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The night driving behavior in a car-following model

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Contents

1 Introduction

Traffic flow modeling is a key area in applied mathematics and transportation science, aiming to capture and predict the movement of vehicles on road networks. Among the modeling approaches, **microscopic models**, particularly **car-following models**, simulate how individual vehicles adjust their behavior based on the relative position and speed of surrounding vehicles [1].

Conventional models typically assume a monotonic decrease in velocity as traffic density increases, which reflects daytime driving. However, **night driving** introduces distinct dynamics: drivers may follow tail lights when visibility is low or drive more cautiously under headlight-limited conditions. This results in a **non-monotonic optimal velocity function**, as proposed by LeVeque [2].

This mini-project re-implements the model presented by Jiang et al. [1], using the **Full Velocity Difference (FVD)** framework, which incorporates both headway and relative velocity to simulate realistic vehicle responses. The simulation explores how night-time traffic evolves under periodic boundary conditions, especially when subjected to **small and large perturbations**.

Using numerical methods (Euler integration), the project examines how traffic stability depends on parameters such as κ, λ, and stochastic disturbances. Observed outcomes include stable clusters, kink–antikink waves, and unstable states, which are compared against the original paper's results. The study aims to assess the model's ability to reproduce night driving phenomena and its applicability to real traffic conditions.

2 Modelling Approach

The modeling approach chosen in this study is a **continuous-time microscopic dynamical system**, implemented via **discrete-time numerical simulation**. The model is based on the Full Velocity Difference (FVD) framework, which describes how each vehicle adjusts its acceleration according to the headway and relative velocity with the preceding vehicle. Mathematically, the system is governed by a set of **ordinary differential equations (ODEs)**, which are solved iteratively using the **Euler integration scheme** with a fixed time step $\Delta t = 0.1$.

While the underlying formulation is continuous in nature, the system's evolution is observed at discrete intervals, making it suitable for numerical analysis. In this framework, the velocity and position of each vehicle are updated step-by-step based on local interactions, forming a high-dimensional nonlinear system. This microscopic modeling approach is particularly appropriate for traffic systems where individual driver behavior and vehicle interaction play a significant role, such as under **night driving conditions** where visibility and perception differ from the daytime.

The FVD model was selected for its ability to realistically capture **nonlinear traffic instabilities**, including cluster formation, stop-and-go waves, and breakdown phenomena. Unlike macroscopic models, which rely on averaged flow parameters, the microscopic nature of FVD enables detailed tracking of vehicle-level dynamics, essential for studying the effects of visibility-driven behavior and stochastic perturbations in traffic flow.

3 Detailed descriptions of the mathematical model

The study of night driving behaviors is conducted using a microscopic traffic flow model, specifically the Full Velocity Difference (FVD) model, which describes the motion of a vehicle based on its interaction with the vehicle ahead [1]. The FVD model improves upon earlier car-following models by incorporating both the headway and the relative velocity between consecutive vehicles, ensuring realistic acceleration dynamics and accurate prediction of start wave speeds.

In the FVD model, the motion of vehicle $n+1$ following vehicle n is governed by the following differential equation:

$$\frac{\mathrm{d}v_{n+1}}{\mathrm{d}t} = \kappa\left[V(\Delta x) - v_{n+1}\right] + \lambda\left(v_n - v_{n+1}\right), \tag{1}$$

where $\Delta x = x_n - x_{n+1}$ is the headway (distance between the positions of vehicles n and $n+1$), v_n and v_{n+1} are the velocities of vehicles n and $n+1$, respectively, κ and λ are sensitivity parameters, and $V(\Delta x)$ is the optimal velocity function.

To simulate the model, the differential equation is discretized using the Euler scheme. The velocity update for vehicle $n+1$ is given by:

$$v_{n+1}(t+\Delta t) = v_{n+1}(t) + \frac{\mathrm{d}v_{n+1}(t)}{\mathrm{d}t}\Delta t, \tag{2}$$

where the acceleration is computed as:

$$\frac{\mathrm{d}v_{n+1}(t)}{\mathrm{d}t} = \kappa\left[V\left(x_n(t) - x_{n+1}(t)\right) - v_{n+1}(t)\right] + \lambda\left(v_n(t) - v_{n+1}(t)\right). \tag{3}$$

The position of the vehicle is updated as follows:

$$x_{n+1}(t+\Delta t) = x_{n+1}(t) + v_{n+1}(t)\Delta t + \frac{1}{2}\frac{\mathrm{d}v_{n+1}(t)}{\mathrm{d}t}(\Delta t)^2. \tag{4}$$

The time step Δt is set to 0.1, and periodic boundary conditions are applied to simulate a closed system of length $L = 500$.

For normal daytime traffic conditions, the optimal velocity function is defined as:

$$V(\Delta x) = \tanh(\Delta x - x_c) + \tanh(x_c), \tag{5}$$

where $x_c = 2$ is a critical headway parameter. This function models a non-increasing speed-density relationship typical of daytime driving.

For night driving conditions, the optimal velocity function is modified to account for the influence of tail lights and headlight-limited visibility, as proposed by LeVeque [2]. It is defined piecewise as:

$$V(\Delta x) = \begin{cases} \tanh(\Delta x - x_c) + \tanh(x_c), & \Delta x < x_{c1}, \\ a - \Delta x, & x_{c1} < \Delta x < x_{c2}, \\ b, & \Delta x > x_{c2}, \end{cases} \tag{6}$$

where $x_c = 2$, $x_{c1} = 3.2$, $x_{c2} = 4$, $a = 5$, and $b = 1$. This function reflects the unique behavior of night driving, where speed increases with density in the range $x_{c1} < \Delta x < x_{c2}$ due to reliance on tail lights, and is constant at low densities ($\Delta x > x_{c2}$) due to headlight limitations.

The linear stability of the FVD model is determined by the condition:

$$V'(\Delta x) < \frac{\kappa}{2} + \lambda, \tag{7}$$

where $V'(\Delta x)$ is the derivative of the optimal velocity function. For the night driving function, $V'(\Delta x)$ is negative in the range $x_{c1} < \Delta x < x_{c2}$, leading to potential instability in traffic flow.

To study traffic stability, perturbations are introduced to an initially homogeneous traffic state. A single vehicle decelerates with a constant deceleration of 1 for n_{dec} time steps. If the vehicle's velocity reaches zero within $m_{\text{dec}} < n_{\text{dec}}$ time steps, it remains stationary for the remaining $n_{\text{dec}} - m_{\text{dec}}$ time steps. Small perturbations are modeled with $n_{\text{dec}} = 1$, while large perturbations use $n_{\text{dec}} = 80$.

To account for stochastic effects in night driving, a random term is added to the velocity update:

$$v_{n+1}(t + \Delta t)^* = v_{n+1}(t) + \frac{\mathrm{d}v_{n+1}(t)}{\mathrm{d}t}\Delta t + \text{rand}() \times A, \tag{8}$$

$$v_{n+1}(t + \Delta t) = \min\left(\max\left(0, v_{n+1}(t + \Delta t)^*\right), v_{\max}\right), \tag{9}$$

$$x_{n+1}(t + \Delta t) = x_{n+1}(t) + \frac{1}{2}\left(v_{n+1}(t) + v_{n+1}(t + \Delta t)\right)\Delta t, \tag{10}$$

where rand() is a uniformly distributed random number between -0.5 and 0.5, A is the magnitude of the stochastic factor, and $v_{\max} = V(3.2)$ is the maximum velocity.

This theoretical framework forms the basis for analyzing the stability and dynamics of night driving behavior in the FVD model, capturing the unique speed-density relationship and the effects of perturbations and randomness on traffic flow.

4 Solving algorithms,implementation & results

The implementation of the night driving behavior in the Full Velocity Difference (FVD) model is carried out using Python, leveraging the NumPy library for numerical computations and Matplotlib for visualization. The code simulates the traffic dynamics described in Jiang et al. [1], focusing on the night driving optimal velocity function with perturbation and its comparison to normal driving conditions. Key components include the initialization of the traffic system, the implementation of the optimal velocity functions, and the numerical integration of the FVD model equations. This section highlights the code for generating the optimal velocity functions and their visualization, as well as the core simulation logic, with accompanying figures to demonstrate the implementation's functionality.

