AWP

SEMESTER : 6

SUBJECT : ANTENNA AND ~~MORE~~ WAVE PROPAGATION

classmate

Date

Page

→ Antenna can be defined in various ways:

- It is an interface between free space and a guarded medium.
- Transducer : converts electronic waves into electromagnetic waves.
- Impedance matching
- It is a device which is used to transmit or receive the EM waves

→ Various applications of antenna:

- Mobile (calling)	- Radio	- WiFi	- Jammers
- Satellite	- Pager	- Radar	- Bluetooth
- TV	- Sensor computer	- Sonar	- Wireless charging
- RFID	- Radio telescopes	- Remote control	- NFC (near field comm.)
- Rovers.			

→ The bandwidth range of antenna is very precise → this is because purchasing of wavelength is very costly.
ITU → International Telecommunication Union
(for India TRI) ⇒ from where bandwidth is purchased and allotted.

Any antenna can be made to work at any frequency. Only problem would be we might not get the desirable parameters

→ How can we distinguish antenna? / Classification

1. Frequency - Antenna can be designed with frequency range VLF (very low frequency) to millimeter wave.

VLF : 3 Hz - 3 kHz.

Millimeter ~~frequency~~ wave : $f = \frac{c}{\lambda} = \frac{3\times10^{8}}{10^{-3}} \cong 10^{11}$

Audio signal frequency range from : 20 Hz to 20 kHz

Voice signal frequency range from : male : 3 kHz, female : 3.6 kHz

Linear polarization and ~~Elliptical~~ Circular polarization are special cases of Elliptical polarization.

2 Aperture :

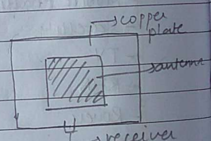
i) Wire Antenna - Simple wire like. (Ex : antenna present in cars) Ex - monopole antenna.

ii) Reflector Antenna - Antenna mounted on a dish / round surface like structure. (Ex: The dish for TV - TV antenna). The round structure prevents any signal from backside being received by antenna. Thus it ensures that there is no interference and also radiated waves travel in one particular direction. Hence has increased directivity.)

iii) Patch :

~~Patch~~ : (Figure)

It is a patch sort of as shown in the figure.



iv) Aperture :

Question: If Electromagnetic waves (EMW) travel from space (air or any other medium) to any other medium say water. Would there be any changes?

Yes. The direction and waves is affected. EMW are made up of : electric field and magnetic field. Hence when its permeability and permitivity changes due to change in medium;

1) velocity changes 2) Direction of propagation changes.

Conditions for circular polarization ⇓ 1) Ex and Ey should be at 90° to each other 2) Ex = Ey (magnitude should be same) Side view

3 Polarization - plane in which electric field vibrates

i) Linear (Horizontal / vertical) [The component of electric field would be either along Ex or Ey]

ii) Circular - good for weather forecasting → sometimes ← even helical [Component of electric field → any direction]

iii) Elliptical . [$E_x \neq E_y$] ⇒

Linear polarization → not used in satellite communication. This is because when EMW travels through different mediums like ionosphere it gets deflected by 90° and plane of polarization changes. Hence linear polarization is not used in satellite communication.

4 Radiation .

It can be divided into 2 types :

- Directional - radiates in only specific directions
- Non directional - radiates in all directions equally.

Two types : i) Isotropic - radiates equally and uniformly in all the planes ⇐ all direc.
- radiation pattern : sphere
- it actually does not exist ⇒ ideal case.

ii) Omnidirectional - radiates uniformly and equally in all directions in any 1 plane. Ex. dipole antenna

- Hemispherical antenna. (radiation in the form of half sphere).

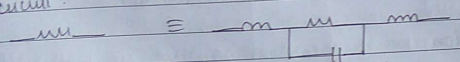
→ Same antenna can be used as a transmitter and receiver.

→ Necessary conditions for radiation :

1) Array increases directivity
2) folded dipole antenna.
3) slot antenna - ~~used~~ a slit on a copper / aluminium plane with other important circuitry
This is used in aeroplanes - The whole body of aeroplane has such slots - hence detection from all the directions is possible.
Slot antenna can detect power signal very efficiently.
4) Surface used copper and aluminium.

