Date 17-Aug

chap-4 Energy and Potential (Hayt)

energy utilized in moving a point charge in an electric field

$$\vec{E} = \frac{\vec{F}}{Q}$$

[when charge is moving in the direction of electric field energy expenditure is (-)]

$$\vec{F_E} = Q\vec{E}$$

↳ force from the field

(component)

$$F_{EL} = \vec{F} \cdot \vec{a_L}$$
$$= Q\vec{E} \cdot \vec{a_L}$$
$$\therefore F_{opp} = -Q\vec{E} \cdot \vec{a_L}$$

usage of energy = F × displacement

small work done $= -Q\vec{E} \cdot \vec{a_L}\, dL$

$$dW = -Q\vec{E} \cdot d\vec{L}$$
$$W = -Q\int_{initial}^{final} \vec{E} \cdot d\vec{L}$$

Example An electric field is given by $\vec{E} = 6y^2z\,\vec{a_x} + 12xyz\,\vec{a_y} + 6xy^2\,\vec{a_z}$ V/m. An incremental path is represent by $d\vec{L} = -3\vec{a_x} + 5\vec{a_y} - 2\vec{a_z}$ micrometer. Find the diff^r work done in moving a 2μC charge along this path if the location of charge is at

(i) $P_A(0, 2, 5)$ (ii) $P_B(1, 1, 1)$

Sol:

$$\vec{E} \cdot d\vec{L} = -18y^2z + 60xyz - 12xy^2$$
$$W = -Q\int_{(0,0,2)}^{P(0,2,5)} (-18y^2z + 60xyz - 12xy^2)$$
$$= 2\times10^{-6} \times 10^{-6} \times (-18 \times 4 \times 5)$$
$$= 720 \text{ pJ}$$

(ii)

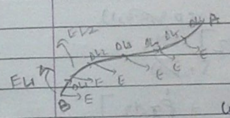
$$W = -Q\int_{(0,0,0)}^{(1,1,1)} (-18y^2z + 60xyz - 12xy^2)$$
$$= -2\times10^{-12}\left[-18\times1\times1 + 60\times1 - 12\times1\right]$$
$$= -60 \text{ pJ} \quad \text{Ans.}$$

line integral:

$$W = -Q\int_{initial}^{final} \vec{E} \cdot d\vec{L}$$

without using vector analysis

$$W = -Q\int_{ini}^{final} E_L\, dL$$



$$W = -Q\left[E_1\Delta L_1 + E_2\Delta L_2 + E_3\Delta L_3 + E_4\Delta L_4 + E_5\Delta L_5 + E_6\Delta L_6\right]$$

OR

$$= -Q\left[\vec{E_1}\cdot\Delta\vec{L_1} + \vec{E_2}\cdot\Delta\vec{L_2} + \dots \vec{E_6}\cdot\Delta\vec{L_6}\right]$$
$$\vec{E_1} = \vec{E_2} = \vec{E_3} = \dots = \vec{E}$$
$$= -Q\vec{E}\left(\Delta\vec{L_1} + \Delta\vec{L_2} + \dots \Delta\vec{L_6}\right]$$
$$= -Q\vec{E}\cdot\vec{L_{BA}}$$
$$= -Q\int_B^A \vec{E}\cdot d\vec{L}$$
$$W = -Q\vec{E}\int_B^A d\vec{L}$$

– work does not depend on path

Example: A Non uniform field is given by $\vec{E} = y\vec{a_x} + x\vec{a_y} + 2\vec{a_z}$ determine the work done in carrying 2C charge from B(1,0,1) to A(0.8, 0.6, 1) along the shorter arc of the circle $x^2+y^2=1$ & $z=1$.

Sol:

$$W = -Q\int_B^A \vec{E}\cdot d\vec{L}$$

$$d\vec{L} = dx\,\vec{a_x} + dy\,\vec{a_y} + dz\,\vec{a_z}$$

$$Q = 2C$$

$$\vec{E} = y\vec{a_x} + x\vec{a_y} + 2\vec{a_z}$$

So, $$W = -2\int_{(1,0,1)}^{(0.8,0.6,1)} y\,dx + x\,dy + 2\,dz$$

$$\cancel{= -2\left[\frac{y^2}{2}\right]}$$

$$\cancel{= -2\left[\int xy + xy + 2z\right]_{(1,0,1)}^{(0.8,0.6,1)}}$$

$$= (-2)\left[\int_1^{0.8} y\,dx + \int_0^{0.6} x\,dy + \int_1^1 2\,dz\right]$$

$$= (-2)\left[\int_1^{0.8}\sqrt{1-x^2}\,dx + \int_0^{0.6}\sqrt{1-y^2}\,dy + 0\right]$$

$$= (-2)\left[\left(\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right)_1^{0.8} + \left(\frac{y}{2}\sqrt{1-y^2} + \frac{1}{2}\sin^{-1}y\right)_0^{0.6}\right]$$

$$\cancel{= (-1)\left[\frac{0.8}{2}\sqrt{1-0.8^2} + \ldots\right]}$$

$$= (-1)\left[0.8\sqrt{1-0.8^2} + \sin^{-1}(0.8) - 1\times 0 - \sin^{-1}(1) + 0.6\sqrt{1-0.6^2} + 1\cdot\sin^{-1}(0.6) - \sin^{-1}(0)\right]$$

$$= -0.96\ J$$

II:

$$y = mx + c$$

$$y - y_B = \frac{y_A - y_B}{x_A - x_B}(x - x_B)$$

$$y = \frac{0.6-0}{0.8-1}(x-1)$$

$$y = -3(x-1)$$

$$x = \frac{-y}{3} + 1;$$

So,

$$W = (-2)\left[\int_1^{0.8} -3(x-1)\,dx + \int_0^{0.6}\left(\frac{-y}{3}+1\right)dy\right]$$

$$= (-2)\left[\left(\frac{-3x^2}{2} + 3x\right)_1^{0.8} + \left(\frac{-y^2}{6} + y\right)_0^{0.6}\right]$$

$$= (-2)\left[-0.06 + 0.54\right]$$

$$= -0.96\ J$$

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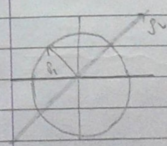
Infinite line charge $\vec{E} = E_\rho\vec{a_\rho} = \frac{\rho_L}{2\pi\epsilon_0\rho}\vec{a_\rho}$

(i)



