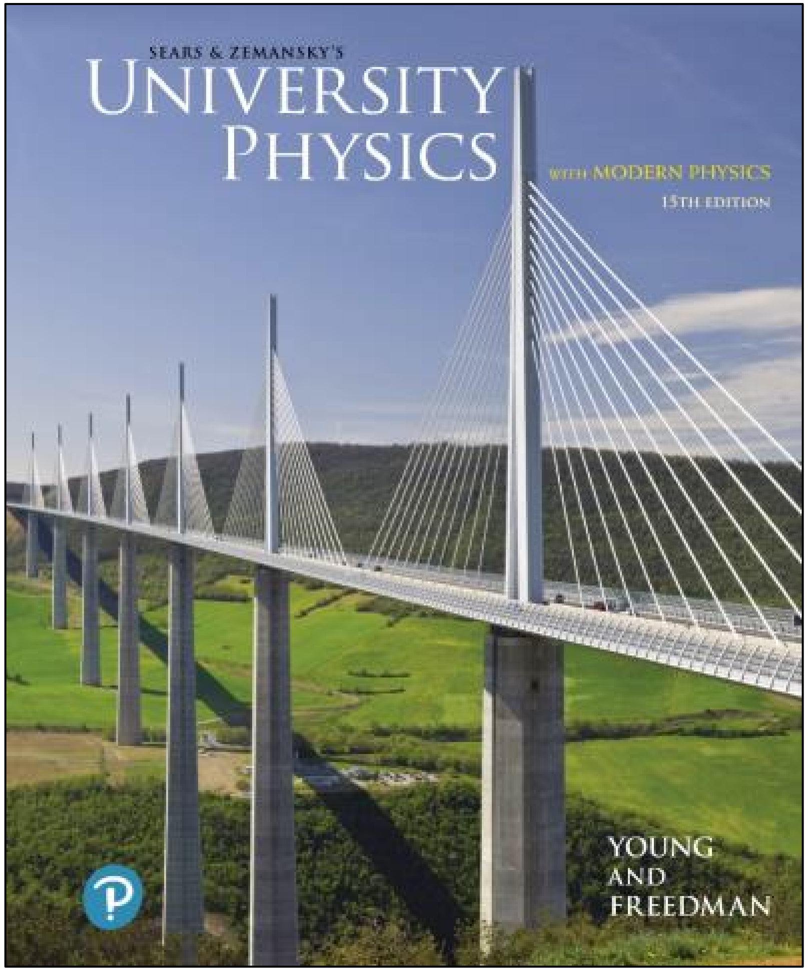
University Physics with Modern Physics

Fifteenth Edition



Chapter 3

Motion in Two or Three Dimensions

Learning Goals for Chapter 3

Looking forward at …

- how to use vectors to represent the position and velocity of a particle in two or three dimensions.
- how to find the vector acceleration of a particle, and how to interpret the components of acceleration parallel to and perpendicular to a particle's path.
- how to solve problems that involve the curved path followed by a projectile.

Learning Goals for Chapter 3

Looking forward at …

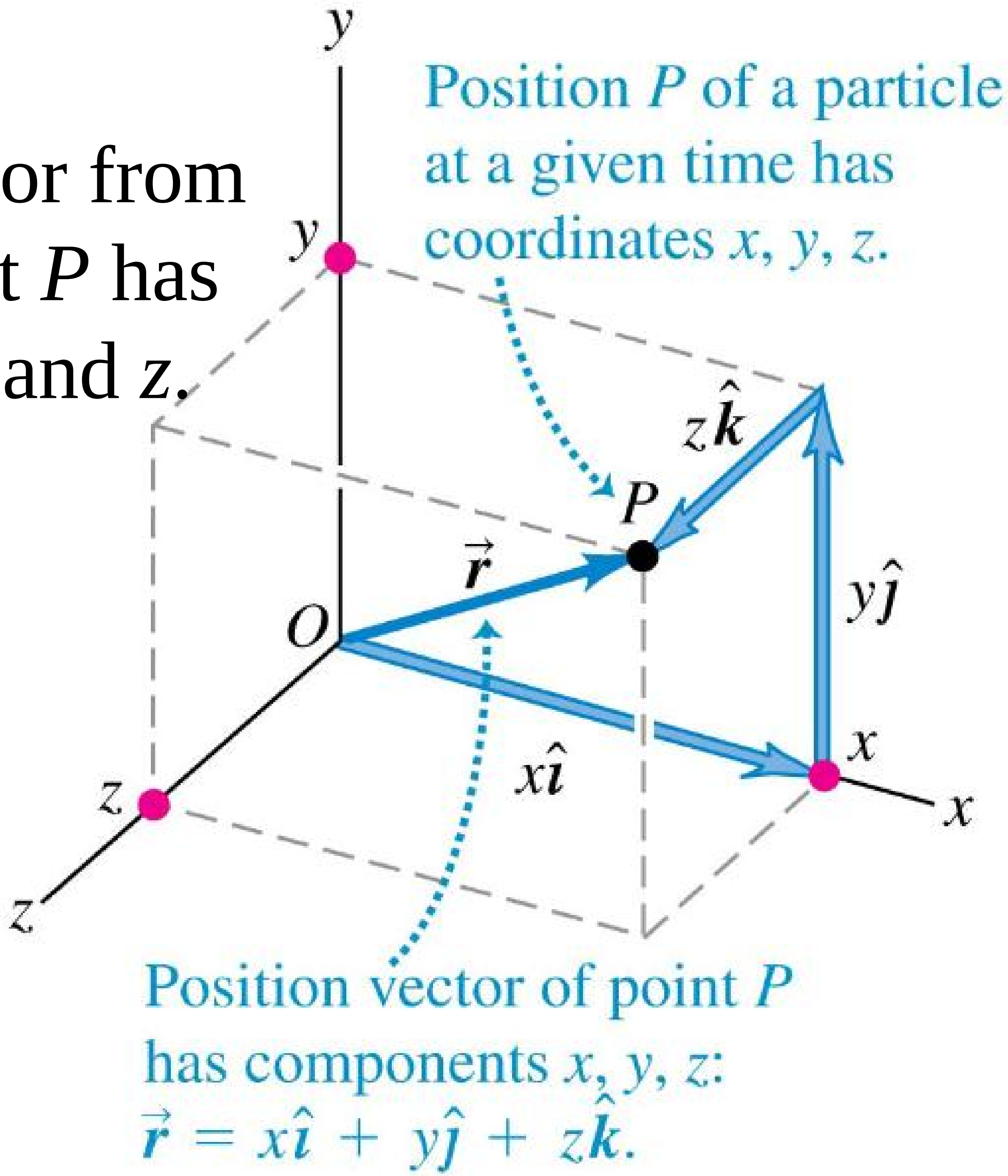
- how to analyze motion in a circular path, with either constant speed or varying speed.
- how to relate the velocities of a moving body as seen from two different frames of reference.

Introduction

- What determines where a batted baseball lands?
- How do you describe the motion of a roller coaster car along a curved track or the flight of a circling hawk?
- We need to extend our description of motion to two and three dimensions.

Position vector

The position vector from the origin to point *P* has components *x*, *y*, and *z*.



Velocity

- We define the **average velocity** as the displacement divided by the time interval and it has the same direction as the displacement.

Change in the particle's position vector

Average velocity vector of a particle during time interval from t_1 to t_2

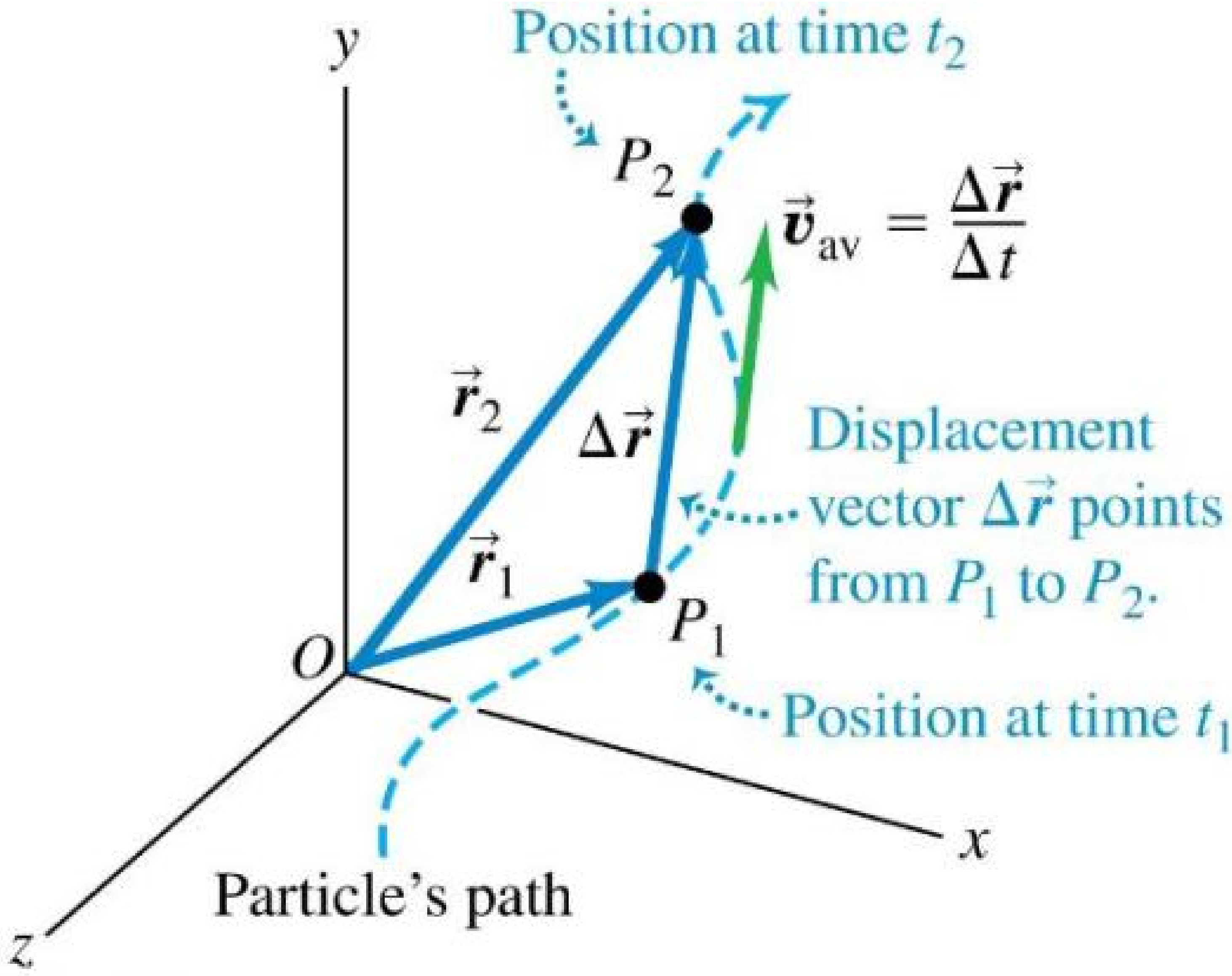
$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

Final position minus initial position

Time interval

Final time minus initial time

Average velocity



Velocity

- **Instantaneous velocity** (a.k.a. "velocity") is the instantaneous rate of change of position with time:

The **instantaneous velocity vector** of a particle ...

$$\vec{\boldsymbol{v}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\boldsymbol{r}}}{\Delta t} = \frac{d\vec{\boldsymbol{r}}}{dt}$$

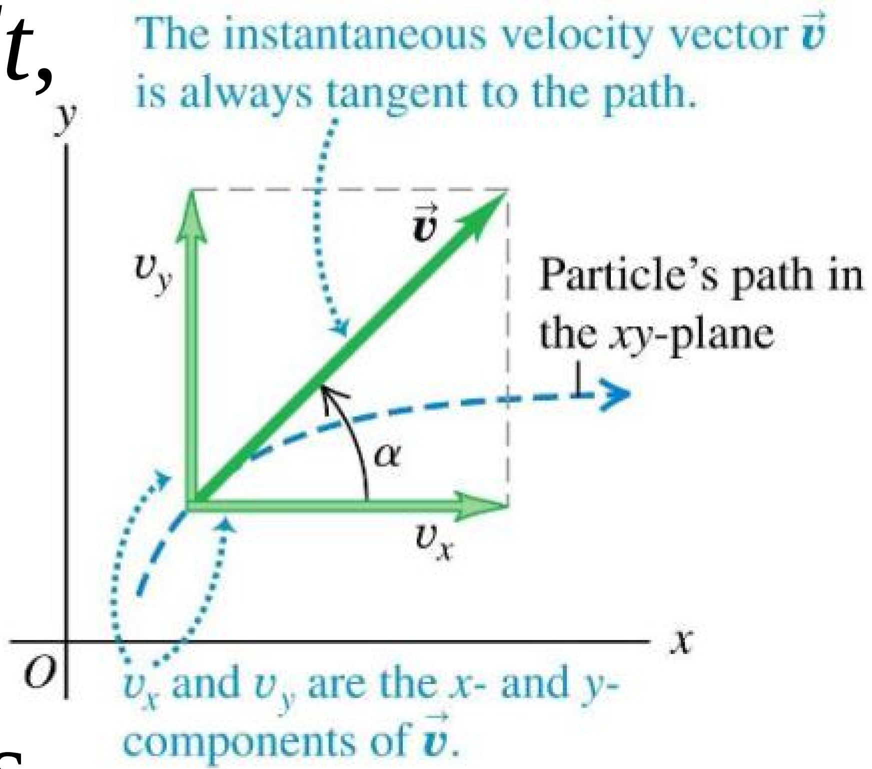
... equals the limit of its average velocity vector as the time interval approaches zero ...

... and equals the instantaneous rate of change of its position vector.

Instantaneous velocity

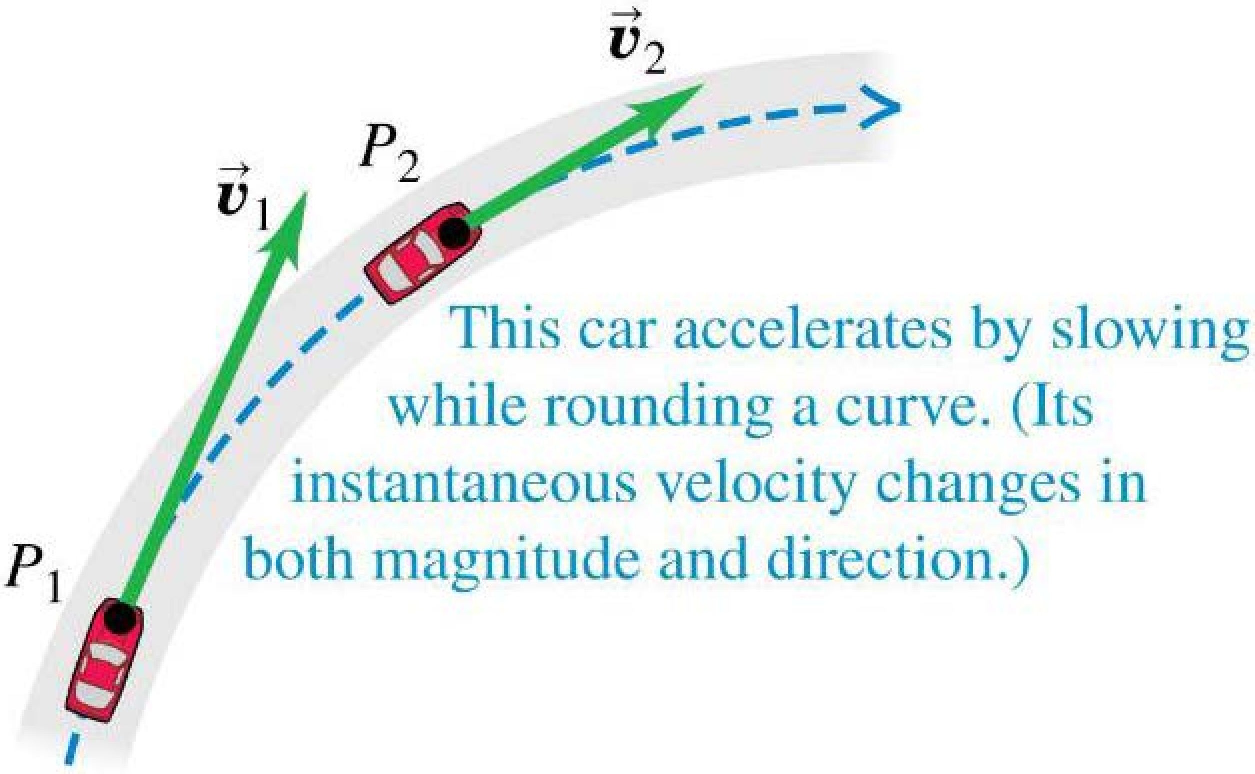
- The components of the instantaneous velocity are $v_x = dx/dt$, $v_y = dy/dt$, and $v_z = dz/dt$.

- The instantaneous velocity of a particle is always tangent to its path.



