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Lecture with Computer Exercises:  
Modelling and Simulating Social Systems with MATLAB

Project Report

**Zombie Outbreak:  
The Effect of Inter-State Cooperation on  
the Survival of Humanity**

by

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## 1 Abstract

This part needs updating once the rest of the report will be finished

Investigation of the application of the SIR model to a zombie outbreak has already been studied, raising the fear of dark days for humanity. However, we would like to deepen this investigation to a multi-state system to see how interactions between subpopulation may brighten the future of the human race. Moreover, we are interested in seeing to what extent the different paradigms of international politics, Realpolitik, Liberalism and Neoconservatism as defined by Daniel W. Drezner in Theories of International Politics and zombies may lead to different outcomes.

## 2 Individual contributions

M.G.M. and B.I.M.W. formulated the question in mathematical terms and discussed the implementation in MATLAB. M.G.M. wrote the code. M.G.M. and B.I.M.W. analysed and discussed the results and B.I.M.W. wrote the report.

## 3 Acknowledgments

We wish to thank Karsten Donnay and Stefano Balietti for their support in our work, fruitful discussions and open-mindedness to accept such a project. We also would like to thank the Chair of Sociology for the computational support provided with simulation time on the ETH cluster Brutus.

## 4 Introduction and Motivations

### 4.1 Zombie Outbreak

While models of international relationship have already been studied under different pressure components, the effect of a zombie outbreak on international collaboration and equilibrium is a question that has been underestimated and was never addressed to the best of our knowledge. It is remarkable that the effect of such an intense event has not been looked at, although the fear of zombies and the threat they represent is vivid for many of us as reflected by the importance of the zombie culture. Zombies, compared to other unnatural creatures such as vampires or aliens, have the very peculiar property not to be a minority inside the human civilization but rather to be in a way a part of it, just not quite as it was, i.e. zombified. Accordingly, zombies cause a much more deep-rooted fear as they not only threaten our lives but also our very sense of identity as it questions our notion of what humanity is. The psychological effect of such a non-standard threat as well as the repercussion on the behaviour of large population systems such as states should be far from trivial. Accordingly, we decided to simulate an inter-state collaboration models in order to see the outcome on a large scale of such an extreme event. While some people might question the validity of such a study (no zombie has been observed so far), we think that the applicability of such a reasoning could extend to a more probable large-scale epidemiological disaster or simply, give a line of reasoning to cope with what former U.S. Secretary of Defense Donald Rumsfeld referred as the "unknown unknowns" of international security. Zombies might not be real, but the threat and stress they could impose on current world politics is.

### 4.2 Epidemiological Studies

Epidemiological models have been well studied to see the evolution dynamics and spread of a disease in a population [REFERENCE](#). They are based on a mass action model where interaction between susceptibles and infected together with an infection constant define the rate of infection. In the simplest case of such models, infected people can move to the immunized category also following a mass action law. These models have been shown to work in numerous cases and when corrections are added [REFERENCE](#), they can very well represent the evolution and spread of an infectious disease in a population. However, an epidemiological model of communicating population has not been tested to the best of our knowledge.

### 4.3 Zombies: a Definition

The origin of the word "zombie" as well as its initial meaning is quite remote from modern pop culture description. The word itself is said to have originated in the voodoo language in the Caribbean [1]. The original description is that of a ritual where the wizard of a tribe would start controlling another person. The fact that this person doesn't have a soul anymore and that it is no longer under the control of another human being, i.e. it has no longer a free-will, makes it a zombie [1]. Some studies suggest that those "zombified" members of the tribe were actually administered a cocktail of two natural drugs, one being a neurotoxin and the other a hallucinogenic drug [REFERENCE](#). In fact, the neurotoxin damages the brain and turns the person into a vegetative state.

Zombies in popular culture differ a lot from this original etymological definition. The canon of the zombie literature have had numerous description and hypothesis on how they may emerge in a human population, as well as what might characterize them. Since Romero's "night of the living dead" [REFERENCE](#), where zombies were said to have raised from some pseudo-magical event, the depiction and origin of zombies has considerably evolved. Recent zombie stories usually describe the genesis of flesh-eating monsters in an epidemiological sense, typically some sort of virus. Recent examples in the zombie culture are numerous and include (non-exhaustively) "Resident Evil", "28 Days Later". For simplicity, we will treat the emergence and zombification events as linked to an infectious disease of some sort, allowing the treatment of the problem with a modified epidemiological model. The big difference with a standard infectious disease where people usually get immunized, being transformed into a zombie is a one way process. The only way out is death and therefore, humans (susceptibles) can only decrease in our system.

### 4.4 Popular Beliefs in the Event of the Zombie Outbreak

It is interesting to notice that most zombie canon predict a very bleak outcome concerning the fate of humanity. Indeed, most movies/film describe an almost total disappearance of humans and the few survivors are rarely in a position that seem to be about to brighten up. While some might argue that the disappearance of humanity might not be such a regretful event and might actually benefit our planet [REFERENCE](#), we decided to see if the usual outcome and fate of the human race in case of a zombie outbreak might differ from those classical scenarios, and if so, under which set of particular conditions.

Mathematical modeling of a zombie outbreak in a single population has previously been simulated [2] but showed very little hope for humans in the case of such an unlikely event. The primary reason for the annihilation of humans in all

of the presented scenarios lies in their models. In contrast to "classical" epidemiological models where infected people can recover and although having changed population statues (going from susceptible to removed in a immunized sense of the term), they do not actually die. This is very different for the zombie scenario, where now "removed" is no longer synonym of "immunized", but is actually a very nice way of putting "dead". Accordingly, under the considerations of the model presented, humans can only day and this eventually happens in every case (some set of parameters can give reprieve the inevitable fate).

#### **4.5 Can a Different Treatment of the Problem Lead to a Different Outcome?**

We rationalized that a more state-based description of the world population might actually help in brightening the outcome of a zombie outbreak. In fact, the world is divided into states and nations that apply their own laws and restrictions in terms of immigration. If immigration applies to humans, it might as well apply to zombies. In such a case, one might envision that in a given state under the threat of a zombie epidemiological disaster, flux of incoming susceptibles to help them kill the zombies or on the other hand the emigration of the survivors to a non-infected state might lead to various outcome. For example, it is imaginable to see the emergence of a zombie-only state where all the remaining survivor would have found shelter in another state. Or even better, that the help of susceptible from another state might help eradicate the new-coming zombie threat.

For those reason we decided to simulate a model of interacting sub-populations, each under an epidemiological-like treatment. The inner-state epidemiological model describes the emergences of zombies from a spreading disease point of view, while the populations fluxes between states would represent immigration/emigration of the populations under concern (humans and/or zombies).

Moreover, we are interested in seeing to what extent the different paradigms of international politics, Realpolitik, Liberalism and Neoconservatism as defined by Daniel W. Drezner in *Theories of International Politics* and zombies may lead to different outcomes. Expected Results

As describe in Drezner's book, we except different equilibrium outcomes depending on the paradigm under consideration. He postulates the possible appearance of zombie states under Realpolitik and Liberalism paradigms while Neoconservatism would not allow such an outcome.



