→ Role of random variables and process in communication:
Tx and Rx results are not same ⇒ noise is added in between → noise is random in nature

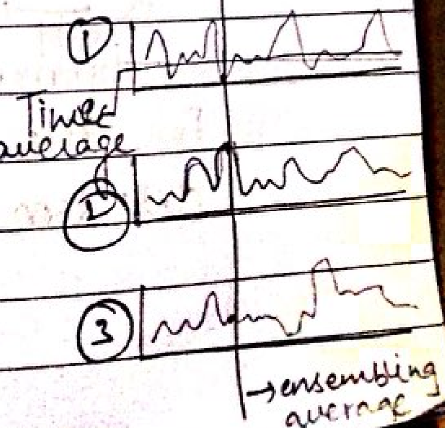
→ Continuous random variable : Example: Temperature variation, change in height of a person.
Discrete random variable → Example: No. of fruits falling of a tree, in a given time frame, tossing a coin.

→ Probability distribution function → it is used to measure the variation in continuous/ discrete random variables.

→ Probability of continuous random variable → unpredictable. Hence 0. ⇒ Area under curve = 1

→ Difference between random variable and random process
Phenomenon of representing random variables on real axis is called random process.

→ Ensembling average - Recording the beats of a music player on graph ⇒ playing it 3 times → Taking the average of values obtained at a particular time instant in all three.



→ Time Average - Average of all the values obtained by experimenting once → average of all the values of graph 1

~~If~~ When time average and ensembling average is same → it is called Ergodic process

If variance ↑ ⇒ consistency ↓ → randomness ↑
(Relation between variance and BER)

→ Maximum Likelihood Detection.

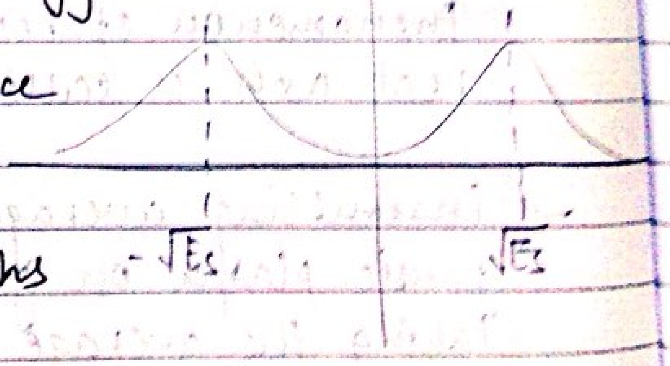
BPSK scheme → information is transmitted in the form of different phases.

Maximum phase change = 180°.

Noise represented in the form of Probability Distribution Function ~~PDF~~ (PDF)
↳ It is never a closed curve.

$$BER = \frac{E_x}{N_0} \quad \frac{(\text{Signal Energy})}{(\text{Noise Energy})}$$

If E_s is increased, distance



If E_s is increased, distance between 2 graphs would increase.

E_s → mean value of a signal energy.

For the curves, lesser the overlap, ~~the~~ easier the detection.

For higher order scheme (32 and above) detection becomes difficult

In wireless communication
Power and Bandwidth → 2 limiting factors in communication.

Power = $(\text{amplitude})^2$
If power ↑s, time required to ~~generat~~ generate such high amplitude is more. Turn on and turn off time taken by amplifier is high. If time increases, frequency decreases. In wireless communication; if time increases, delay spread also increases.

Delay spread.
Amount of time after which the second part of the signal is received ~~as~~ once the first (previous) part is received.

Time instance between first and last amplitude → delay spread.

If we increase the time period (width of pulse being sent from Tx), delay spread increases ⇒ ISI also increases. as at receiver chances of interference also increases.

Hence after a certain limit we cannot expand / increase the time period of a pulse as the delay spread and bandwidth both increases in case of wireless communication.

Wireless Channel Characteristics

Four important factors:
1) Delay spread
2) Coherence Bandwidth
3) Doppler spread
4) Coherence Time

Coherence Bandwidth.

Channel should remain same while a signal is being transmitted through Tx to Rx.

Frequency flat fading channel.

The coherence bandwidth of the channel is larger than the bandwidth of the signal. All frequency components would experience same magnitude of fading.

① Selective fading
② Coherent Bandwidth
③ Flat fading

Frequency selective fading channel.