Acceleration

- Acceleration describes how the velocity changes.



Average acceleration

- The change in velocity between two points is determined by vector subtraction.
- We define the **average acceleration** as the change in velocity divided by the time interval:

Change in the particle's velocity

Average acceleration vector of a particle during time interval from t_1 to t_2

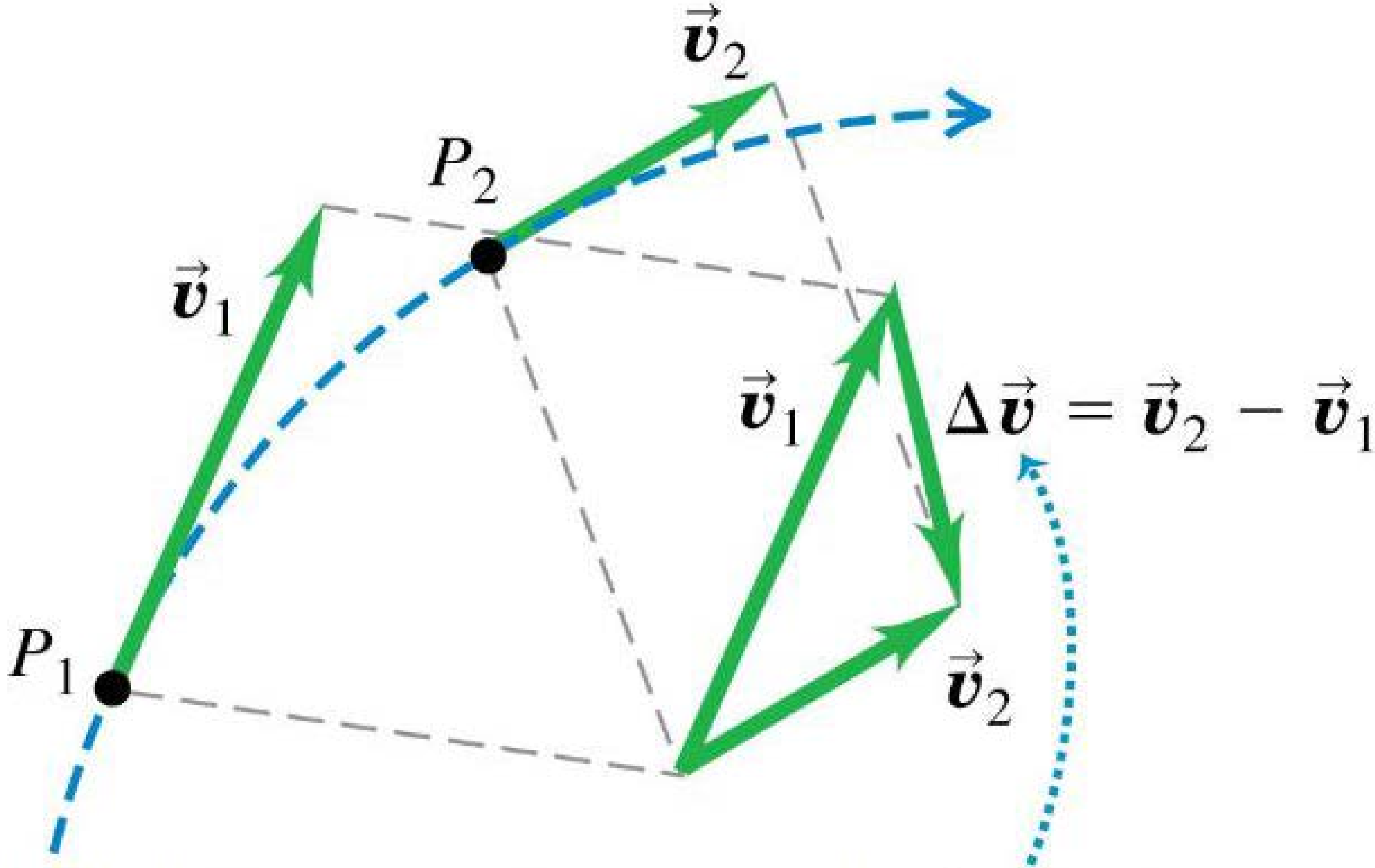
$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

Final velocity minus initial velocity

Time interval

Final time minus initial time

Average acceleration



To find the car's average acceleration between P_1 and P_2, we first find the change in velocity $\Delta\vec{v}$ by subtracting $\vec{v}_1$ from $\vec{v}_2$. (Notice that $\vec{v}_1 + \Delta\vec{v} = \vec{v}_2$.)

Average acceleration

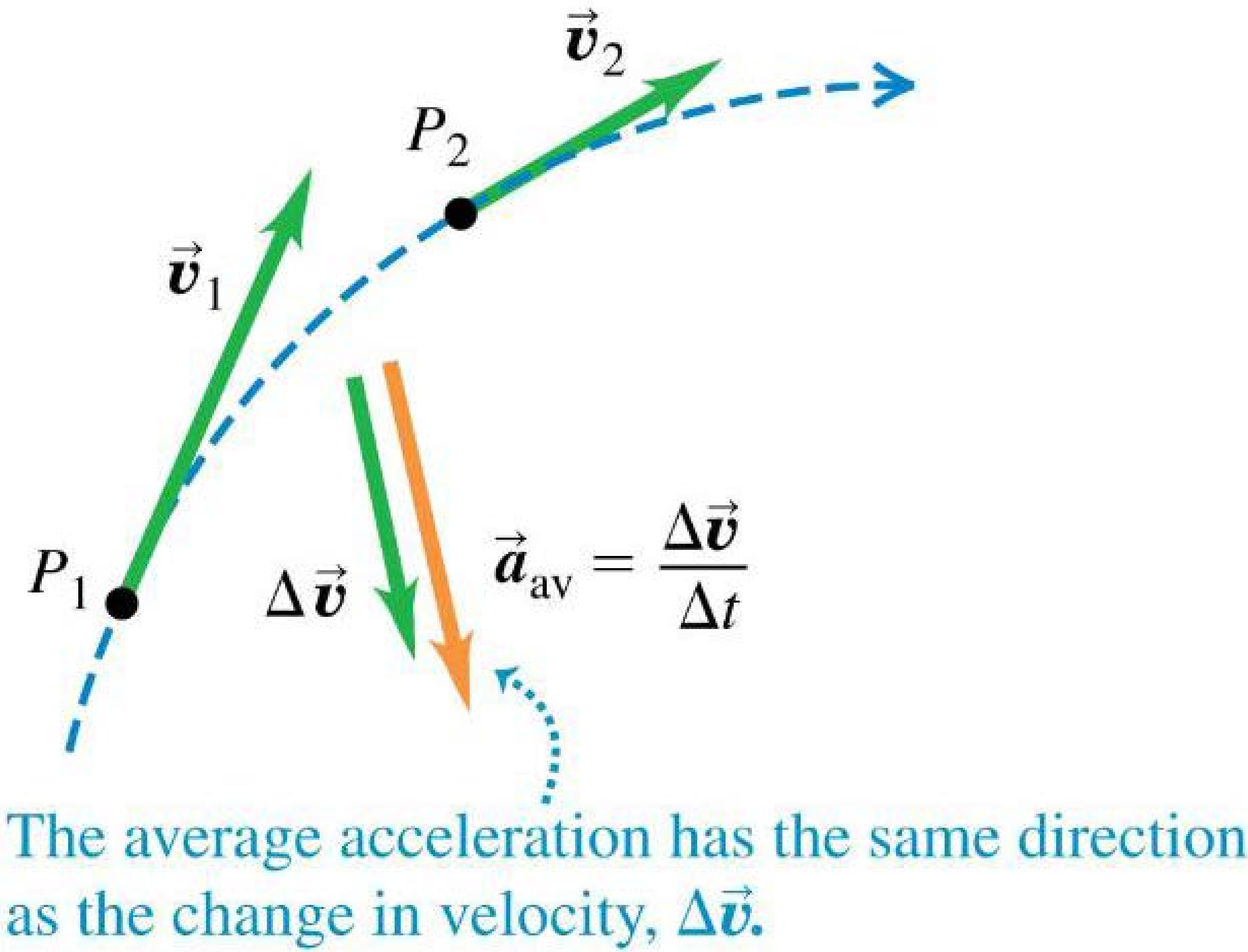
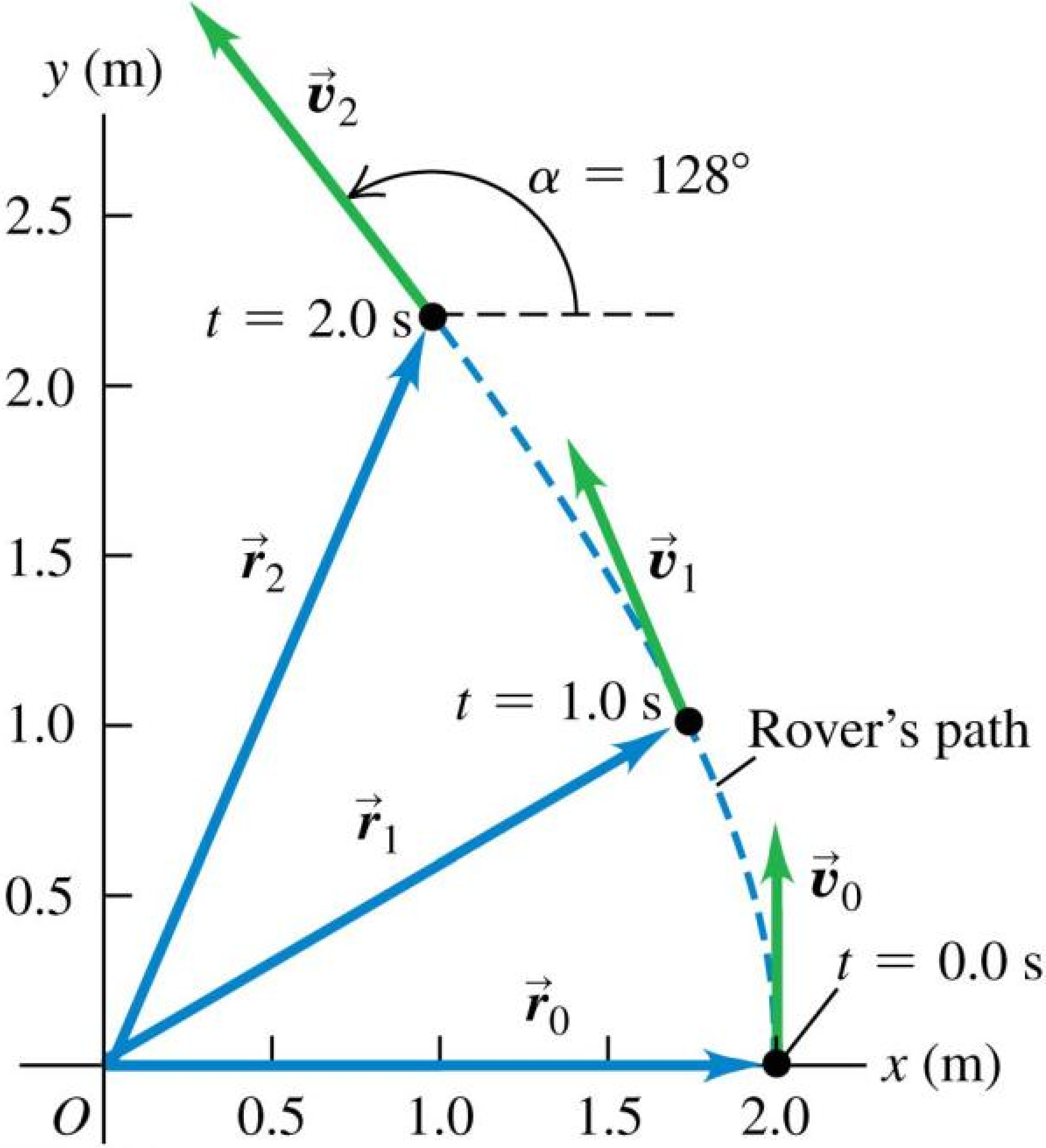


Figure 3.5



y (m)
$\vec{v}_2$
$\alpha = 128°$
2.5
$t = 2.0$ s
2.0
1.5
$\vec{r}_2$
$\vec{v}_1$
$t = 1.0$ s
1.0
Rover's path
$\vec{r}_1$
0.5
$\vec{v}_0$
$t = 0.0$ s
$\vec{r}_0$
x (m)
O
0.5
1.0
1.5
2.0

Instantaneous acceleration

- The velocity vector is always tangent to the particle's path, but the instantaneous acceleration vector does *not* have to be tangent to the path.

- **Instantaneous acceleration** (a.k.a. "acceleration") is the instantaneous rate of change of velocity with time:

The **instantaneous acceleration vector** of a particle ...

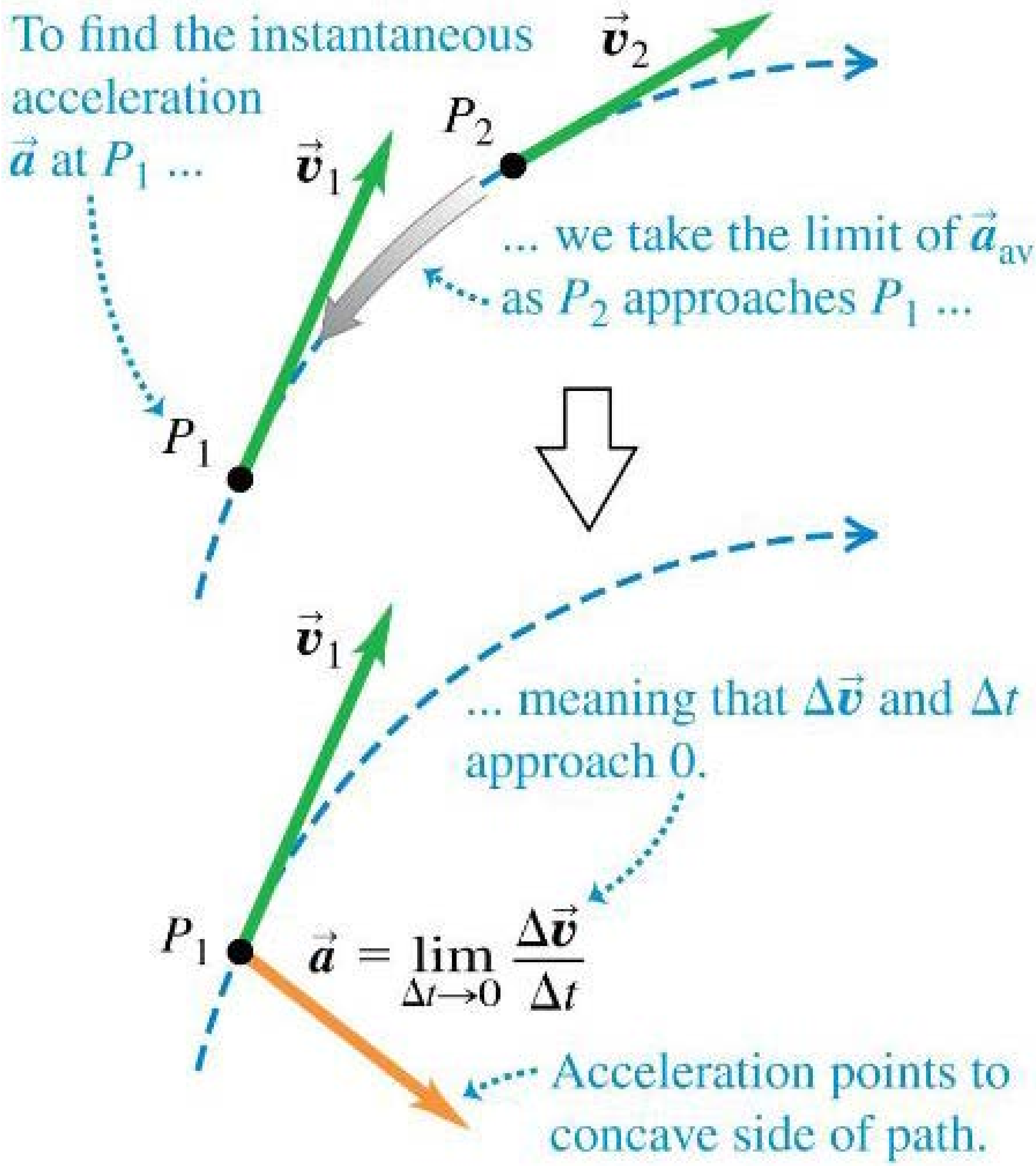
$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

... equals the limit of its average acceleration vector as the time interval approaches zero ...