Moving along the circle

$$dL = \rho_1\,d\phi\,\vec{a_\phi}$$

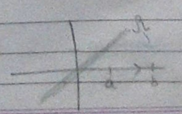
$$W = -Q\int_{ini}^{final} \vec{E_\rho}\cdot d\vec{L}$$

$$= -Q\int_{ini}^{f} \frac{\rho_L}{2\pi\epsilon_0\rho}\vec{a_\rho}\cdot\rho_1\,d\phi\,\vec{a_\phi}$$

$(\because \vec{a_\rho}\cdot\vec{a_\phi} = 0)$

$$\boxed{W = 0}$$

(ii)



moving along radial path a to b

$$W = -Q\int_{ini}^{final} \frac{\rho_L}{2\pi\epsilon_0 \rho}\, \vec{a_\rho} \cdot d\rho\, \vec{a_\rho}$$

$$= -Q\int_{ini}^{final} \frac{\rho_L}{2\pi\epsilon_0} \cdot \frac{d\rho}{\rho}$$

$$\boxed{W = \frac{-Q\rho_L}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right)}$$

Potential & potential Difference:

Potential difference is defined as workdone to move a ~~th~~ +ve point charge from one point to another in direction of E.

$$W = -Q\int_{ini}^{final} \vec{E}\cdot\vec{dL} \qquad , V = W/Q$$

pot. diffr $\quad V = -\int_{ini}^{final} \vec{E}\cdot\vec{dL} \qquad$ [V or J/C]

$$V_{AB} = -\int_B^A \vec{E}\cdot\vec{dL}$$

– Let $W = \frac{Q\rho_L}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right)$

$$V_{ab} = \frac{W}{Q} = \frac{\rho_L}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right)$$

– $\vec{E} = E_r \vec{a_r} = \frac{Q}{4\pi\epsilon_0 r^2} \vec{a_r}$

$$dL = dr\, \vec{a_r}$$

$$V_{AB} = -\int_B^A \frac{Q}{4\pi\epsilon_0 r^2} \vec{a_r} \cdot dr \cdot \vec{a_r}$$

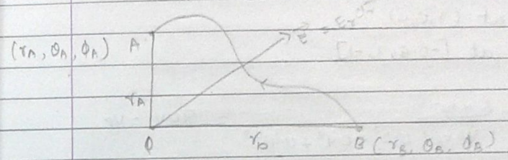
$$= -\int_B^A \frac{Q}{4\pi\epsilon_0} \cdot \frac{dr}{r^2}$$

$$V_{AB} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) = V_A - V_B$$

The potential field of a point charge

$$V_{AB} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$$= V_A - V_B$$



$$\vec{dL} = dr\, \vec{a_r} + r\,d\theta\, \vec{a_\theta} + r\sin\theta\, d\phi\, \vec{a_\phi}$$

$$V_{AB} = -\int_{r_B}^{r_A} E_r\, dr = -\int_{r_B}^{r_A} \frac{Q}{4\pi\epsilon_0 r^2}\, dr$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

∴ potential difference is independent of path but dependent on distance of charge to that point.

Let $V = 0$ at ∞ point

$\Rightarrow r = r_B$ is at ∞

$$\therefore V = \frac{Q}{4\pi\epsilon_0 r_A} = \frac{Q}{4\pi\epsilon_0 r}$$

∴ $Q/4\pi\epsilon_0 r$ J of work is to be done to bring 1C charge from ∞ to a point at distance r from Q charge.

⇒ **Equipotential surfaces:**

potential difference between two points in a surface = 0 ; Hence W = 0

Eg: A point charge of 6nC is located at the origin in free space. find V_P if point P is located at $[0.2, -0.4, 0.4]$ and

i) $V = 0$ at ∞

ii) $V = 0$ at $(1,0,0)$

(iii) $V = 20V$ at $[-0.5, 1, -1]$

Sol:

i) $V_{PQ} = \frac{6\times10^{-9}}{4\pi\varepsilon_0 (0.2^2+0.4^2+0.4^2)^{1/2}} = 90V = V_P$

ii) $V_{PQ} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_P} - \frac{1}{r_Q}\right) = 6\times9\left(\frac{5}{3} - 1\right) = 36V = V_P$

$V_{PQ} = V_P - V_Q$

iii) $V_{PQ} = 6\times9\left(\frac{5}{3} - \frac{2}{3}\right) = 54V$

$V_{PQ} = V_P - V_Q$

$54 = V_P - 20$

$V_P = 74V$

⇒ Potential field of a system of charges - Conservative property

$V(\vec{r}) = \frac{Q_1}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|}$

$V(\vec{r}) = \frac{Q_1}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|} + \frac{Q_2}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2|}$

n-point charges

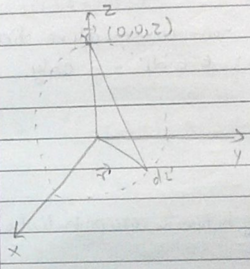
$V(\vec{r}) = \frac{Q_1}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|} + \frac{Q_2}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2|} + \dots + \frac{Q_n}{4\pi\varepsilon_0|\vec{r}-\vec{r}_n|}$

$= \sum_{m=1}^{n} \frac{Q_m}{4\pi\varepsilon_0|\vec{r}-\vec{r}_m|}$

$V(\vec{r}) = \frac{\rho_v(\vec{r}_1)\Delta v_1}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|} + \frac{\rho_v(\vec{r}_2)\Delta v_2}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2|} + \dots$

$= \int_{vol} \frac{\rho_v(\vec{r}')\,dv}{4\pi\varepsilon_0|\vec{r}-\vec{r}'|}$

⇒ potential of a uniform line charge (in ring shape)



$|\vec{r} - \vec{r}'| = \sqrt{\rho^2+z^2}$

$dL' = a\,d\phi'$

$\rho = a$; $z = 0$

ρ_L at $\vec{r}'$

$V(\vec{r}) = \int \frac{[\rho_L(\vec{r}')]\,dL'}{4\pi\varepsilon_0|\vec{r}-\vec{r}'|}$

$dL' = a\,d\phi'$; $\vec{r} = z\,\vec{a_z}$; $\vec{r}' = a\,\vec{a_\rho}$

$|\vec{r} - \vec{r}'| = \sqrt{a^2+z^2}$

$V(\vec{r}) = \int_0^{2\pi} \frac{\rho_L \cdot a \cdot d\phi'}{4\pi\varepsilon_0 (a^2+z^2)^{1/2}}$

