TRANFORM :

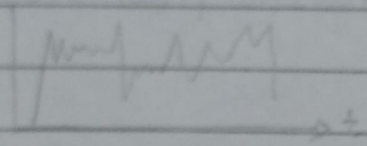
time domain Signal → Fourier transform → Spectrum (freq. domain)

$$\Downarrow$$

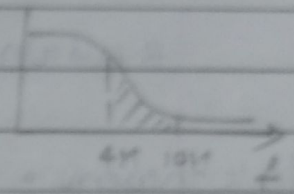
$$\int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$$

Filtering concept:

g(t)



g(f)



if we have to need only till 4M spectrum, then we need Low Pass filter for that we have designed such a filter that is

$$g(t) \overset{IFT}{\longleftrightarrow} \int_{0}^{2\pi f} g(\omega)\, e^{j\omega t}\, d\omega \qquad \because f = 4M$$

$\Rightarrow$

Time Domain	Freq. Domain
CT	AP
P	DT
DT	P
AP	CP

DTFT ⟶ FT ⟶ FS

$$x(K) = \int_{-T/2}^{T/2} x(t)\, e^{-j\omega t K}\, dt$$

⟶ Kernel Basis func

$\Downarrow$

$K = -\infty$ to ∞

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

Analog [max. freq. = ∞ ; $t = 0 \rightarrow dc$
discrete [max. freq. = π

Discrete sinusoid concept :

(Analog) $\cos \Omega t$

↓

(discrete) $\cos(\Omega n T_s)$ put $t = nT_s$

$= \cos \omega n$

$\omega \triangleq \Omega T_s$

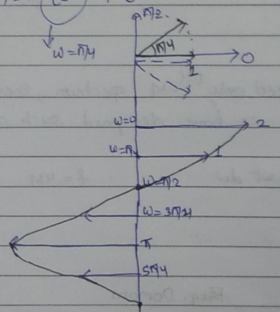
$\omega \rightarrow$ discrete freq. in rad. / sample

(how much phase is covered rad. in per sample)

$2\cos \omega n = e^{j\omega n} + e^{-j\omega n}$

$\omega = \pi/4$

Euler's theorem :



$\omega = \Omega \times \frac{1}{f_s}$; $T_s = 1/f_s$

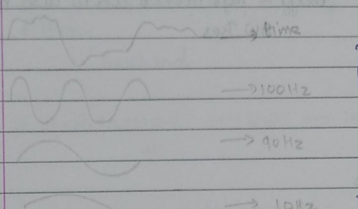
discrete $\leftarrow$ $\boxed{\omega = \frac{2\pi F}{f_s}}$ $\rightarrow$ continuous signal freq. $\rightarrow$ sampled freq.

for max. discrete freq. $\omega_{max} = \pi$ $\therefore f_s = 2F$

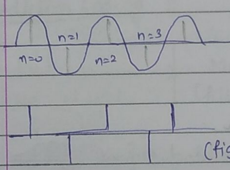
$2\pi f = \frac{2\pi F}{F_s}$

$\boxed{f = \frac{F}{F_s}}$

↳ max. freq. is always π, no matter whatever be the max. F of a signal.

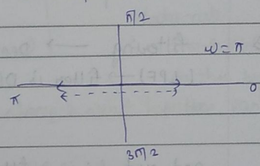


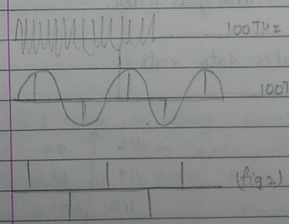
all are sampled with same frequency.



100Hz is sampled with 200Hz

(fig1)





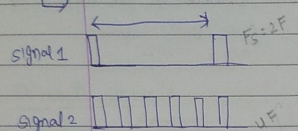
(fig2)

(fig1) & (fig2) are same for both signals 100Hz and 100THz but one thing that differs them is T_s value.

- Basic component of digital s/s is clock
  $\Rightarrow$ clock speed differentiates the two systems.