4.1 FVD model with no pertubation

The optimal velocity functions for normal and night driving conditions, as defined in Eqs. (5) and (6), are implemented to model the speed-density relationship. The normal driving function uses a hyperbolic tangent form, while the night driving function incorporates a piecewise structure to account for headlight-limited speeds at low density, tail-light dependence at moderate density, and congestion at high density.

```python
headway = np.linspace(0, 10, 100)
def normal_velocity(Dx, x_c=x_c):
    return np.tanh(Dx - x_c) + np.tanh(x_c)
def night_velocity(Dx, x_c=x_c, x_c1=x_c1, x_c2=x_c2, a=a, b=b):
    V = np.zeros_like(Dx)
    mask1 = Dx < x_c1
    V[mask1] = np.tanh(Dx[mask1] - x_c) + np.tanh(x_c)
    mask2 = (Dx >= x_c1) & (Dx < x_c2)
    V[mask2] = a - Dx[mask2]
    mask3 = Dx >= x_c2
    V[mask3] = b
    return V
```

Listing 1: Optimal Velocity functions

Figure 1 shows the night driving function which exhibits a non-monotonic structure: a decreasing region at low headways, an increasing linear segment between x_{c1} and x_{c2}, and a flat profile at high headways. This shape captures how driver behavior adapts to headlight visibility and reliance on tail lights at night, as proposed by LeVeque [2].

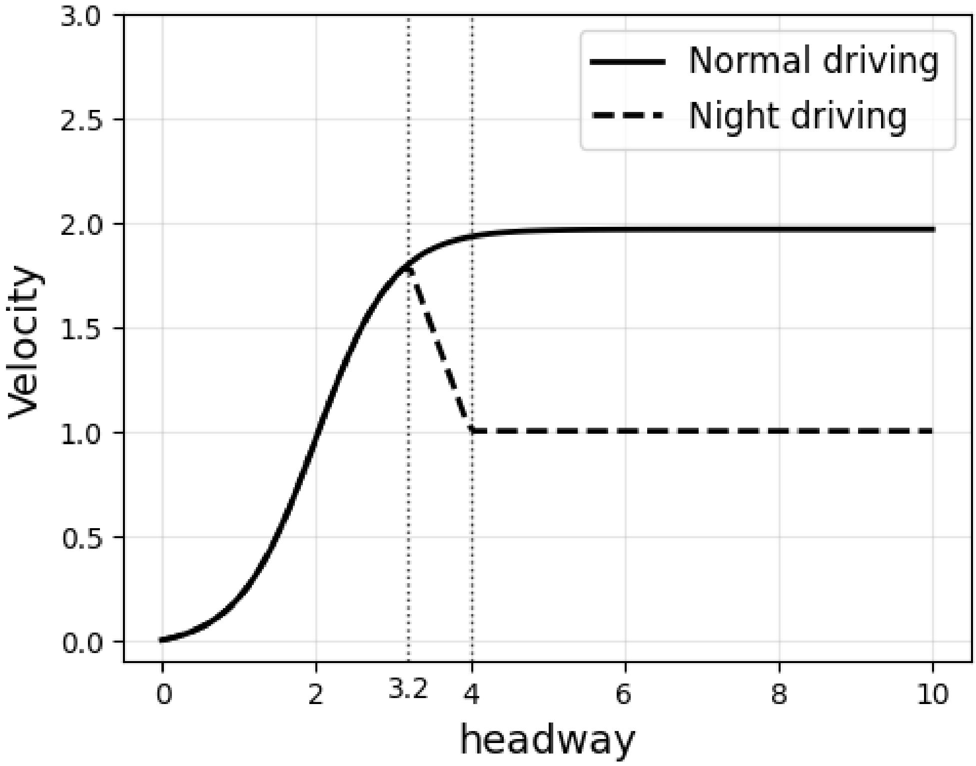


Figure 1: Optimal velocity functions for normal driving (solid line) and night driving (dashed line) as functions of headway, with parameters $x_c = 2$, $x_{c1} = 3.2$, $x_{c2} = 4$, $a = 5$, and $b = 1$. Vertical dotted lines indicate x_{c1} and x_{c2}.

Figure 2 shows the derivatives of the corresponding optimal velocity functions. Notably, the derivative of the night driving function becomes negative in the range $x_{c1} < \Delta x < x_{c2}$, indicating a region of linear instability according to the stability criterion $V'(\Delta x) < \frac{\kappa}{2} + \lambda$. This property is key to the emergence of clusters in night traffic flow. When $\kappa = 1.0$, $\lambda = 0.2$, $V'(\Delta x)$ will smaller than threshold 0.7 in some parts.

```
def normal_velocity_derivative(Dx, x_c=x_c):
    return np.square(1.0 / np.cosh(Dx - x_c))
def night_velocity_derivative(Dx, x_c=x_c, x_c1=x_c1, x_c2=x_c2):
    V_prime = np.zeros_like(Dx)
    mask1 = Dx < x_c1
    V_prime[mask1] = np.square(1.0 / np.cosh(Dx[mask1] - x_c))
    mask2 = (Dx >= x_c1) & (Dx < x_c2)
    V_prime[mask2] = -1.0
    mask3 = Dx >= x_c2
    V_prime[mask3] = 0.0
```

```
    return V_prime
```

Listing 2: Derivative velocity functions

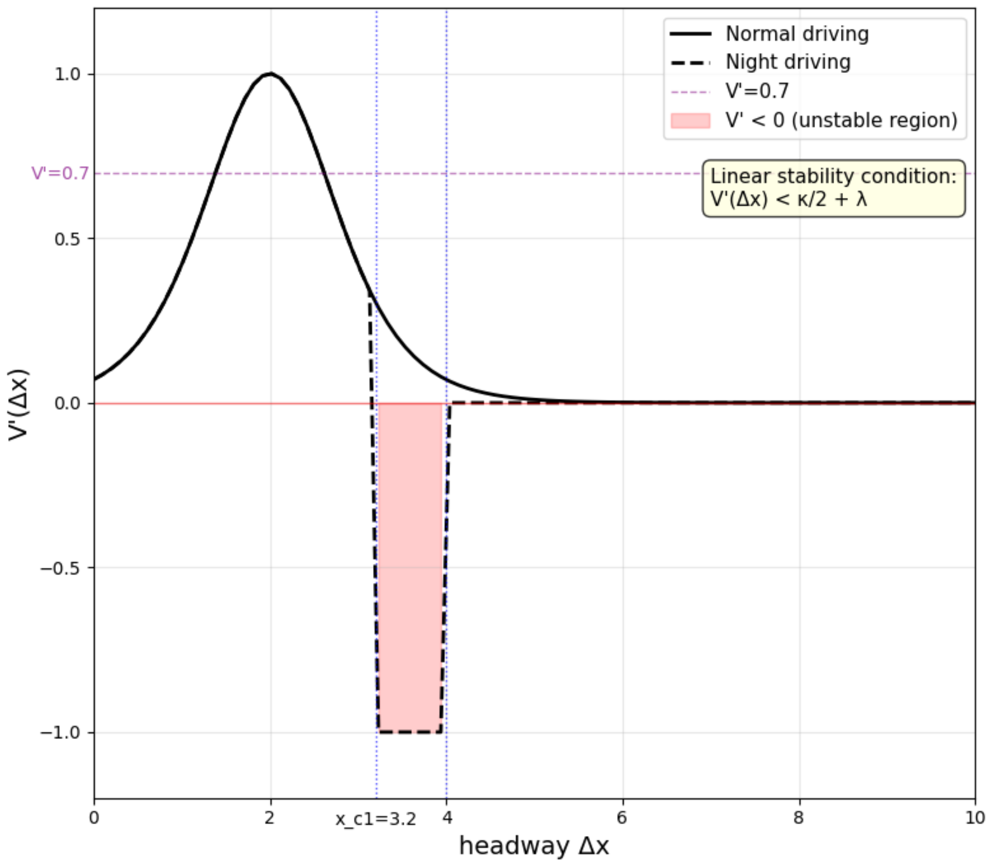


Figure 2: Derivatives of the optimal velocity functions for normal driving (solid line) and night driving (dashed line) as functions of headway, with parameters $x_c = 2$, $x_{c1} = 3.2$, and $x_{c2} = 4$. Vertical dotted lines indicate x_{c1} and x_{c2}.