$= \frac{\rho_L \cdot a}{4\pi\varepsilon_0 (a^2+z^2)^{1/2}} \cdot (2\pi)$

$V(\vec{r}) = \frac{\rho_L \cdot a}{2\varepsilon_0 (a^2+z^2)^{1/2}}$

- any field which satisfies $\oint \vec{E}\cdot d\vec{L} = 0$ is a conservative field as work done is 0

Eg: Gravitational field

Eg:- $\vec{E} = \sin(\pi\rho)\,\vec{a_\phi}$

Sol: we are assuming circular path of $\rho = \rho_1$

$d\vec{L} = \rho\,d\phi\,\vec{a_\phi}$

$W = \oint \vec{E}\cdot d\vec{L}$

$= \oint_0^{2\pi} \sin\pi\rho \cdot \rho\,d\phi$

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$= \phi \, 2\pi\rho \sin\pi\rho$

at $\phi = 1$; $w = 2\pi\rho \sin\pi\rho$

$$w = \oint \vec{E}\cdot d\vec{l} = \begin{cases} 0 \; ; & \rho = 0, 2, 4 \ldots \\ 1 \; ; & \rho = 1/2, 1/4 \ldots \end{cases}$$

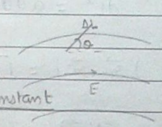
above is example of non-conservative field.
in non-conservative field ; $\oint \vec{E}\cdot d\vec{l} = 0$ only possible at some points.

potential Gradient :

rate of change of spacing between equipotential surfaces is is called potential gradient.

Gradient: rate of change of one thing w r t to other.

$V = -\int \vec{E}\cdot d\vec{L}$



$\Delta\vec{L}, \vec{E}$

$\Delta V = -\vec{E}\cdot\Delta\vec{L}$ $\quad \vec{E} \to$ constant

$\Delta\vec{L} = \Delta L \, \vec{a_L}$

$\Delta V = -E \Delta L \cos\theta$

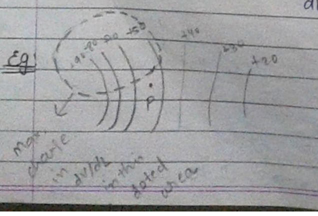
$$\boxed{\frac{\Delta V}{\Delta L} = -E\cos\theta = \frac{dV}{dL}}$$

- $\left.\frac{dV}{dL}\right|_{max}$; when $\cos\theta = -1$; it will may possible when dV & dL are in opp direction to each other.

$$\left.\frac{dV}{dL}\right|_{max} = E$$

$$\vec{E} = -\left.\frac{dV}{dL}\right|_{max} \vec{a_N}$$ direction of E, normal to the equipotential surface

Eg:



spherical: $\frac{\partial V}{\partial r}\vec{a_r} + \frac{1}{r}\frac{\partial V}{\partial\theta}\vec{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\vec{a_\phi}$

cyln: $\frac{\partial V}{\partial\rho}\vec{a_\rho} + \frac{1}{\rho}\frac{\partial V}{\partial\phi}\vec{a_\phi} + \frac{\partial V}{\partial z}\vec{a_z}$

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$$\left.\frac{dV}{dL}\right|_{max} = \frac{dV}{dN}$$

$$\therefore \boxed{\vec{E} = -\frac{dV}{dN}\vec{a_N} = -\text{grad } V}$$

- Gradient of T = grad T
$= \frac{dT}{dN}\vec{a_N}$

- $\vec{E} = -\text{grad } V$

$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$ ——— ①

$dV = -\vec{E}\cdot d\vec{L}$
$= -E_x dx - E_y dy - E_z dz$ ——— ②

comparing ① & ②

$E_x = -\frac{\partial V}{\partial x}$; $E_y = -\frac{\partial V}{\partial y}$; $E_z = \frac{\partial V}{\partial z}$

so $\vec{E} = -\left(\frac{\partial V}{\partial x}\vec{a_x} + \frac{\partial V}{\partial y}\vec{a_y} + \frac{\partial V}{\partial z}\vec{a_z}\right)$

$\text{grad } V = \frac{\partial V}{\partial x}\vec{a_x} + \frac{\partial V}{\partial y}\vec{a_y} + \frac{\partial V}{\partial z}\vec{a_z}$

- $$\boxed{\nabla = \frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}}$$

Eg: given the potential field $V = 2x^2y - 5z$ and a point $P(-4,3,6)$. find the potential V at point P, the electric field intensity E, the dirc of E, electric flux density D and the volume charge density at point P.

Sol:

$V)_{-4,3,6} = 2(-4)^2(3) - 5(6)$
$= 2\times16\times3 - 30 = 66 \text{ V}$

$\vec{E} = -\text{grad } V = -4xy\,\vec{a_x} - 2x^2\vec{a_y} + 5\vec{a_z}$
$\vec{E}|_P = +48\,\vec{a_x} - 32\,\vec{a_y} + 5\,\vec{a_z}$ V/m

$$\vec{E} = \frac{48\vec{a_x} \cancel{+} 32\vec{a_y} + 5\vec{a_z}}{57.90}$$

$$= 0.82\vec{a_x} \cancel{+} 0.55 a_y + 0.086\vec{a_z} \quad V/m$$

$$\vec{D} = \epsilon_0 \vec{E} \qquad = \epsilon_0(-4xy\vec{a_x} + 2x^2\vec{a_y} + 5\vec{a_z})$$

$$= \epsilon_0(48, -32, 5) \quad = 35.4\,xy\vec{a_x} - 17.71x^2\vec{a_y} + 44.3\vec{a_z}$$

PC/m^2

Volume charge density $\rho_v = div\,\vec{D}$

$$= \epsilon_0[-4y + 0 + 0]$$

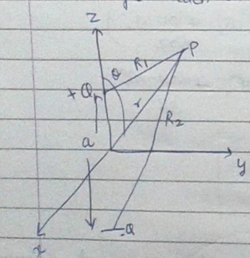
$$= \cancel{-4y\epsilon_0}$$

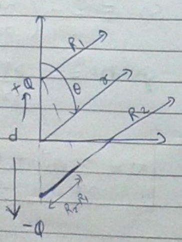
$$= \cancel{-4 \times 3 \times 8.85 \times 10^{-12}}$$

$$= 35.4 \times y$$

$$= 106.2 \ PC$$

Dipole: two opposite charges are placed at certain distance from each other.