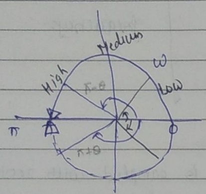
DSP: ADC $\rightarrow$ Processor $\rightarrow$ DAC

↳



energy is high in (2) & cost is also high
BW ↑ses

⇒

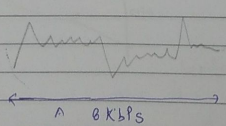


↳ filtering ⟶ Decoder
(LPF) ⟶ filter is Digital filter
all real times signals are base band signal

and antialiasing filter is analogous filter. ⟶ sampling (CT)

➡ $\boxed{\text{Signal's B.W. } \propto \text{ Signal's data rate}}$

signal A:



1/2	00
1/4	01
1/8	10
0	11

- signal spectrum is changed by the filter.
  minimize the bPS ⟶ BW ↓es ⟶ cost ↓es

- We don't tramit our actual signal or another form of signal. we only transmit the base band signal with the help of filters. and also we detect our data back with the help of filter (base band filter). we only transmit the parameters of filters.

- In cellphones, for transmit the data, we don't transmit the any form of data but we only transmit the parmeters of filters.

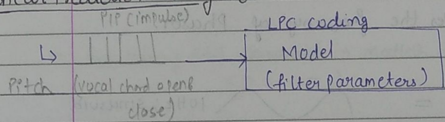
Baseband

*

dc signal (constant) ⟶ slw ⟶ impulse $\xrightarrow{FT}$ having all freq. compo
air

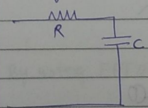
required data ⟵ By filters
(mouth cavity)

Linear Prediction Coding :



data rate
3 to 4 KbPS
in this by filters parameters

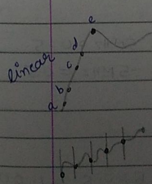
analog filter (Low Pass filter)



parameters are R & C

- But when we doing sampling, Quantization, encoder - - - then data rate is 64 KbPs which is very high as compare to (model method) Linear prediction coding.

- we predict the filter parameter in LPC.



$e = K_1 a + K_2 b + K_3 c + K_4 d$
(Linear prediction)

By sampling we can predicted it.

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- antialiasing filter is analog filter & LPF, HPF, BPF, are digital filters.

Transforms :

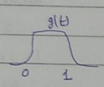
when we talk about transmission or signal processing then ~~spectrum~~ ~~we talk about the~~ spectrum.

We only consider on the spectrum

fourier transform: $G(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$

$G(\omega)$; its gives the frequency of Phasor.

as example:



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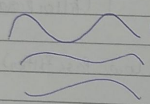


10Hz; sinusoid
$\omega = 2\pi \times 10$

$g(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$

Pos. Phase: $g(2\pi \times 10) = \int_{0}^{1} 1 \cdot e^{-j2\pi \times 10 t}\, dt$ — ①

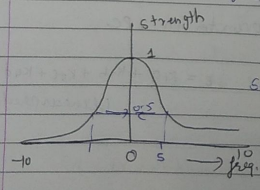
its gives the strength of Phasor at $f = 10Hz$

because $2\cos(2\pi \times 10)t = e^{j(2\pi\times 10)t} + e^{-j(2\pi \times 10)t}$

nego Phase: $G(-2 \times 10 \times \pi) = \int_{0}^{1} 1 \cdot e^{j2\pi \times 10 t}\, dt$ — ②

its gives the strength of phasor at $f = -10$

$G(\omega) + G(-\omega) \rightarrow$ gives the strength of sinusoidal signal.



Strength of Phasor at 5MHz = 0.5
" " " at -5MHz = 0.5

strength of sinusoidal signal = 0.5 + 0.5 = 1.00

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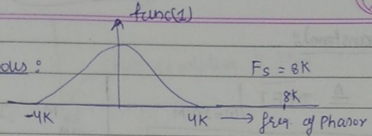
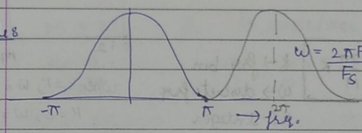
↳ continous :



$F_s = 8K$

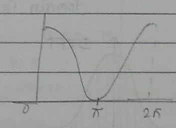
spectrum will remain same by doing sampling on the func (1). if its not then it may disorted.

discrete:



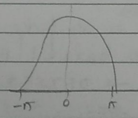
$\omega = \frac{2\pi F}{F_s}$

- By using fft command in matlab, we get fourier transform 0 to 2π
(0 to 2π)



- By using FFT SHIFT

(−π to π)



* $x(t - t_0) \rightarrow e^{j\omega t_0} X(f)$
↳ phase change
magnitude same

* $x(t) \rightarrow X(f - f_0)$

- When we shifted the signal in time domain, the spectrum of Phase also change / shift in freq. domain.