5) vivald antenna - wide band antenna → not used now

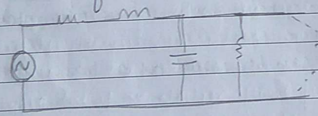
→ Effect of high frequency on components of the circuit:



(Resistance at low frequency) (Resistance at high frequency)

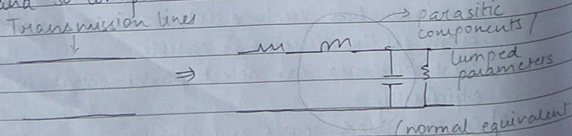
[At high frequency, the nature of wire changes. 2 leads adjacent to resistor acts as ~~indicat~~ inductor. 2 metals separated by a dielectric → capacitor formed)

At high frequencies two parallel transmission lines will have equivalent circuit as:

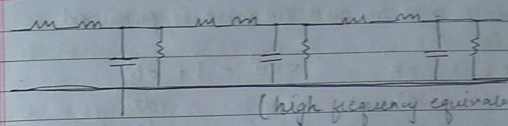


This happens because at high frequency X_L and X_C values are considered.

At high frequency, wavelength (λ) is very low (∵ frequency and wavelength are inversely proportional) and so components are distributed.



(normal equivalent circuit)



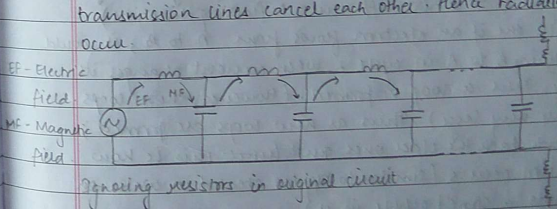
(high frequency equivalent circuit).

The components gets distributed because the phase of signal changes very rapidly at high frequency ⇒ hence the values of capacitor and inductor changes accordingly.

→ Every conductor which conducts currents can work as antenna but the parameters are uncontrollable. So in antennas, in the transmission lines are

Why

→ If ~~others~~ all current carrying conductors ~~are~~ can work as antennas, why can't electric cables radiate! Electric cables have transmission lines closely packed. In such cases, the radiation field ~~of~~ generated by each transmission lines cancel each other. Hence radiation does not occur.



Ignoring resistors in original circuit

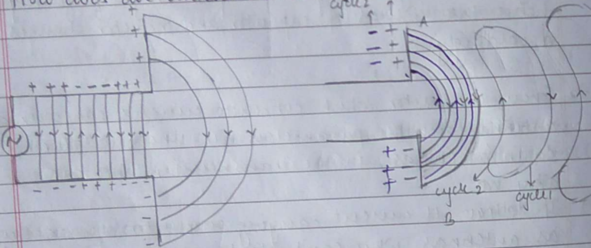
Here, time-varying current is generated by the source which gives rise to time-varying magnetic field → by maxwell's equation → time varying magnetic field would be produced by time varying electric field displacement current which.

This cycle goes on and EM wave is produced

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{dt} \qquad \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{dt}$$

↳ displacement current.

How does an antenna radiate?



An AC source is providing the current. Hence due to alternate current; polarities would change (cycle 1 and cycle 2 → two diff. current flowing directions)
In cycle one if an electron flows from A to B; it would flow from B to A in cycle 2 when current changes its polarity. Thus a loop is formed. (However, these loops are very very large). Thus as new loops are formed; they push back the older ones and ~~hence~~ this is how radiation occurs. (Similar to ripples created in water on throwing a stone/pebble in it).