Coherent bandwidth of channel is smaller than the bandwidth of the signal. Different frequency component experience different and uncorrelated fading

Large scale fading

Result of signal attenuation due to signal propagation over large distances and diffraction around large objects in propagation path.

Small scale fading

Due to large obstruction like hill or a large building between the sender and receiver, there is drop in received power

→ Reflection, refraction, scattering → affect delay spread.

$\delta(t) \rightarrow \boxed{} \rightarrow$ response would be : $\delta(t-\tau_i)$

delay $= \tau_i$ amplitude : $\sum a_i(\delta(t-\tau_i))$

Signal bandwidth → B_s Coherence bandwidth → B_c

$B_s > B_c \longrightarrow$ Signal distorted (selective fading)

$B_s < B_c \longrightarrow$ Flat fading

$B_c \propto \frac{1}{T_d} \Rightarrow B_c = \frac{1}{2T_d}$.

→ Doppler Spread.

Doppler spread is a measure of the spectral broadening caused by the time rate of change of mobile radio channel and is defined as the range of frequencies over which the received doppler spectrum is essentially non zero.

Doppler effect → due to relative motion → different frequencies at different places and hence we hear different sounds (voice levels).

Frequency variations → due to interaction with the moving objects in the path.

This frequency variation → at a given time is known as spectral shift.

Difference in doppler shifts between different signal components contributing to a single fading channel tap is called doppler spread.

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Maximum doppler spread → when moving along the source in line of sight.

Effect of doppler spread on receiver's side → more doppler effect → spreading of spectrum → distortion.

Doppler Spread $\propto \frac{1}{\text{Coherence time}}$

Coherence time $= \frac{1}{4f_d}$ → doppler spread.

TDM – Time Division Multiplexing

- Multiplexing and Multiple Access.
  Multiplexing → method by which multiple analog or digital signals are combined into one signal over a shared medium.

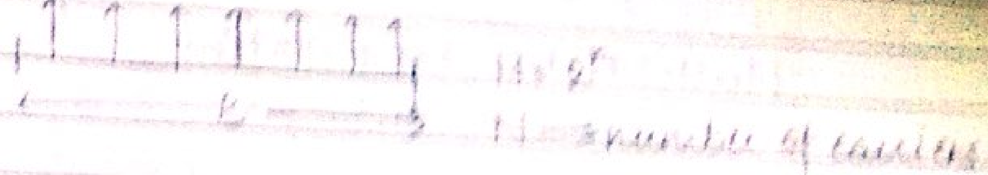
- Multiple Access – A channel access method / multiple access method allows several terminals connected to the same multi-~~port~~point transmission medium to transmit over it and to share its capacity

→ Single carrier modulation and Multi Carrier Modulation ⇒ different number of carriers.

~~No~~ Number of bits transmitted in 1 sec → bit rate
Number of symbols transmitted in 1 sec → symbol rate
Symbol → group of bits

Single carrier

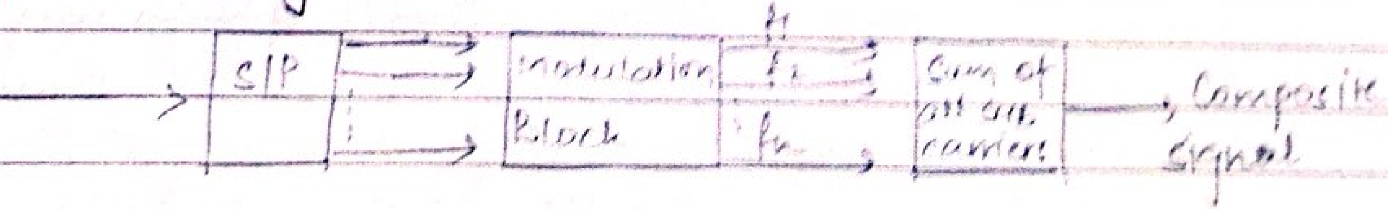
Multi carrier.



In single carrier → 1 bandwidth for one signal.
In multi carrier → 1 bandwidth divided between many signals → hence bandwidth per signal decreases → in frequency domain ↓s → time domain ↑s.

Transmitting frequency > Coherence → goes to frequency selective

Block diagram of multicarrier modulation.