DTFT: discrete time fourier transform

$X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n}$

discrete freq. : −π to π

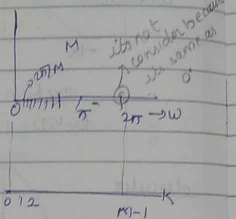
here discrete freq. are infinite. So By doing DFT, we can get finite frequencies

DFT (Discrete fourier transform):

$$DFT \triangleq DTFT \Big|_{\omega = 2\pi/M}$$

M-Point DFT ;
$0 \leq K \leq M-1$

$$\boxed{X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{M}Kn}}$$

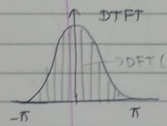


$$\boxed{\omega = \frac{2\pi}{M} K}$$

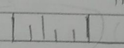
K → freq. bin
ω → discrete freq
K ∈ Integer

when $K = 1$; $\omega = 2\pi/M$
$K = 2$; $\omega = \frac{2\pi}{M} \times 2$

— sampling in one domain is repeatation in another domain is occurs.



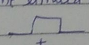
freq. domain
DFT (by cutting this → sampling)
inverse → in time domain we get $x(n)$; which is repeatative in time domain.

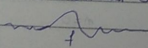


* $x(n) \rightarrow DTFT \rightarrow DFT \rightarrow x(n)$
↓ repeatation (aliasing effect)

— for remove or overcome from the antialiasing filter ;
$\boxed{M \geq N}$
how many parts between 0 to 2π ↓
in orignal signal, how much samples are there

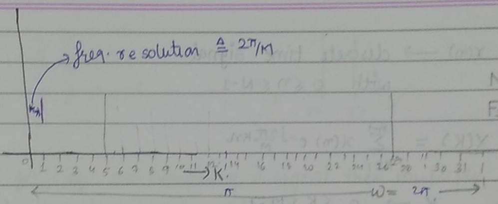
— time limited Signal → Band unlimited signal





Example: Rahul samples the unknown signal at 64 sample / second. He selects 32 samples and computes 32 points DFT.

DFT is used to find the unknown frequency.
K → freq. bin



N = 32
Fs = 64 sam/sec

M = 32 $\quad f_s$ = 64 sample / sec $\quad$ For 32 samples Hzs

$\omega_1 = \frac{2\pi}{32} \times 5$ $\qquad \omega_2 = \frac{2\pi}{32} \times 27$

$F_1 = \frac{2\pi \times 5 \times 64}{32 \times 2\pi}$ $\qquad F_2 = \frac{2\pi}{32} \times 27 \times \frac{64}{2\pi}$

$= 10 Hz$ $\qquad = 54 Hz$

max. discrete freq. = π ; and range between 0 to π
only one ω_1 is satisfy $\omega = \frac{10\pi}{32}$;

analogus freq. F = 10 Hz

Example: Which freq. bin, what do u expect have max. Power. If the I/P was at 11 Hz instead of 10 Hz ?

Sol:-
$F = 11$ Hz
$$\omega = \frac{2\pi F}{f_s} = \frac{2\pi \times 11}{64} = \frac{11\pi}{32}$$

$$\omega = \frac{2\pi}{M} \cdot k$$

$$\frac{11\pi}{32} = \frac{2\pi}{32} \cdot k$$

$$k = \frac{11}{2} = 5.5 \notin I$$

* freq. resolution $\triangleq \frac{2\pi}{M} = \frac{2\pi}{32}$

* DFT → 0 to 2π

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$x(n) \longrightarrow$ discrete time signal
with $0 \le n \le N-1$

$$X(K) = \sum_{K=1}^{M-1} x(n)\, e^{-j\frac{2\pi}{M}Kn}$$

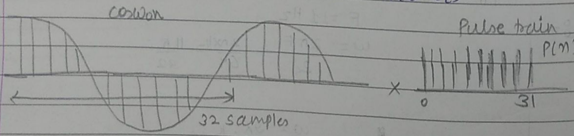
$K \longrightarrow 0 \le K \le M-1$
$N \rightarrow$ in time domain (how many samples)
$M \rightarrow$ in freq. domain (how many parts in DFT, 0 to 2π)

* K can never be fractional no. It should be integer always.

– DFT missed the spectrum, because DFT always takes the value of K as integer.
the graph/spectrum acheived is 0 at all points.

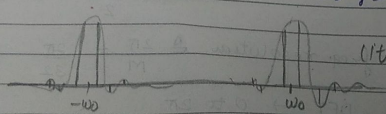
Modulation property :
to get the spectrum, modulation property of fourier transform is used.