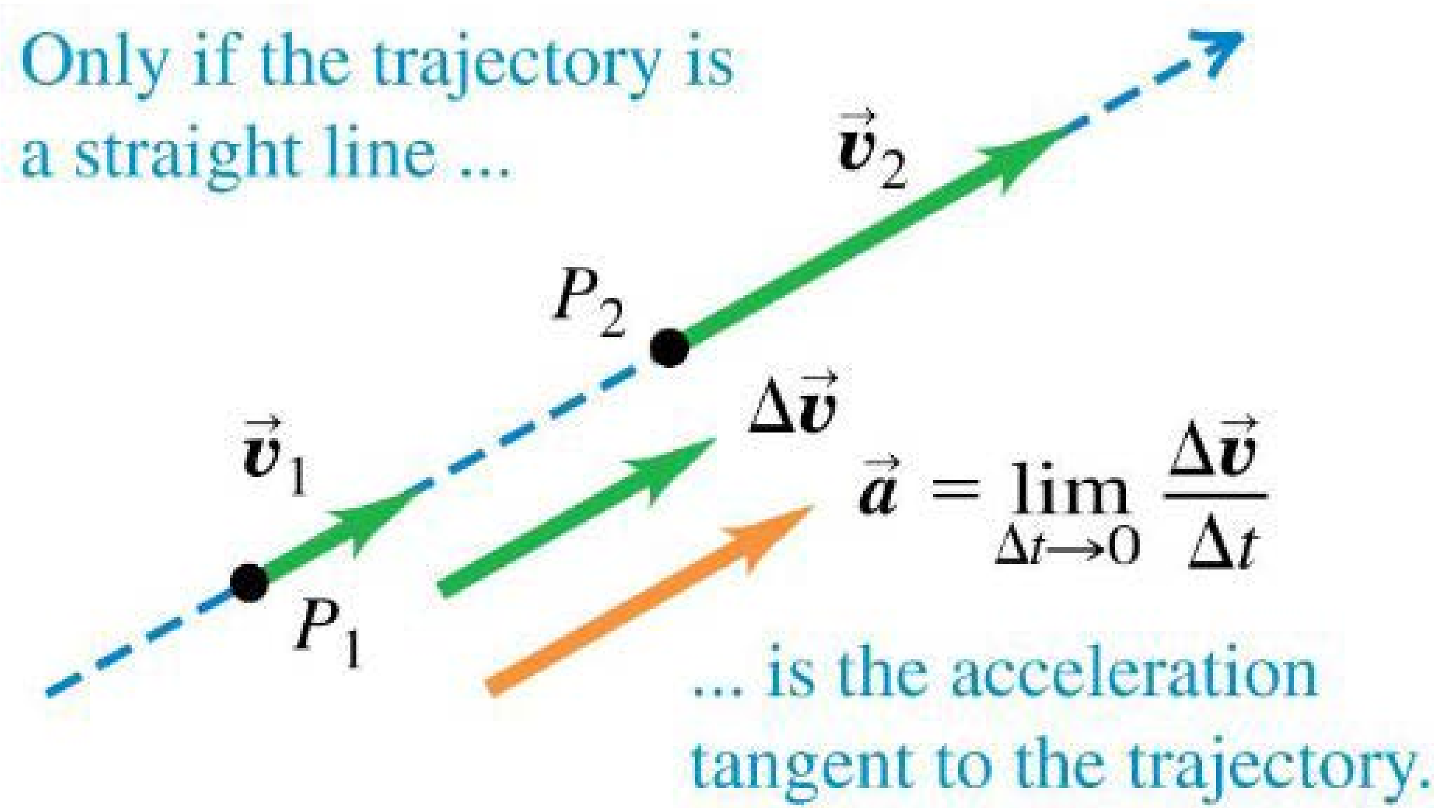
... and equals the instantaneous rate of change of its velocity vector.

Instantaneous acceleration

- If the path is curved, the acceleration points toward the concave side of the path.



Instantaneous acceleration



Components of acceleration

Each **component of** a particle's **instantaneous acceleration vector** ...

$$a_x = \frac{dv_x}{dt} \qquad a_y = \frac{dv_y}{dt} \qquad a_z = \frac{dv_z}{dt}$$

... equals the instantaneous rate of change of its corresponding velocity component.

- Shooting an arrow is an example of an acceleration vector that has both *x*- and *y*-components.

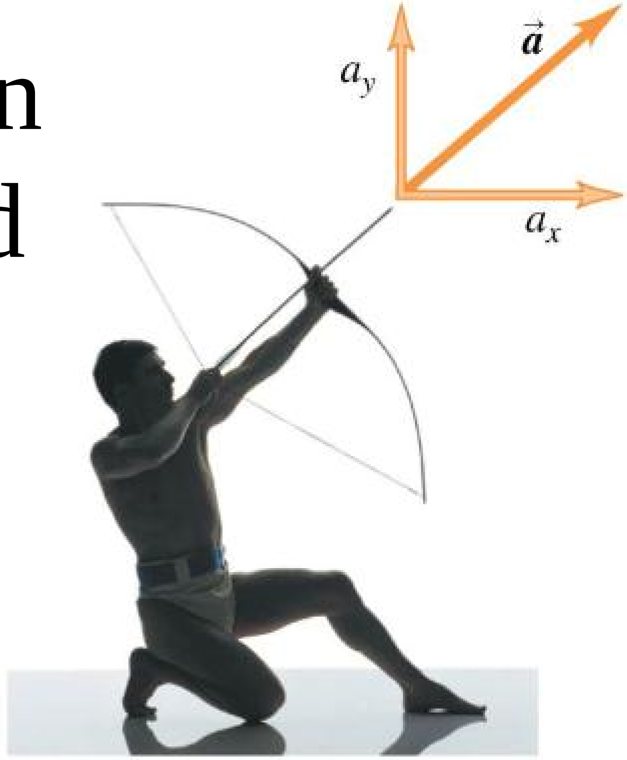


Figure 3.10

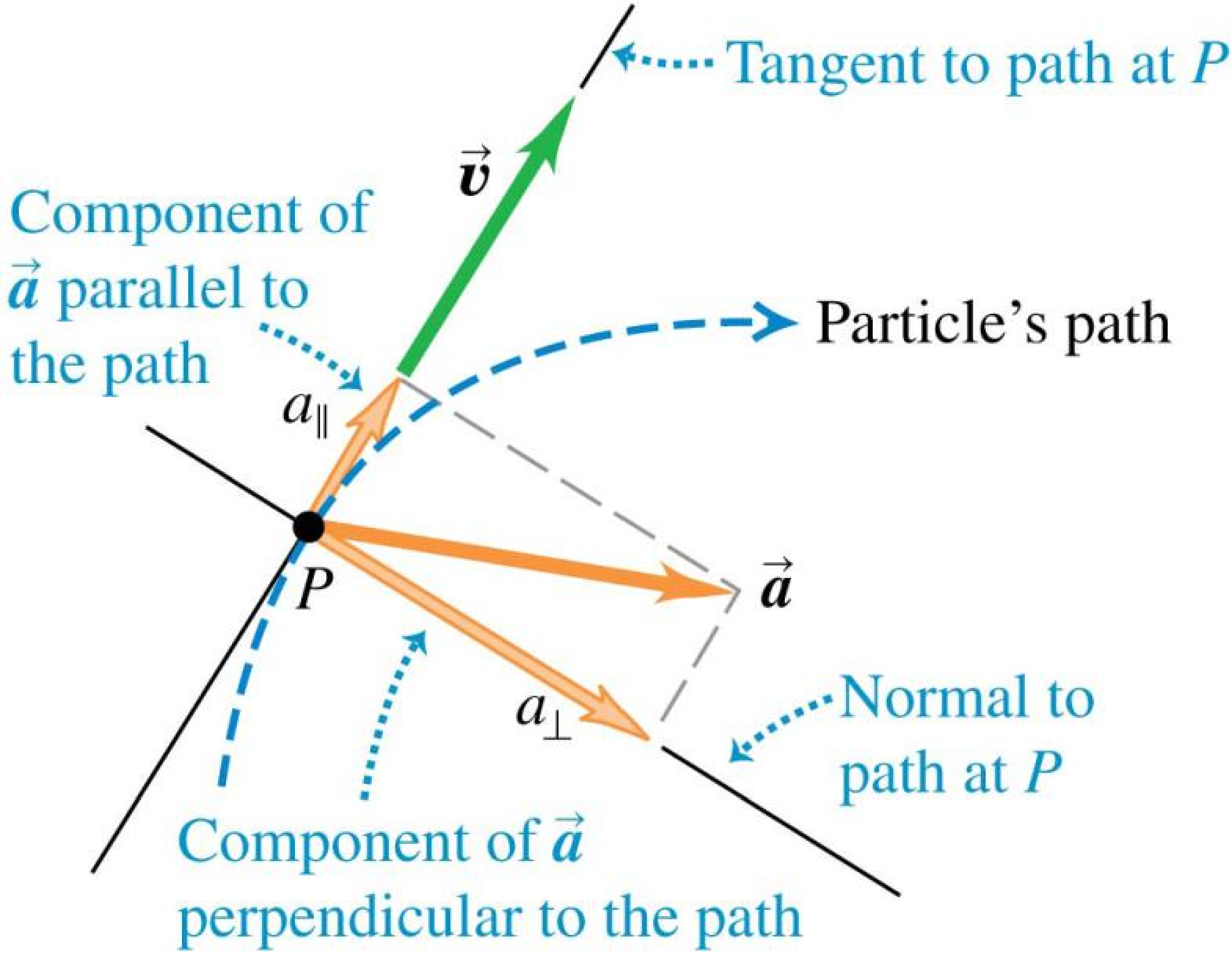

Tangent to path at P
$\vec{v}$
Component of
$\vec{a}$ parallel to
the path
Particle's path
$a_{\parallel}$
P
$\vec{a}$
$a_{\perp}$
Normal to
path at P
Component of $\vec{a}$
perpendicular to the path

Figure 3.11a

(a) Acceleration parallel to velocity

Changes only *magnitude* of velocity: speed changes; direction doesn't.

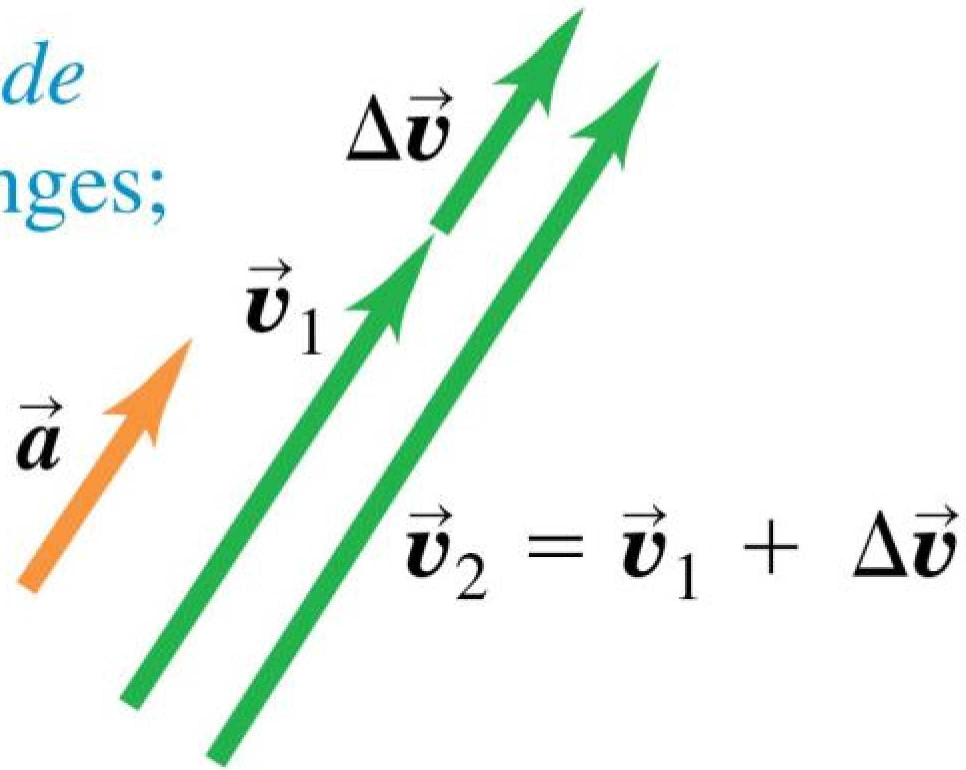
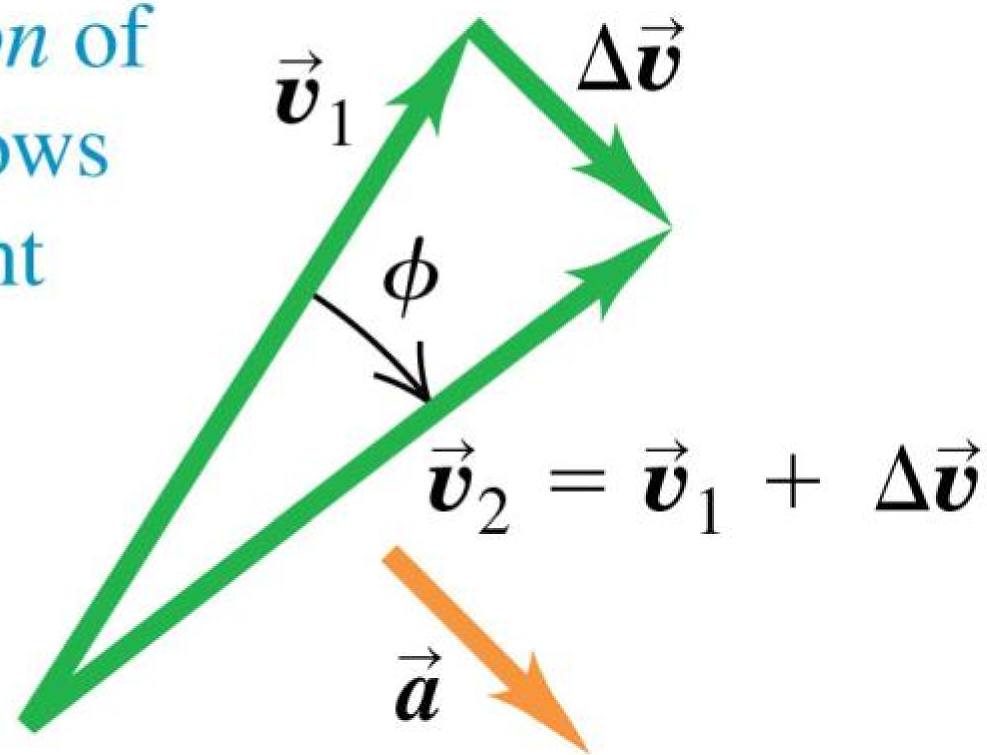


Figure 3.11b

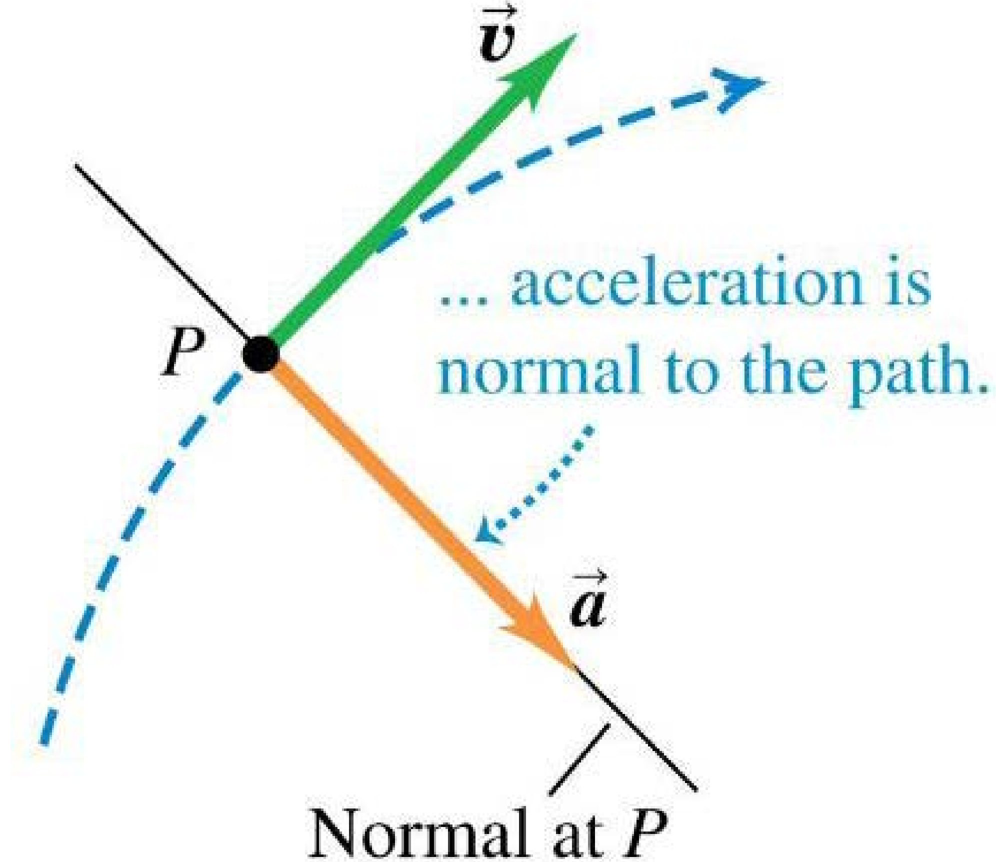
(b) Acceleration perpendicular to velocity

Changes only *direction* of velocity: particle follows curved path at constant speed.