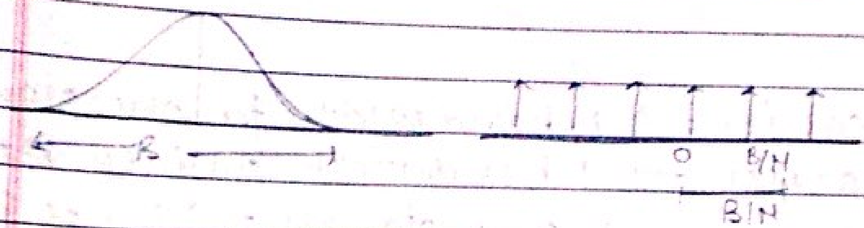
Data rate of single carrier and multiple carrier is same

Single : $\frac{1}{1/B} = B$ Multi : $\frac{N}{N/B} = B$.

Advantage of multicarrier – we can convert freq. selective to flat freq.

limitation of modulation block → it requires oscillations = number of frequency → to remove this limitation → we use OFDM → orthogonal signals → by DFT and FFT.

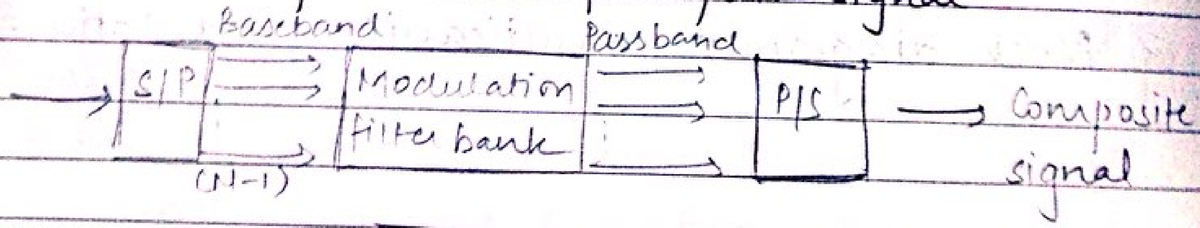
Multicarrier Modulation.

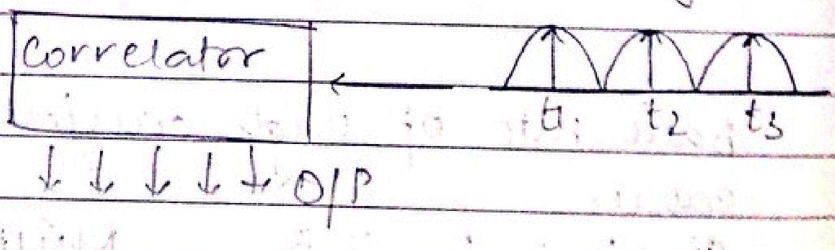


Wide bandwidth → less time
Narrow bandwidth → more time

limitation → large number of carriers required.

Passband representation of composite signal





High frequency → Pass band.

Baseband → Passband : Done in filter bank

⇓ used for mathematical analysis

⇓ signal transmission in this form is easy.

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$s_p(t) = Re\{ s_b(t) e^{j2\pi f_c t} \}$

Passband   Baseband

Composite signal → having all the frequencies.

Composite signal : $s(t) = \sum_i X_i e^{j2\pi i \frac{B}{N} t}$

On sampling this:

$$s(t) = \sum_i X_i e^{j2\pi i B/N \times u/B}$$

$$s(ut) = \sum_i X_i e^{j2\pi i u/N}$$ → signals are orthogonal to each other.

We just sample the composite signal → discretize the horizontal axis → Result : Inverse Discrete Fourier Transform.

$$s(uT_s) = \sum_i X_i e^{j2\pi (u/N) i}$$ ⇒ OFDM signal.

If channel is frequency selective, spreading would be large.

$$\text{Delay Spread} = \frac{1}{2\ \text{Coherent time}}$$

↳ This means presence / occurrence of ISI and hence frequency selective. → modelled as FIR filter → present signal depends on previous signal → FIR filters have z^{-1} element → delay element. Hence as delay increases, spread also increases.

Flat fading → no ISI → small delay spread

In multicarrier modulation → no relation between carriers.

$x[k] \rightarrow \boxed{\text{IDFT}} \rightarrow x(n)$ → this composite signal needs to be transmitted through a wireless channel.