Modulation: $x(t)\cos w_c t \longleftrightarrow X(\omega + w_c) + X(\omega - w_c)$



32 samples = $[\cos w_0 n] \cdot P(n)$ = spectrum of ~~phasor~~ Pulse is shifted to at w_0 and $-w_0$ freq.
spectrum of pulse is sinc func

=



(It's continuous) but

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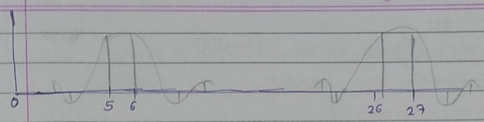
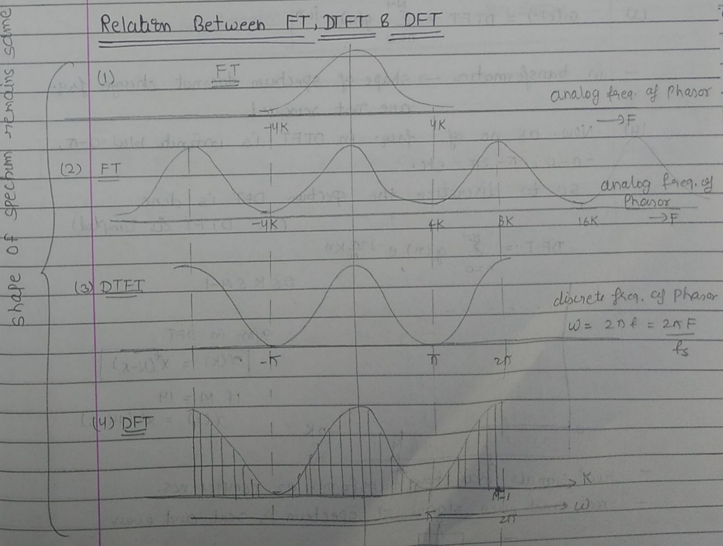


to get only 32 samples of a signal then multiply it with pulse train (32 samples 0 to 31)
$\Rightarrow$ 32 samples of signal is acheived.

– The DFT of impulse train is discrete sinc. as we get max. peak at K = 5 & 6

Relation Between FT, DTFT & DFT



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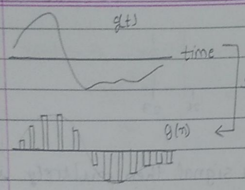
(1)



(2)

$g(n)$ / $g(t)$ is quantized $\Rightarrow$ $g(n)$ is acheived (discrete signal)

$$FT = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$$

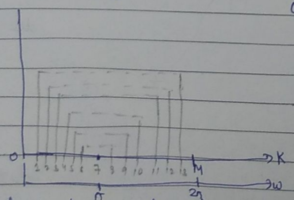
(3) $$G(e^{j\omega}) = DTFT = \sum_{n=0}^{N-1} g(n)\, e^{-j\omega n}$$

- in transformation $\rightarrow$ shape of spectrum doesnot change freq. are not removed

(4) Now, as no. of freq. in DTFT is infinite b/w $0-\pi$, $-\pi-0$, $\pi-2\pi$ etc.
so to discretize the spectrum DFT is done
(i.e. DTFT is sampled)

$$DFT = \sum_{n=0}^{N-1} g(n)\, e^{-j\frac{2\pi}{M}Kn}$$

$0 \leq K \leq M-1$



sym. in DFT

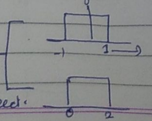
$$\boxed{x(K) = X^*(N-K)}$$

if $M = 14$
$x(8) = X^*(14-8)$

- real signals may have transforms as complex nos.
- real and even signal $\Rightarrow$ spectrum is real and even

real

- Same magnitude spectrum
- different phase spect.



$\longleftrightarrow$ complex as shifting a signal in time domain

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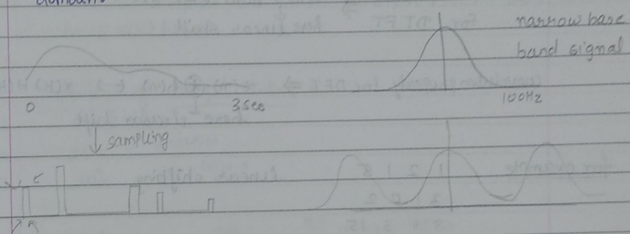
DSP

- sampling in one domain results into periodicity in other domain.