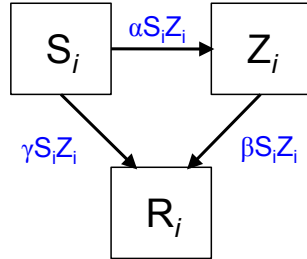
## 5 Description of the Model

In order to simulate a multi-state system under epidemiological evolution we needed to define a clear mathematical framework treating both the population fluxes within states and among states. Such a treatment was necessary to represent the two-fold problem of a zombie outbreak at an international level:

- i. Intra-state fluxes have a fixed physical definition and are invariable among states. They represent the true epidemiological part of our model
- ii. Inter-state fluxes do not have a similar physical meaning and will depend greatly on the paradigm of international politics under consideration (*Realpolitik*, Liberal, Neoconservatorism). They do not represent epidemiological variation *per se* but modelisation of immigration/emigration fluxes.

With such an approach, we would be able to separate the population evolutions at the international and domestic level. We decided to simulate the microstate level under a modification of the classical SIR model REFERENCE called the SZR model (S for susceptible, Z for Zombies, R for removed) [2]. Contrary to the

### 5.1 Microstate Description



**Figure 1:** Description of population fluxes at the microstate level.

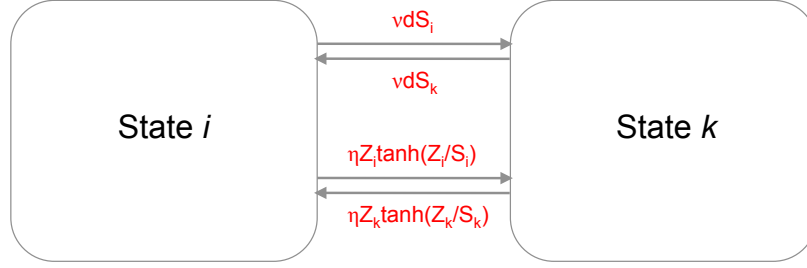
the first equation is (1), the second is (2) and so on

$$\Delta S_i^{micro} = -\alpha S_i Z_i - \gamma S_i Z_i = -(\alpha + \gamma) S_i Z_i \quad (1)$$

$$\Delta Z_i^{micro} = +\alpha S_i Z_i - \beta S_i Z_i = (\alpha - \beta) S_i Z_i \quad (2)$$

$$\Delta R_i^{micro} = +\beta S_i Z_i + \gamma S_i Z_i = (\beta + \gamma) S_i Z_i \quad (3)$$

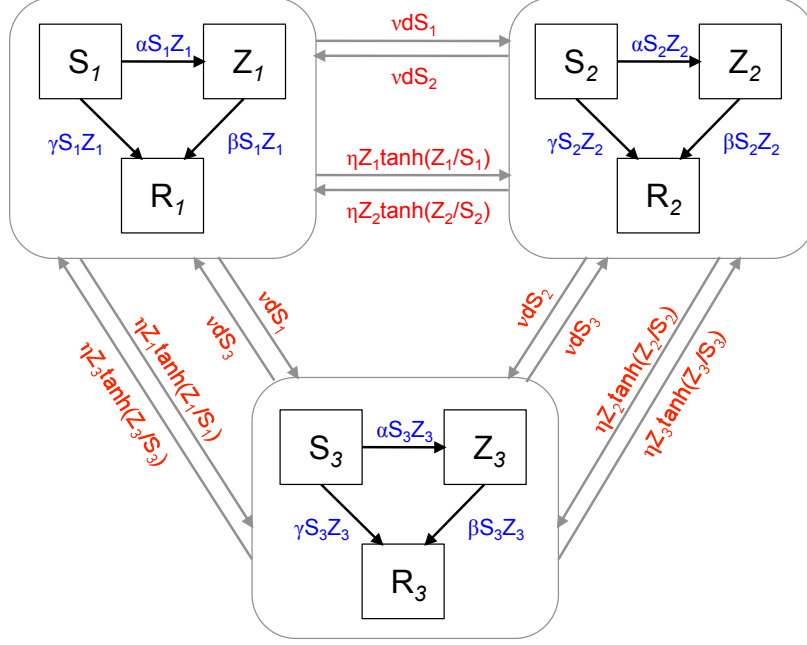
## 5.2 Macrostate Description



**Figure 2:** Description of population fluxes at the macrostate level.

$$\Delta S_i^{macro} = \sum_{j \neq i} (\nu \langle \Delta S_j \rangle - \nu \langle \Delta S_i \rangle) \quad (4)$$

$$\Delta Z_i^{macro} = \sum_{j \neq i} \left( \eta Z_j \tanh \left( \frac{Z_j}{S_j} \right) - \eta Z_i \tanh \left( \frac{Z_i}{S_i} \right) \right) \quad (5)$$



**Figure 3:** Schematic of our 3-state model including all variables. The parts in blue represent the fluxes at the microstate level while the parts in orange represent the macrostate population fluxes. This schematic represent our simulated system where the interaction of three states (microstates) is modeled

## 6 Implementation

The implementation in MatLab works on two levels: one taking care of the simulation of the epidemiological model, the other one taking care of the sweeping of the epidemiological parameters. We will discuss the key parts of the implementation as well as some of the interesting details that had to ensure the coherence of the model.

### 6.1 The Outbreak Function

The outbreak function is the wrapping function of the epidemiological model. It first evaluates the different parameters, initialize the different matrices used throughout the simulation, calls the update function of our model and is finally responsible for the validation of the new data generated after each update cycle. The validation of the data allows to test if it worth continuing the simulation or if it should be shut down. This allows to save computational time. We implemented three exit policies in the simulation loop. First of all, we implemented two obvious safeguards when either the susceptible or the zombie population reaches zero. Continuing the simulation in either case would not make sense since the system

already reached its final state. The third one is triggered when the different population evolution becomes too slow, indicating a relative steady state. Even though it might not be a true dynamical equilibrium, very little population variation over a very long time would not be of any significance within the framework of our model (e.g. a single zombie killing thousands of remaining humans over the course of a very long simulation simply because the infection to death rate as a particular value). In order to avoid unphysical convergences, we define the following control:

$$\langle |\Delta S| \rangle < x \ \&\& \ \langle |\Delta Z| \rangle < x \quad (6)$$

Where  $x$  is the threshold value. The mean of the absolute variation of susceptibles and zombies on the last 100 steps is calculated using a sliding window and compared to a threshold  $x = 0.1$ . If the values are smaller than the threshold, we assume a steady state or "quasi-equilibrium" and exit the simulation. Furthermore, a maximum number of step can be defined (default:  $10^8$  steps) as last exit condition. Details of the code can be found in Appendix 10.1.