OFDM frame: $\underbrace{\tilde{x}(0) \ldots \tilde{x}(n-2)\ \tilde{x}(n-1)}_{\text{Block 2}}\ \underbrace{x(0)\ x(1) \ldots x(n-1)}_{\text{Block 1}}$

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$

(FIR filter)

$h(0)\ h(1) \ldots h(L-1)$ $L \rightarrow$ tapped

Received signal:

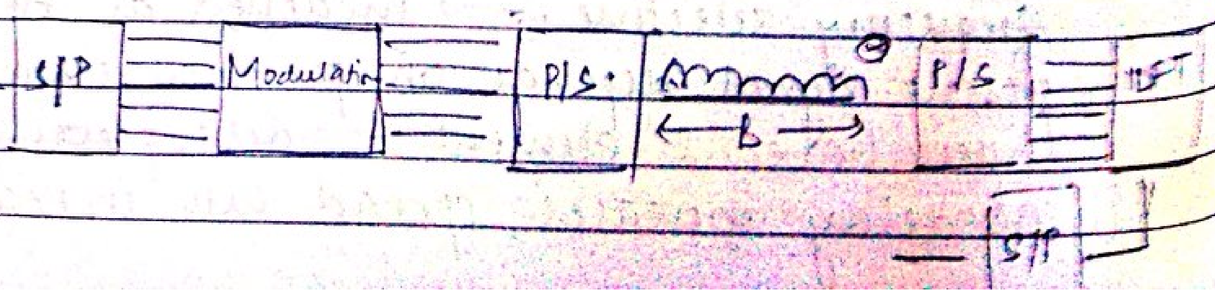
$y(0) = h(0)\,x(0)\ h(1)\,\tilde{x}(n-1)\,h(2)\,\tilde{x}(x-2)$

But the problem in this is delay spread → solved by adding cyclic prefix.

→ Cyclic prefixing

Received signal

$y(0) = h(0)\,x(0) + h(1)\,\tilde{x}(n-1) + h(2)\,\hat{x}(x-2) + \ldots$

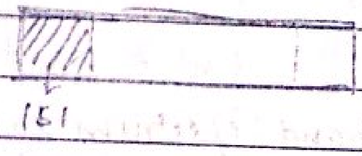


1) $x(0)\, x(1) \ldots x(N-1) \longrightarrow \boxed{L\text{-tapped}} \longrightarrow$

$$y(0) = x(0)h(0) + h(1)\tilde{x}(n-1) + h(2)\tilde{x}(n-2) + \ldots h(L-1)\,\tilde{x}(n-(L-1))$$

$$y(1) = x(1)h(0) + x(0)h(1) + \tilde{x}(N-1)h(2) + \ldots \tilde{x}(N-L-2)h(L-1)$$

More the delay spread $\rightarrow$ more the ISI.



To remove/avoid ISI; if we copy the last part before the first part; it interference with its own self sort $\rightarrow$ hence no disruption of ~~bits~~ other bits.

This method of adding last bit before is called cyclic prefixing.

Disadvantages :

- ↳ Increase in data size
- ↳ Redundancy
- ↳ Loss in throughput
- ↳ reduction in efficiency.

$$\text{Loss in effeciency} = \frac{\text{Cyclic Prefix}}{\text{Total OFDM Symbol length}}$$

$$= \frac{(L-1)}{(N+L-1)}$$

Cyclic prefix ≥ delay spread.

If we increase N; spread in time domain; effect of ISI will be less.

limitation of OFDM → cyclic prefix technique cannot be use in live streaming → takes time in loading.

loss in effeciency in 4G standard.

No. of carriers. $N = 256$

Freq. 4 MHZ → BW

CP (cyclic prefix) = 12.5% of symbol time.

Symbol bandwidth := $B_s = \frac{B}{N} = \frac{4000}{256} = 15.625$ kHz

Justify OFDM converts wide band spectrum to a narrow band spectrum (frequency selective changed to freq. flat).

$N = 256$ $\quad$ $B = 4$ MHz

$B = 4$ MHz $\quad$ $B_C = 300$ kHz

$B_s = 15.625$ kHz $\quad$ as $B_s \cancel{<} B_C \Rightarrow$ it has been converted.

Total time $= \frac{1}{4} \times 256$ → symbol time before cyclic prefix $\frac{1}{4} = T_s$ ($\because$ 4 is freq.)