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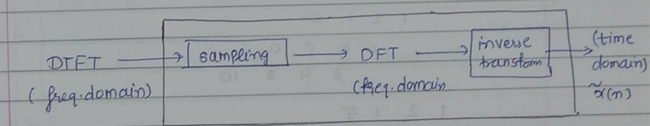
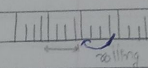
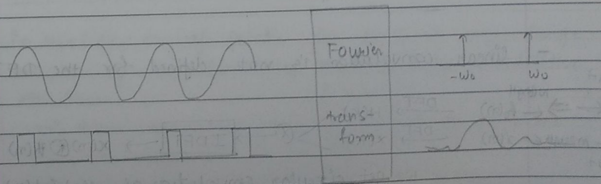


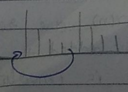


(?) Periodicity in one-domain results in sampling (?)



its always not true.

- DFT is always periodic in both domain.
  - circular shift
  - in dFT not linear shift, it rotating shift, [which comes out that comes in]



- $x((n-n_0)_N)$ $\downarrow$ circular shift

$g(t-t_0) \longleftrightarrow G(\omega)\, e^{-j\omega t_0}$
$\downarrow$ linear shift $\qquad \downarrow$ rotating / circular shift.

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- $x(n)$ — $\boxed{h(n)}$ — $y(n) = x(n) * h(n)$

convolution property for DTFT $\Rightarrow$ $x(n) * h(n) \longleftrightarrow X(e^{j\omega}) H(e^{j\omega})$
here linear shift

convolution property for DFT $\Rightarrow$ $x(n) \circledast h(n) \longleftrightarrow X(K) H(K)$
here circular shift

<u>for example</u>

linear shifting

```
        1  2  1  5
        3  1  0  2
        ----------
        3  6  3  15
           1  2  1  5
              0  0  0  0
                 2  4  2  10
```

circular shifting

```
        1  2  1  5
        3  1  0  2
        ----------
        3  6  3  15
        5  1  2  1
        0  0  0  0
        10 2  4  2
        ----------
```

- linear convolution is not defined for the DFT.

$N+M-1$ $\longleftrightarrow \Rightarrow$ N length $h(n) \xrightarrow{DFT} H(K)$

M length $x(n) \xrightarrow{DFT} X(K)$ → ⊗ → $\boxed{IDFT}$ → $x(n) \circledast h(n) = y(n) = ?$

M+N-1

X(K) H(K) → ⊗ → IDFT → y(n)

we get circular convolution of $x(n)$ & $h(n)$ but as we know that we always have linear convolution. So we have to change this circular to linear

- same length of $h(n)$ & $x(n)$ are required for linear convolution. so we added the $n+m-1$
- O/P of any filter $(h(n))$ is convolution.

$x(n) \rightarrow \boxed{h(n)} \rightarrow y(n) = x(n) * h(n)$ $\quad \Sigma\, x(K) h(n-K)$

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— By doing $\Sigma\, x(k) h(n-k)$, we also get the linear convolution but its requirment of multipication and division is more as compare to DFT. So processor work will more in $\Sigma\, x(K) h(n-K)$.

Notation of Sanjit Mitra :

$\omega_N = e^{-j2\pi/N}$

$\omega_N^4 = e^{-2\pi/N \cdot 4}$

can be written in matrix form

DFT $\Rightarrow$ $X(K) = \sum_{n=0}^{M-1} x(n)\, e^{-j\frac{2\pi}{M}Kn}$

$0 \le K \le M-1$

IDFT $\Rightarrow$ $x(n) = \frac{1}{N} \sum_{K=0}^{M-1} X(K)\, e^{j\frac{2\pi}{M}Kn}$

$X = x \cdot D \Rightarrow D = \begin{bmatrix} - & \omega_N & - \\ - & \omega_N^{1} & - \\ - & \cdots & - \end{bmatrix}$

<u>Inverse of processing speed of processor :</u>

- in reality no. of points for DFT is very large so no. of operations of multipication & addition etc. ↑ses
  - ⇒ time taken and power consumed by processor was more.
  - ⇒ we need to ↓se the time taken.

- $X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{M}Kn}$

for $N = 2$; $M = 2$ → min. M for N = 2 as $M \ge N$

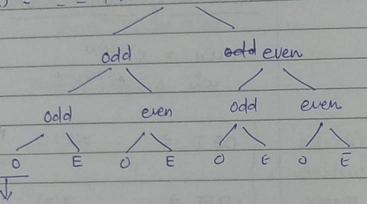
$X(K) = x(0)\, e^{-j\frac{2\pi K(0)}{M}} + x(1)\, e^{-j\frac{2\pi}{M}K(1)}$

$X(0) = x(0) + x(1)$

$X(1) = x(0) + x(1)\, e^{-j\pi}$

- DFT of 2 points is most simple.
  So the basic idea to decrease time was to convert 1000 samples to 2 samples and then computing.

