1 Rules design explanation

1.1 remark 1

Question:

Why not generalising $\vdash^-$ by introducing $\vdash^{-k}$ for $k > 1$ with the following meaning: $\Gamma \vdash^{-k} M : A$ when variable in Γ have orders at least $\mathsf{ord(M) - k}$?

Answer: The only rule that really exploit the judgment $\vdash^-$ is the weakening rule. We can see that this rule is only necessary for $k = 1$ and therefore there is no need for generalization.

Consider $M : \overline{A}|B$. Because of homogeneity of the partitioned type $\overline{A}|B$ we have $\mathsf{ord(M)} = 1 + \mathsf{ord(A)}$.

Note that there are only two cases where we need to weaken a judgment $\Gamma \vdash M : \overline{A}|B$ to $\Gamma, x : C \vdash^- M : \overline{A}|B$ while forming an unsafe term. In both cases, the hope is that eventually we will form a safe term. The two cases are:

- we want to form a safe term by abstracting a variable. The requirement here is to respect type homogeneity therefore we need to have $\mathsf{ord(x)} \geq \mathsf{ord(A)}$. Hence the weakening should allow the case when $\mathsf{ord(x)} = \mathsf{ord(A)} = \mathsf{ord(M)} - 1$.

- we want to form a safe term by apply another term N to M such that N has a free variable that do not appear M. This weakening can always be postpone until we really need it : just before instancing the (App) rule with M and N as the premises.

  The only way to recover the unsafety of $\Gamma, x : C \vdash^- M : \overline{A}|B$ is to applied the safe term N to it (in that way the unsafe term M does not appear at an operand position).

  But since x occurs freely in the safe term $\Gamma, x : C \vdash N$ we should have $\mathsf{ord(x)} \geq \mathsf{ord(N)} = \mathsf{ord(A)} = \mathsf{ord(M)} - 1$

Hence the only weakening rule that we need is $\vdash^-$ 1.