$$V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \frac{Q}{4\pi\epsilon_0}\frac{(R_2 - R_1)}{R_1 R_2}$$

here $R_2 - R_1 = d\cos\theta$

$R_1 = R_2 = r$

$$\boxed{V = \frac{Q \cdot d\cos\theta}{4\pi\epsilon_0 r^2}}$$

$$\vec{E} = -grad\,V$$

$$= -\left(\frac{\partial V}{\partial r}\vec{a_r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\vec{a_\phi}\right)$$

$$= -\frac{Qd\cos\theta(-2)}{4\pi\epsilon_0 r^3}\vec{a_r} + \frac{Qd\sin\theta}{4\pi\epsilon_0 r^3}\vec{a_\theta} + \frac{1}{r\sin\theta} \times 0$$

$$\vec{E} = \frac{2Qd\cos\theta}{4\pi\epsilon_0 r^3}\vec{a_r} + \frac{Qd\sin\theta}{4\pi\epsilon_0 r^3}\vec{a_\theta}$$

$$\boxed{\vec{E} = \frac{Qd}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\vec{a_r} + \sin\theta\,\vec{a_\theta}\right)}$$

$\therefore$ we can say that $V \propto \frac{1}{r^2}$ & $E \propto \frac{1}{r^3}$

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Dipole movement:

$$\vec{P} = Q\vec{d} \qquad cm$$

$$\vec{d} \cdot \vec{a_r} = d\cos\theta$$

$$V = \frac{\vec{P} \cdot \vec{a_r}}{4\pi\epsilon_0 r^2}$$

$$V = \frac{1}{4\pi\epsilon_0 |\vec{r} - \vec{r'}|^2} \cdot \vec{P} \cdot \frac{\vec{r} - \vec{r'}}{|\vec{r} - \vec{r'}|} \qquad r \rightarrow$$

$r' \rightarrow$ determine the dipole centered

Ex: Two dipoles with dipole moment $-5\vec{a_z}$ ncm & $9\vec{a_z}$ nc/m are located at points $(0, 0, -2)$ & $(0, 0, 3)$ respectively. find the potential at the origin.

Sol:

$$V_1 = \frac{P_1 r_1}{4\pi\epsilon_0 (r_1)^3}$$

$r_1 = (0,0,0) - (0,0,-2)$

$= 2\vec{a_z}$

$$= \frac{-5\vec{a_z} \cdot 2\vec{a_z}}{4\pi\epsilon_0 (2)^3} = \frac{-10}{4\pi\epsilon_0 \cdot 8}$$

$|r_1| = 2$

$$V_2 = \frac{9\vec{a_z}(-3\vec{a_z})}{4\pi\epsilon_0 (3)^3} = \frac{-27}{4\pi\epsilon_0 (27)}$$

$r_2 = (0,0,0) - (0,0,3) = -3\vec{a_z}$

$|r_2| = |3\vec{a_z}| = 3$

$$V = -225 \ V$$

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{\vec{p}_1 \cdot \vec{r}_1}{|\vec{r}_1|^3} + \frac{\vec{p}_2 \cdot \vec{r}_2}{|\vec{r}_2|^3}\right]$$

Energy density in the electrostatic field:

work done to position Q_2 = $Q_2 \cdot V_{21}$

work done to position Q_3 = $Q_3 \cdot V_{3,1} + Q_3 V_{3,2}$

(Potential at Q_3 because of Q_1)

total work

$$W_E = Q_2 V_{2,1} + Q_3 (V_{3,1} + V_{3,2}) \quad \text{--- (1)}$$

$$Q_3 \cdot V_{3,1} = Q_3 \cdot \frac{Q_1}{4\pi\epsilon_0 R_{13}} = \frac{Q_1 Q_3}{4\pi\epsilon_0 R_{31}} = Q_1 \cdot V_{1,3}$$

$$\text{So; } W_E = Q_1 \cdot V_{1,2} + Q_1 V_{1,3} + Q_2 V_{2,3} + Q_2 V_{2,4} \quad \text{--- (2)}$$

(1) + (2)

$$2W_E = Q_1 (V_{1,2} + V_{1,3} + V_{1,4} + \ldots) + Q_2 (V_{2,1} + V_{23} + \ldots) + Q_3(\ldots) + Q_n$$

Let assume $V_{1,2} + V_{1,3} + V_{1,4} \ldots = V_1$

$$\text{So; } W_E = \frac{1}{2}\left(Q_1 V_1 + Q_2 V_2\right)$$

$$W_E = \frac{1}{2} \sum_{m=1}^{N} Q_m V_m$$

continous charge distribution $\quad W_E = \frac{1}{2}\int_{\text{Volume}} \rho_v \, dV$. $V \rightarrow$ Pottential

$$\rho_v = \nabla \cdot \vec{D}$$

$$\nabla \cdot (V\vec{D}) = V(\nabla \cdot \vec{D}) + \vec{D}(\nabla V)$$

$$\text{So; } W_E = \frac{1}{2}\int_{vol} (\nabla \cdot \vec{D}) V \, dV$$

$$= \frac{1}{2}\int_{vol} \left[\nabla \cdot (V\vec{D}) - \vec{D}(\nabla V)\right] dV$$

divergence theorem $\oint_S \vec{D} \cdot d\vec{S} = \int \nabla \cdot \vec{D} \, dV$

(Volume integral converted into surface integral)

$$\text{So; } W_E = \frac{-1}{2}\int_{vol} \vec{D}(\nabla V) dV + \frac{1}{2}\oint_S V\vec{D} \cdot d\vec{S}$$

$1/r$ $\quad$ $1/r^2$ $\quad$ $r \rightarrow \infty$ $\quad$ $1/r^2 \rightarrow 0$

$$W_E = \frac{-1}{2}\int_{vol} \vec{D}(\nabla V) dV$$

$$\vec{E} = -\nabla V$$

$$\text{So; } W_E = \frac{1}{2}\int_{vol} \vec{D}\vec{E} \, dV$$

$$\boxed{W_E = \frac{1}{2}\int_{vol} \epsilon_0 E^2 dV} \qquad D = \epsilon_0 E$$

energy density $= \frac{dW_E}{dV}$

$$= \frac{1}{2}\vec{D} \cdot \vec{E}$$

$$= \frac{1}{2}\epsilon_0 E^2 \Big/ \frac{D^2}{2\epsilon_0}$$

$$= \frac{\epsilon_0 E^2}{D^2}$$

$\frac{1}{2}\epsilon_0 E^2 = W$

Ex:- point charge $-1nC$, $4nC$ and $3nC$ are located at $(0,0,0)$, $(0,0,1)$ & $(1,0,0)$ respectively. find the energy in the s/s.