$= 64$ µsec.

After adding cyclic prefixing $= T_{cp} = \frac{12.5}{100} = 8$ µsec

→ OFDM

Data Source → QAM modulator → 1:N Mux → N symbol streams → IFFT → OFDM streams → N:1 demux → QAM demodulator → Dest.

Different signals of different frequencies → combined → composite signal does not have the same power.

cyclic prefix → copying last prefix in front.

Ncp → no. of samples adding (cyclic prefix)   Bc → coherence bandwidth
Ts → time of each sample
Td → Delay spread

$N_{cp} \times T_s \geq T_d$

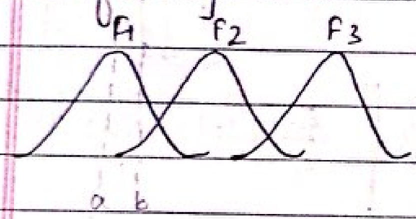
Delay spread → directly related to frequency selective. Hence directly related to no. of cyclic prefix symbols and total time period ⇒ else ISI and noise would be added. ~~But to thi~~

$N_{cp} \geq \frac{T_d}{T_s} = B \times \frac{1}{2B_c}$

$N_{cp} \geq \frac{1}{2}\frac{B}{B_c}$ ← Bandwidth of system.

For efficiency:
$N >> N_{cp} \geq \frac{1}{2}\frac{B}{B_c}$

→ Frequency and Timing offset (Challenge in OFDM)



→ Now, even if the positioning of 1 frequency shifts (from a→b); orthogonality of all signals would be distrubed and this would cause interference. This is the first challenge in OFDM.

→ When we have say 5 carriers → orthogonal to each other in OFDM signal → ~~po~~ average power would have high variations.

if ratio of $\frac{\text{Max. Power}}{\text{Avg power}}$ → high → signal would go in saturation → get clipped ~~Eff~~ $\boxed{\text{PAPR effect}}$

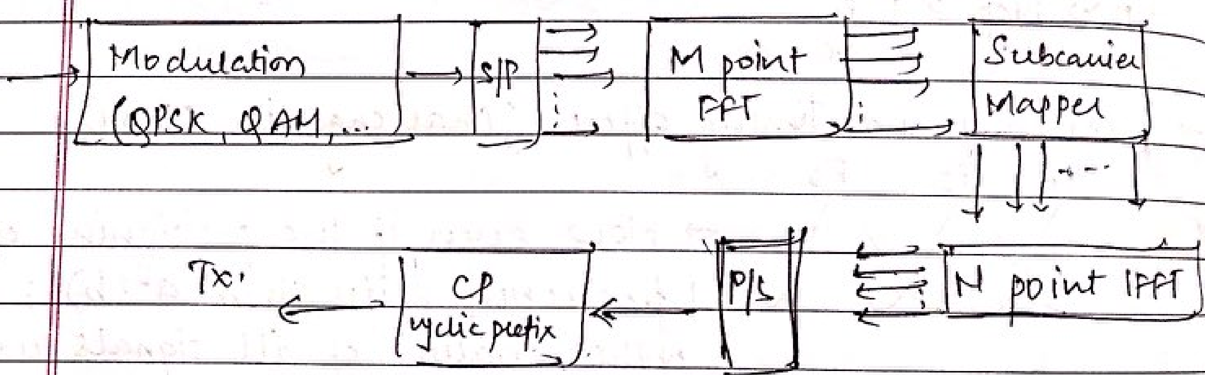
PAPR → other disadvantage / challenge of OFDM
↳ peak to average power.
For this → to rectify → amplifiers required are very ~~cost~~ expensive.

OFDM → 1 user is given one frame of several frequencies orthogonal to each other → all frequencies carrying different data.

Access technology

OFDMA → has freq. time grid → particular user is given a particular slot of time and a particular frame of frequencies. User 1 Frame 1 Time slot 1 & ~~Frame 2~~ User 2 Frame 2 Time slot 2.

SC-OFDMA



Zero padding.

If we transmit signal at F1 and F4 and not on F2 & F3:
- reduction in spectral efficiency
- reduction in ISI
- reduction in PAPR.

Multiple carrier → LTE → uplink → SDMA
OFDM → downlink

CHAPTER - MULTIPLE INPUT MULTIPLE OUTPUT (MIMO) SYSTEMS

Wired : $\boxed{Tx} \longrightarrow \boxed{Rx}$

~~Tx~~ x ~~Rx~~ y.