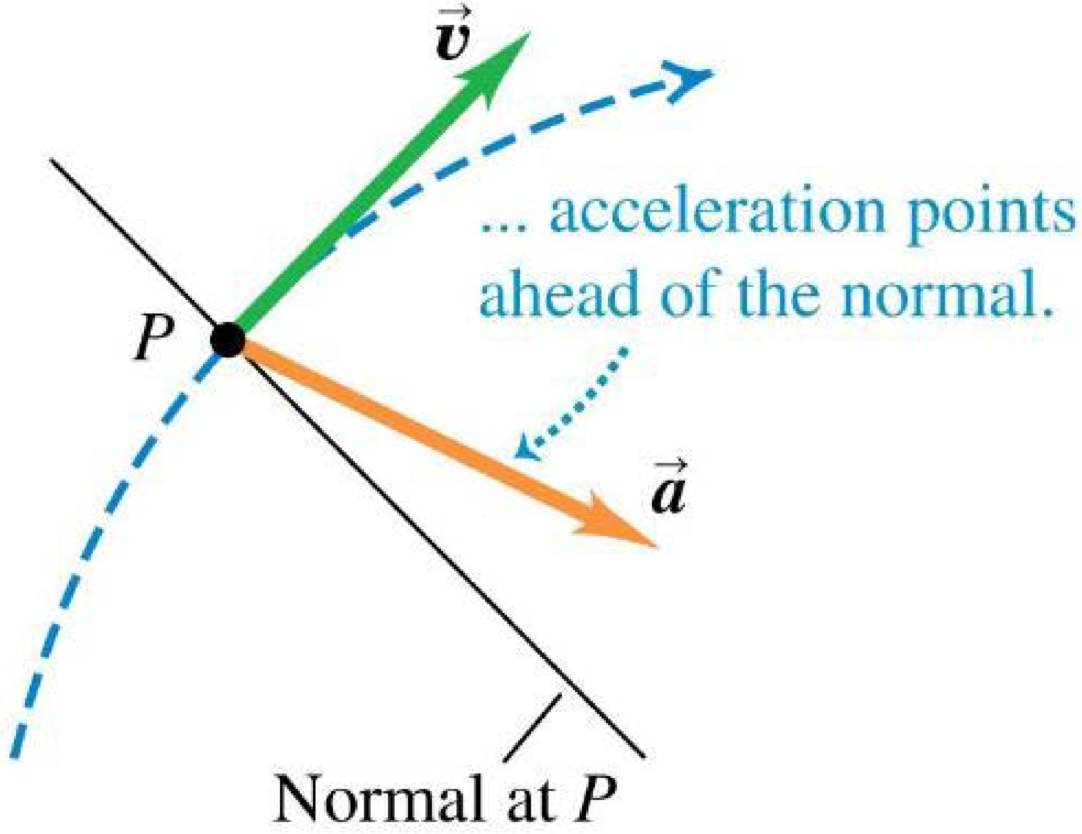
Parallel and perpendicular components of acceleration

Velocity and acceleration vectors for a particle moving through a point *P* on a curved path with *constant speed*



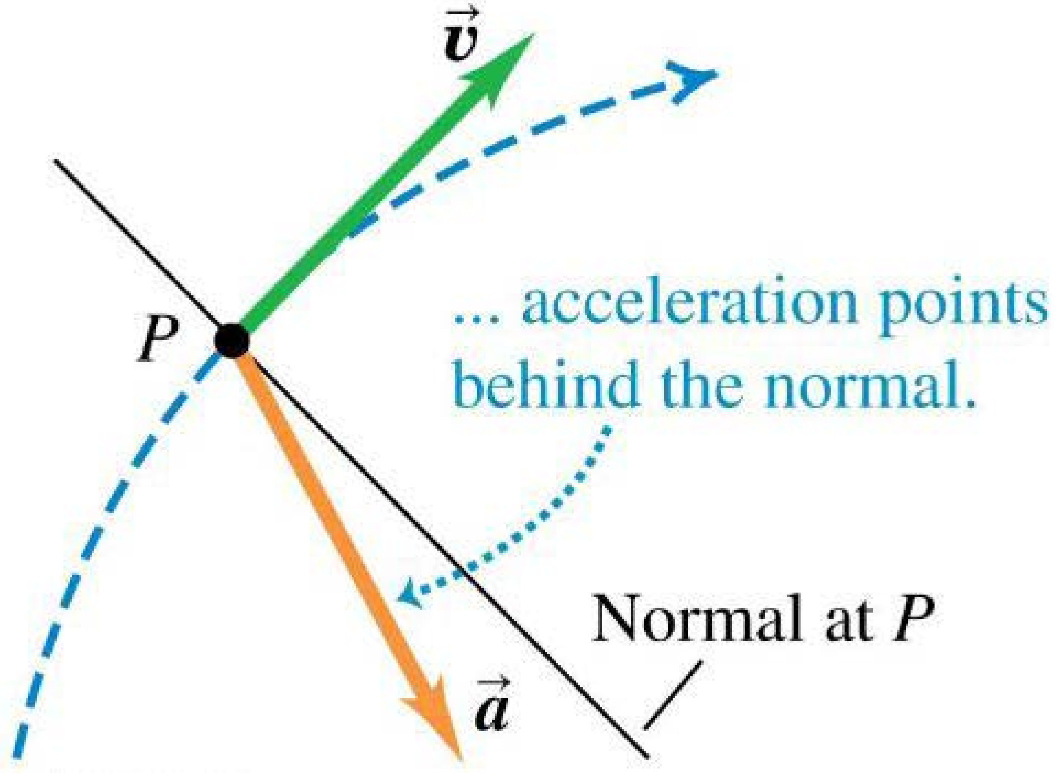
Parallel and perpendicular components of acceleration

Velocity and acceleration vectors for a particle moving through a point *P* on a curved path with *increasing speed* (v_2 *speed* > v_1 *speed)*



Parallel and perpendicular components of acceleration

Velocity and acceleration vectors for a particle moving through a point *P* on a curved path with *decreasing speed* (v_2 *speed* < v_1 *speed*)

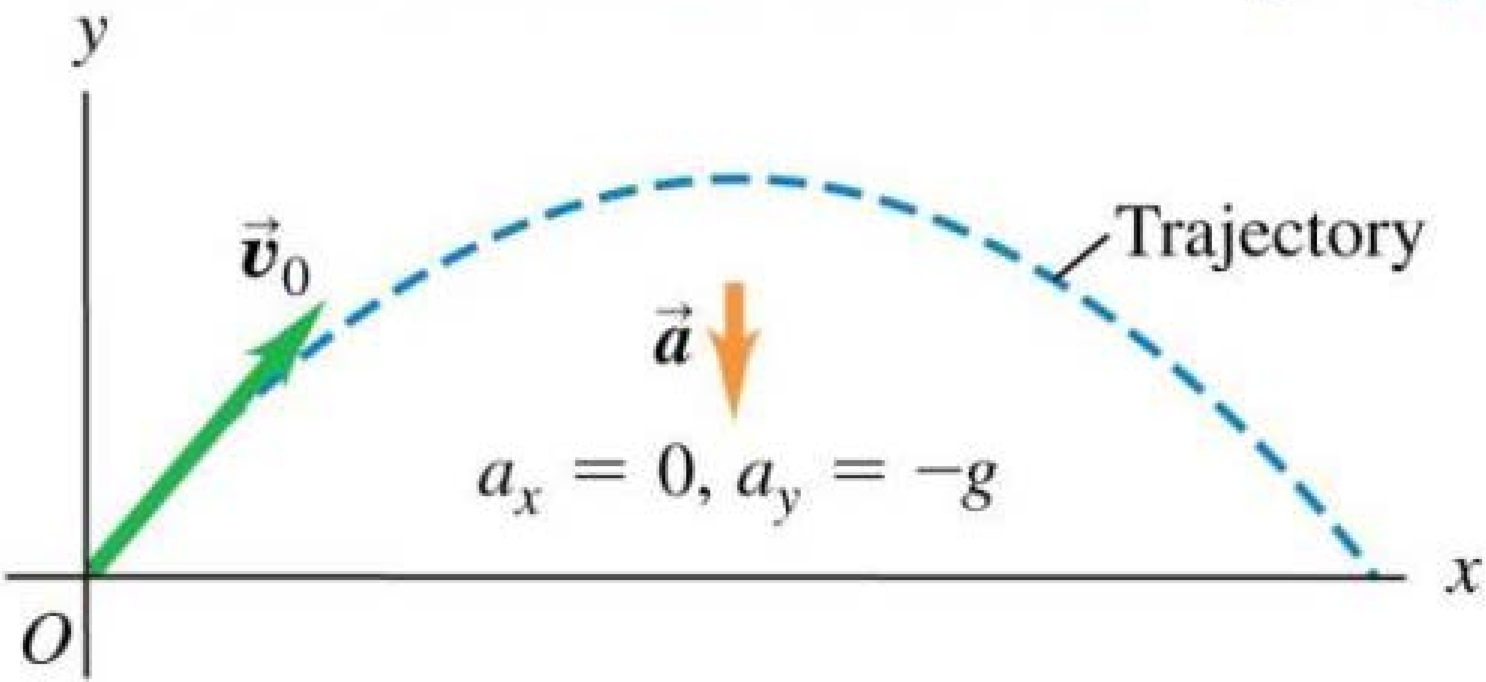


Projectile motion

- A **projectile** is any body given an initial velocity that then follows a path determined by the effects of gravity and air resistance.
- Begin by neglecting resistance and the curvature and rotation of the earth.

Projectile motion (2-D)

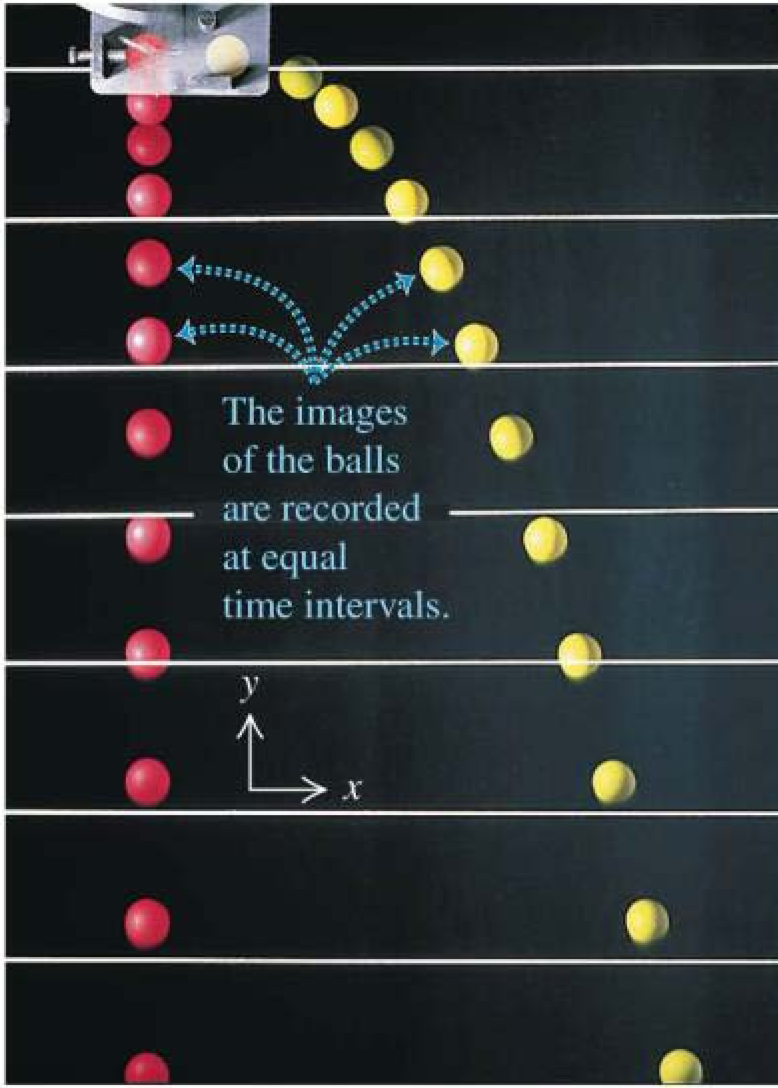
- A projectile moves in a vertical plane that contains the initial velocity vector $\vec{v}_0$.
- Its trajectory depends only on $\vec{v}_0$ and on the downward acceleration due to gravity.



The *x*- and *y*-motion are separable

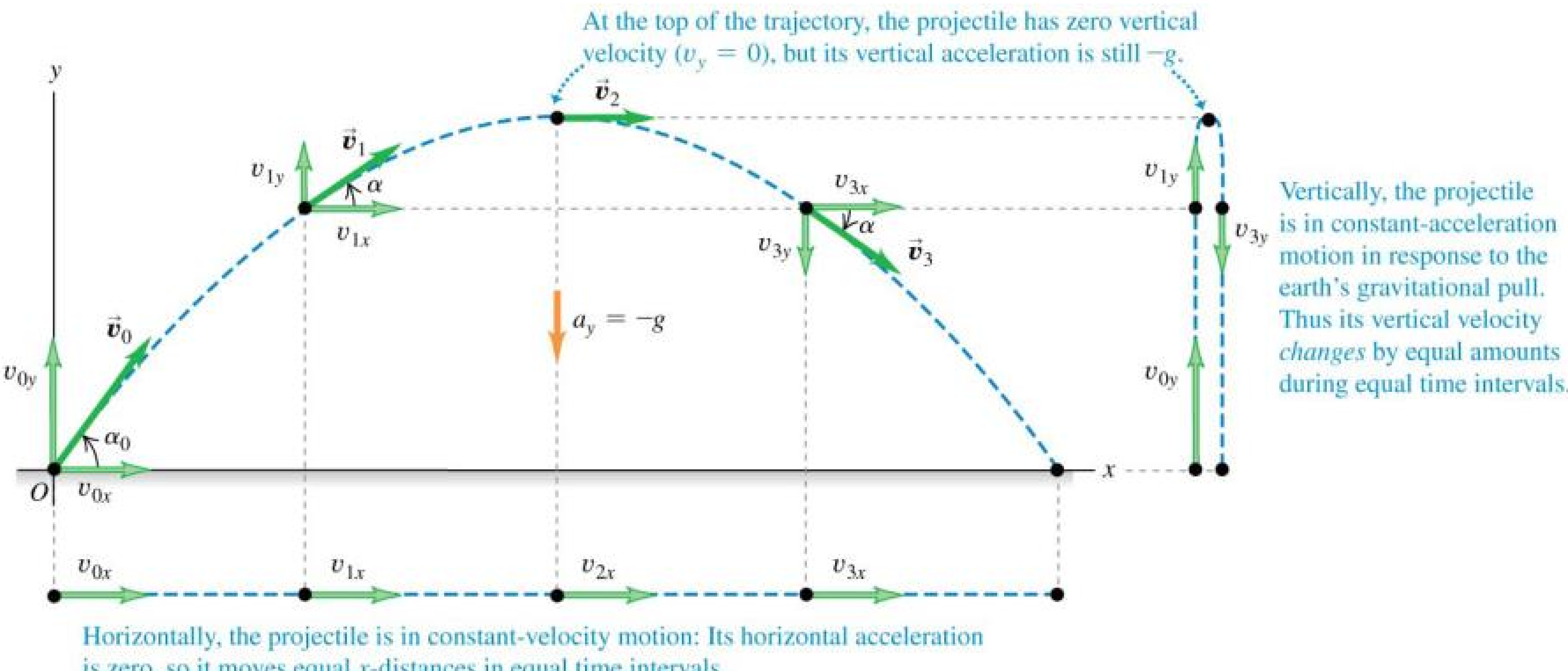
- The red ball is dropped at the same time that the yellow ball is fired horizontally. The strobe marks equal time intervals.

- We can analyze projectile motion as horizontal motion with constant velocity and vertical motion with constant acceleration: $a_x = 0$ and $a_y = -g$.