Sol:

$$W_1 = 0$$

$$W_2 = Q_2 V_{2,1} = \frac{Q_2 \cdot Q_1}{4\pi\varepsilon_0 R_{1,2}}$$

$R_{12} = (0,0,1) - (0,0,0) = (0,0,1)$

$$W_3 = Q_3 V_{3,1} + Q_3 V_{3,2} = \frac{Q_3 Q_1}{4\pi\varepsilon_0 R_{13}} + \frac{Q_3 Q_2}{4\pi\varepsilon_0 R_{23}}$$

$R_{13} = (1,0,0) - (0,0,0) = (1,0,0)$

$R_{23} = (1,0,0) - (0,0,1) = (1,0,-1)$

$$\text{So; } W_E = 0 + \frac{(-4)\times10^{-9}\times10^{-9}}{4\pi\varepsilon_0(1)} - \frac{3\times10^{-18}}{4\pi\varepsilon_0(1)} + \frac{12\times10^{-18}}{4\pi\varepsilon_0(\sqrt{2})}$$

$$= \frac{-7\times10^{-18}}{4\pi\varepsilon_0} + \frac{12\times10^{-18}}{4\pi\varepsilon_0(\sqrt{2})}$$

$$= \frac{1.48\times10^{-18}}{4\pi\varepsilon_0}$$

$$= 13.36\times10^{-9}\ J$$

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Ch-5.6

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CURRENT AND CONDUCTORS

Convection & Conduction Currents:

$$I = \frac{dQ}{dt}$$

current density $\vec{J} = \frac{\Delta I}{\Delta S}$

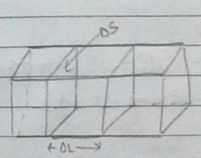
$$\Delta I = \vec{J}\,\Delta S \rightarrow ($$

$$\Delta I = \vec{J}\cdot\Delta\vec{S}$$

$$I = \int_S \vec{J}\cdot d\vec{S}$$

Convection current: it does not involve any conductor and not satisfy ohm's law. it occurs when current flow ~~from in~~ ~~inductor~~ insulator as vaccume, liquid etc.

Example: $e^{\ominus}$ beam in vaccume tube



$$\vec{u} = u_y \vec{a_y}$$

$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \frac{\Delta V}{\Delta t} = \rho_v \cdot \Delta S \frac{\Delta L}{\Delta t}$$

$$\Delta I = \rho_v \Delta S\, u_y$$

$$J_y = \frac{\Delta I}{\Delta S} = \rho_v u_y$$

$$\boxed{\vec{J} = \rho_v \vec{u}}$$

convection density

Conduction current: its involves conductor and satisfy the ohm's law.

$$\vec{F} = -e\vec{E}$$

$$\frac{m\vec{u}}{\tau} = -e\vec{E} \quad \tau \to \text{avg. time of collision}$$

$$\vec{u} = -\frac{e\tau\vec{E}}{m}$$

$$\rho_v = -ne$$

$$\vec{J} = \rho_v \vec{v}$$

$$= (-ne)\left(-\frac{e\tau\vec{E}}{m}\right)$$

$$\vec{J} = \frac{ne^2\tau\vec{E}}{m}$$

$$\boxed{\vec{J} = \underbrace{\sigma}_{\text{conductivity}}\vec{E}} \rightarrow \text{point form of ohm' law}$$

<u>Num.</u> if current den. $\vec{J} = \frac{1}{r^3}(2\cos\theta\, \vec{a_r} + \sin\theta\, \vec{a_\theta})$ A/m². Calculate current passing through

i) A hemi-spherical shell of radius 20 cm.

ii) A spherical shell of radius 10 cm.

<u>Sol^n</u>

i)
$$I = \int_s \vec{J}\cdot d\vec{s}$$

$$d\vec{s} = r^2 \sin\theta\, d\theta\, d\phi\, \vec{a_r}$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} \frac{1}{r^3}\left(2\cos\theta \cdot r^2 \sin\theta\, d\theta\, d\phi\right)$$

$$= \frac{1}{r}(2\pi)\left[\frac{-\cos 2\theta}{2}\right]_0^{\pi/2}$$

$$= \frac{2\pi}{2\cdot 20\times10^{-2}}\left[-\cos\frac{2\times\pi}{2} + \cos 0\right]$$

$$= \frac{\pi\times10^2}{20}\left[2\right] = \pi\times10^2$$

$$= 31.4\text{ A}$$

ii)
$$I = \frac{2\pi}{r\times2}\left[\frac{-\cos 2\theta}{}\right]_0^{\pi}$$

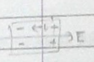
$$= \frac{-\pi}{10\times10^{-2}}(1-1) = 0$$

<u>Materials:</u>

conductors, insulator, semiconductor / dieletric material

very few e⁻ are present for conduction of current

conductors ↓



i) $E = 0$

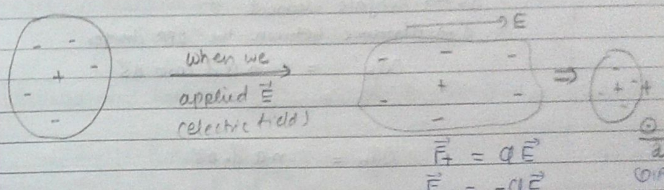
$E - E' = 0$ bcuz $E = \vec{J}/\sigma$, $\sigma \to \infty$

ii) $v_{drift} = 0$

iii) conductivity $= \infty$

<u>polarization in Dielectric:</u>

(179)



$$\vec{F_+} = q\vec{E}$$
$$\vec{F_-} = -q\vec{E}$$

bcuz (+) & (−) charges are seperated because of E, then it will created dipole (align ~~of~~ in dir^n of $\vec{E}$ field)

$$\ominus \xrightarrow{\vec{d}} \oplus$$

$$P = Q\vec{d}$$

$$Q_1\vec{d_1} + Q_2\vec{d_2} + Q_3\vec{d_3} + \dots + Q_n\vec{d_n} = \sum_{k=1}^{N} Q_k\vec{d_k}$$

Polar intensity: $\vec{P} = \lim_{\Delta v\to 0} \frac{\sum_{k=1}^{n} Q_k\vec{d_k}}{\Delta v}$ C/m²

Non-Polar mat. : Hydrogen, Oxygen (where dipole is not present, but by applying $\vec{E}$, then dipole is created)

Polar material : H_2O, (dipole is already available in the material

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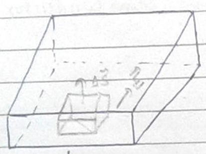
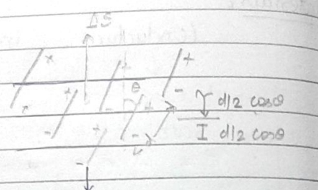
polarization in dilectrics :