1000 samples $\longrightarrow X(K) = \_\ \_ + \_\ \_ + \_\ \_ + \_\ \_ + \_\ \_$



Here $X(K)$ is divided into many 2 samples pair (even & odd)

for N-Points

$$\left.\begin{array}{l} X(K) \longrightarrow 2N^2(\times) \longrightarrow N/2 \log_2 N \\ 2N^2\,(+) \longrightarrow N \log_2 N \end{array}\right] \text{— FFT}$$

and if no. of samples are odd then a zero valued sample is added $\longrightarrow$ called as zero padding.

DTFT : $X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n}$

Date 3-Aug

* $\Rightarrow$ multiplication of spectrum by $e^{-j\omega t_0}$ in freq. domain, this cause real spectrum will change into conjugate spectrum

***Ex:-** DTFT $X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n}$; what is the condition for $X(e^{j\omega})$ DTFT to exist ?

Sol: i.e. magnitude is finite for existence of DTFT

$$X(e^{j\omega}) < \infty$$

$$X(e^{j\omega}) = x(0)e^{-j\omega 0} + x(1)\, e^{-j\omega(1)} + x(2)\, e^{-j\omega(2)} + x(3)\, e^{-j\omega 3}$$

magnitude: $|X(e^{j\omega})| = |x(0)| + |x(1)| + |x(2)| + |x(3)| + \dots$

So; $\boxed{\sum_{n=0}^{N-1} |x(n)| < \infty}$ — ① for existence of DTFT

$X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n}$ $\longrightarrow$ how to make this convergence or absolute summable for existence of DTFT

- if we don't know about the s/s then how can we find out its response. By giving know i/p & see the o/p we can evaluate the s/s response.
- i/p signals :- Unit step , ramp
  but Unitstep signal is $\sum_{n=0}^{N-1} |x(n)| < \infty$ not satisfy this

By doing z-transform, we can satisfy this eqn ①

z-transform:

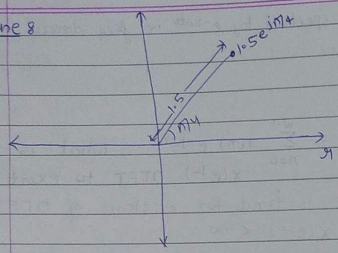
added one series when $n \to \infty$, $r^{-n} \to 0$

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n} \longrightarrow \sum_{n=0}^{N-1} r^{-n}\, e^{-j\omega n}\, x(n)$$

$$= \sum_{n=0}^{N-1} x(n)\, (r e^{j\omega})^{-n}$$

$\boxed{X(z) = \sum_{n=0}^{N-1} x(n)\, z^{-n}}$ "z-transform" $z = r e^{j\omega}$

z-plane :



$Z = r e^{j\omega}$
$= 1.5 e^{j\pi/4}$

To analyze, implementation the digital signal, z-transform is used.

* In laplace $s = \sigma + j\omega$

$\sigma =$?

* By putting $r = 1$
  z-transform = DTFT

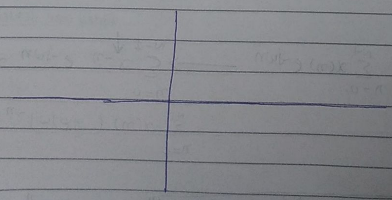
ROC (Reigon of Convergence) :

for what value of z, the z-transform exist ?

Example :

$$H(z) = \frac{(z-1)(z-0.5)}{z(z+1)} \begin{array}{l}\longrightarrow \text{zeros} \\ \longrightarrow \text{poles}\end{array}$$

H(z) is the s/s response of z-plane.



concept of zero – sensors + 50Hz ($\omega = 2\pi F/f_s$) → for supressed the 50Hz freq. → for this at this freq. $H(z) = 0$; zeros →
$z_1 = r e^{j\omega}$ → converted into z-plane →
$H(z) = (z - z_1)$

– Digital s/s design : if we want to suppress any freq. we find the 'z' of the freq., we keep at zero
The nearby surrounding freq. which is unwantedly suppressed is lifted by assigning poles at those frequencies.