## 6.2 The Update Function

The update function takes care of the evolution of the different populations at each cycle. It applies the different equations of the models and uses safeguards to avoid unphysical results. It first calculates the the variation-to-be of each population of each state based on the current populations. It does this in a sequential manner, first the susceptibles and then the zombies. After the update, it checks for the actual variations obtained. This is done in order to ensure that variations larger than the actual population size cannot occur. The first constraint is applied during the computation of the flux exiting each states. Due to the structure of the program, negatives population values can arise. Though these populations will be extremely small ( $> -10^{-4}$ ), resulting in small negative exiting flux, they will lead to negative entering flux in the other states. These flux being considered as positive into the next control procedure, negative values have to be avoided.

The second constraint is applied after the computation of all flux. There is the possibility for the negative flux (death, contamination, emigration) to be bigger than the population plus the positive flux (immigration). In such case, we apply a simple algorithm to avoid the negative population:

1. the negative flux are normalized and multiplied by the population plus the positive flux,
2. the incoming flux of the two other states are corrected to fit the values,
3. the validation is applied to the two other states.

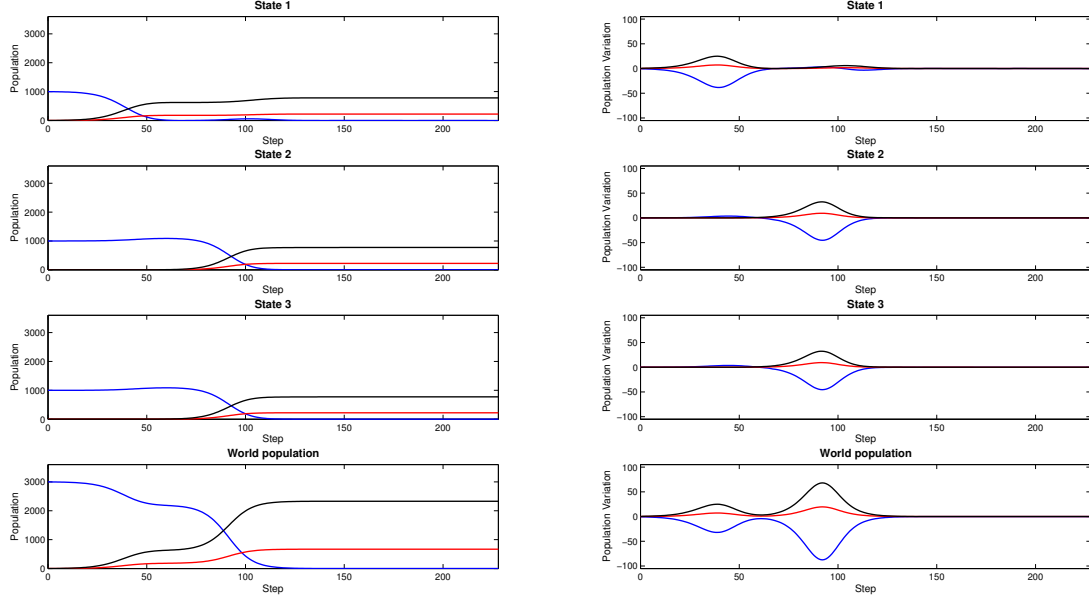
This algorithm is applied until all discrepancy is removed. Note that negative values up to  $10^{-4}$  are allowed and that during the first step of the algorithm, in the case where the sum of the population and incoming flux is negative, this value is zeroed. This method allows a relatively fast convergence of the values while retaining physical significance of the model and its implementation. Details of the code can be found in Appendix 10.2.

### **6.3 The Sweep Function**

The sweep function acts as a wrapper function for the sweeping of each parameters between two values using defined steps. The results of each individual simulation is stored in a dedicated folder. It also assigns a id string to the sweep. This id allows to resume the sweep with the first non completed simulation, this feature was added due to the duration of the sweep with sufficient resolution. Details of the code can be found in Appendix 10.3.

## 7 Simulation Results and Discussion

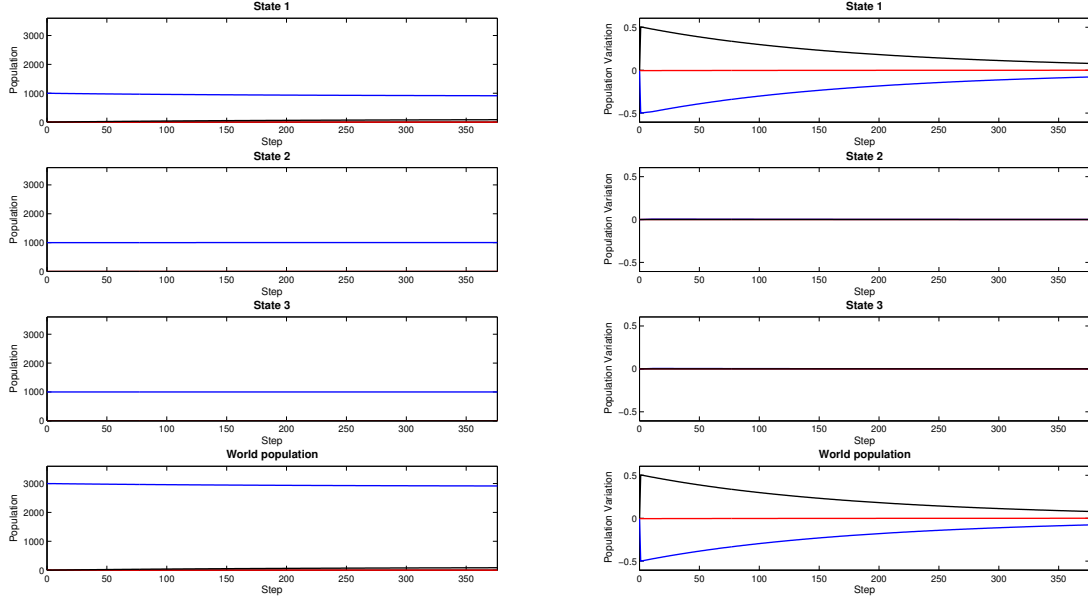
### 7.1 Population Time-Evolution under Different Regimes



**Figure 4:** Doomsday scenario. Emergence of zombies in state 1 leads to full contamination of the world's population (blue = susceptibles, red = zombies, black = removed,  $\alpha = 1.5 \cdot 10^{-4}$ ,  $\beta = 5 \cdot 10^{-6}$ ,  $\gamma = 5 \cdot 10^{-4}$ ,  $\nu = 0.1$ ,  $\eta = 1.5 \cdot 10^{-4}$ ).