↓ dilectric material



↓ dipoles when E is appeid

$\theta \rightarrow$ angle between ΔS & dipole

$n \rightarrow$ no. of molecules

$Q \rightarrow$ charge

$\Delta S \rightarrow$ surface element

$d \rightarrow$ distance between the opp. charges

$$\underbrace{\Delta Q_b}_{\text{bound charges}} = n Q d \cos\theta \, \Delta \vec{S}$$

$$\Delta Q_b = n Q \vec{d} \cdot \Delta \vec{S}$$

$$\Delta Q_b = \vec{P} \cdot \Delta \vec{S}$$

$$Q_b = -\oint_S \vec{P} \cdot \Delta \vec{S}$$

-ve sign just indicates the the charges are present inside the surface, not outside the surface.

$$Q_T = \oint_S \epsilon_0 \vec{E} \cdot d\vec{S} = \oint \vec{D} \cdot d\vec{S}$$

$$Q_T = \underset{(\text{free})}{Q} + \underset{(\text{bounded})}{Q_b}$$

$$Q = Q_T - Q_b$$

$$= \oint_S \epsilon_0 \vec{E} \cdot d\vec{S} + \oint_S \vec{P} \cdot d\vec{S}$$

$$\boxed{Q = \oint_S (\epsilon_0 \vec{E} + \vec{P}) \cdot d\vec{S}}$$

so, $\boxed{\vec{D} = \epsilon_0 \vec{E} + \vec{P}}$ → dilectric flux density

flux density of dilectric material, is dependent on electric field and polarization.

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By divergence theorm $Q_T = \int_V \rho_T \, dv$

$$Q = \int_V \rho_v \, dv$$

$$\nabla \cdot \vec{P} = -\rho_b$$

$$\nabla \cdot \epsilon_0 \vec{E} = \rho_T$$

$$\nabla \cdot \vec{D} = \rho_v$$

* (184) linear material: when D varies linearily with E.

isotropic material: material for which $\underset{\text{flux density}}{D}$ & $\underset{\text{electric field intensity}}{E}$ are in same direction.

Homogenous material: material which ϵ is constant.

* (182) Relationship between polarization & D for isotropic material

$$\vec{P} = \chi_e \epsilon_0 \vec{E}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$= \epsilon_0 \vec{E} + \chi_e \epsilon_0 \vec{E}$$

$$= \epsilon_0 \vec{E} \, (1 + \chi_e) \rightarrow \epsilon_r \rightarrow \text{relative Permitivity}$$

$$= \epsilon_0 \epsilon_r \vec{E}$$

$$\boxed{\vec{D} = \epsilon \vec{E}}$$ → this relationship is true only for isotropic

Ex:- Electric field instensity in Polysterene $\epsilon_r = 2.55$, filling the space between the plates of a parallel plate cap. is 10 KV/m. The distance between the plate is 1.5 mm. Calculate flux density and Polarization. ($\chi_e = 1.55$)

Sol:

$\chi_e = 1.55$; $\epsilon_r = 2.55$

$d = 1.5\text{mm}$; $E = 10 \text{ KV/m}$

$\vec{D} = \epsilon_0 E + P$

polarization: $\vec{P} = \cancel{Qd} \ \chi_e \epsilon_0 \vec{E}$

$= \cancel{1.6 \times 10^{-19} \times 1.5 \times 10^{-3}}$

$= 1.55 \times 8.85 \times 10^{-12} \times 10 \times 10^{3} = 13.75 \times 10^{-8}$

$= 137.5 \text{nC m}$

Relaxation time

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flux density $D = \varepsilon_0 \varepsilon_r E$

$= 8.85 \times 10^{-12} \times 10 \times 10^3 \times 2.55$

$= 22.56\ n$

Potential diffr $V = E \times d$

$= (10 \times 10^3)(1.5\,mm)$

$= 15\ V$

1) D of free charges on plates $= \vec{D} \cdot \vec{a_N} = D$

Continuity Equation:

principle of charge convergence:

-ve sign bcuz change moving out, hence decreases in the materials

$$I_{out} = \oint \vec{J} \cdot d\vec{s} = -\frac{dQ_{in}}{dt} \quad \text{--- (1)}$$

divergence theorem

$$\oint_s \vec{J} \cdot d\vec{s} = \int \nabla \cdot \vec{J}\, dV \quad \text{--- (2)}$$

$$-\frac{dQ_{in}}{dt} = -\frac{d}{dt}\int_v \rho_v\, dV = -\int \frac{\partial \rho_v}{\partial t}\, dV$$

$$\int \nabla \cdot \vec{J}\, dV = -\int \frac{\partial \rho_v}{\partial t}\, dV$$

$$\boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}}$$

for steady current $\nabla \cdot \vec{J} = 0$; is no movement of charges. Hence no change in density.

Boundary Conditions:

$\oint \vec{E} \cdot d\vec{l} = 0$ (work done in a closed path = 0)

$\oint \vec{D} \cdot d\vec{s} = Q_{enclosed}$

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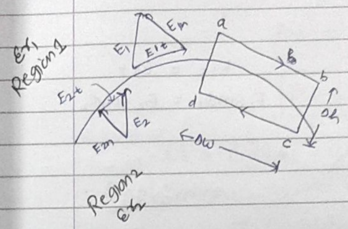
(i) Dielectric - dielectric B.C.



$\vec{E_1} = \vec{E_{1t}} + \vec{E_{1n}}$ (tang. & normal component)

$\vec{E_2} = \vec{E_{2t}} + \vec{E_{2n}}$

$0 = E_{1t} \cdot \Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\Delta w + E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2}$

$E_t = |\vec{E_t}|$ & $E_n = |\vec{E_n}|$

$0 = (E_{1t} - E_{2t})\Delta w$

$\Delta h \to 0$

$\boxed{E_{1t} = E_{2t}}$ → continuous

$\vec{D} = \varepsilon \vec{E} = \vec{D_t} + \vec{D_n}$

$\frac{D_{1t}}{\varepsilon_1} = E_{1t} = E_{2t} = \frac{D_{2t}}{\varepsilon_2}$

or

$\boxed{\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}}$ → discontinuous

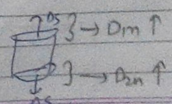
$\Delta Q = \rho_s \Delta S = D_{1n}\Delta S - D_{2n}\Delta S$