A space-time diagram is generated to illustrate vehicle positions over time in the density range $x_{c1} < \Delta x < x_{c2}$, where clustering is expected due to the negative derivative of the optimal velocity function.

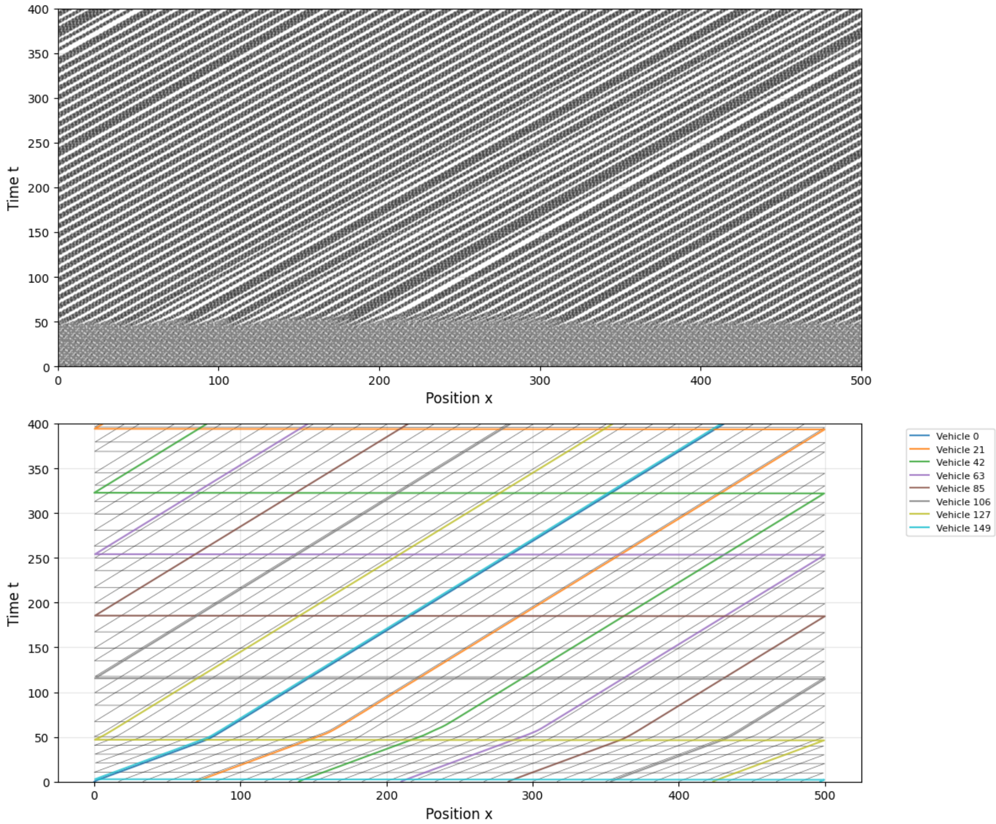
```
initial_spacing = L / N
x = np.array([i * initial_spacing for i in range(N)])
initial_optimal_v = night_velocity(np.array([initial_spacing]))[0]
v = np.ones(N) * initial_optimal_v

# Simulation loop
for step in range(n_steps):
    t = step * dt
    headway = np.zeros(N)
    for i in range(N):
        prev_i = (i - 1) % N
        dx = x[prev_i] - x[i]
```

```
        if dx < 0:
            dx += L
        headway[i] = dx
    V_opt = night_velocity(headway)
    acceleration = np.zeros(N)
    for i in range(N):
        prev_i = (i - 1) % N
        acceleration[i] = kappa * (V_opt[i] - v[i]) + lambda_param * (v[prev_i] - v
[i])
    v = v + acceleration * dt
    v = np.maximum(v, 0)
    x = x + v * dt + 0.5 * acceleration * dt**2
    x = x % L
```

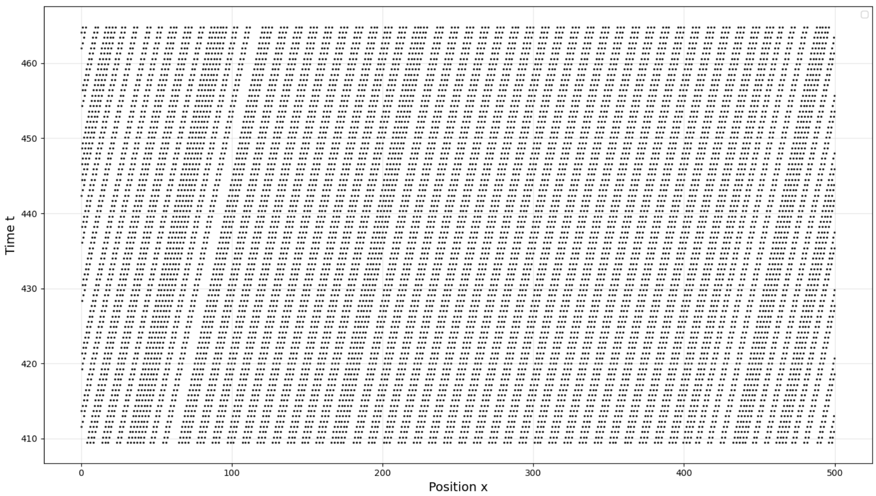
Listing 3: Compute space-time of vehicles under periodic boundary conditions

Figure 3. In subfigure 3a, cluster formation begins early in the simulation as vehicles adjust to local headway differences. These clusters persist and grow over time, eventually forming well-defined stop-and-go waves, as shown in subfigure 3b. The formation of such clusters, despite the absence of external perturbations, highlights the intrinsic instability introduced by the night driving optimal velocity function in specific headway ranges.

One can see that the formation of clusters in the density range $x_{c1} < \Delta x < x_{c2}$, the headway of the leading vehicle of each cluster is larger than x_{c2}. As a result, the leading vehicles move with velocity 1.0. Consequently, all the vehicles move with velocity 1.0.



(a) Early stage: showing cluster formation in the range $x_{c1} < \Delta x < x_{c2}$.



(b) Late stage: same parameters, showing developed clusters.

Figure 3: Space-time diagrams for night driving with $N = 150$, $\kappa = 1.0$, $\lambda = 0.5$ in the range $x_{c1} < \Delta x < x_{c2}$.