### 7.2 Phase Transition at the Microstate

### 7.3 Phase Transition at the Macrostate Level



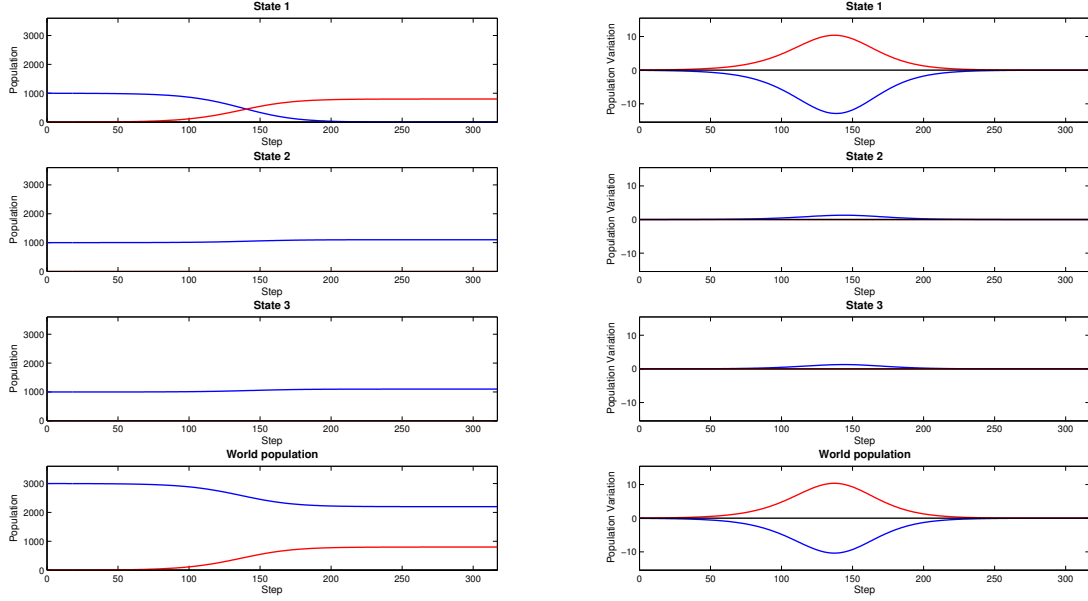
**Figure 5:** Survival of humanity by zombie eradication. As soon as zombies appear in state 1, they get killed until the outbreak is contained (blue = susceptibles, red = zombies, black = removed,  $\alpha = 1.5 \cdot 10^{-8}$ ,  $\beta = 5 \cdot 10^{-6}$ ,  $\gamma = 5 \cdot 10^{-4}$ ,  $\nu = 0.01$ ,  $\eta = 1.5 \cdot 10^{-4}$ ).

## 8 Summary and Outlook

### 8.1 Regimes that Allowed the Survival of Humanity

### 8.2 Further Work: Inter-State fluxes under Game-Theoretical Treatment

For each state (microstate), we defined a SZR model that evaluates the evolution of the different populations under studies: susceptibles (S), zombies (Z) and removed (R). Epidemiological-like (mass-action) transfer of populations between the states (at the macrostate level, *i.e.* international level) also occurs as defined above and models the refugees and zombie transfer across states. Those transfers are dependent of the parameter  $\nu$  for susceptible transfer and parameter  $\eta$  for the zombies. Within the scope of this semester work, we decided to simulate the different paradigms of inter national politics by simply fixing values of both  $\eta$  and  $\nu$  at the beginning of each simulation and observe the effect on the outcome. Both



**Figure 6:** Survival of humanity and emergence of a zombie state. Emergence of zombies in state 1 leads to the exodus of the remaining population to states 2 and 3. State 1 then becomes fully infested of zombies and the remaining of the world’s population is in state 2 and 3 (blue = susceptibles, red = zombies, black = removed,  $\alpha = 5 \cdot 10^{-5}$ ,  $\beta = 5 \cdot 10^{-10}$ ,  $\gamma = 5 \cdot 10^{-10}$ ,  $\nu = 0.1$ ,  $\eta = 0$ ).

the relative and absolute values of those parameters are supposed to represent the paradigm under consideration (*e.g.* a very small value of  $\nu$  would be representative of neoconservatism within the framework of our model since it drastically limits the possibility of immigration of refugees into a given state). Of course, such a treatment is very static and does not take into account the likely variation over time of international cooperation elements (change in immigration politics, military action on foreign soil, etc...). This change in foreign policies is even likely to be more pronounced for the case of a zombie outbreak since such an event would be unprecedented and decision-makers would have a hard time figuring out in such a short time what position to adopt.

Therefore, a nice way to make the macrostate parameters ( $\nu$ ,  $\eta$ ) dynamic in order to represent such changes would be to make them functions of a game-theoretical treatment. In such a model, population fluxes would still be treated within the framework of the current model, with the only variation that the macrostate parameters would be time-evolving. Redefining  $\nu$  and  $\eta$  would be under a cost-hypothesis model as defined in game-theory and then the cost-hypothesis themselves would become fixed parameters over the course of a simulation, defined



in order to represent the different paradigms of international relationships. Accordingly, macrostate fluxes could vary over the course of the simulation, possibly having an effect on the outcome but those variation would be anisotropic and depend on foreign policies bias. The apparition of zombies in one state would start the game. Each state would then evolve on the domestic and international level. The domestic level will follow a standard SZR model, whereas the international level will introduce exchange in the population of susceptible and zombies between the states. These exchanges will be influenced by the state decisions on foreign policies such as humanitarian or military actions determined by our game theory framework. The Game-Theoretical framework is defined as the possibility of undertaking military action of foreign soil (exporting S) or changing the refugee politics by modifying the  $\mu$  parameter (allowing more S to come into one's state, and with a collateral cost of having more zombies crossing as well). Each action will be defined with a specific payoff, which in turn will depend on the international cooperation system under scrutiny. For simplicity, models should be treated homogenously, i.e. all states should adopt the same international politics paradigm. Finally, a “feedback” loop on the payoff depending on the success of a previously undertaken action (positive or negative affectation of the payoffs) could be introduced. This effect could be a modeling of the psychological effect of a successful or unsuccessful action on future action, for example the effectiveness of a military attack. This effect will be made as to converge after a certain time to model the wearing out of the psychological effect over time. The system will be implemented as a step-based update. This implies the ignorance of the actors (the states) of the action of the other actors. This rationalisation comes as the idea that the outbreak would occur over a short period of time, forcing for rapid decision-making and therefore not allow a reaction-based decision-making process.