$y = x + n$

n → random - gaussian ~~quassian~~ in nature. $N(0, \sigma_n^2)$

Wireless $\boxed{Tx} \cdots\cdots \boxed{Rx}$ → the link can be considered as that due to antenna → antenna at both ends forming link.

x y.

$y = x + n$

$y = hx + n$.

- attenuation
- impulse response
- doppler effect
- reflection, etc

} wireless channel response h.

h → Rayleigh fading. ~~A~~ - fading.

↳ directly affects the transmitted signal

Here we have not considered impulse noise.

$h = ae^{i\phi}$ $|h^*|^2 = |a^2|$

Bit error rate → criteria to check the performance of channel.

$$\boxed{BER = Q\left(\sqrt{\frac{P}{\sigma_N^2}}\right)} \longrightarrow \frac{\text{Bit energy}}{\text{Noise}} \rightarrow \text{Wired.}$$

Received SNR in wired channel = $\dfrac{\text{Received signal power}}{\text{Noise power}}$

Received SNR in wireless channel = $\dfrac{\text{Signal Power}}{\text{Noise power}}$

Received signal power = Transmitted power × gain of the channel.

$= Pa^2$

$SNR = \frac{Pa^2}{\sigma n^2} \Rightarrow$ wireless channel.

$$\boxed{BER = Q\left(\sqrt{\frac{a^2 P}{\sigma n^2}}\right)} \longrightarrow \text{~~Received~~ Wireless.}$$

In wired

$SNR = \frac{P}{\sigma n^2}$ $\quad BER = Q\left(\sqrt{\frac{P}{\sigma n^2}}\right) = Q\sqrt{SNR}$

In wireless

$SNR = \frac{Pa^2}{\sigma n^2}$ $\quad BER = Q\left(\sqrt{\frac{Pa^2}{\sigma n^2}}\right) = \frac{1}{2}\left[1 - \sqrt{\frac{SNR}{2+SNR}}\right]$

The 'a' term in wireless → random in nature → attenuation.

	Wired	Wireless
BER	$BER = Q\sqrt{SNR}$	$BER = \left[1 - \sqrt{\frac{SNR}{2+SNR}}\right] \approx \frac{1}{2SNR}$
	BER for 10 ~~B~~dB $BER = 7.8 \times 10^{-4}$	For SNR = 20dB $BER = 5 \times 10^{-4}$ 10,000 times more power signal.

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Three different diversities :

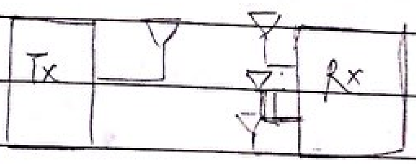
- Time
sending signal again and again → same signal at 1st, 2nd, 3rd sec

- Frequency
sending same signal at different frequencies
performace improves, bandwidth ~~wasted~~ required is more

- Space.
more than one antenna used at Tx and Rx to receive the same signal that is being transmitted
Disadvantage:
- complex circuitry
- ~~Rx~~ cost increases
- Power consumption increases.

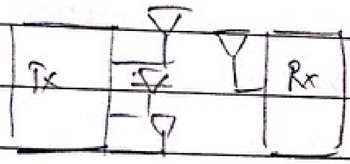
Space diversity.

SIMO



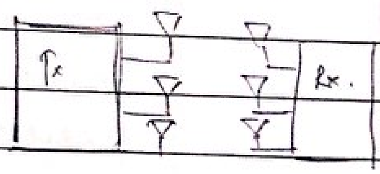
1 antennae at Tx
many at Rx.

MISO



many to 1

MIMO



Coherent detection → Channel properties are known at receiver's side.

CSI → Channel State information
- Available
- Unavailable
- Partial available / ~~quasi available~~

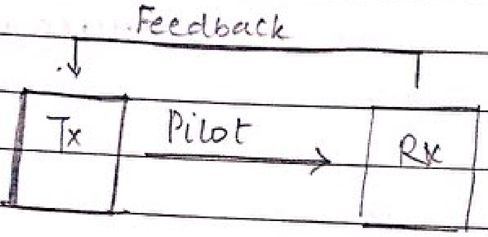
Pilot bits sent in case of coherent detection → disadvantage is : too much delay.

non CSI → detection with the help of energy.