4.2 FVD model with small perturbation

To investigate the relationship between traffic density and flow, a fundamental diagram is generated, comparing normal driving, theoretical night driving, and night driving with small perturbations

($n_{\text{dec}} = 1$). The diagram is computed by simulating the FVD model across a range of vehicle numbers to vary density, calculating the resulting flow as $Q = \kappa \cdot \bar{v}$, where $\kappa = N/L$ and $\bar{v}$ is the average velocity.

```python
def update_arrays(x, v, N, kappa, lambda_param, use_night_driving,
                  perturbation_active, perturbation_counter, vehicle_stopped,
                  perturbed_vehicles, n_dec, deceleration=1, A=0):
    headways = get_headways_arrays(x, N)
    accelerations = np.zeros(N)
    for i in range(N):
        prev_veh = (i + 1) % N
        headway = headways[i]
        optimal_velocity = (night_velocity if use_night_driving else normal_velocity)(np.array([headway]))[0]
        velocity_diff = v[prev_veh] - v[i]
        accelerations[i] = kappa * (optimal_velocity - v[i]) + lambda_param * velocity_diff

    if perturbation_active and perturbation_counter < n_dec:
        for pv in perturbed_vehicles:
            if not vehicle_stopped:
                accelerations[pv] = -deceleration
                if v[pv] + accelerations[pv] * 0.1 <= 0:
                    v[pv] = 0
                    vehicle_stopped = True
            else:
                accelerations[pv] = 0
                v[pv] = 0
        perturbation_counter += 1
        if perturbation_counter >= n_dec:
            perturbation_active = False

    v_old = v.copy()
    v_max = night_velocity(np.array([3.2]))[0] if use_night_driving else normal_velocity(np.array([10.0]))[0]
    for i in range(N):
        if not (perturbation_active and i in perturbed_vehicles and vehicle_stopped):
            noise = (np.random.uniform(-0.5, 0.5) * A) if A else 0
            v_new = v[i] + accelerations[i] * 0.1 + noise
            v[i] = min(max(0, v_new), v_max)

```

```
    for i in range(N):
        if A == 0:
            x[i] += v[i] * 0.1 + 0.5 * accelerations[i] * 0.01
        else:
            x[i] += 0.5 * (v_old[i] + v[i]) * 0.1
        x[i] %= L
    return x, v, perturbation_active, vehicle_stopped, perturbation_counter
```

Listing 4: Main update function for position and velocity

The above code produces Figure 4, which displays the fundamental diagram for traffic flow. Vertical dotted lines mark the critical densities points $\kappa_{c1} = 1/x_{c2}$ and $\kappa_{c2} = 1/x_{c1}$, highlighting the unstable density range where clustering occurs. For this case, the traffic is unstable in the density range kc3okokc4 from the stability condition. Moreover, it is also unstable in the density range $\kappa_{c3} < \kappa < \kappa_{c4}$. In this density range, the traffic evolves into kink–antikink waves.

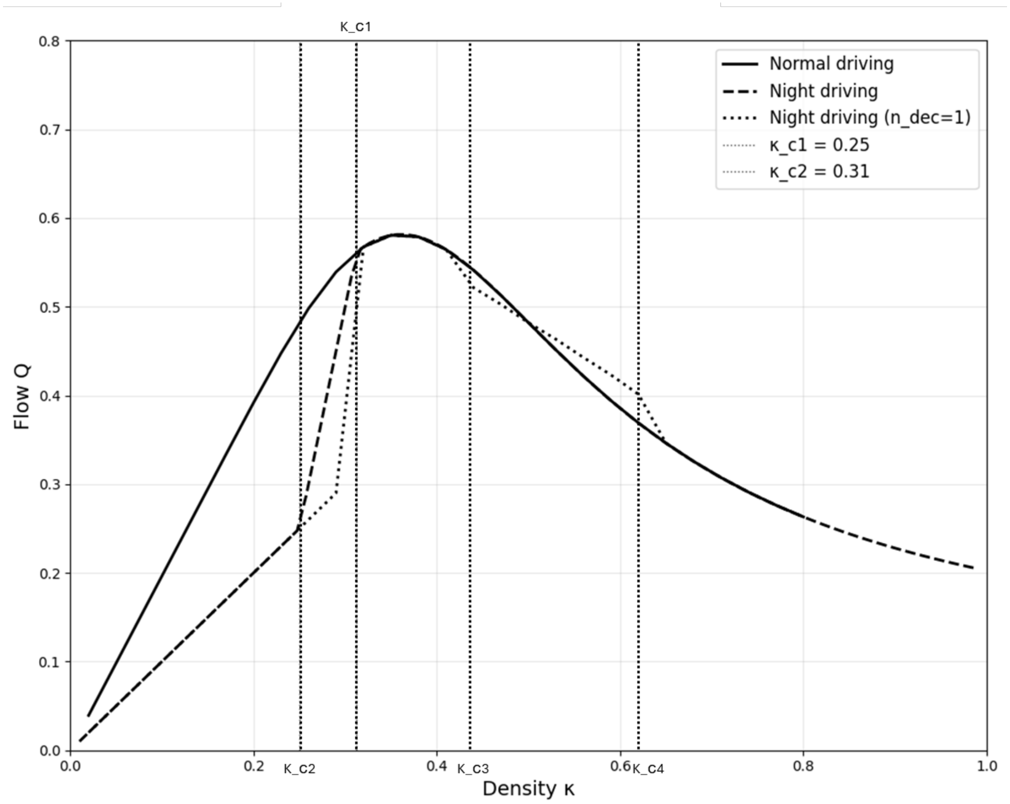


Figure 4: Fundamental diagram for traffic flow with $\kappa = 1.0$, $\lambda = 0.2$, showing normal driving (solid line), theoretical night driving (dashed line), and night driving with small perturbations ($n_{\text{dec}} = 1$, dotted line).

Figure 5 shows the simulation corresponds to a density range $\kappa_{c3} < \kappa < \kappa_{c4}$, where the traffic flow becomes unstable despite the relatively large sensitivity parameters. The resulting pattern forms kink–antikink waves—alternating high and low-density regions—that propagate through the system.

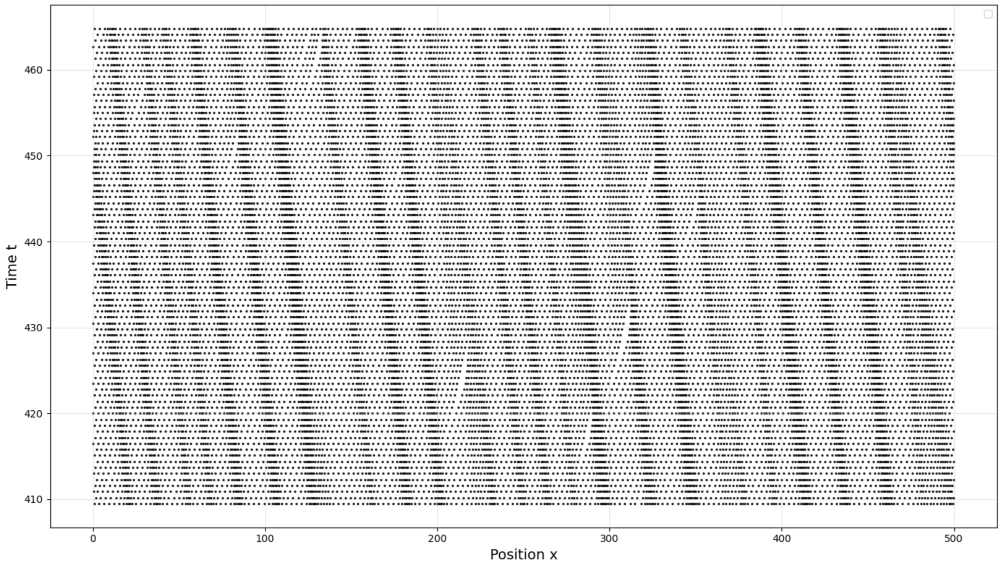


Figure 5: Space-time diagram for night driving with $N = 250$, $\kappa = 1.0$, $\lambda = 0.2$, showing vehicle positions in the late stage of the simulation in the density range $\kappa_{c3} < \kappa < \kappa_{c4}$.

This behavior aligns with the findings in Jiang et al. [1], highlighting the transition from stable to oscillatory dynamics as the system enters the linearly unstable regime. According to the linear stability condition, traffic becomes unstable in two key density intervals: $\kappa_{c5} < \kappa < \kappa_{c6}$ and $\kappa_{c2} < \kappa < \kappa_{c1}$ (Figure 6). In the latter range, the traffic exhibits cluster formation similar to that observed in Figure 3, where instabilities arise due to negative slope in the optimal velocity function.

However, within the interval $\kappa_{c5} < \kappa < \kappa_{c7}$, the system exhibits a different dynamic. Rather than evolving into kink–antikink waves, the vehicles form **stable clusters**, as visualized in Figure 7. In this regime, leading vehicles within each cluster maintain a headway greater than x_{c2}, allowing them to travel at maximum speed. As a result, the entire cluster propagates forward with an average velocity close to unity. Notably, unlike earlier observations, a persistent density wave is visible, indicating the presence of structured but stable congestion patterns.