*Microstate treatment  $\Rightarrow$  SZR model*

$$Macrostate\ treatment \Rightarrow \begin{cases} \Delta S_i^{macro} = \sum_{j \neq i} (\nu \langle \Delta S_j \rangle - \nu \langle \Delta S_i \rangle) \\ \Delta Z_i^{macro} = \sum_{j \neq i} \left( \eta Z_j \tanh \left( \frac{Z_j}{S_j} \right) - \eta Z_i \tanh \left( \frac{Z_i}{S_i} \right) \right) \end{cases}$$

where  $\nu = f(GT)$  ,  $\eta = f(GT)$

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## 10 Appendix

### 10.1 outbreak.m

```
1 function dump = outbreak( varargin )
2
3 %
4 % OUTBREAK runs the simulation of a zombie outbreak in a 3 states
5 %   system.
6 %
7 %   DUMP = OUTBREAK( ARGS )
8 %       ARGS are name-value pairs. Possible arguments are :
9 %           params: structure containing rate parameters (alpha, beta,
10 %                gamma, eta, nu) as arrays (3x1 for alpha, beta and
11 %                gamma or 3x2 for eta and nu).
12 %           paramfile: path to a mat-file storing the parameter
13 %                structure.
14 %           zombies: 3x1 array containing the number of initial zombie
15 %                per state (default: 0, 0, 0).
16 %           population: 3x1 array containing the number of initial
17 %                susceptible population per state (default: 1000,
18 %                1000, 1000).
19 %           steps: maximal number of steps for the simulation (default:
20 %                1e8).
21 %           silent: level of output for the simulation:
22 %                   0 = all outputs
23 %                   1 = all outputs excepts graphs
24 %                   2 = no outputs
25 %           dslength: size of the sliding window used for the
26 %                calculation of the mean of dS (used as weight
27 %                factor for the inter-state susceptible transfer).
28 %
29 %   DUMP is a structure containing the following fields:
30 %       S: 4xn array containing the evolution of the susceptible
31 %           population.
32 %       dS: 4xn array containing the evolution of the susceptible
33 %           population's variation.
34 %       Z: 4xn array containing the evolution of the zombie
35 %           population.
36 %       dZ: 4xn array containing the evolution of the zombie
37 %           population's variation.
38 %       R: 4xn array containing the evolution of the removed
39 %           population.
40 %       dR: 4xn array containing the evolution of the removed
41 %           population's variation.
42 %       step: number of steps performed.
43 %       alpha, beta, gamma, eta, nu: transfer rate parameters.
44 %       time: simulation time.
45 %
46
47
```

```

48 %% Variable initialization
49 silent = 0;
50 dslength = 10;
51 maxSteps = 1e8;
52 rates = struct( 'alpha', [ 0.00095 ; 0.00095 ; 0.00095 ], 'beta', ...
    [0.00025 ; 0.00025 ; 0.00025 ], 'gamma', [ 0.00005 ; 0.00005 ; ...
    0.00005 ], 'mu', [ 0.00000005 , 0.00000005 ; 0.00000005 , ...
    0.00000005 ; 0.00000005 , 0.00000005 ], 'nu', [ 0.00000005 , ...
    0.00000005 ; 0.00000005 , 0.00000005 ; 0.00000005 , 0.00000005 ], ...
    'eta', [ 0.00000005 , 0.00000005 ; 0.00000005 , 0.00000005 ; ...
    0.00000005 , 0.00000005 ] );
53 zombies = [ 0 ; 0 ; 0 ; 0 ];
54 populations = [ 1e3 ; 1e3 ; 1e3 ; 3e3 ];
55
56 % Structure storing populations and populations variation
57 states = struct( 'pop', zeros( 4, 3 ), 'dpop', zeros( 4, 3 ) );
58
59 % See exit condition at the end of the loop.
60 exitThreshold = 1e-1;
61
62 p = [ 'alpha' ; 'beta ' ; 'gamma' ; 'eta ' ; 'mu ' ; 'nu ' ];
63 reverseStr = '';
64
65
66 %% Argument parsing
67 % All argument are optional.
68 for i = 1:2:nargin
69
70     % Argument are passed in name-value pairs. If i + 1 does not exist,
71     % then the value is absent and the program stops.
72     if( nargin < i + 1 )
73
74         error( [ 'Missing argument for parameter "' varargin{ i } ...
75             '","' ] );
76     end
77
78     switch varargin{ i }
79
80         % params excepts a structure containing the rates values for the
81         % epidemiologic model. Not all fields have to be present. If a
82         % field is missing, the default value is used.
83         case 'params'
84             for j = 1:6
85
86                 if isfield( varargin{ i + 1 }, strtrim( p( j, : ) ) )
87
88                     rates.( strtrim( p( j, : ) ) ) = varargin{ i + 1 ...
89                         }.( strtrim( p( j, : ) ) );
90
91                     end
92             end
93
94         % paramfile excepts the path to a mat-file containing a
95         % parameter structure (refer to parms).

```

```

93     case 'paramfile'
94         pfile = load( varargin{ i + 1 }, '-mat' );
95         for j = 1:6
96
97             if isfield( pfile{ i + 1 }, strtrim( p( j, : ) ) )
98
99                 rates.( strtrim( p( j ) ) ) = pfile{ i + 1 }. ( ...
100                     strtrim( p( j ) ) );
101             end
102         end
103
104         % zombies excepts an array containing the initial zombie
105         % populations in each state.
106         case 'zombies'
107             temp = varargin{ i + 1 };
108             for j = 1:3
109
110                 zombies( j ) = temp( j );
111                 zombies( 4 ) = zombies( 4 ) + temp( j );
112             end
113
114         % zombies excepts an array containing the initial susceptible
115         % population in each state.
116         case 'population'
117             temp = varargin{ i + 1 };
118             populations( 4 ) = 0;
119             for j = 1:3
120
121                 populations( j ) = temp( j );
122                 populations( 4 ) = populations( 4 ) + populations( j );
123             end
124
125         % steps excepts the maximum number of steps allowed for the
126         % simulation.
127         case 'steps'
128             maxSteps = varargin{ i + 1 };
129
130         % silent except a int defining the output type :
131         % 2 : No output
132         % 1 : All outputs but the graphs
133         % 0 : Normal output
134         case 'silent'
135             silent = varargin{ i + 1 };
136
137         % dslength excepts the size of the sliding window used for the
138         % calculation of the mean of dS (used as weight factor for the
139         % inter-state susceptible transfer).
140         case 'dslength'
141             dslength = varargin{ i + 1 };
142
143         otherwise
144             error( [ 'Unknown parameter "', varargin{ i }, "'. ' ] );
145     end