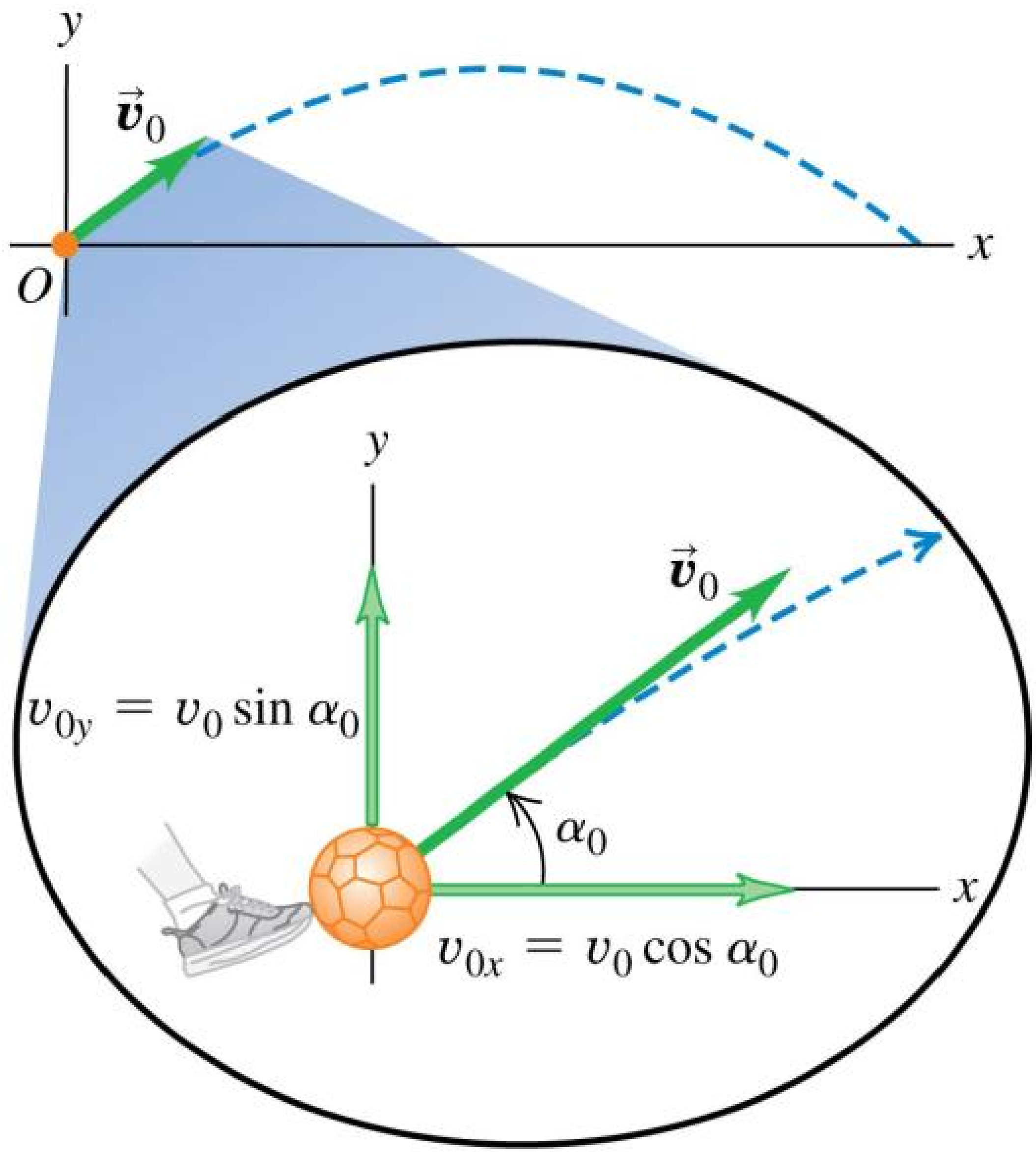
Projectile motion

- If air resistance is negligible, the trajectory of a projectile is a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.



Projectile motion – Initial velocity

The initial velocity components of a projectile (such as a kicked soccer ball) are related to the initial speed and initial angle.



The equations for projectile motion

- If we set $x_0 = y_0 = 0$, the equations describing projectile motion are shown below:

Coordinates at time t of a **projectile** (positive y-direction is upward, and $x = y = 0$ at $t = 0$)

$$x = (v_0 \cos \alpha_0)t$$

$$y = (v_0 \sin \alpha_0)t - \tfrac{1}{2}gt^2$$

Speed at $t = 0$

Direction at $t = 0$

Time

Acceleration due to gravity: Note $g > 0$.

Velocity components at time t of a **projectile** (positive y-direction is upward)

$$v_x = v_0 \cos \alpha_0$$

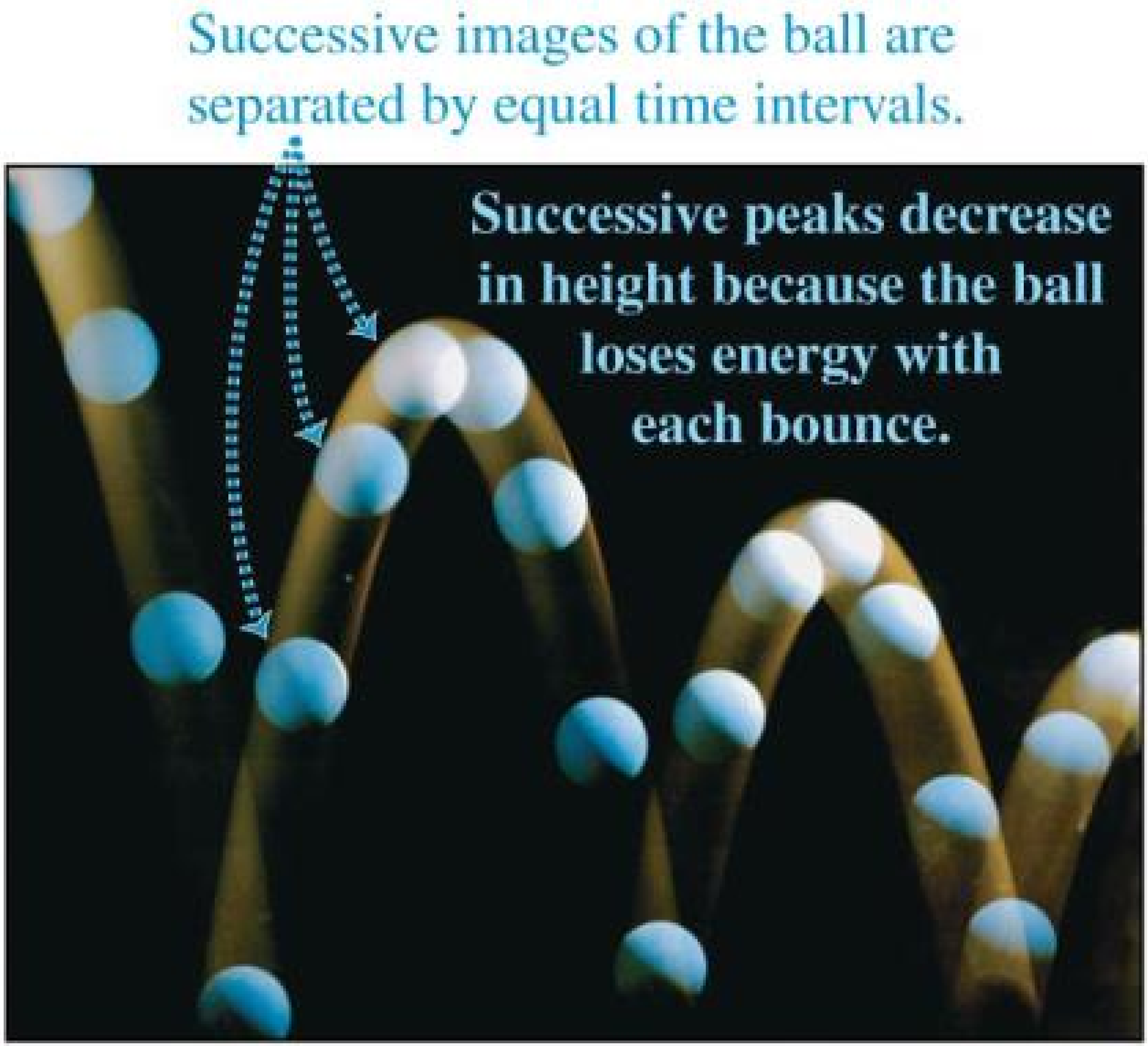
$$v_y = v_0 \sin \alpha_0 - gt$$

Speed at $t = 0$

Direction at $t = 0$

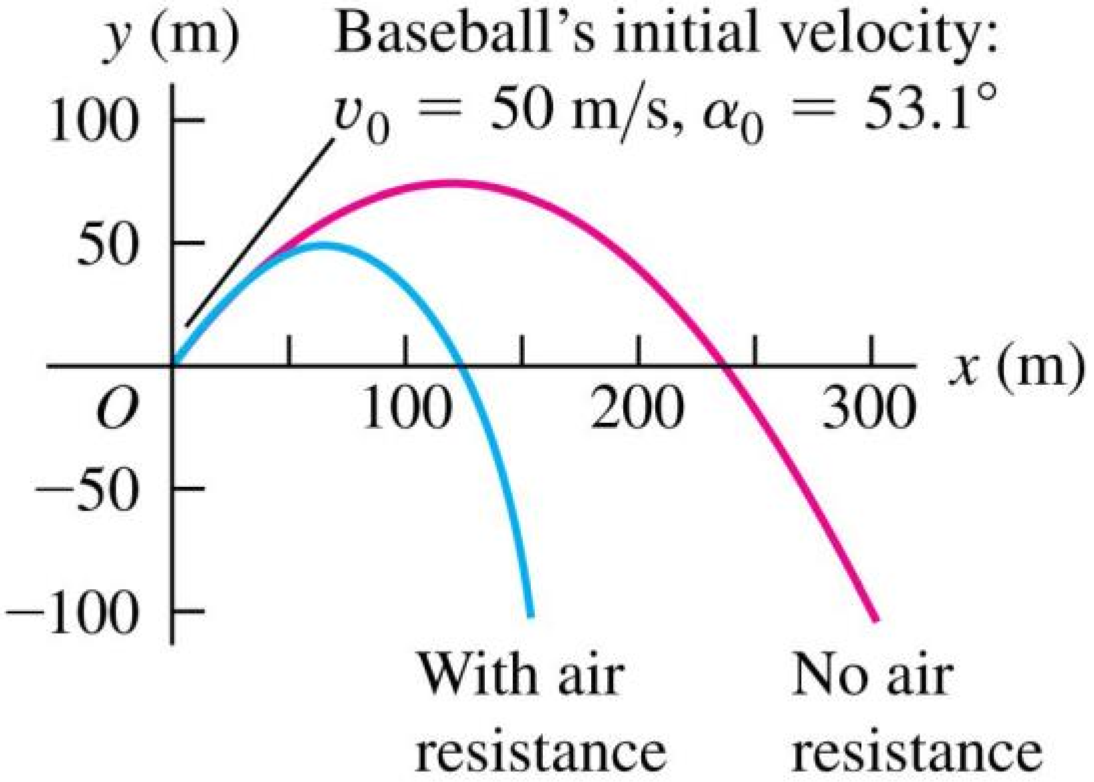
Time

Parabolic trajectories of a bouncing ball



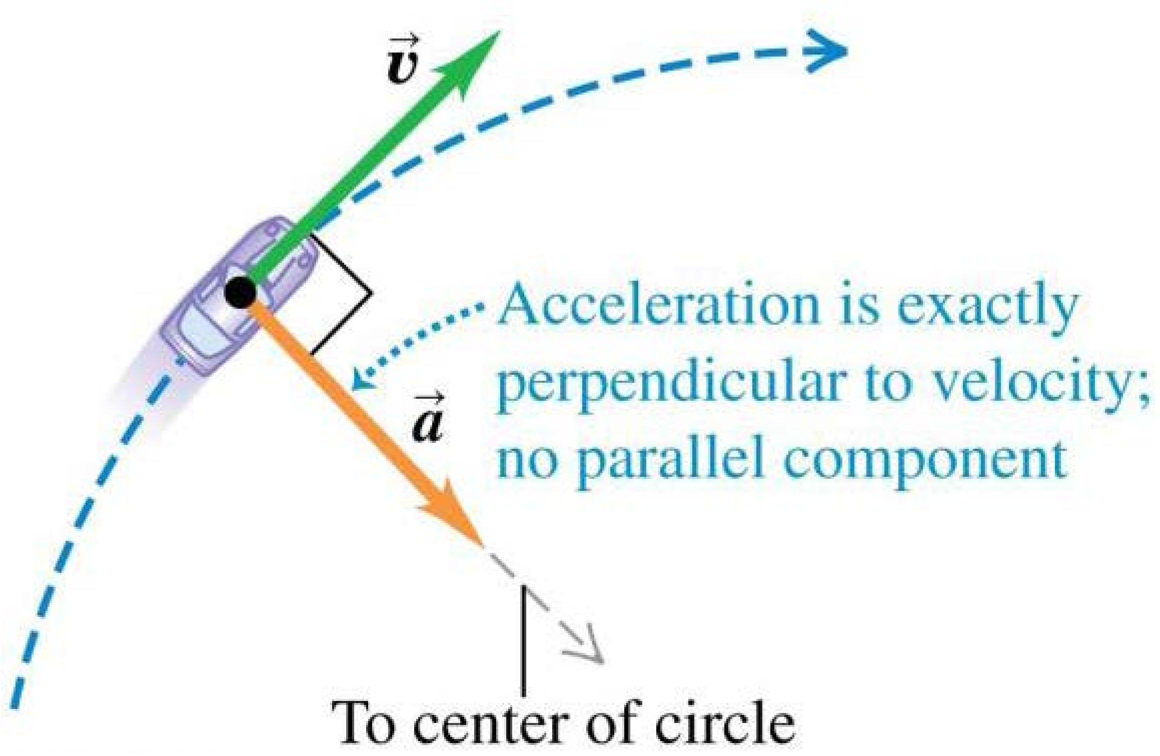
For the record: The effects of air resistance

- Calculations become more complicated.
- Acceleration is not constant.
- Effects can be very large.
- Maximum height and range decrease.
- Trajectory is no longer a parabola.



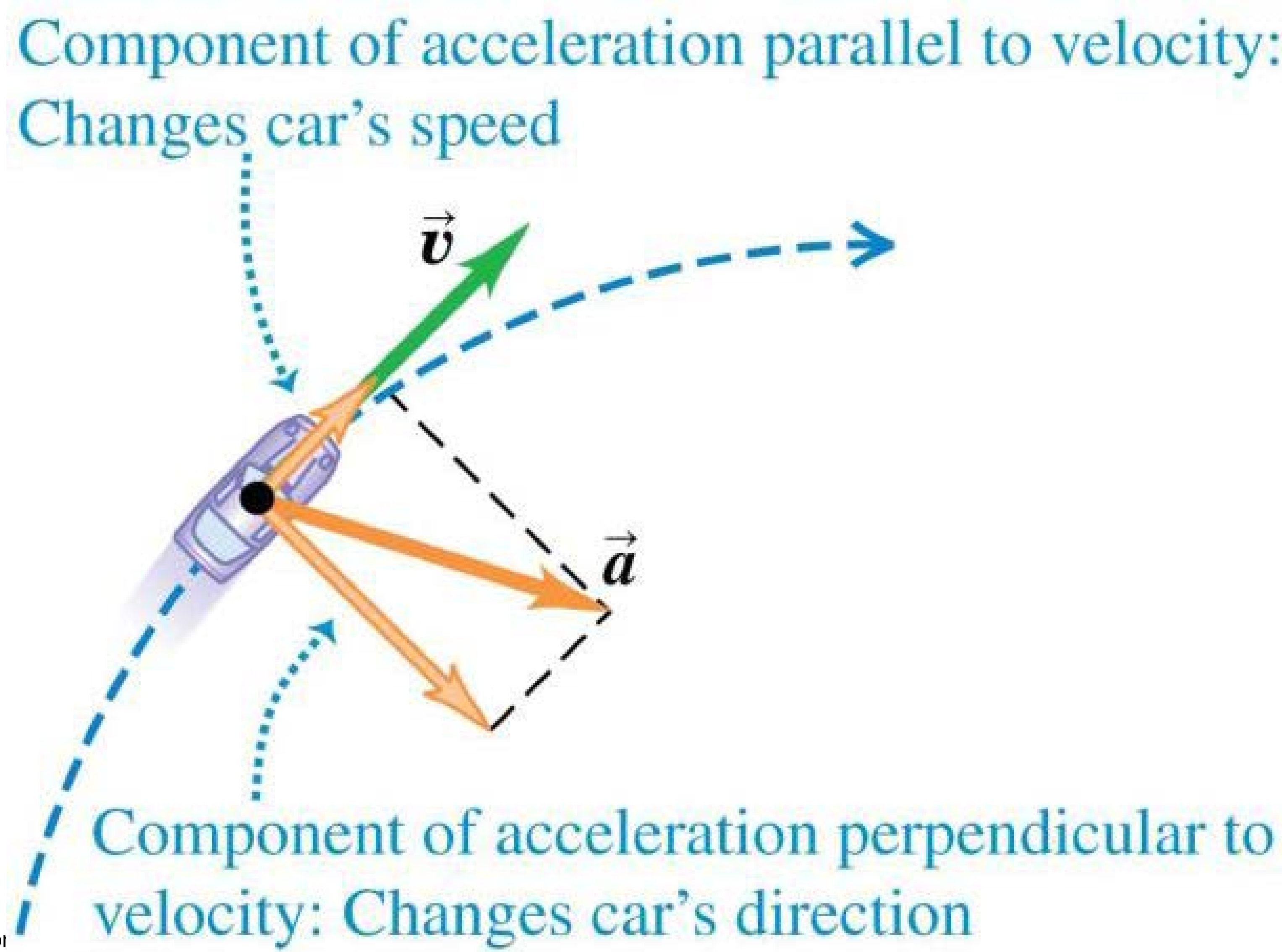
Motion in a circle

- **Uniform circular motion** is constant speed along a circular path.