As the overall density increases beyond κ_{c7}, inter-cluster spacing diminishes, leading to the breakdown of this stable configuration. Figure 8 illustrates the resulting scenario: **unstable clusters** begin to emerge. These clusters are transient in nature—appearing and dissolving periodically—reflecting a regime where the traffic system fails to maintain coherent structure under sustained high density and low sensitivity.

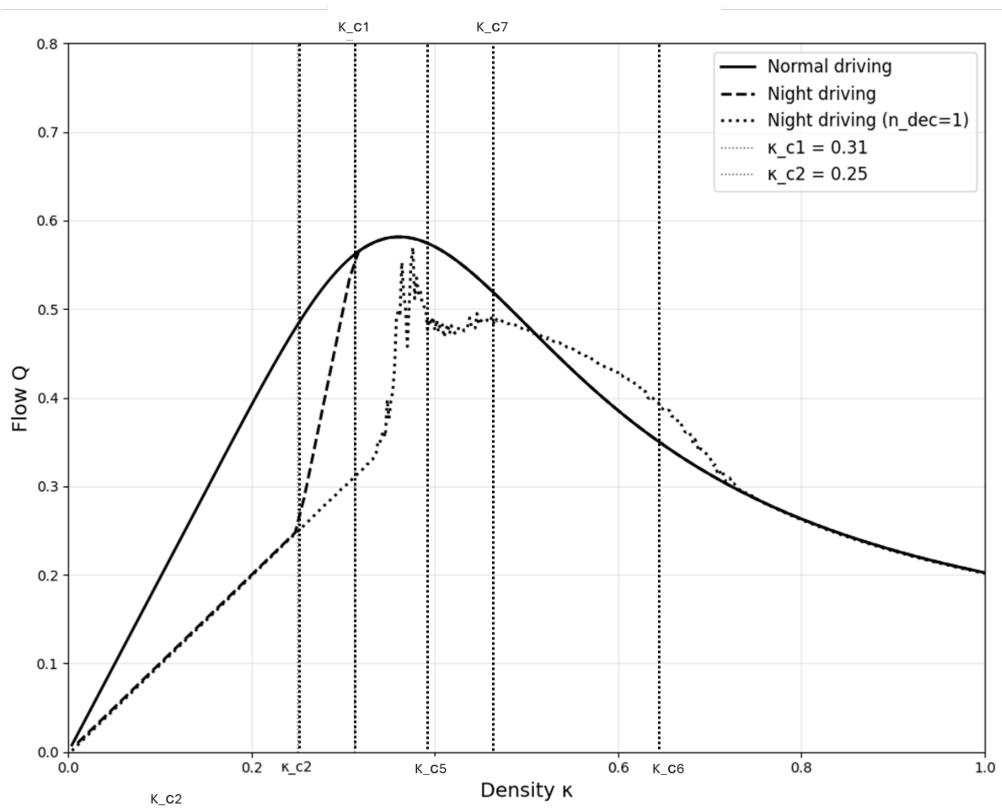


Figure 6: Fundamental diagram for traffic flow with $\kappa = 1.0$, $\lambda = 0.1$ with small perturbations ($n_{\text{dec}} = 1$).

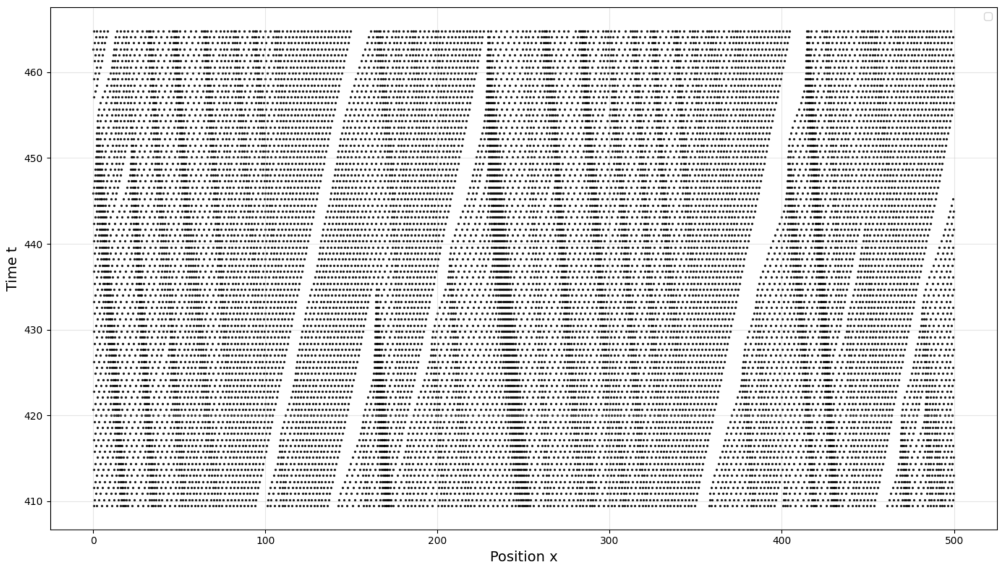


Figure 7: Space-time diagram for night driving with $N = 230$, $\kappa = 1.0$, $\lambda = 0.1$, showing vehicle positions in the late stage of the simulation in the density range $\kappa_{c5} < \kappa < \kappa_{c7}$.

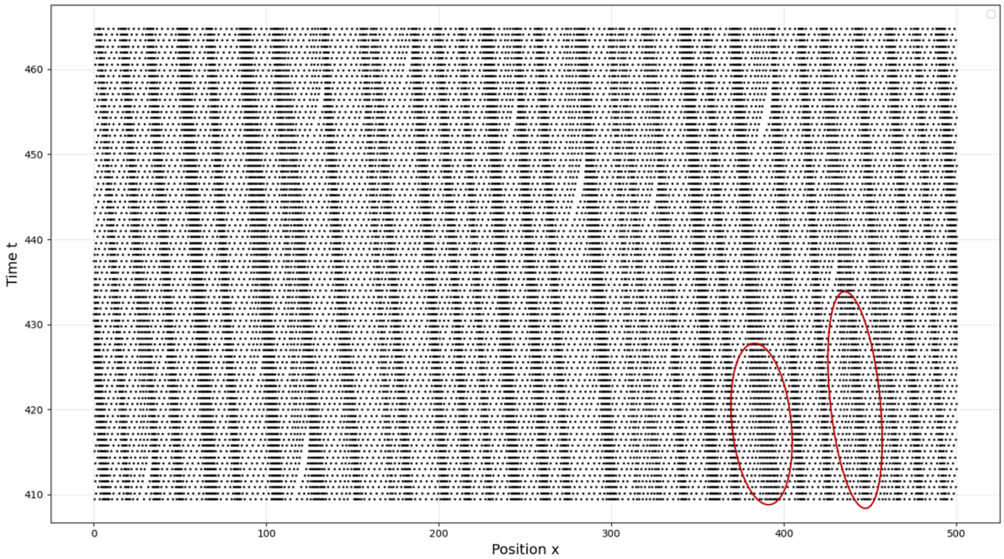
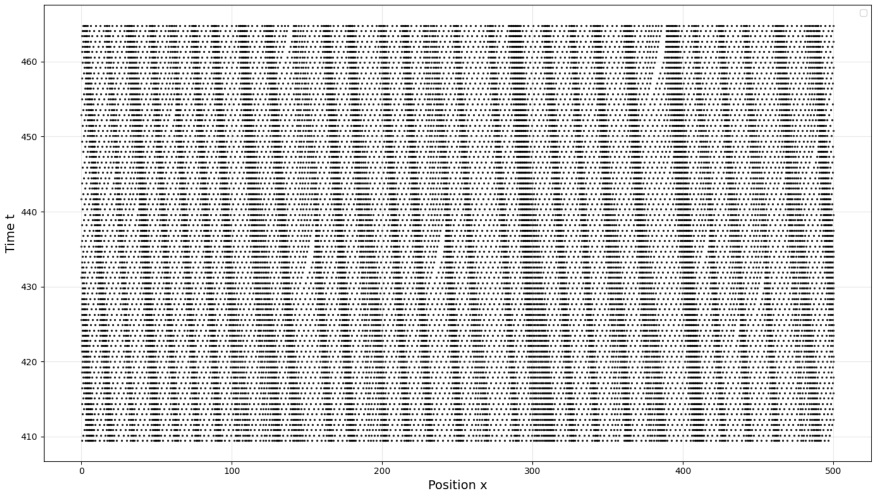