Quasi static channel - static for a small period of time

Now as the channel will not change for a small period → the signal won't be hampered → h parameter remains same.

Transmit Beam forming - feedback is ~~received~~ required from receiver to transmitter. → to enable the transmitter know the channel properties → channel must remain constant throughout



Based on pilot bits → we can estimate the characteristics of channel.

h → channel characteristics at Tx's side

Feedback → h' → changed channel / channel present at receiver's side.

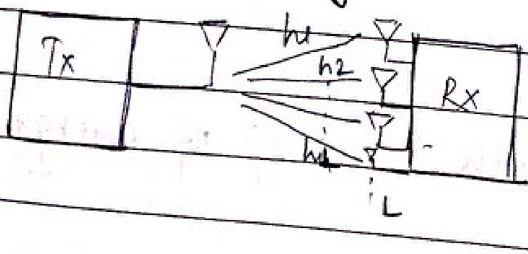
NPTEL lecture 4 to 6 derivations →

1) Maximal Ratio Combining
2) Selection combining
3) Equal Gain Combining.

} Methods of calculating SNR at receiver's side of multiple antenna → Receiver diversity techniques.

→ MIMO System.

Receiver density.



$$\bar{y} = \bar{h}x + \bar{n}$$

Received signal

$y_1, y_2, y_3, y_4 \ldots y_n$ → the ones received at our receiver's antenna.

$\downarrow \quad \downarrow \quad \downarrow$

$w_1^* y_1 \quad w_2^* y_2 \quad w_3^* y_3 \ldots$

w^* → wave function. (multiplied it with y)

$\therefore \bar{w}^* \bar{y}$ → Beamformer.

Output of receiver $= w_1^* y_1 + w_2^* y_2 + w_3^* y_3 + \ldots$

$\bar{y} = \bar{h}x + \bar{n}$

$$\therefore \bar{w}^* [\bar{h}x + \bar{n}] = [\underbrace{\bar{w}^* \bar{h}x}_{\text{signal}} + \underbrace{\bar{w}^* \bar{n}}_{\text{noise}}]$$

$$SNR = \frac{\text{Signal Power}}{\text{Noise Power}}.$$

Signal power $|\bar{w}^* \bar{h}| P$ (P → power of signal x transmitted)

Noise power $= |(\bar{w}^* \bar{n})^2|$

$$= |(\bar{w}^* \bar{n})(\bar{w}^* \bar{n})^*|$$

$$= |\bar{w}^* \bar{n} \; \bar{w} \bar{n}^*|$$

→ $w_1^* y_1 + w_2^* y_2 + \ldots + w_n^* y_n$

$$[w_1^* \ldots \ldots w_n^*] \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \bar{w}^H \bar{y}$$ → Hermitian Matrix.

Date
Page

Hermitian Matrix

$$\bar{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_L \end{bmatrix}$$

Transpose and conjugate
$[w^1 \; w^2 \ldots w_L] = \bar{w}^H$

→ Noise power $= E[(\bar{w}^H \bar{n})(\bar{w}^H \bar{n})^*]$

E

→ $SNR = \dfrac{|\bar{w}^H \bar{h}|^2 P}{\sigma_n^2 \, \bar{w}^H \bar{w}}$

$$= \underbrace{\frac{|\bar{w}^H \bar{h}|^2}{\underbrace{\bar{w}^H \bar{w}}_{||w||^2}}} \underbrace{\left(\frac{P}{\sigma_n^2}\right)}_{\gamma_c}$$

This value needs to selected such that $||w||^2 = 1$

$SNR = \gamma = (\;\;) \gamma_c$

$\bar{w} = c\bar{h}$ → vector at receiver side

$||w||^2 = c^2 ||h||^2$

$1 = c^2 ||h||^2$

$c = \dfrac{1}{||h||}$ $\qquad \bar{w}_{opt} = \dfrac{1}{||h||} \bar{h}$

$SNR = \underbrace{|\bar{w}^H \bar{h}|^2}_{\text{↳ optimize}} \gamma_c$

$= \left| \dfrac{\bar{h}^H \bar{h}}{||w||} \right|^2 \gamma_c = ||h||^2 \gamma_c$

wireless comm ⟶ Simon
lect : OFDM / MIMO → Aditya jagannam — IIT kanpur
3G, 4G → Wireless comm. (4, 5, 6, 25 ... 28).