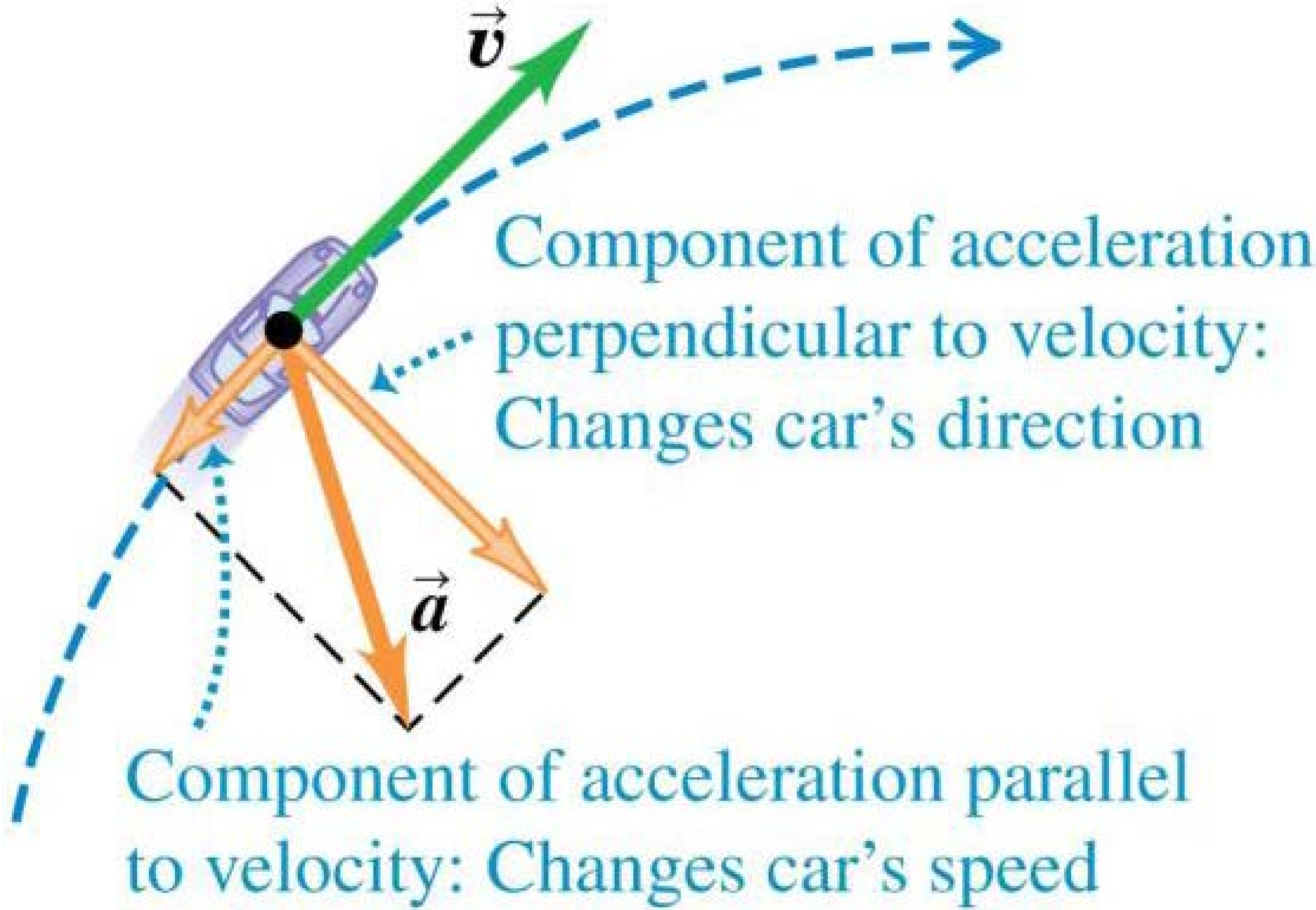
Motion in a circle

- Car speeding up along a circular path



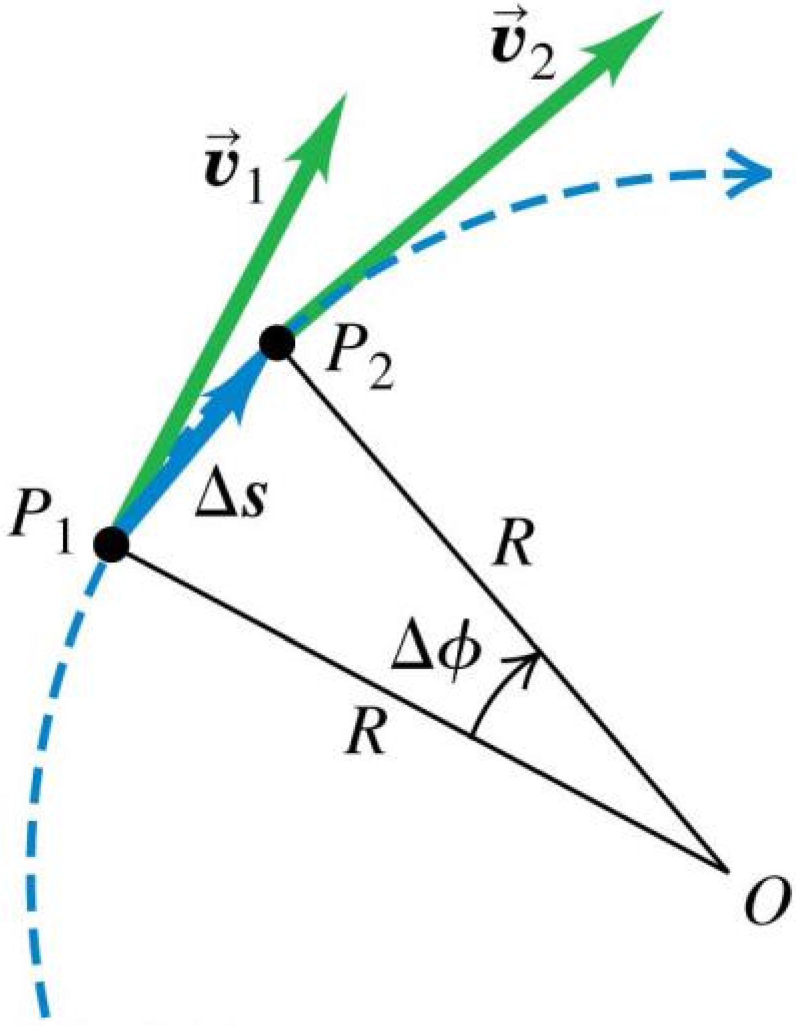
Motion in a circle

- Car slowing down along a circular path



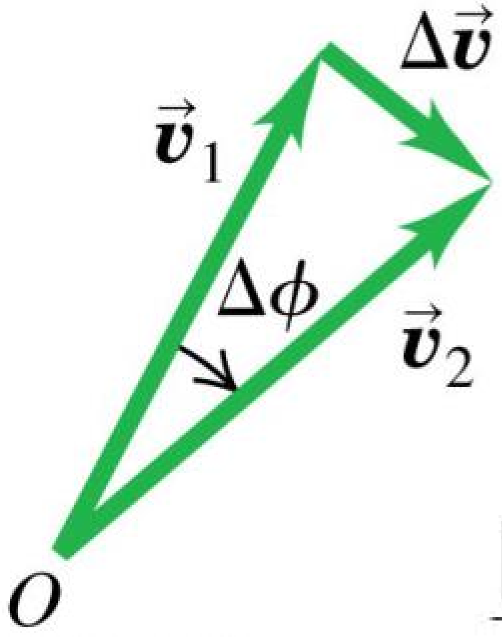
Acceleration for uniform circular motion

(a) A particle moves a distance Δs at constant speed along a circular path.



Acceleration for uniform circular motion

(b) The corresponding change in velocity and average acceleration



$$\frac{|\Delta\vec{v}|}{v_1} = \frac{\Delta s}{R} \quad \text{or} \quad |\Delta\vec{v}| = \frac{v_1}{R}\Delta s$$

Acceleration for uniform circular motion

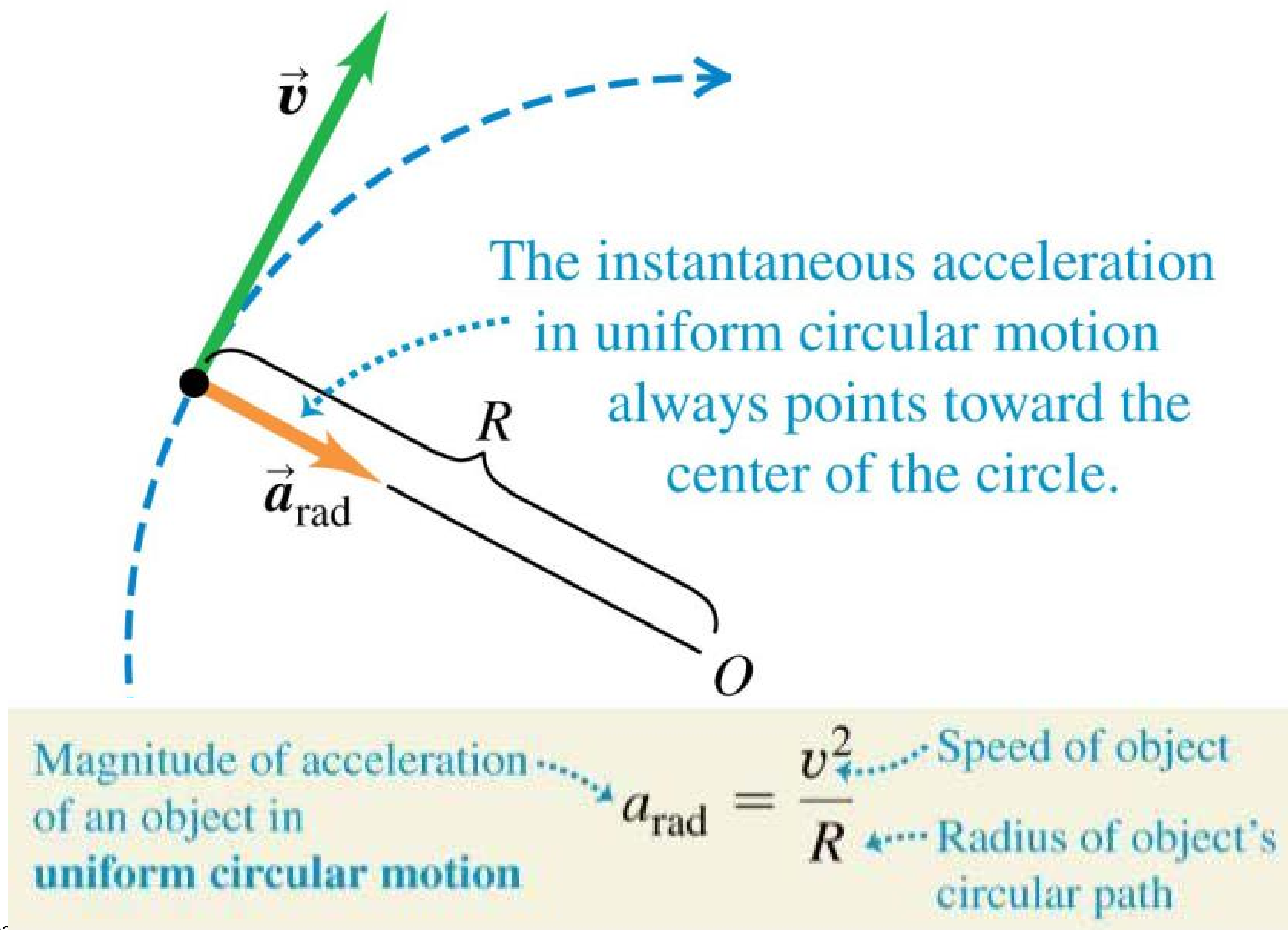
$$\frac{|\Delta\vec{v}|}{v_1} = \frac{\Delta s}{R} \qquad \text{or} \qquad |\Delta\vec{v}| = \frac{v_1}{R}\Delta s$$

$$a_{\mathrm{av}} = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v_1}{R}\frac{\Delta s}{\Delta t}$$

$$a = \lim_{\Delta t \to 0} \frac{v_1}{R}\frac{\Delta s}{\Delta t} = \frac{v_1}{R}\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t}$$

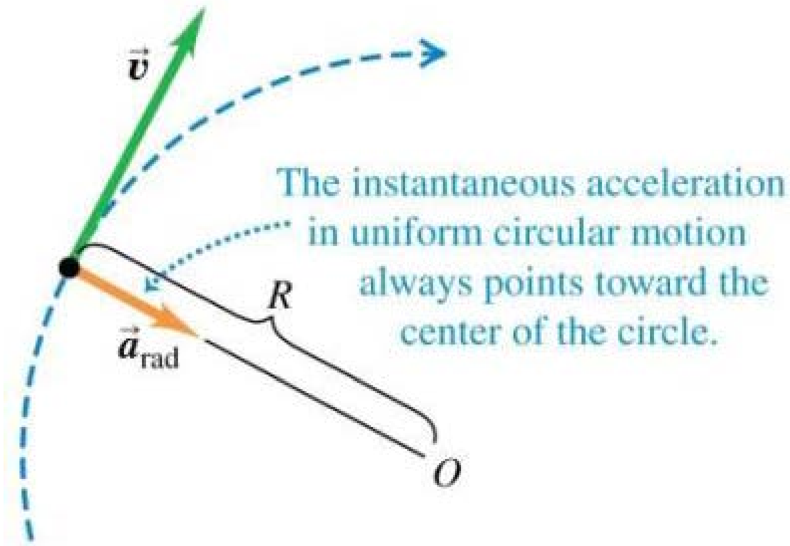
Figure 3.28c

(c) The instantaneous acceleration



Acceleration for uniform circular motion

- For uniform circular motion, the instantaneous acceleration always points toward the center of the circle and is called the **centripetal acceleration**.



- The magnitude of the acceleration is $a_{rad} = v^2/R$.

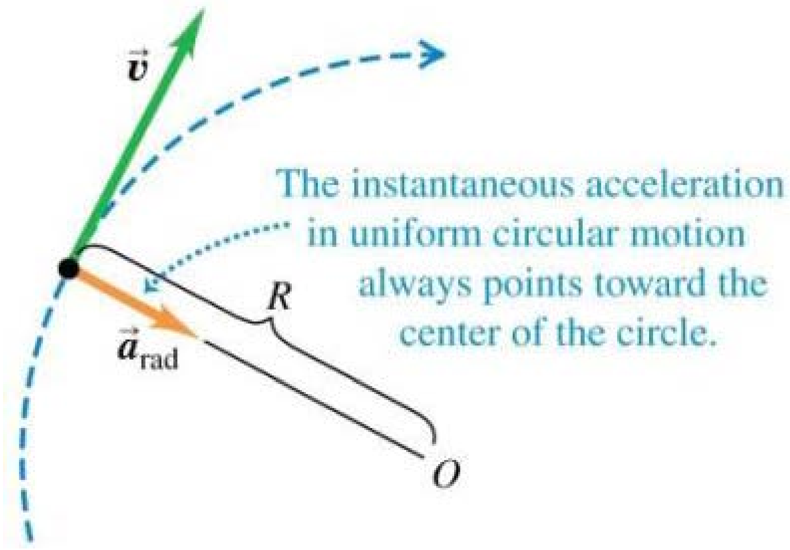
Acceleration for uniform circular motion

- The magnitude of the acceleration is $a_{\text{rad}} = v^2/R$.
- The *period T* is the time for one revolution, and as