$D_{1n} - D_{2n} = \rho_s$

$\rho_s = 0 \Rightarrow \boxed{D_{1n} = D_{2n}}$ → continuous

$\boxed{\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}}$

direction of D_{1n} & D_{2n} are same



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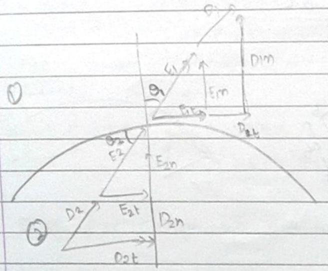


$$E_1t = E_2t$$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2$$

$$D_{1n} = D_{2n}$$

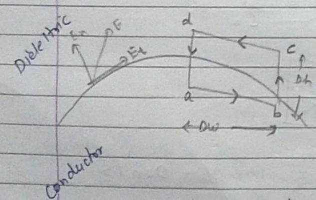
$$\varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2$$

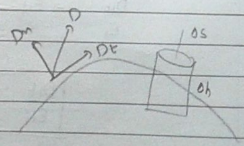
$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

Or

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

(ii) <u>Conductor - Dielectric B.C.</u>





- in conductor $\vec{E}=0$ (equipotential surface)

$$0 = 0\cdot\Delta w + 0\cdot\frac{\Delta h}{2} + E_n\frac{\Delta h}{2} - E_t\Delta w - E_n\frac{\Delta h}{2} - 0\cdot\frac{\Delta h}{2}$$

$\Delta h \to 0$

$$\boxed{E_t = 0}$$

$$\Delta Q = D_n \Delta S - 0\cdot D_n \Delta S$$

$$\vec{D} = \varepsilon\vec{E} = 0$$

$$D_n = \frac{\Delta Q}{\Delta S} = \rho_s$$

$$\boxed{D_n = \rho_s}$$

$\varepsilon E_n = \rho_s$

$E_n = \frac{\rho_s}{\varepsilon}$

- $D_t = \varepsilon_0 \varepsilon_r E_t = 0$ & $D_n = \varepsilon_0 \varepsilon_r E_n = \rho_s$

<u>Application:</u>

- shielding

<u>Example</u> two extensive homogenous isotropic dielectric meet on plane $z=0$. for $z>0$, $\varepsilon_{r1}=4$ for $z<0$, $\varepsilon_{r2}=3$ a uniform field $\vec{E_1} = 5, -2, 3$ kV/m exists for $z \geq 0$

(a) find $\vec{E_2}$ for $z \leq 0$

(b) angle $\vec{E_1}$ & $\vec{E_2}$ makes with interference

(c) the energy density in both the di-electric

(d) the energy within a cube of side 2m centered at $(3, 4, -5)$ → in second region $z<0$

<u>Sol:</u>

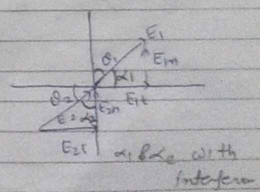
(a)

$$E_{1n} = \vec{E_1}\cdot\vec{a_n}$$

$$= \vec{E_1}\cdot\vec{a_z}$$

$$= 3 \text{ kV/m}$$

$$\vec{E_{1n}} = 3\vec{a_z} \text{ kV/m}$$



$$\vec{E} = \vec{E_n} + \vec{E_t}$$

$$\vec{E_{1t}} = 5\vec{a_x} - 2\vec{a_y} \text{ kV/m}$$

$$\vec{E_{2t}} = 5\vec{a_x} - 2\vec{a_y} = \vec{E_1t}$$

$$\vec{E_{2n}} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}\vec{E_{1n}} = \frac{4}{3}(3) = 4 \text{ kV/m}$$

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$\vec{E_2} = \vec{E_{2t}} + \vec{E_{2n}}$

$\vec{E_2} = 5\vec{a_x} - 2\vec{a_y} + 4\vec{a_z}$ kV/m **Ans.**

(b)

$\tan\theta_1 = \frac{|E_{1t}|}{|E_{1n}|} = \frac{(5^2+2^2)^{1/2}}{3} \Rightarrow \theta_1 = 60'$

$\alpha_1 = 29.12$

$\tan\theta_2 = \frac{|E_{2t}|}{|E_{2n}|} = \frac{(5^2+2^2)^{1/2}}{4} \Rightarrow \theta_2 = 53.37$

$\alpha_2 = 36.6°$

(c)

energy density 1 $= \frac{1}{2}\varepsilon_0 E_1^2 \varepsilon_{r_1}$

$= \frac{1}{2} \times 8.85\times10^{-12} \times 4 \times (|5,-2,3|)^2$

$= \cancel{1094.4 \times 10^{-12}}$

en. den 2 $= \frac{1}{2}\varepsilon_0 \varepsilon_r E_2^2$

$= \frac{1}{2} \times 8.85 \times 3 \times (|5,-2,4|)^2 \times 10^{-12}$

$= 597$ PJ

for ②

(d) E_1; $W = \oint \vec{E}\cdot d\vec{l}$ $(3,4,-5) \rightarrow$ Centre

$= \int_2^4 E_x\,dx + \int_3^5 E_y\,dy + \int_{-6}^{-4} E_z\,dz$

$= 5(2) + (-2)(5-3) + 4(-4+6)$

$= 10 - 4 + 8 = 14$ J

E_2; $= 5(2) - 2(2) + 3(2) = 10+6-4 = 12$ J

(iii) Conductor - free space BC :

$E_t = 0$

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$\vec{D} = \varepsilon_0\vec{E} = 0$

$\varepsilon_r = 1$

$D_t = \varepsilon_0 E_t = 0$

$D_n = \varepsilon_0 E_n = \rho_s$

$D_n = \rho_s = \Delta Q/\Delta S$

$\boxed{D_n = \rho_s}$

$E_n = \rho_s/\varepsilon$

$\boxed{E_n = \frac{\rho_s}{\varepsilon_0}}$

(PN) 207

$\Rightarrow$ Derivation of poisson's and laplace eqn :

$\nabla\cdot\vec{D} = \rho_v$ — ①

$\vec{D} = \varepsilon\vec{E}$ — ②

$\vec{E} = -\nabla V$ — ③

$\nabla\cdot(\varepsilon\vec{E}) = -\nabla\cdot(\varepsilon\nabla V) = \rho_v$

OR

poisson's eqn $\nabla\cdot\nabla V = -\frac{\rho_v}{\varepsilon}$ — ④

divergence of gradient of V

rect. cord. :

$\nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$\nabla V = \frac{\partial V}{\partial x}\vec{a_x} + \frac{\partial V}{\partial y}\vec{a_y} + \cdots$

$\nabla\cdot\nabla V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) + \cdots$

$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\varepsilon}$

if charge & potential is given at particular boundary then we can only used poisson & laplace eqn.