$r = \|h_1\|^2 + \|h_2\|^2 + \dots \|h_L\|^2 r_c$

→ Maximal ~~s~~ ratio combining

$r = \|h\|^2 r_c$

$= [|h_1|^2 + |h_2|^2 + \dots |h_L|^2] r_c$

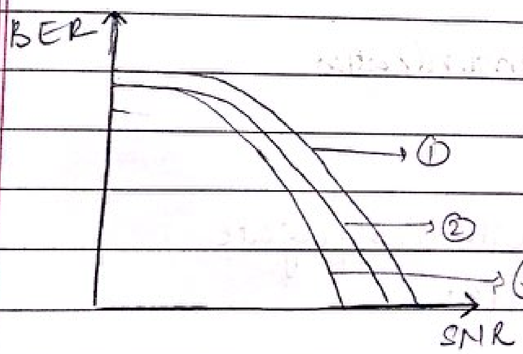
→ Equal gain combining

$r = [|h_1| + |h_2| + \dots |h_L|]^2 r_c$ $\quad$ $\max[h_1 \dots h_L]$

→ Selection combining

$r = [|h_{max}|] r_c$

→ Performance at receiver: (5 criteria to check performance):
1) SNR
2) BER
3) Outage Probability
4) Channel capacity
5) Diversity Order



① and ② → same slopes ⇒ parallel to each other → same diversity order.
slope → represents diversity.

Trade off between power and bit error rate.
Bit error rate of ③ < ① ~~but~~ ~~at~~ Hence better performance of ③. But at the same time, from the power point of view → ① > ③ ⇒ 1 is better.

Two types of systems:
→ Close - Channel is available at transmitter side
CSIT → Channel State Information Transmission.
Disadvantages : 1) Channel characteristics to be known

by feedback.
2) lengthy complex process

→ TRANSMIT DIVERSITY

Alamouti.

MRC → receiver diversity scheme → many antenna at receiver's side

But in applications like mobile handsets; we cannot keep more than 1 antenna at receiver's side. In such case; more than one antenna is present at transmitter.

Hence Alamouti → transmitter diversity scheme

Alamouti scheme :

- Improves performance
- Data rate
- Capacity of wireless communication.

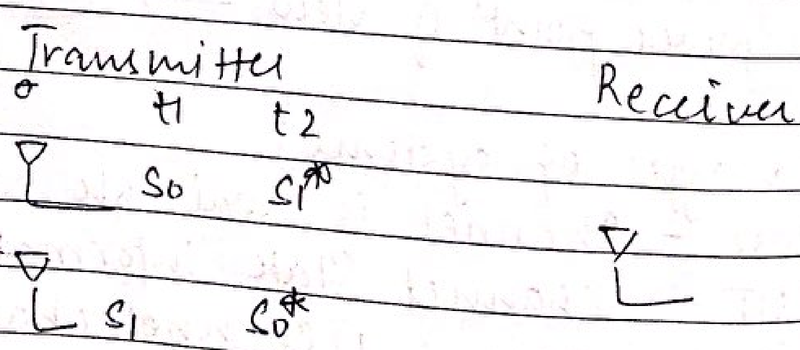
Rx Revisit → MRRC

Signal → multiplied with conjugate

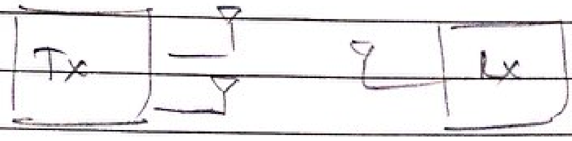
Add → decoder

At x number of antenna at transmitter side and 1 at receiver side.

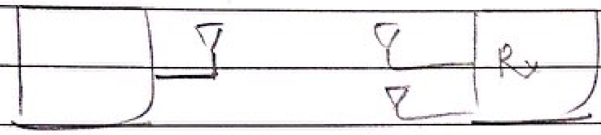
So :



Hence at time instance t_1; s_0 and s_1 is transmitted by antenna 0 and antenna 1 ~~at~~ ~~t~~ and at t_2; s_1^* and s_0^* by 0 and 1 antenna.



⇒ Alamouti



⇒ MRC