$\vec{a} = (a_1, a_2, a_3)$ is in the plane, want to see it it's $\perp$ to $(-3, -6, 1)$

$(a_1, a_2, a_3) \cdot (-3, -6, 1)$

$= -3a_1 - 6a_2 + a_3$

So formula for plane is

$$\boxed{-3a_1 - 6a_2 + a_3 = 0} \quad \square$$

$\vec{B}$

Ex: plane not through $\vec{0}$

Find formula for plane containing $(2, -1, 3)$ at same angle.

— consists of vectors

$(2, -1, 3) + (1, 0, 3)s + (-2, 1, 0)t$

parametric

parameters

(a_1, a_2, a_3) is in plane $\mathcal{P}$

$(a_1, a_2, a_3) - (0, -1, 3)$ is $\perp (-3, -6, 1)$

$(a_1, a_2+1, a_3-3) \perp (-3, -6, 1)$

$-3a_1 - 6(a_2+1) + (a_3-3) = 0$

OR

proj of (a_1, a_2, a_3) onto $(-3, -6, 1)$

$$\frac{(a_1, a_2, a_3) \cdot (-3, -6, 1)}{|(-3, -6, 1)|^2} (-3, -6, 1)$$

$\vec{v} \cdot \vec{w} = |\vec{v}||\vec{w}|\cos\theta$

$$\frac{-3a_1 - 6a_2 + a_3}{9 + 36 + 1} (-3, -6, 1)$$

proj of $(0, -1, 3)$ onto $(-3, -6, 1)$

$$\frac{-3 \cdot 0 - 6 \cdot (-1) + 3}{46} \quad (-3, -6, 1)$$

$$= \frac{9}{46} (-3, -6, 1) = \frac{-3a_1 - 6a_2 + a_3}{46} (-3, -6, 1)$$

(a_1, a_2, a_3) on plane if

$$9 = -3a_1 - 6a_2 + a_3$$

Prove formula $-3a_1 - 6(a_2 + 1) + (a_3 - 3) = 0$

$$-3a_1 - 6a_2 - 6 + a_3 - 3 = 0$$

$$-3a_1 - 6a_2 + a_3 = 9 \quad \square$$

Does plane contain $(0, 1, 15)$

$$-3 \cdot 0 - 6 \cdot 1 + 15 = -6 + 15 = 9 \quad \text{YES}$$

$(4, 0, 11)$

$$-3 \cdot 4 - 6 \cdot 0 + 11 = -12 + 11 = -1 \quad \text{NO}$$

Find closest point on plane

Want $(4, 0, 11) + (-3, -6, 1) r$ ← real number scalar
on plane

↖ line

$(4-3r, 0-6r, 11+r) \cdot (-3, -6, 1)$ want $=9$

$-12 + 9r + 36r + 11 + r = 9$

$46r = 9 + 12 - 11 = 10$

$r = \dfrac{10}{46} = \dfrac{5}{23}$

$(4, 0, 11) + (-3, -6, 1) \dfrac{5}{23} = (4, 0, 11) + \left(-\dfrac{15}{23}, -\dfrac{30}{23}\right)$

$= \left(4 - \dfrac{15}{23}, -\dfrac{30}{23}, 11 + \dfrac{5}{23}\right)$ □

$Ax^2 + By^2 + Cz^2 + Dz + Ey + Fz + G = 0$

Ex eq'n sphere centre $(2, 3, -1)$
radius 5

$$\left| (a_1, a_2, a_3) - (2, 3, -1) \right| = 5$$

$$\left| (a_1 - 2, a_2 - 3, a_3 + 1) \right| = 5$$

$$\sqrt{(a_1 - 2)^2 + (a_2 - 3)^2 + (a_3 + 1)^2} = 5$$

$$(a_1 - 2)^2 + (a_2 - 3)^2 + (a_3 + 1)^2 = 25$$

$$a_1^2 - 4a_1 + 4 + \cdots \cdots = 25$$