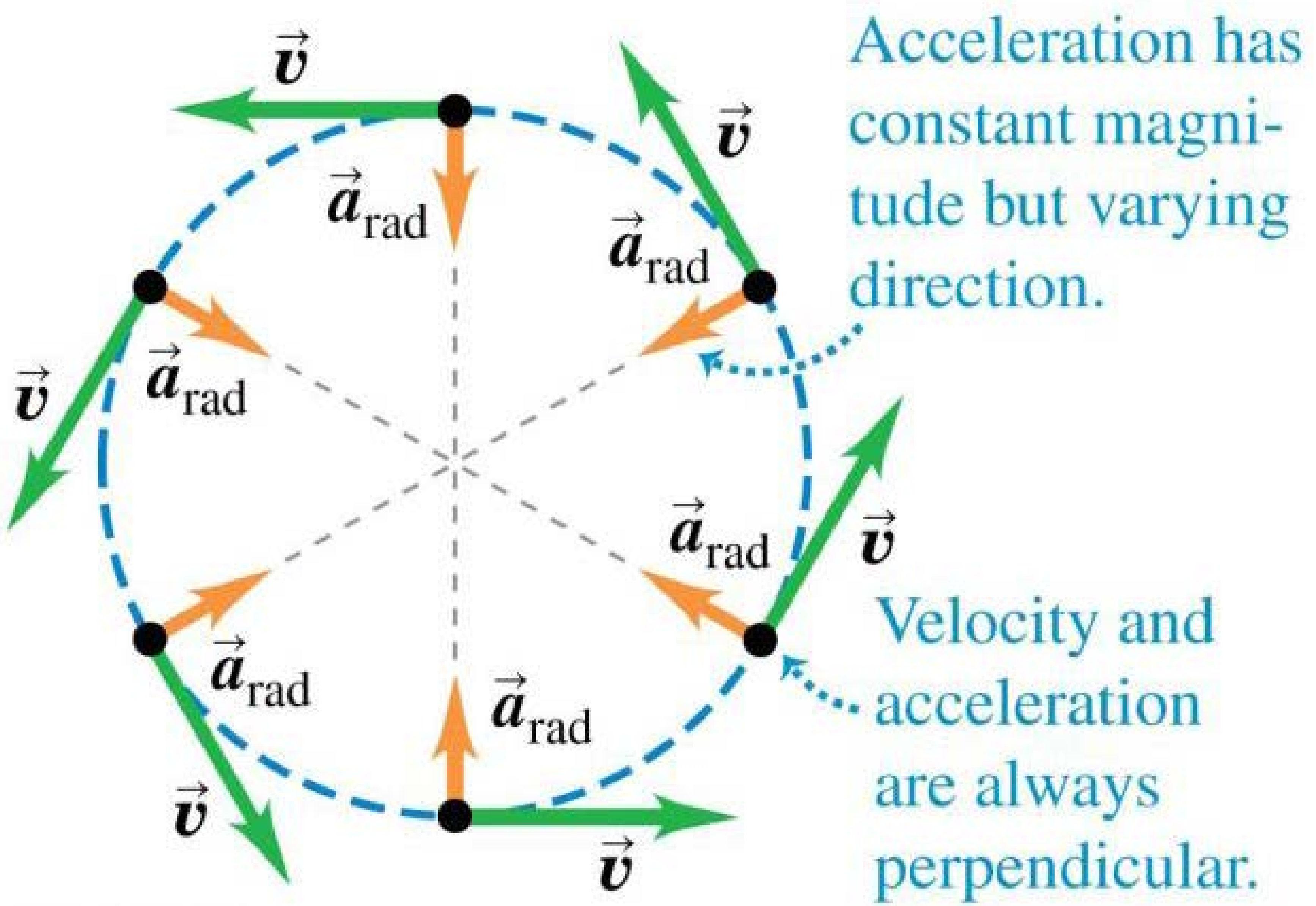
$$v = 2\pi R/T$$

then

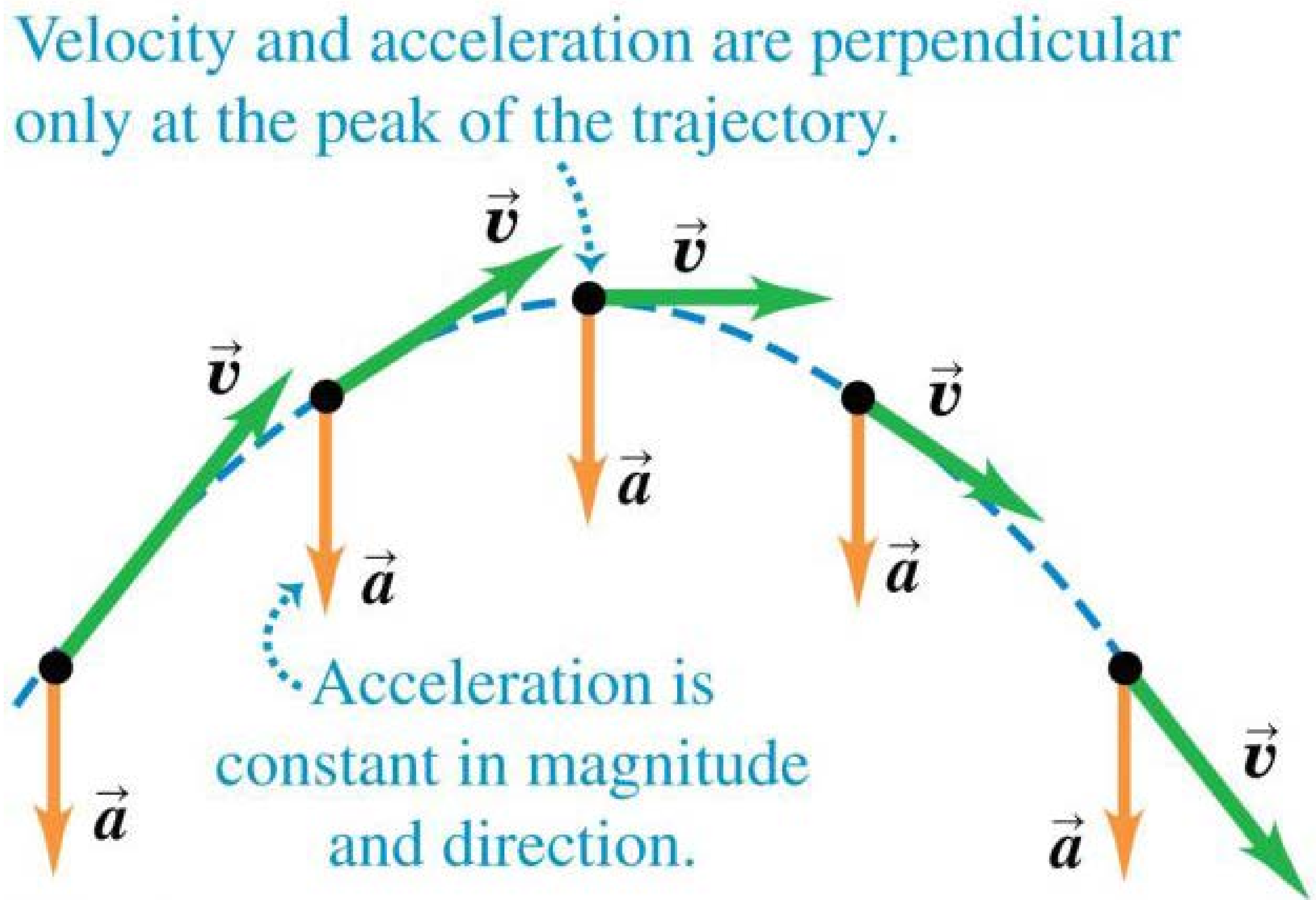
$$a_{\text{rad}} = 4\pi^2 R/T^2.$$



Uniform circular motion



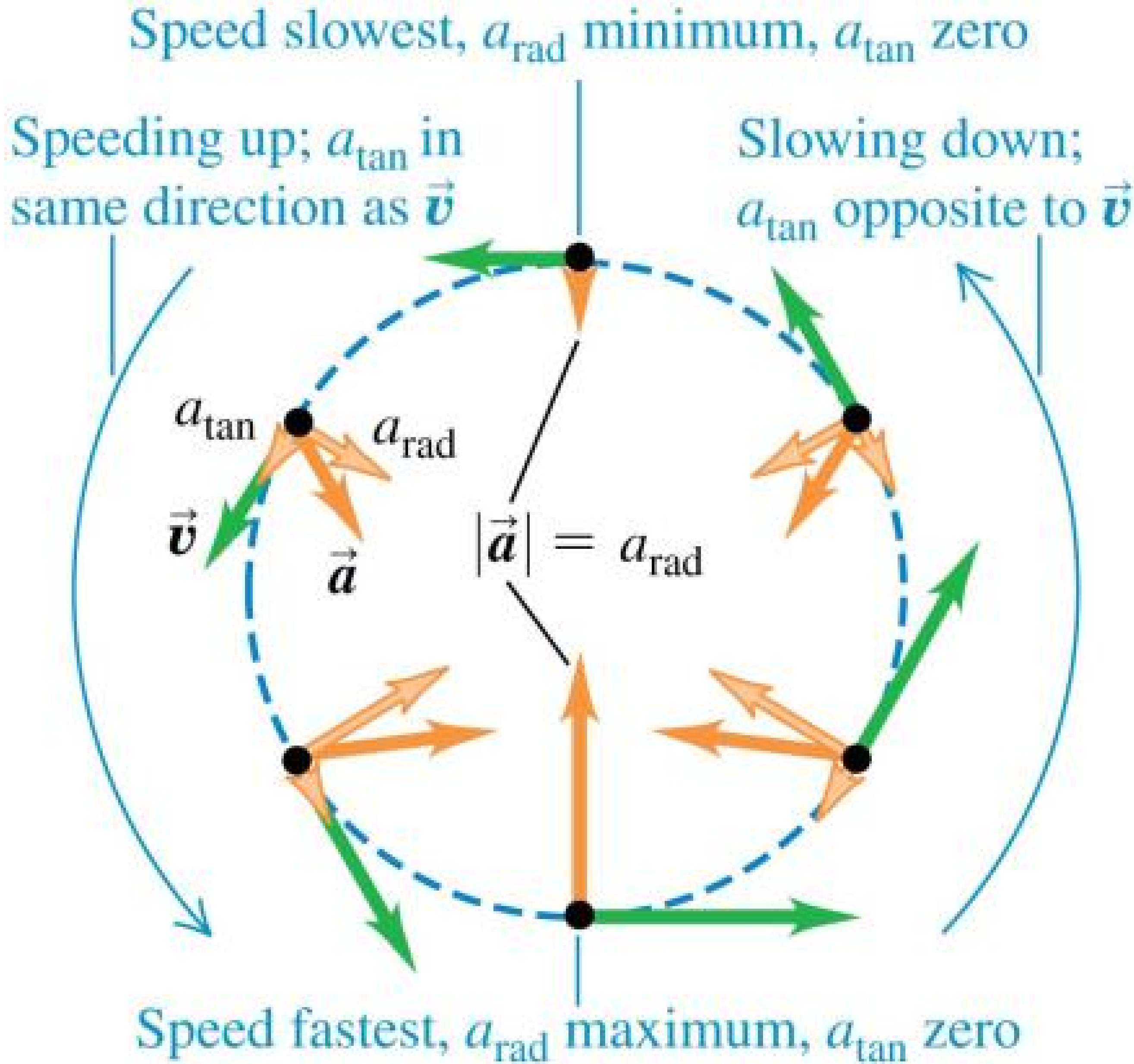
Projectile motion



Nonuniform circular motion

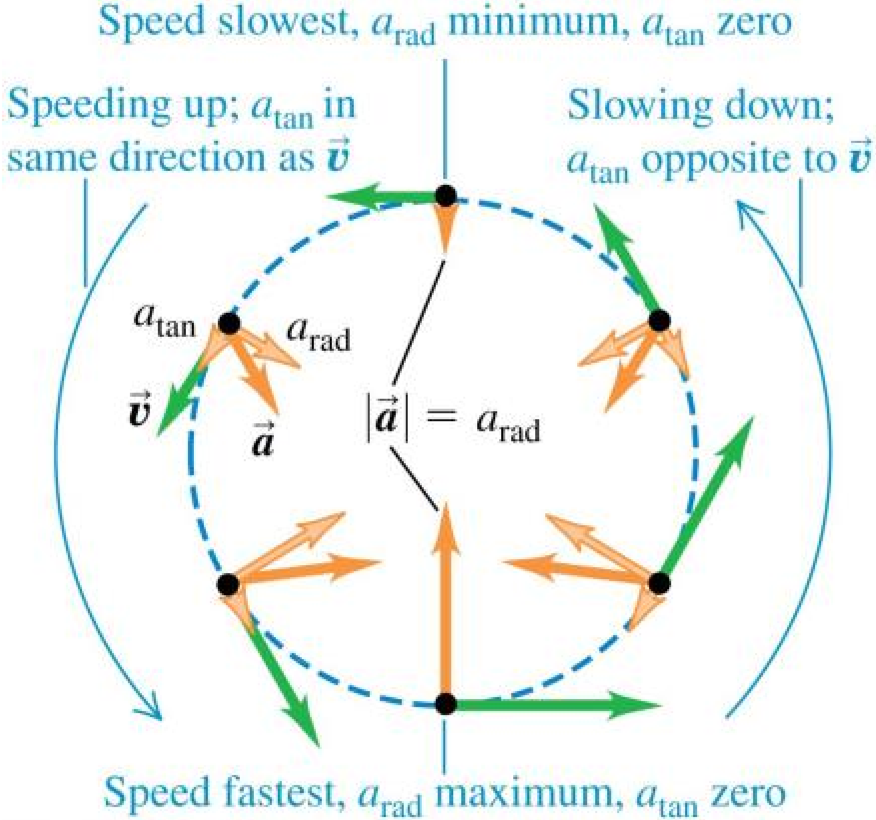
If the speed varies, the motion is *nonuniform circular motion*.

Nonuniform circular motion



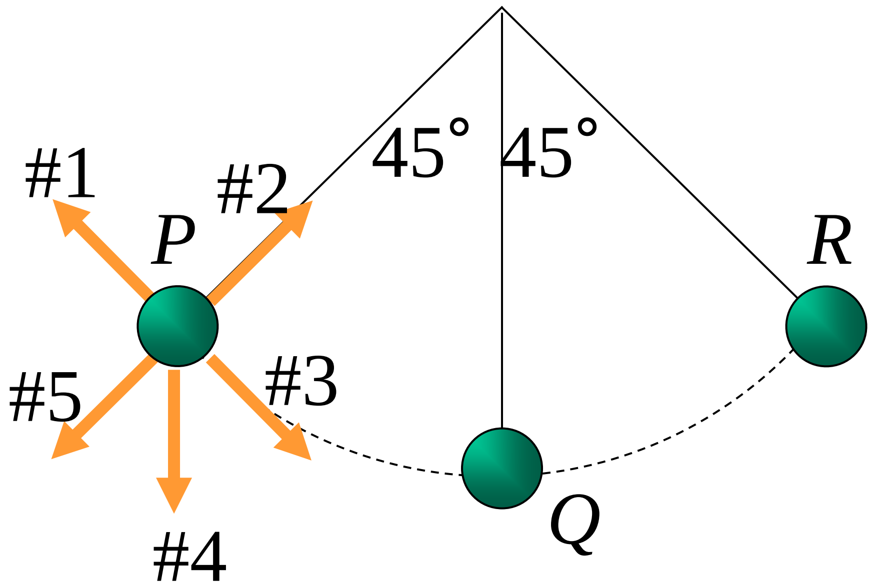
Nonuniform circular motion

The radial acceleration component is still $a_{\text{rad}} = v^2/R$, but there is also a tangential acceleration component a_{tan} that is *parallel* to the instantaneous velocity.



Q3.9

A pendulum swings back and forth, reaching a maximum angle of 45° from the vertical. Which arrow shows the direction of the pendulum bob's acceleration at *P* (the far left point of the motion)?

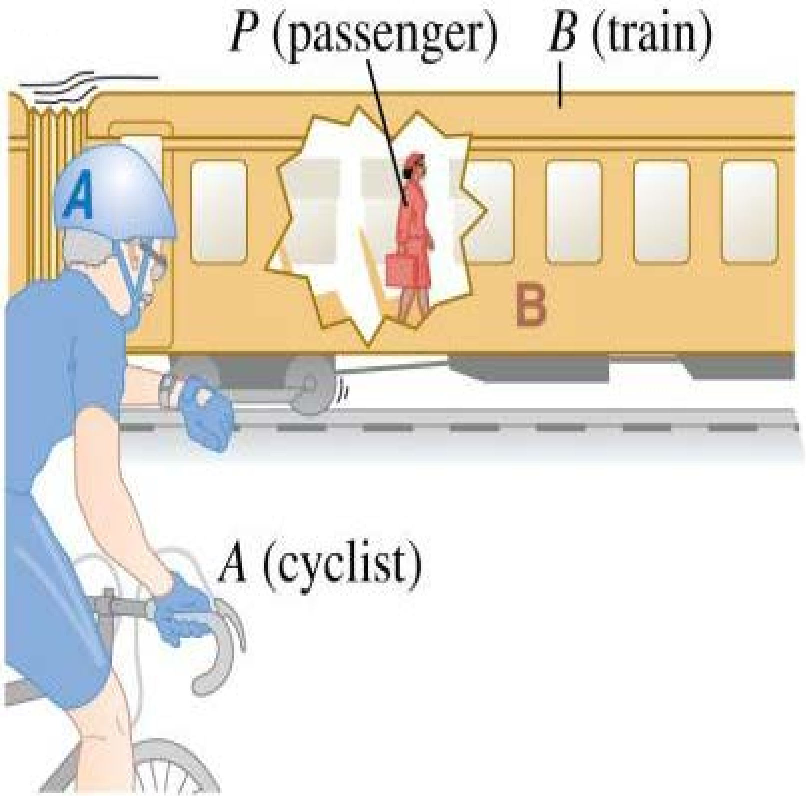


Relative velocity

- The velocity of a moving body seen by a particular observer is called the velocity *relative* to that observer, or simply the **relative velocity**.
- A **frame of reference** is a coordinate system plus a time scale.
- In many situations relative velocity is extremely important.

