Real Analysis I
Homework #1

Please prepare your solutions as a compiled LaTeX document.
Due date September 17.

(1) Complete the fill in the blank practice problems 2.1.14 (page 45), 2.1.15 (page 46) and 2.3.17 (page 70). Please type out the complete paragraphs underlining the filled in text.
(2) Suppose that B is a set and $\{A_i : i \in I\}$ is an indexed family of sets. Prove that

$$B \setminus \bigcup_{i \in I} A_i = \bigcap_{i \in I} B \setminus A_i$$

(3) Do problem 3.1#24: Prove by induction on $n \in \mathbb{N}$ that for any real number $x \geq -1$,

$$(1+x)^n \geq 1 + nx$$

(4) Defining the rationals from the integers: Let $\equiv$ be the relation on $Q = \mathbb{Z} \times \mathbb{N}$ defined by $(n,m) \equiv (n',m')$ if $n \cdot m' = n' \cdot m$. Prove the following
  (a) $\equiv$ is an equivalence relation on Q
  Remark: Your proof should only depend on the operations $+$ and $\cdot$ defined on the integers, however, you may use the fact that the integers satisfy: If $ab = cd$ and $b = d$ then it follows that $a = c$).
  (b) Let $\mathbb{Q} = \{[(n,m)] : (n,m) \in Q\}$ be the set of equivalence classes and define the operation $\color{red}{+}$ on $\mathbb{Q}$ by $[(n,m)] \color{red}{+} [(k,l)] = [(ln + km, ml)]$. Prove that $\color{red}{+}$ is *well-defined*. Just to clarify: $+$ is the given operation of addition on the integers and red coloured plus, $\color{red}{+}$, is the newly defined operation of addition on the equivalence classes of Q.