→ lumped and parasitic parameters
→ distributed parameters → come into picture due to change in phase
→ change in electric field generates magnetic field and change in magnetic field generates electric field.

power pattern ← $P_d(\theta, \phi) = \frac{1}{2} \frac{(E(\theta,\phi))^2}{\eta_0}$ → field / ~~elect~~ E-pattern

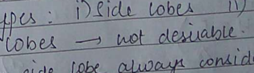
↳ intrinsic impedance (120π or $377\ \Omega$)

→ Radiation pattern
- Defined as a mathematical function or a graphical representation of the radiation properties of antenna as the function of space coordinates
- Coordinate system considered: Spherical.
- $r = 0$ to ∞ $\quad \phi = 0$ to 2π $\quad \theta = 0$ to π
- Radiation properties include power flux density, radiation intensity, field strength, polarization.
- Power pattern - normalized power v/s spherical coordinate position OR Trace of received power at constant radius
[Normalised: We have 10 readings. Dividing all by the maximum reading. Hence we then obtain the values in a particular range]

→ Two planes considered when talking about Antenna:
1) Azimuth plane - When radiation pattern is considered in horizontal ~~plain~~ plane (x-y plane) $\theta = 90°$
2) Elevation plane - When radiation pattern is considered in vertical plane (y-z plane) $\phi = 90°$

→ Two types of principal patterns:
1) E-plane (Electric plane)
2) H-plane → plane consisting of magnetic field.

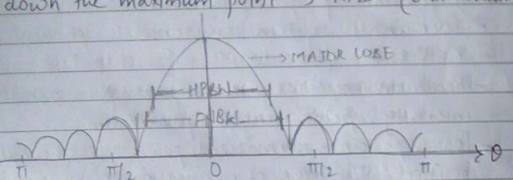
→ Radiation pattern lobes.
i) Major lobe - radiation lobe containing the direction of maximum radiation. (radiation in intended direction).
ii) Minor lobe - other than major lobe is the minor lobe.
Two types: i) Side lobes ii) Back lobes (at 180°)
Minor lobes → not desirable.
Power in side lobe always considered as a ratio of amount of power in major lobe and that of side lobe.



Radians in circle = 2π
Steradians in sphere = 4π

→ Half Power Beam Width (HPBW) / Half Power Bandwidth

3dB down the maximum point → HPBW. (unit · radians)



FNBW - First Null Beam / Band Width (unit radians)
occurs at the ~~first side~~ lobe point where it becomes 0.

~~Normalized field pattern~~ :

Patterns - expressed in terms of power per unit area. (or Poynting vector $S(\theta, \phi)$ → tells us the direction in which the vector is moving)

$$P_n(\theta, \phi)_n = \frac{S(\theta, \phi)}{S(\theta, \phi)_{max}} \quad \text{(dimensionless)}$$

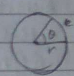
$$S = \frac{1}{\mu}(\vec{E} \times \vec{H})$$
$$= \vec{E}\vec{H} \sin\theta$$
$$= \vec{E}\vec{H} \sin 90 \quad (E \perp H)$$
$$= \vec{E}\vec{H}$$
$$= \frac{|E|^2}{Z_0} \quad \left(H = \frac{E}{Z_0}\right)$$

Power per unit area → Poynting vector.

$$\eta_0 = \frac{|E|}{|H|}$$

$$\therefore S = \frac{E_\theta^2(\theta, \phi) + E_\phi^2(\theta, \phi)}{Z_0}$$; where Z_0 → free space impedance

→ Measure of plane angle is a radian.



Radian → circle Steradian → sphere
One radian is defined as the angle with it's vertex at the centre of a ~~sphere~~ circle of radius r. H

(Ω_A) Solid angle → the element of solid angle $d\Omega$ of a sphere can be written as

$$d\Omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\phi \quad (sr)$$

Beam area - Solid angle through which all of the power radiated by the ~~atenna~~ antenna would be majority concentrated (major lobe)

Beam area = $\phi_{HP} \cdot \theta_{HP}$.

$$\text{Beam area} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} P_n(\theta, \phi) \sin\theta \, d\theta \, d\phi$$

If $\phi = 30^\circ$, $\theta = 2^\circ$; Beam area = 60°.

→ Radiation intensity - Power radiated by an antenna per unit solid angle is called radiation intensity U (watts per steradian / per square degree)
↳ not depended on distance.