↳ when $\rho_v = 0$

$\nabla^2 V = 0 \longrightarrow$ its called as laplace eqn

laplacian operator

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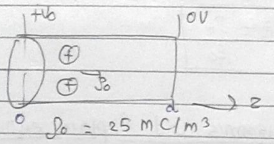
Rectangular: $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

cylinderical: $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2}$

Ex:



$\rho_0 = 25\ mC/m^3$

$V_0 = 22kV$

here we can't use laplace eqn bcuz ($\rho_V \neq 0$ given)

Sol: $\nabla^2 V = -\frac{\rho_v}{\varepsilon}$ poisson's eqn

charge in z direction only $\longrightarrow$

$\frac{d^2 V}{dz^2} = -\frac{\rho_v}{\varepsilon}$

$\frac{dV}{dz} = -\frac{\rho_v}{\varepsilon} z + A$

$V = \frac{-\rho_v z^2}{2\varepsilon} + Az + B$

At $z = 0$; $V = V_0$ & $z = d$; $V = 0$

$B = V_0 = 22kV$ | $0 = -\frac{\rho_v d^2}{2\varepsilon} + A \cdot d + 22kV$

$B = V_0$; | $A = \frac{\rho_v d}{2\varepsilon} - \frac{V_0}{d}$

So $V = -\frac{\rho_v z^2}{2\varepsilon} + \left(\frac{\rho_v d}{2\varepsilon} - \frac{V_0}{d}\right) z + V_0$

field intensity $E = -\nabla V = -\frac{dV}{dz}\vec{a_z}$

$= \left(\frac{\rho_v z}{\varepsilon} - A\right)\vec{a_z}$

$= \left(\frac{\rho_v z}{\varepsilon} - \frac{\rho_v d}{2\varepsilon} + \frac{V_0}{d}\right)\vec{a_z}$

Prob: 6.1, 6.2, 6.3 } 9 pm

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net force $\vec{F} = Q\vec{E}$

$F = \int \rho_v E\, dV = \rho_s \int ds \int_{z=0}^{d} E\, dz$

$= \rho_v \cdot S \left[\frac{V_0 z}{d} + \frac{\rho_v}{2\varepsilon}(z^2 - dz)\right]_0^d \vec{a_z}$

$F = \rho_v \cdot S \cdot V_0\ \vec{a_z}$ N $= \rho_v \cdot S\left[V_0 + \frac{\rho_v}{2\varepsilon}\times 0 - 0 - \frac{\rho_v}{2\varepsilon}\times 0\right]$

force per unit area or pressure is

$P = F/S$

$= \rho_v V_0 = 25\times 10^{-3} \times 22\times 10^3$

$= 550\ N/m^2$

Uniqueness Theorem:

Solution of laplace & poisson's eqn is V, is that unique or not?

- its unique

V_1 & V_2 are soln of lapalace or poisson's eqn

$\nabla^2 V_1 = 0$ and $\nabla^2 V_2 = 0$

$\nabla^2 (V_1 - V_2) = 0$

$V_b \rightarrow$ pottential at boundary

V_1 at the boundary $= V_{1b}$

V_2 " " $= V_{2b}$

$V_{1b} = V_{2b} = V_b$ (bcuz at bound. potentials are same

Or

$V_{1b} - V_{2b} = 0$

$\nabla \cdot (V\vec{D}) \cong V(\nabla \cdot \vec{D}) + \vec{D} \cdot (\nabla V)$

$V \rightarrow V_1 - V_2$ | $V_1 - V_2$ = scalar (Potential is scalar)

$\vec{D} \rightarrow \nabla(V_1 - V_2)$ | $\nabla(V_1 - V_2)$ = Vector

$\int_{vol} \nabla \cdot ((V_1 - V_2)\nabla(V_1 - V_2))\, dv \cong \int (V_1 - V_2)\ \nabla \cdot (\nabla(V_1 - V_2)) + \int \nabla(V_1 - V_2) \cdot (\nabla(V_1 - V_2))$

~~ingre~~ integrating the eqⁿ

(By divergence theorem $\oint_S A \, ds = \int_V \nabla A \, dV$)

$$\text{LoHoSo} = \oint_S (V_{1b} - V_{2b}) \nabla (V_{1b} - V_{2b}) \, d\vec{S}$$

now $V_{1b} = V_{2b}$ (By the boundary condition)

$$= 0$$

RoHoSo

$$\nabla \nabla (V_1 - V_2)$$

but $V_{1b} = V_{2b}$

$$\therefore \nabla^2 (V_1 - V_2) = 0$$

$$\int_{vol} [\nabla (V_1 - V_2)]^2 dV = 0$$

$$[\nabla (V_1 - V_2)]^2 = 0$$

$$\nabla (V_1 - V_2) = 0$$

$$V_1 - V_2 = 0$$

$$V_1 = V_2$$

- importance of the theorem ⓟ
and how we can say that the solⁿ of laplace & poission's eqⁿ are unique

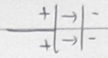
Capacitance

- Capacitor holds charges and stored electric energy.

$$Q \propto V \quad \text{—} ①$$

$$C = Q/V$$

$C \rightarrow$ the measure of capacity of capacitor, capacitance



parallel Plate　　spherical　　cylinderical

$$\vec{E} = \sigma / \varepsilon_0 \qquad \sigma \rightarrow \text{charge density} = q/A$$

$$\vec{E} = \frac{Q}{A \varepsilon_0}$$

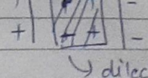
$$V = \vec{E} d \longrightarrow \text{Potential across capacitor}$$

$$= \frac{Q d}{A \varepsilon_0}$$

So in eqⁿ ① $\boxed{C = \frac{A \varepsilon_0}{d}}$

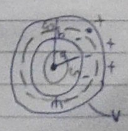
$A \rightarrow$ area of Plate
$d \rightarrow$ distance between plates

Here, we have to maintain the min distance between capacitor plates, so that Plates can't attract each other

-  $\longrightarrow$ Pseudo cap.

↳ dilectric material

spherical :



$$\frac{Q}{\varepsilon_0} = \oint E \cdot dA$$

due to $G_1 \Rightarrow Q = 0$

due to $G_2 \Rightarrow Q \neq 0$