```

```

145     end
146
147
148
149     %% Initialization of simulation
150
151     % Setting the initial populations.
152     states.pop( :, 1 ) = populations;
153     states.pop( :, 2 ) = zombies;
154
155     % Initialization of the sliding window for inter-state population
156     % transfer.
157     dshistory = zeros( 3, dslength );
158
159     % Setting up the initial dump structure. Large matrices are used to
160     % avoid the need of dynamic memory allocation to improve code
161     % efficiency.
162     d = zeros( 4, max( 200, .0001 * maxSteps ) );
163     dump = struct( 'S', d, 'dS', d, 'Z', d, 'dZ', d, 'R', d, 'dR', d, ...
        'step', 1, 'alpha', mean( rates.alpha ), 'beta', mean( rates.beta ...
        ), 'gamma', mean( rates.gamma ), 'eta', mean( mean( rates.eta ) ...
        ), 'nu', mean( mean( rates.nu ) ), 'time', 0 );
164     dump.S( :, 1 ) = states.pop( 1:4, 1 );
165     dump.Z( :, 1 ) = states.pop( 1:4, 2 );
166     dump.R( :, 1 ) = states.pop( 1:4, 3 );
167
168     % Initialize timer.
169     tic;
170
171     %% Update loop.
172     for i = 1:maxSteps
173
174         % Output of step # and current population count.
175         if silent < 2
176
177             msg = sprintf('Processing step %d... Human population %d, ...
                Zombie population %d\n', i, dump.S( 4, dump.step ), ...
                dump.Z( 4, dump.step ) );
178             fprintf([ reverseStr, msg]);
179
180             reverseStr = repmat(sprintf('\b'), 1, length(msg));
181         end
182
183         % Update function (see update.m for details).
184         [ states, rates, dshistory ] = update( states, rates, dshistory );
185
186         % Dumping of the latest state of the simulation.
187         % Update of the step number.
188         dump.step = i + 1;
189
190         % Dumping of the various population and population variation.
191         dump.S( :, dump.step ) = states.pop( :, 1 );
192         dump.dS( :, dump.step ) = states.dpop( :, 1 );

```

```

193     dump.Z( :, dump.step ) = states.pop( :, 2 );
194     dump.dZ( :, dump.step ) = states.dpop( :, 2 );
195     dump.R( :, dump.step ) = states.pop( :, 3 );
196     dump.dR( :, dump.step ) = states.dpop( :, 3 );
197
198     % Stops simulation if S is equal to 0.
199     if states.pop( 4, 1 ) == 0
200
201         if silent < 2
202
203             disp( ' ' );
204             disp( 'End of the human race.' );
205         end
206         break;
207     end
208
209     % Stops simulation if Z is equal to 0.
210     if states.pop( 4, 2 ) == 0
211
212         if silent < 2
213
214             disp( ' ' );
215             disp( 'Humanity survived.' );
216         end
217         break;
218     end
219
220     % Stops simulation if the average fluctuation of both dS and dZ
221     % over a sliding window is lower than the threshold (simulation
222     % reached equilibrium in the limit of our time frame).
223     if i > 100 && mean( abs( dump.dS( 4, ( i - 100 ):i ) ) ) < ...
        exitThreshold && mean( abs( dump.dZ( 4, ( i - 100 ):i ) ) ) < ...
        exitThreshold
224
225
226         if silent < 2
227
228             disp( ' ' );
229             disp( 'Equilibrium reached.' );
230         end
231         break;
232     end
233 end
234
235 % Store simulation time.
236 dump.time = toc;
237
238 % Resizing of the dump matrices before plotting.
239 dump.S = dump.S( :, 1:dump.step );
240 dump.dS = dump.dS( :, 1:dump.step );
241 dump.Z = dump.Z( :, 1:dump.step );
242 dump.dZ = dump.dZ( :, 1:dump.step );
243 dump.R = dump.R( :, 1:dump.step );

```

```

244     dump.dR = dump.dR( :, 1:dump.step );
245
246     if ~silent
247
248         % Plot the simulation history (see plotResults.m for details).
249         plotResults( dump );
250     end
251 end

```

## 10.2 update.m

```

1     function [ states, rates, dshistory ] = update( states, rates, ...
2         dshistory )
3
4     %
5     % UPDATE compute the population evolution for one step of the
6     % epidemiological model.
7     %
8     % [ STATES, RATES, DSHISTORY, DUMP ] = UPDATE( STATES, RATES, ...
9     % DSHISTORY, DUMP )
10    %
11    % STATES is the structure storing the populations and population
12    % variations as defined in the OUTBREAK function.
13    % RATES is the structure containing the epidemiological transfer
14    % rates as defined in the OUTBREAK function.
15    % DSHISTORY is the array containing the sliding window for
16    % inter-state susceptible transfer.
17    %
18    %% Initialization of parameters :
19
20    s = states.pop( 1:3, 1 );
21    z = states.pop( 1:3, 2 );
22
23    alpha = rates.alpha;
24    beta = rates.beta;
25    gamma = rates.gamma;
26    nu = rates.nu;
27    eta = rates.eta;
28
29    % Permutation matrix used to set the input inter-state flux from the
30    % inter-state output flux to avoid redundant calculations. See flux
31    % update.
32    permutation = [ 0 0 1 ; 1 0 0 ; 0 1 0 ];
33
34    % Matrix containing the coordinates of the input inter-state flux from
35    % each state (line) to the other two (block of two colons). See flux
36    % correction.
37    iFluxCoor = [ 2 2 3 1 ; 3 2 1 1 ; 1 2 2 1 ];

```



```

37 %% Update of the susceptible population.
38
39 % Depletion of the susceptible population goes according to :
40 %
41 %   S  -> Z  = - alpha * s * z
42 %   S  -> R  = - gamma * s * z
43 %   S1 -> S2 = nu * dSmean
44 %   S1 -> S3 = nu * dSmean
45 %
46 % Immigration is the only source of susceptible population.
47 %
48 % dS is a 3x6 matrix containing for each state :
49 %
50 %   Row 1 : n( S  -> Z  )
51 %   Row 2 : n( S  -> R  )
52 %   Row 3 : n( S1 -> S2 ) | n( S2 -> S3 ) | n( S3 -> S1 )
53 %   Row 4 : n( S1 -> S3 ) | n( S2 -> S1 ) | n( S3 -> S2 )
54 %   Row 5 : n( S2 -> S1 ) | n( S3 -> S2 ) | n( S1 -> S3 )
55 %   Row 6 : n( S3 -> S1 ) | n( S1 -> S2 ) | n( S2 -> S3 )
56 %
57 % The sum of each line gives the population variation for the state.
58
59 % Mean variation of the susceptible in the sliding window.
60 dsmean = min( mean( dshistory, 2 ), 0 );
61
62 dS = zeros( 3, 6 );
63 dS( :, 1 ) = min( - alpha .* s .* z, 0 );
64 dS( :, 2 ) = min( - gamma .* s .* z, 0 );
65 dS( :, 3 ) = nu( :, 1 ) .* dsmean;
66 dS( :, 4 ) = nu( :, 2 ) .* dsmean;
67 dS( :, 5 ) = - permutation * permutation * dS( :, 4 );
68 dS( :, 6 ) = - permutation * dS( :, 3 );
69
70
71 % There is the possibility that any of the |-dSi| is larger than the
72 % actual population in the state (negative population fluctuation larger
73 % than the actual population). If so, a correction is applied to
74 % avoid negative population.
75 % As long as any state falls into this category:
76 while sum( sum( dS( :, 1:4 ), 2 ) + s + sum( dS( :, 5:6 ), 2 ) < ...
77         -1e-6 )
78     for i = 1:3
79
80         % Correct the state that falls into this category:
81         if( sum( dS( i, 1:4 ), 2 ) + ( s( i ) + sum( dS( i, 5:6 ), 2 ...
82             ) ) < -1e-6 )
83
84             % Population is considered equal to the sum of previous
85             % population and the inter-state input transfer. The
86             % negative contributions to the dS are reduced so that it
87             % equals this population.
88             dS( i, 1:4 ) = dS( i, 1:4 ) / abs( sum( dS( i, 1:4 ) ) ) ...

