSize - 1 ) );
170     dshistory( :, 1 ) = states.dpop( 1:3, 1 );
171
172     % Update of the current populations
173     states.pop( 1:3, : ) = states.pop( 1:3, : ) + states.dpop( 1:3, : );
174
175     % Update of the total populations
176     states.pop( 4, : ) = sum( states.pop( 1:3, : ) );
177     states.dpop( 4, : ) = sum( states.dpop( 1:3, : ) );
178 end
```

10.3 sweep.m

```
1 function params = sweep( params )
```

```
2
3       %
4       % SWEEP allows to run multiple simulation sequentially varying the
5       %   parameters in a defined range with defined steps.
6       %
7       %   PARAMS = SWEEP( PARAMS )
8       %       PARAMS is a structure storing for each parameter alpha, beta,
9       %           gamma, eta and nu a 1x3 matrix with the minimal value, the
10      %           step and the maximal value for the parameter.
11      %           The return value also contains the ID of the sweep as well
12      %           as the number of simulations done.
13      %
14      %   PARAMS = SWEEP( ID )
15      %       ID is the ID string of a previous sweep. The sweep will start
16      %           over where the sweep stopped the last time.
17      %
18
19
20      %% Parsing of the argument
21      switch class( params )
22
23          case 'struct'
24              % If the parameter set has never been employed, the restart
25              % fields are created.
26              if length( params.alpha ) == 3
27
28                  params.alpha( 4 ) = params.alpha( 1 );
29                  params.beta( 4 ) = params.beta( 1 );
30                  params.gamma( 4 ) = params.gamma( 1 );
31                  params.eta( 4 ) = params.eta( 1 );
32                  params.nu( 4 ) = params.nu( 1 );
33
34              else
35                  if ¬isfield( params, 'step' )
36
37                      params.alpha( 4 ) = params.alpha( 1 );
38                      params.beta( 4 ) = params.beta( 1 );
39                      params.gamma( 4 ) = params.gamma( 1 );
40                      params.eta( 4 ) = params.eta( 1 );
41                      params.nu( 4 ) = params.nu( 1 );
42                  end
43              end
44
45              % Initialization of the number of simulation ran
46              if ¬isfield( params, 'step' )
47
48                  params.step = 1;
49              end
50
51              % If an ID is assigned, the restart file is loaded, otherwise,
52              % an ID is created as well as a folder for the simulation
53              % results.
54              if isfield( params, 'id' )
```

```
55
56                  if exist( [ params.id '.restart' ], 'file' )
57
58                      params = load( [ id '.restart' ], '-mat' );
59                  end
60              else
61
62                  params.id = num2hex( ceil( 1e20*rand ) );
63                  mkdir( [ '../results/' params.id ] );
64              end
65
66          case 'char'
67              % If an ID is provided, the restart file is loaded.
68              params = load( [ params '.restart' ], '-mat' );
69      end
70
71      %% Display of the simulation details
72      disp( [ 'Id : ' params.id '.' ] );
73      disp( 'Use the id as parameter to resume the sweep anytime.' );
74      disp( '--------------------------------------------------------------------' );
75      disp( params );
76      disp( '--------------------------------------------------------------------' );
77
78      %% Variables initialization
79      cParams = struct( 'alpha', zeros( 3, 1 ), 'beta', zeros( 3, 1 ), ...
            'gamma', zeros( 3, 1 ), 'eta', zeros( 3, 2 ), 'nu', zeros( 3, 2 ) );
80      output = struct( 'S', 0, 'Z', 0, 'R', 0, 'alpha', 0, 'beta', 0, ...
            'gamma', 0, 'eta', 0, 'nu', 0, 'e', 0, 'step', 0 );
81      reverseStr = '';
82      step = 1;
83
84      %% Parameter sweeping
85      for nu = params.nu( 1 ):params.nu( 2 ):params.nu( 3 )
86
87          for eta = params.eta( 1 ):params.eta( 2 ):params.eta( 3 )
88
89              for gamma = params.gamma( 1 ):params.gamma( 2 ):params.gamma( ...
                    3 )
90
91
92                  for beta = params.beta( 1 ):params.beta( 2 ):params.beta( ...
                        3 )
93
94                      for alpha = params.alpha( 1 ):params.alpha( 2 ...
                            ):params.alpha( 3 )
95
96                          % If the current step number is larger than the
97                          % step number in the parameter file, the simulation
98                          % has already be done and can be skipped.
99                          if step > params.step
100
101                             % Update of the current process status.
102                             msg = sprintf('Step %d.\nAlpha = %f, Beta = ...
```

```
                        %f,\nGamma = %f, Eta = %f, Nu = %f.', ...
                        step, alpha, beta, gamma, eta, nu );
103                     fprintf([ reverseStr, msg]);
104                     reverseStr = repmat(sprintf('\b'), 1, ...
                            length(msg));
105
106                     % Preparation of the simulation parameters
107                     cParams.alpha = alpha * ones( 3, 1 );
108                     cParams.beta = beta * ones( 3, 1 );
109                     cParams.gamma = gamma * ones( 3, 1 );
110                     cParams.eta = eta * ones( 3, 2 );
111                     cParams.nu = nu * ones( 3, 2 );
112
113                     % Simulation
114                     dump = outbreak( 'silent', 2, 'params', ...
                            cParams, 'zombies', [ 1 0 0 ] );
115
116                     % Preparation of the output structure
117                     output.S = dump.S;
118                     output.R = dump.R;
119                     output.Z = dump.Z;
120                     output.step = dump.step;
121                     output.alpha = alpha;
122                     output.beta = beta;
123                     output.gamma = gamma;
124                     output.eta = eta;
125                     output.nu = nu;
126
127                     % Saving of the results of the simulation
128                     save( [ '../results/' params.id '/output.' ...
                            int2str( params.step ) '.mat' ], ...
                            '-struct', 'output' );
129
130                     % Update of the parameter structure and saving
131                     % of the restart file.
132                     params.step = params.step + 1;
133                     params.alpha( 4 ) = alpha + params.alpha( 2 );
134                     save( [ params.id '.restart' ], '-struct', ...
                            'params' );
135                 end
136
137                 % Update of the current step.
138                 step = step + 1;
139             end
140
141             % Parameters update
142             params.alpha( 4 ) = params.alpha( 1 );
143
144             params.beta( 4 ) = beta + params.beta( 2 );
145         end
146
147         params.beta( 4 ) = params.beta( 1 );
148
```

```
149                 params.gamma( 4 ) = gamma + params.gamma( 2 );
150             end
151
152             params.gamma( 4 ) = params.gamma( 1 );
153
154             params.eta( 4 ) = eta + params.eta( 2 );
155         end
156
157         params.eta( 4 ) = params.eta( 1 );
158
159         params.nu( 4 ) = nu + params.nu( 2 );
160     end
161 end
```