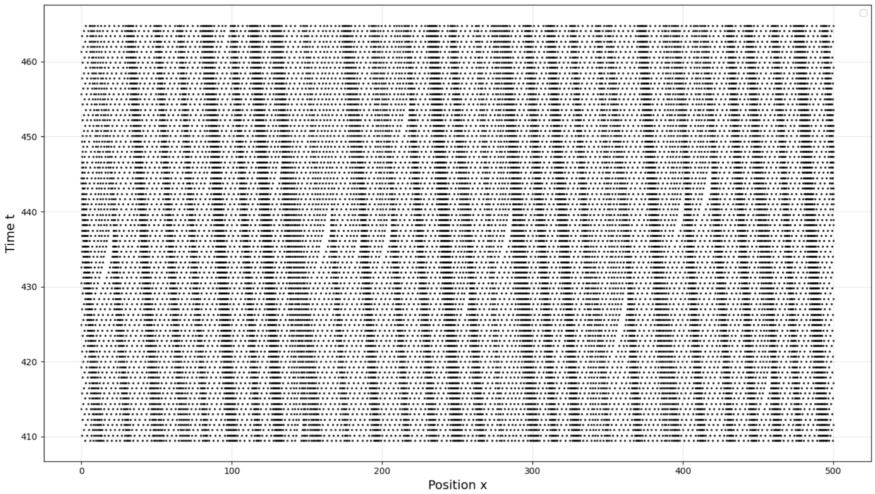
Figure 8: Space-time diagram for night driving with $N = 300$, $\kappa = 1.0$, $\lambda = 0.1$, showing vehicle positions in the late stage of the simulation in the density range $\kappa_{c7} < \kappa < \kappa_{c6}$.

4.3 FVD model with small perturbation and randomness

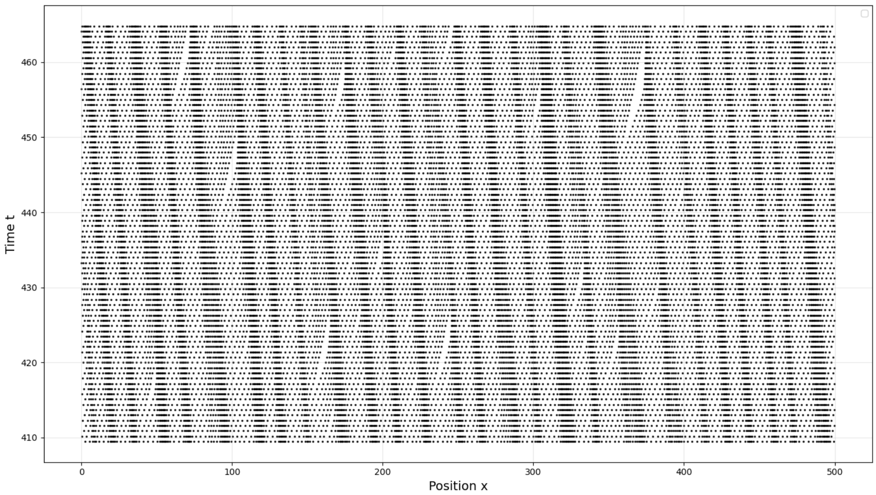
Figure 9 illustrates the impact of stochastic fluctuations on night driving traffic flow. At low noise levels ($A = 0.01$), clusters remain narrow and stable, resembling deterministic behavior. As randomness increases to $A = 0.05$, the system exhibits temporal instability and cluster fragmentation. When $A = 0.1$, the dynamics become fully disordered, with the emergence of a dense, macroscopic congestion zone. These results suggest that moderate randomness can destabilize otherwise stable traffic patterns, while high randomness leads to persistent breakdown of flow.



(a) $A = 0.01$: Stable clusters with reduced width.



(b) $A = 0.05$: Clusters become unstable and start oscillating.



(c) $A = 0.1$: Highly unstable flow with macroscopic high-density region.

Figure 9: Space-time diagrams for night driving with $N = 300$, $\kappa = 1.0$, $\lambda = 0.1$, under increasing levels of randomness A. As the magnitude of stochastic noise increases, traffic patterns shift from stable to unstable, with increasingly irregular clustering.

4.4 FVD model with large perturbation

Figures 10 to 14 illustrate the behavior of night traffic under large perturbations ($n_{\text{dec}} = 80$) for decreasing values of the sensitivity parameter λ. As shown in Figure 10, when $\lambda = 0.5$, the flow remains stable at higher densities but collapses below a critical density, confirming nonlinear instability. The corresponding space-time diagram in Figure 11 reveals the formation of a single large cluster with a free-flowing leading vehicle. When λ is reduced to 0.2 (Figure 12), the unstable density region widens, and the cluster structure becomes more complex, as seen in Figure 13, where internal density waves appear within the jam. At $\lambda = 0.1$, Figure 14 shows that the system exhibits widespread instability, characterized by persistent formation and dissolution of traffic clusters. These results indicate that lower sensitivity amplifies the impact of large disturbances, leading to more severe congestion and complex dynamic patterns.

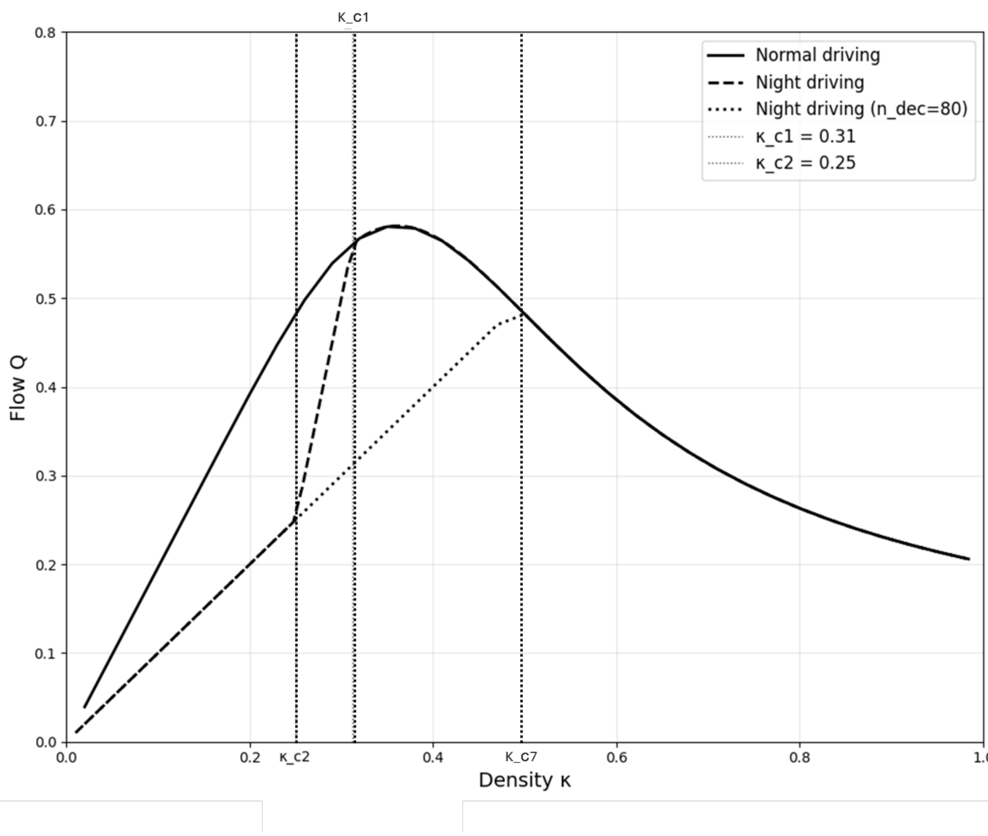


Figure 10: Fundamental diagram under large perturbation ($n_{\text{dec}} = 80$), $\kappa = 1.0$, $\lambda = 0.5$. Flow collapses below critical density.