```

```

88         * max( sum( dS( i, 5:6 ) ) + s( i ), 0 );
89
90         % The effect of the correction on the output flux is
91         % applied to the other state input flux.
92         dS( iFluxCoor( i, 1 ), 4 + iFluxCoor( i, 2 ) ) = - dS( i, ...
93             3 );
94         dS( iFluxCoor( i, 3 ), 4 + iFluxCoor( i, 4 ) ) = - dS( i, ...
95             4 );
96     end
97 end
98
99 % Note that the output flux can only be lower than the original flux.
100 % Therefore, the input flux for the two other states can only
101 % be smaller than before the correction. Accordingly, the
102 % process has to be repeated until all states have a population
103 % differential such that it doesn't lead to a negative population.
104 end
105
106 %% Update of the zombie population.
107
108 % Depletion of the zombie populations goes according to :
109 %
110 % S -> Z = alpha * s * z
111 % Z -> R = - beta * s * z
112 % Z1 -> Z2 = - eta * z * tanh( z / s )
113 % Z1 -> Z3 = - eta * z * tanh( z / s )
114 %
115 % Conversion from susceptible and "immigration" are the two sources of
116 % zombies.
117 %
118 % dZ is a 3x6 matrix containing for each states :
119 %
120 % Row 1 : n( S -> Z )
121 % Row 2 : n( Z -> R )
122 % Row 3 : n( Z1 -> Z2 ) | n( Z2 -> Z3 ) | n( Z3 -> Z1 )
123 % Row 4 : n( Z1 -> Z3 ) | n( Z2 -> Z1 ) | n( Z3 -> Z2 )
124 % Row 5 : n( Z2 -> Z1 ) | n( Z3 -> Z2 ) | n( Z1 -> Z3 )
125 % Row 6 : n( Z3 -> Z1 ) | n( Z1 -> Z2 ) | n( Z2 -> Z3 )
126 %
127
128 dZ = zeros( 3, 6 );
129 dZ( :, 1 ) = max( - dS( :, 1 ), 0 );
130 dZ( :, 2 ) = min( - beta .* s .* z, 0 );
131 dZ( :, 3 ) = min( - eta( :, 1 ) .* z .* tanh( z ./ s ), 0 );
132 dZ( :, 4 ) = min( - eta( :, 2 ) .* z .* tanh( z ./ s ), 0 );
133 dZ( :, 5 ) = - permutation * permutation * dZ( :, 4 );
134 dZ( :, 6 ) = - permutation * dZ( :, 3 );
135
136 % The same control procedure as for the susceptible population is
137 % applied to avoid negative population of zombies. See susceptible
138 % correction for details on the procedure.
139 while sum( ( dZ( :, 1 ) + sum( dZ( :, 5:6 ), 2 ) + z ) + sum( dZ( :, ...
140     2:4 ), 2 ) < - 1e-6 )

```

```

137
138     for i = 1:3
139
140         if( ( dZ( i, 1 ) + sum( dZ( i, 5:6 ), 2 ) + z( i ) ) + sum( ...
141             dZ( i, 2:4 ) ) < - 1e-6 )
142
143             dZ( i, 2:4 ) = dZ( i, 2:4 ) * max( sum( dZ( i, 5:6 ) ) + ...
144                 dZ( i, 1 ) + z( i ), 0 ) / abs( sum( dZ( i, 2:4 ) ) );
145
146             dZ( iFluxCoor( i, 1 ), 4 + iFluxCoor( i, 2 ) ) = - dZ( i, ...
147                 3 );
148             dZ( iFluxCoor( i, 3 ), 4 + iFluxCoor( i, 4 ) ) = - dZ( i, ...
149                 4 );
150         end
151     end
152 end
153
154 %% Update of the removed population
155
156 % Variation in the removed population is strictly positive and comes
157 % from both the susceptible and zombie population:
158 %
159 %   S -> R = alpha * s * z
160 %   Z -> R = beta * s * z
161 %
162 % The value obtained for dS and dZ are used:
163
164 dR = - [ dS( :, 2 ), dZ( :, 2 ) ];
165
166 %% Update each population
167
168 % Compilation of the populations variations
169 states.dpop( 1:3, : ) = [ sum( dS, 2 ), sum( dZ, 2 ), sum( dR, 2 ) ];
170
171 % Update of the sliding window matrix for the susceptible.
172 [ ~, dsSize ] = size( dshistory );
173 dshistory( :, 2:dsSize ) = dshistory( :, 1:( dsSize - 1 ) );
174 dshistory( :, 1 ) = states.dpop( 1:3, 1 );
175
176 % Update of the current populations
177 states.pop( 1:3, : ) = states.pop( 1:3, : ) + states.dpop( 1:3, : );
178
179 % Update of the total populations
180 states.pop( 4, : ) = sum( states.pop( 1:3, : ) );
181 states.dpop( 4, : ) = sum( states.dpop( 1:3, : ) );
182 end

```

### 10.3 plotResults.m

```

1 function plotResults( dump )

```

```

2
3 % PLOTRESULTS plots the populations' evolution during the simulation. It
4 % plots the different populations (S, Z, R) for each state as well as
5 % the population variation. Furthermore the global populations and
6 % population variations are also displayed.
7 %
8 % PLOTRESULTS( DUMP )
9 % DUMP is the dumping structure as defined in the OUTBREAK function.
10 %
11
12
13 set(0, 'DefaultAxesColorOrder', [ 0, 0, 1 ; 1, 0, 0 ; 0, 0, 0 ], ...
14     'DefaultLineLineWidth', 1.45 );
15
16 x = 0:( length( dump.S ) - 1 );
17
18 % For each of the three states, plot the populations and
19 % populations' variations.
20 for i = 1:3
21     subplot( 4, 2, 1 + ( i - 1 ) * 2 );
22     plot( x, dump.S( i, : ), x, dump.Z( i, : ), x, dump.R( i, : ) );
23     ylim( [ 0 dump.S( 4 ) ] );
24     xlabel( 'Step', 'fontsize', 11 );
25     ylabel( 'Population', 'fontsize', 11 );
26     title( [ 'State ', int2str( i ) ], 'fontweight', 'b', 'fontsize', ...
27           12 );
28
29     subplot( 4, 2, 2 + ( i - 1 ) * 2 );
30     plot( x, dump.dS( i, : ), x, dump.dZ( i, : ), x, dump.dR( i, : ) );
31     xlabel( 'Step', 'fontsize', 11 );
32     ylabel( 'Population Variation', 'fontsize', 11 );
33     title( [ 'State ', int2str( i ) ], 'fontweight', 'b', 'fontsize', ...
34           12 );
35
36 end
37
38 % Plot the total populations and total populations' variations.
39 subplot( 4, 2, 7 );
40 plot( x, dump.S( 4, : ), x, dump.Z( 4, : ), x, dump.R( 4, : ) );
41 ylim( [ 0 dump.S( 4 ) ] );
42 xlabel( 'Step', 'fontsize', 11 );
43 ylabel( 'Population', 'fontsize', 11 );
44 title( 'World population', 'fontweight', 'b', 'fontsize', 12 );
45
46 subplot( 4, 2, 8 );
47 plot( x, dump.dS( 4, : ), x, dump.dZ( 4, : ), x, dump.dR( 4, : ) );
48 xlabel( 'Step', 'fontsize', 11 );
49 ylabel( 'Population Variation', 'fontsize', 11 );
50 title( 'World population', 'fontweight', 'b', 'fontsize', 12 );
51 end