Relative velocity in one dimension

- If point P is moving relative to reference frame A, we denote the velocity of P relative to frame A as $v_{P/A}$.

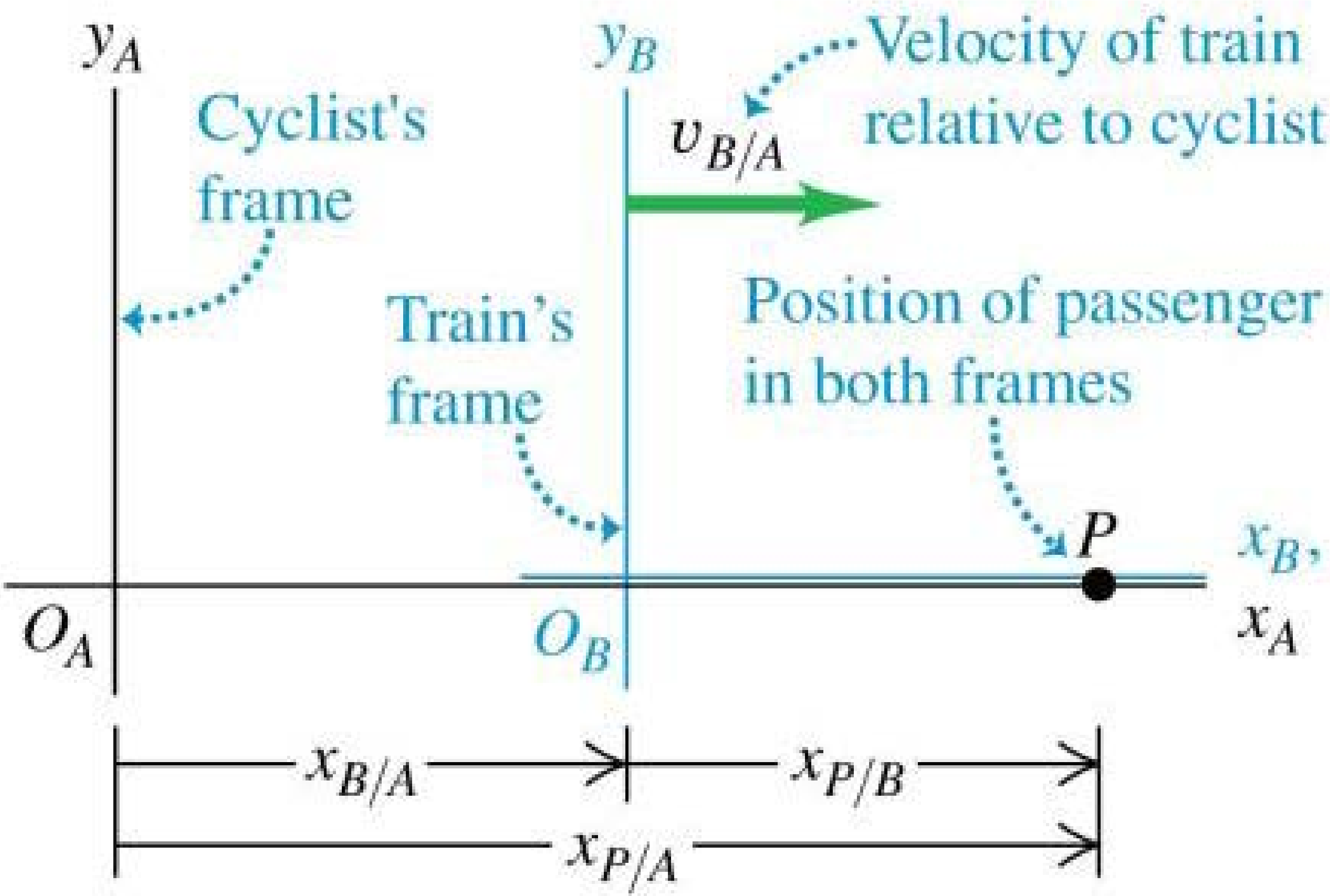


Relative velocity in one dimension

- If P is moving relative to frame B and frame B is moving relative to frame A, then the x-velocity of P relative to frame A is
- $v_{P/A\text{-}x} = v_{P/B\text{-}x} + v_{B/A\text{-}x}$.
- Recall that velocity is simply the rate of change of displacement with time (a differential v = dx/dt.)

Relative velocity in one dimension

- $v_{P/A\text{-}x} = v_{P/B\text{-}x} + v_{B/A\text{-}x}$.

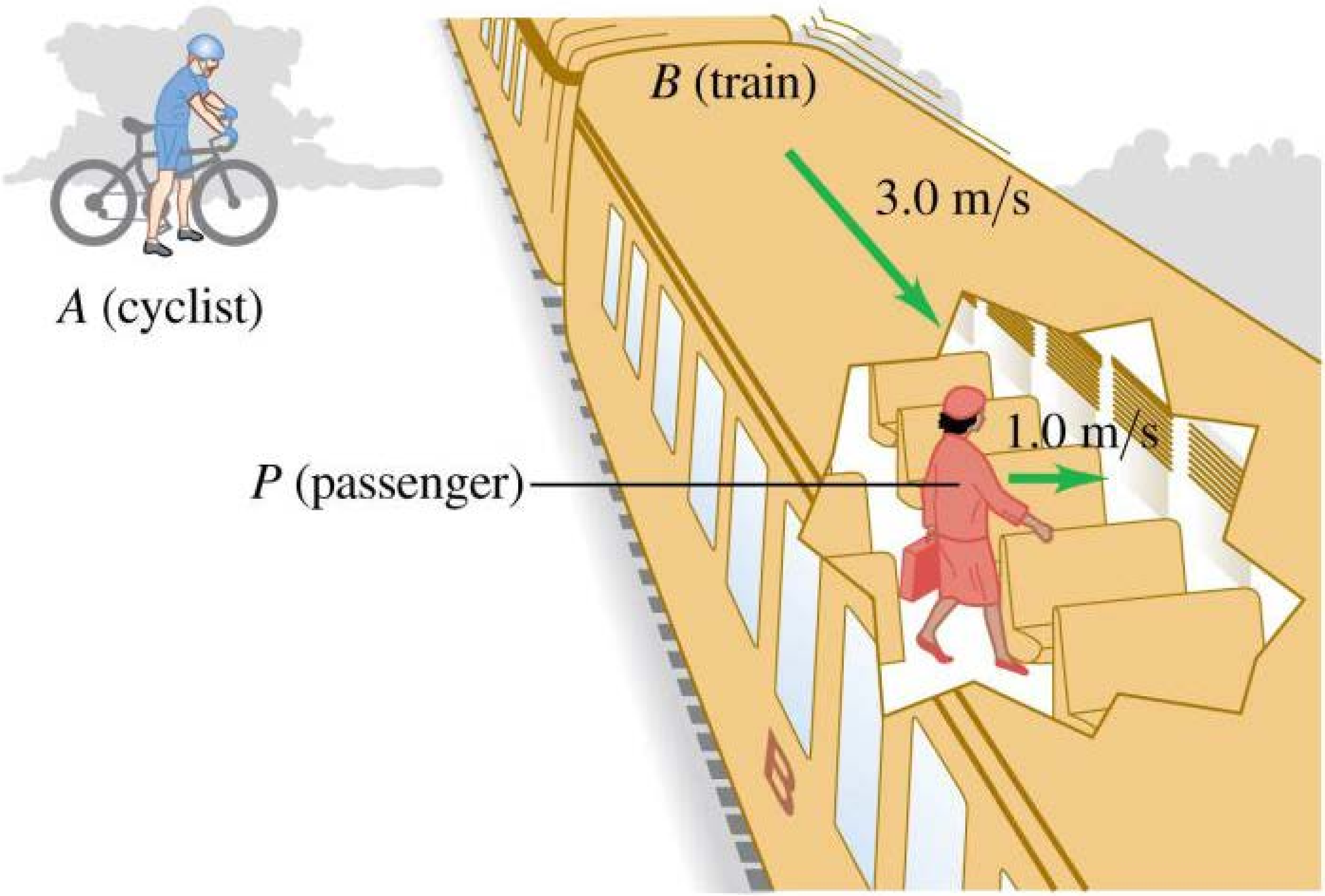


Relative velocity in one dimension

- In general, if A and B are any 2 frames of reference
- $v_{A/B\text{-}x}$. = - $v_{B/A\text{-}x}$.

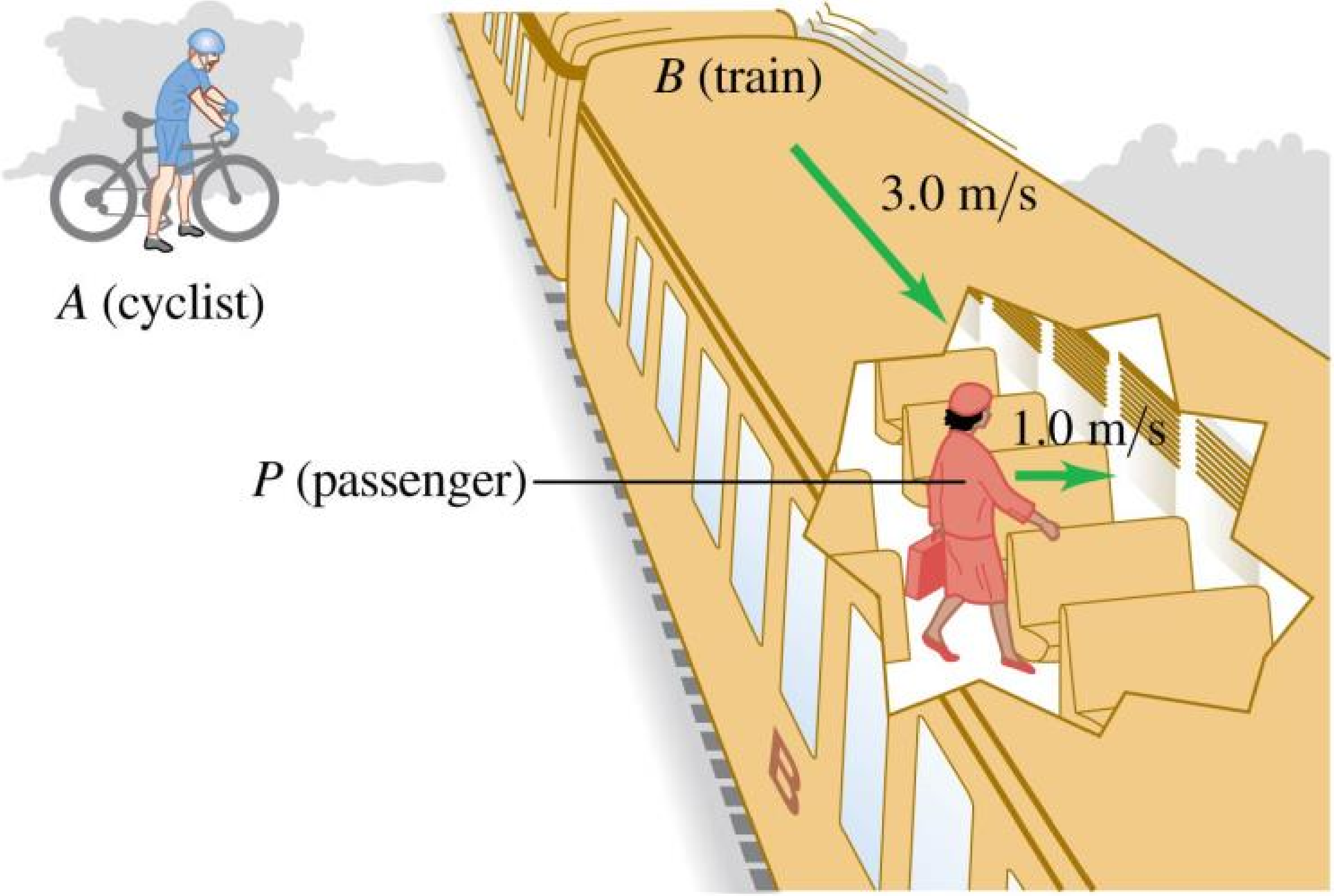
Relative velocity in two or three dimensions

- We extend relative velocity to two or three dimensions by using vector addition to combine velocities.

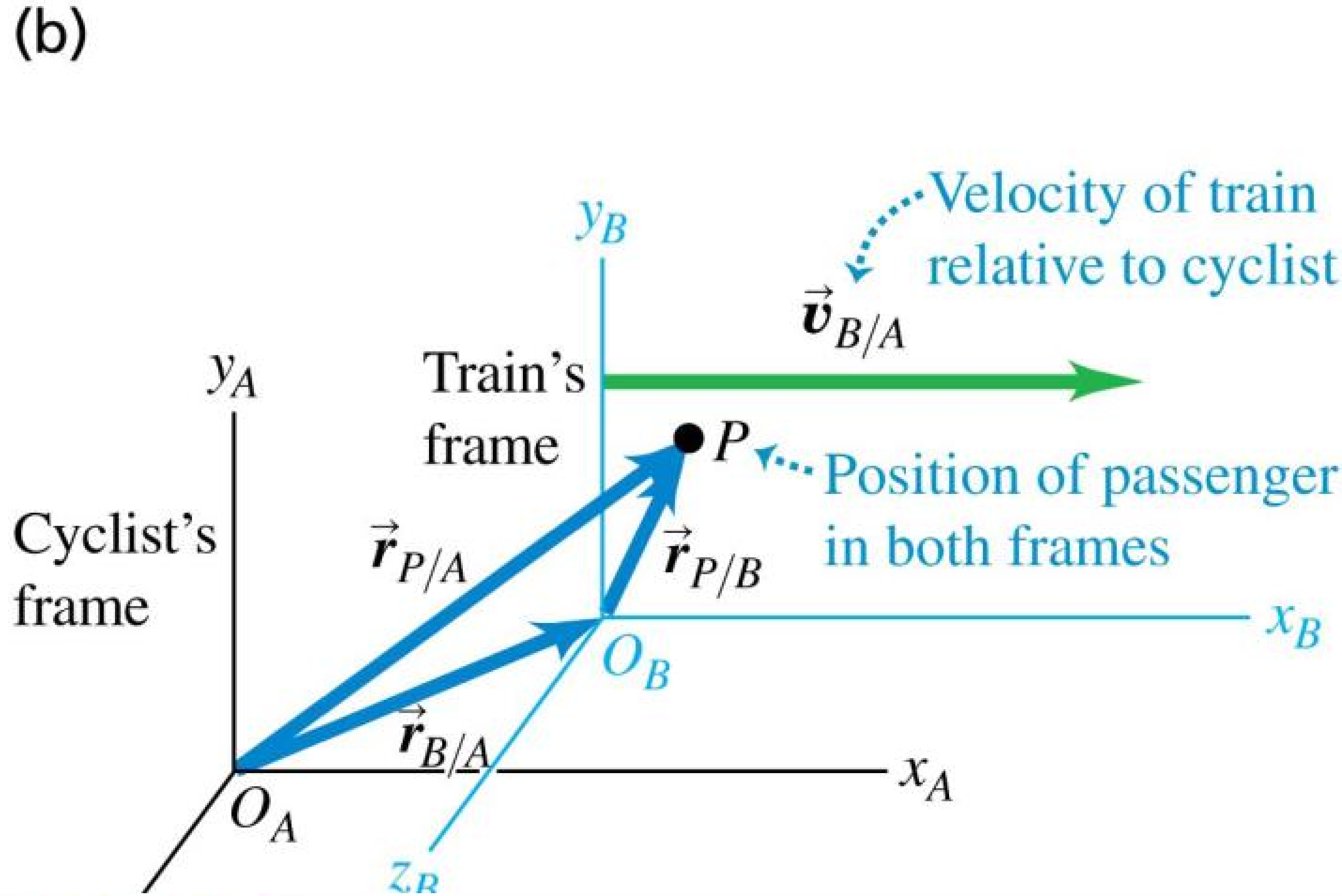


Relative velocity in two or three dimensions

(a)



Relative velocity in two or three dimensions



Relative velocity in space:

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

Velocity of P relative to A

Velocity of P relative to B

Velocity of B relative to A

Relative velocity in two or three dimensions

- Galilean Velocity Transformation

Relative velocity in space:

$$\underbrace{\vec{v}_{P/A}}_{\text{Velocity of } P \text{ relative to } A} = \underbrace{\vec{v}_{P/B}}_{\text{Velocity of } P \text{ relative to } B} + \underbrace{\vec{v}_{B/A}}_{\text{Velocity of } B \text{ relative to } A}$$

- Does not apply at relativistic velocities as demonstrated by Einstein and his theory of Relativity