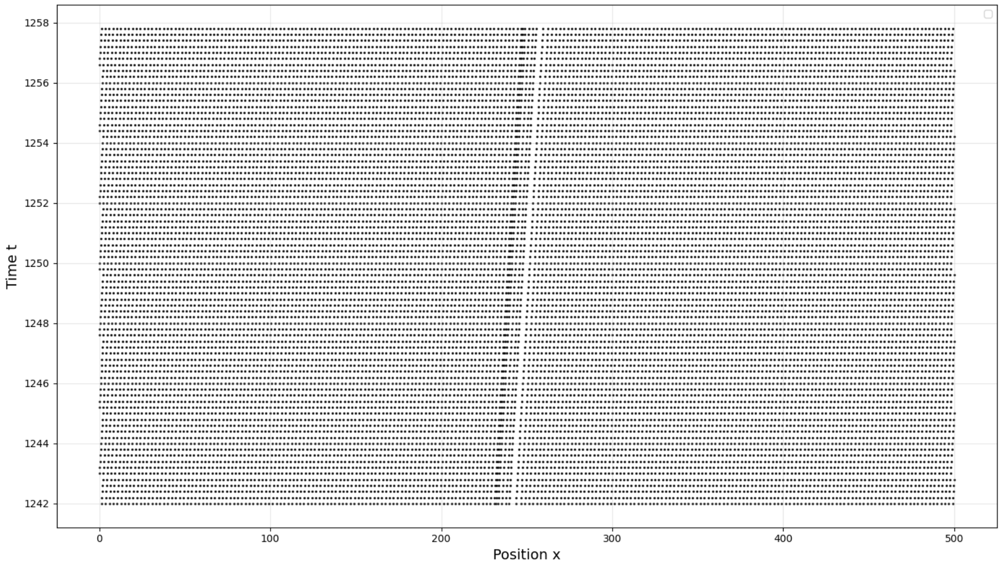


Figure 11: Space-time diagram with $N = 220$, $\kappa = 1.0$, $\lambda = 0.5$. One large cluster forms post-perturbation.

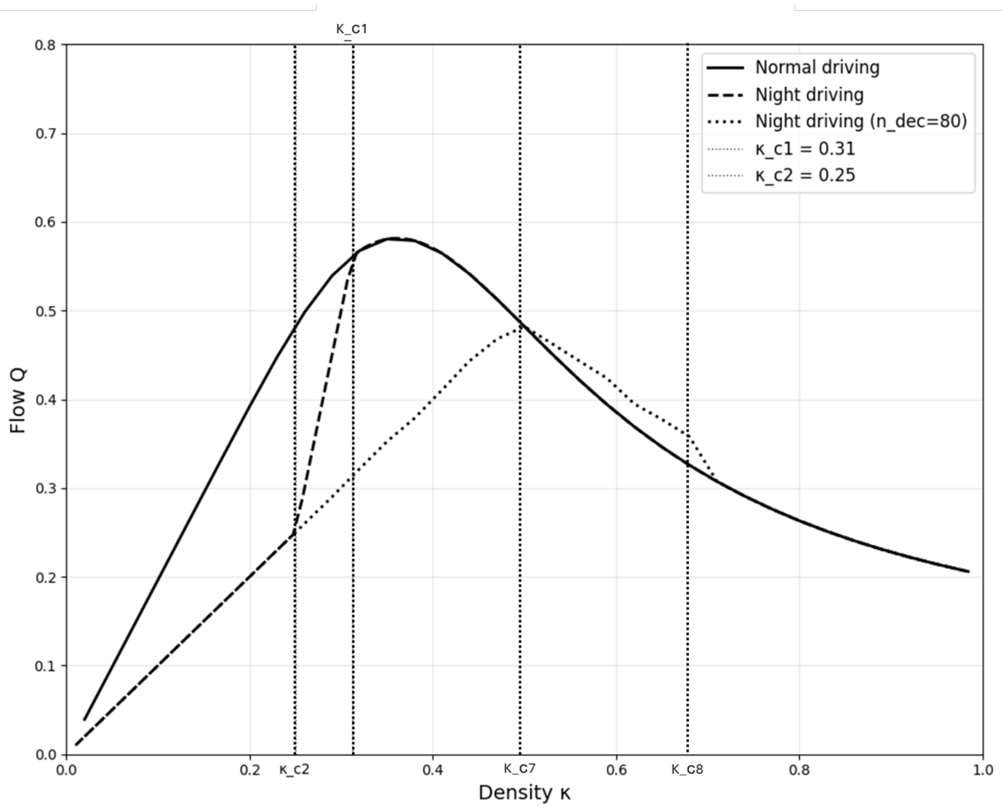


Figure 12: Fundamental diagram with $\lambda = 0.2$. Lower sensitivity leads to wider unstable region.

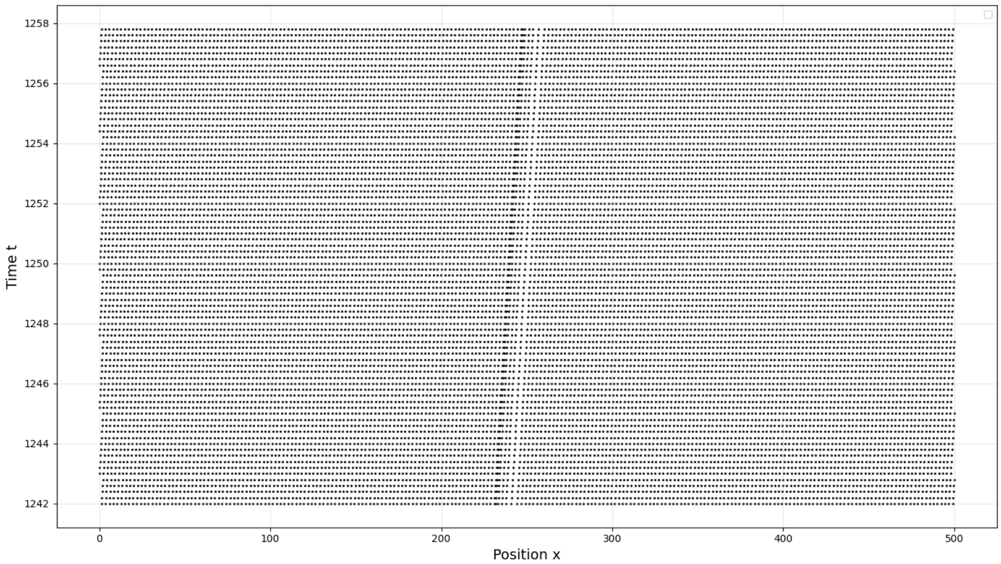


Figure 13: Space-time diagram for $\lambda = 0.2$. Cluster contains internal density waves.