```

## 10.4 sweep.m

```
1 function params = sweep( params )
2
3 %
4 % SWEEP allows to run multiple simulation sequentially varying the
5 % parameters in a defined range with defined steps.
6 %
7 % PARAMS = SWEEP( PARAMS )
8 %     PARAMS is a structure storing for each parameter alpha, beta,
9 %     gamma, eta and nu a 1x3 matrix with the minimal value, the
10 %     step and the maximal value for the parameter.
11 %     The return value also contains the ID of the sweep as well
12 %     as the number of simulations done.
13 %
14 % PARAMS = SWEEP( ID )
15 %     ID is the ID string of a previous sweep. The sweep will start
16 %     over where the sweep stopped the last time.
17 %
18
19
20 %% Parsing of the argument
21 switch class( params )
22
23     case 'struct'
24         % If the parameter set has never been employed, the restart
25         % fields are created.
26         if length( params.alpha ) == 3
27
28             params.alpha( 4 ) = params.alpha( 1 );
29             params.beta( 4 ) = params.beta( 1 );
30             params.gamma( 4 ) = params.gamma( 1 );
31             params.eta( 4 ) = params.eta( 1 );
32             params.nu( 4 ) = params.nu( 1 );
33
34         else
35             if ~isfield( params, 'step' )
36
37                 params.alpha( 4 ) = params.alpha( 1 );
38                 params.beta( 4 ) = params.beta( 1 );
39                 params.gamma( 4 ) = params.gamma( 1 );
40                 params.eta( 4 ) = params.eta( 1 );
41                 params.nu( 4 ) = params.nu( 1 );
42             end
43         end
44
45         % Initialization of the number of simulation ran
46         if ~isfield( params, 'step' )
47
48             params.step = 1;
49         end
50
```

```

51         % If an ID is assigned, the restart file is loaded, otherwise,
52         % an ID is created as well as a folder for the simulation
53         % results.
54         if isfield( params, 'id' )
55
56             if exist( [ params.id '.restart' ], 'file' )
57
58                 params = load( [ id '.restart' ], '-mat' );
59             end
60         else
61
62             params.id = num2hex( ceil( 1e20*rand ) );
63             mkdir( [ '../results/' params.id ] );
64         end
65
66         case 'char'
67             % If an ID is provided, the restart file is loaded.
68             params = load( [ params '.restart' ], '-mat' );
69     end
70
71     %% Display of the simulation details
72     disp( [ 'Id : ' params.id '.' ] );
73     disp( 'Use the id as parameter to resume the sweep anytime.' );
74     disp( '_____' );
75     disp( params );
76     disp( '_____' );
77
78     %% Variables initialization
79     cParams = struct( 'alpha', zeros( 3, 1 ), 'beta', zeros( 3, 1 ), ...
80         'gamma', zeros( 3, 1 ), 'eta', zeros( 3, 2 ), 'nu', zeros( 3, 2 ) );
81     output = struct( 'S', 0, 'Z', 0, 'R', 0, 'alpha', 0, 'beta', 0, ...
82         'gamma', 0, 'eta', 0, 'nu', 0, 'e', 0, 'step', 0 );
83     reverseStr = '';
84     step = 1;
85
86     %% Parameter sweeping
87     for nu = params.nu( 1 ):params.nu( 2 ):params.nu( 3 )
88
89         for eta = params.eta( 1 ):params.eta( 2 ):params.eta( 3 )
90
91             for gamma = params.gamma( 1 ):params.gamma( 2 ):params.gamma( ...
92                 3 )
93
94                 for alpha = params.alpha( 1 ):params.alpha( 2 ...
95                     ):params.alpha( 3 )
96
97                     % If the current step number is larger than the
98                     % step number in the parameter file, the simulation
99                     % has already be done and can be skipped.

```

```

99         if step > params.step
100
101             % Update of the current process status.
102             msg = sprintf('Step %d.\nAlpha = %f, Beta = ...
                        %f,\nGamma = %f, Eta = %f, Nu = %f.', ...
                        step, alpha, beta, gamma, eta, nu );
103             fprintf([ reverseStr, msg]);
104             reverseStr = repmat(sprintf('\b'), 1, ...
                        length(msg));
105
106             % Preparation of the simulation parameters
107             cParams.alpha = alpha * ones( 3, 1 );
108             cParams.beta = beta * ones( 3, 1 );
109             cParams.gamma = gamma * ones( 3, 1 );
110             cParams.eta = eta * ones( 3, 2 );
111             cParams.nu = nu * ones( 3, 2 );
112
113             % Simulation
114             dump = outbreak( 'silent', 2, 'params', ...
                        cParams, 'zombies', [ 1 0 0 ] );
115
116             % Preparation of the output structure
117             output.S = dump.S;
118             output.R = dump.R;
119             output.Z = dump.Z;
120             output.step = dump.step;
121             output.alpha = alpha;
122             output.beta = beta;
123             output.gamma = gamma;
124             output.eta = eta;
125             output.nu = nu;
126
127             % Saving of the results of the simulation
128             save( [ './results/' params.id '/output.' ...
                        int2str( params.step ) '.mat' ], ...
                        '-struct', 'output' );
129
130             % Update of the parameter structure and saving
131             % of the restart file.
132             params.step = params.step + 1;
133             params.alpha( 4 ) = alpha + params.alpha( 2 );
134             save( [ params.id '.restart' ], '-struct', ...
                        'params' );
135         end
136
137         % Update of the current step.
138         step = step + 1;
139     end
140
141     % Parameters update
142     params.alpha( 4 ) = params.alpha( 1 );
143
144     params.beta( 4 ) = beta + params.beta( 2 );

```

```

145         end
146
147         params.beta( 4 ) = params.beta( 1 );
148
149         params.gamma( 4 ) = gamma + params.gamma( 2 );
150     end
151
152     params.gamma( 4 ) = params.gamma( 1 );
153
154     params.eta( 4 ) = eta + params.eta( 2 );
155 end
156
157 params.eta( 4 ) = params.eta( 1 );
158
159 params.nu( 4 ) = nu + params.nu( 2 );
160 end
161 end

```