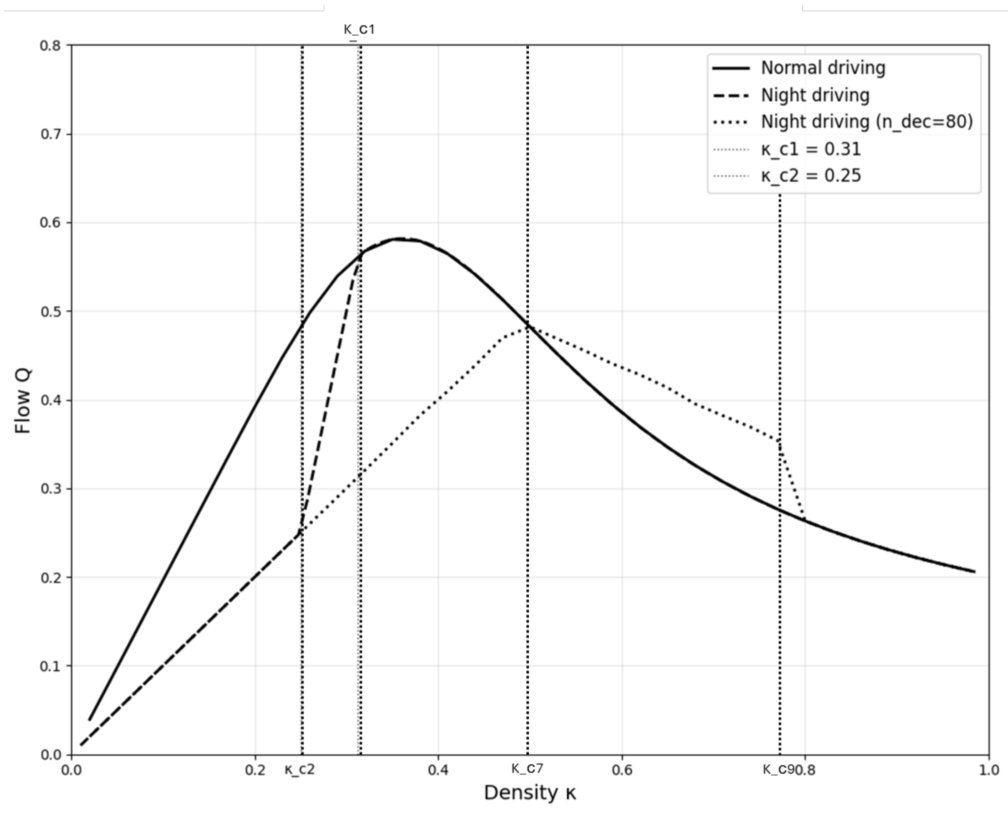


Figure 14: Fundamental diagram for $\lambda = 0.1$. Unstable clusters dominate at low sensitivity.

5 Sensitivity Analysis and Model Robustness

The behavior of the night driving FVD model is strongly influenced by the choice of sensitivity parameters, perturbation magnitude, and randomness. This section reflects on the model's sensitivity and robustness based on the simulation results presented earlier.

Sensitivity to parameters κ and λ: The sensitivity parameters κ and λ control how strongly a vehicle responds to the optimal velocity and the velocity difference with the vehicle in front. As observed in Figures 10–14, reducing λ increases the range of densities over which instability occurs. When λ is large (e.g., 0.5), the system tends to recover from perturbations and form a single stable cluster. However, for smaller values (e.g., 0.1), persistent unstable clusters or internal density waves emerge, suggesting the system is more fragile under low sensitivity. Thus, the model is highly sensitive to the **responsiveness of drivers**.

Effect of perturbation scale n_{dec}: The model also responds differently to small vs. large perturbations. Small perturbations ($n_{\text{dec}} = 1$) often lead to kink–antikink waves or stable clusters, depending on the density and parameter regime. In contrast, large perturbations ($n_{\text{dec}} = 80$) more easily disrupt the traffic flow and induce long-lived clusters or jams, as seen in Figures 11 and 13. This indicates that even linearly stable configurations can become unstable under strong enough disturbances—highlighting a form of **nonlinear sensitivity**.

Impact of randomness A: Introducing stochastic noise to the velocity update helps assess the model's robustness under uncertainty. As shown in Figure 9, a small amount of randomness ($A = 0.01$) compresses clusters and mimics realistic driver variation without breaking system stability. However, higher levels of noise ($A = 0.1$) quickly lead to chaotic, disordered motion and the loss of structured clustering. Therefore, the model is **moderately robust** to small fluctuations but deteriorates under significant noise.

Overall Robustness

The night driving FVD model is constructed based on several simplifying assumptions, yet simulation results suggest that its **core qualitative behaviors remain robust** even when some of these assumptions are relaxed.

First, the model adopts **periodic boundary conditions**, meaning vehicles move on a closed loop without entry or exit. This abstraction is widely used to eliminate boundary effects and focus purely on the internal dynamics of traffic flow. If open boundaries were used (e.g., finite roads with in/out flow), the clustering dynamics could be influenced by vehicle injection and removal, but the essential instability mechanism—arising from the optimal velocity function—would persist.

Second, vehicles in the model follow a **single-lane, non-overtaking** scheme, where each vehicle only responds to the one in front. This is a reasonable approximation for night-time driving, where limited visibility discourages frequent lane changes or overtaking. In reality, overtaking or lane-switching would introduce additional complexity, but the dominant feedback loop between headway and acceleration would still exist, suggesting that the model's conclusions would hold under extended scenarios.

Third, while the perturbation mechanism is modeled via a fixed deceleration over n_{dec} steps, the simulation also implicitly accounts for the fact that the **leading vehicle may experience variability in velocity**, not just due to initial perturbations but also due to road-dependent conditions such as curves, slopes, or changes in visibility. These effects can be interpreted as external inputs to the leading car and act as sources of instability that propagate backward. Therefore, **leader-induced variability**—whether from initial conditions or environmental effects—is well captured by the current model structure.

Moreover, the model uses a **non-monotonic optimal velocity function** tailored to night driving: constant at low density (limited by headlight), increasing at intermediate densities (following tail lights), and decreasing at high density (due to congestion). This structure is critical to the emergence of clusters and stop-and-go waves. Even if the precise shape of this function is changed—as long as the non-monotonic characteristic remains—the model will retain its capacity to reproduce key phenomena.

Lastly, the model assumes homogeneous vehicles and drivers with uniform sensitivity parameters. In reality, driver behaviors are heterogeneous. While introducing such diversity would add randomness and potentially blur clean bifurcations or cluster boundaries, the simulations already include **stochastic noise** in the update rules, which partially mimics behavioral variation. The presence of stable and unstable regimes despite such randomness suggests the conclusions are robust to moderate heterogeneity.

In summary, although the model relies on idealized assumptions such as periodic boundaries, non-overtaking dynamics, and homogeneous agents, its **main findings remain robust**. The emergence of clustering, kink–antikink waves, and sensitivity to driver responsiveness all stem from the fundamental structure of the velocity–headway feedback loop, which continues to operate under more realistic conditions. This validates the model as a useful framework for understanding night-time traffic behavior and designing control strategies.

6 Conclusion on the Reference Study

This mini-project successfully re-implemented the night driving behavior model proposed by Jiang et al. [1], based on the Full Velocity Difference (FVD) framework. The simulation captured the core dynamics of night-time traffic, including the impact of tail-light following at moderate densities and headlight-limited behavior at low densities, as described through a non-monotonic optimal velocity function. While the reproduced results show minor deviations from the original paper, the key patterns—such as cluster formation, wave propagation, and instability under perturbation—were consistently observed.

The study confirmed the FVD model's ability to reflect complex traffic phenomena through the use of sensitivity parameters κ and λ, and revealed its response to different perturbation scales. Practical challenges included maintaining numerical stability in the Euler integration and enforcing periodic boundary conditions, both of which were addressed through careful implementation and parameter tuning.

Overall, the re-implementation reinforces the theoretical findings of the original study and demonstrates the FVD model's capability to replicate essential features of night driving traffic. This project provided hands-on experience in simulating nonlinear systems, and highlighted the importance of both deterministic rules and stochastic effects in capturing real-world traffic dynamics.

References

[1] R. Jiang, Q.-S. Wu, Z.-N. Zhu, D.-Y. Huang, and B.-H. Wang. The night driving behavior in a car-following model. *Physica A: Statistical Mechanics and its Applications*, 375(1):297–306, 2007. doi: 10.1016/j.physa.2006.12.007. URL https://ideas.repec.org/a/eee/phsmap/v375y2007i1p297-306.html.

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Github repository: https://github.com/bluff-king/Night-driving-behavior.git