→ Beam efficiency (ε_M)
Total beam area = Main beam area + Minor lobe area ($\Omega_A = \Omega_M + \Omega_m$)

$$\text{Beam efficiency } (\varepsilon_M) = \frac{\Omega_M}{\Omega_A} \quad \text{(dimensionless)}$$

Ratio of minor lobe area to the total beam area is called stray factor $\varepsilon_m = \frac{\Omega_m}{\Omega_A}$ $\varepsilon_M + \varepsilon_m = 1$

Scanned by CamScanner

Em should be higher than Em because we want maximum power to be concentrated in our major lobe.

Directivity (D) (dBi)

- tells us about the direction of radiation by antenna.
- Defined as: Ratio of maximum power density to its average value over a sphere as observed in the far field of an antenna.

$$D = \frac{P(\theta,\phi)_{max}}{P(\theta,\phi)_{av}} \quad \text{(dimensionless)} \quad D \geq 1$$

$$= \frac{P(\theta,\phi)}{\frac{1}{4\pi}\iint P(\theta,\phi)\, d\Omega}$$

$$= \frac{1}{\frac{1}{4\pi}\iint [P(\theta,\phi) / P(\theta,\phi)_{max}]\, d\Omega}$$

$$= \frac{4\pi}{\iint P_n(\theta,\phi)\, d\Omega} = \frac{4\pi}{\Omega_A} \rightarrow \text{directivity from beam area } \Omega_A$$

~~Hig~~ Greater the increase in directivity; larger the distance the radiation would be able to travel.

Smaller the beam area, larger the directivity.

$$\boxed{D = \frac{4\pi}{\Omega_A}}$$ → This equation proves that directivity and beam area are inversely proportional.

Lowest possible directivity = 1. [idealized isotropic antenna ($\Omega_A = 4\pi$ sr)].

→ Gain - Gain of an antenna is a quantity which is less than the directivity D due to ohmic losses in the antenna.

During transmission, losses - involve power fed to the antenna which is not radiated but heats the antenna structure.

Impedance matching affects the losses → if the impedance of channel and receiver matches → no reflection
However, in case of mismatch; some waves are reflected back in the channel. Hence transmitted and reflected waves meet constructively and destructively, giving rise to formation of standing waves.

VSWR - Voltage Standing Wave Ratio.

$$= \frac{max}{min} = 1 \text{ (ideally)}$$

VSWR ↑ ⇒ reflected waves ↑ → low impedance matching.

When we speak of 'antenna' we need it to be more lossy as it has to radiate.

$$\text{Gain} = \frac{P_{max}(AUT)}{P_{max}(\text{ref. ant.})} \times G(\text{ref. ant.})$$ AUT - Antenna Under Test.

$$\text{Stray factor} = \varepsilon_m = \frac{\Omega_m}{\Omega_A}$$

Scanned by CamScanner

→ Antenna Apertures.

Aperture of an antenna is the area that captures energy from a passing radio wave.

Two types of ~~antenna~~ aperture
1) Physical aperture 2) Effective aperture.

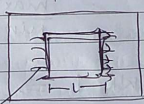
Effective aperture is not always equal to the physical aperture.

Aperture efficiency = $\varepsilon_{ap} = \frac{A_e}{A_p}$ (dimensionless)

Physical apertures → The whole body.
Effective apertures → The part of the antenna which radiates.

In patch antenna, effective ~~stat~~ aperture > Physical
In microstrip antenna; physical > effective

microstrip antenna



fringing field

~~First~~
~~Assuming~~
Aperture beam area relation:
$\lambda^2 = A_e \Omega_A$ (m^2)

Directivity from aperture = $D = \frac{4\pi A_e}{\lambda^2}$ $A_e \rightarrow$ effective aperture.

∴ Directivity is directly proportional to effective aperture of the antenna.

$G = D$ (ideally) $\because k = 1$ $\underline{\underline{G = kD}}$.

$V = Eh$ $S = \frac{E}{Z}$ $P = \frac{V^2}{4R} = \frac{E^2h^2}{4R}$ $P = SA = \frac{E^2A}{2}$

→ Different regions of antenna.
1) Far field 2) Near field.

Threshold - Largest dimension of antenna. (D)
length / width / etc → which dimension?

→ Effective Height
- Ratio of induced voltage to incident field.
Boundary between near and far field = $\frac{2d^2}{\lambda}$

$h = \frac{V \text{ (induced voltage)}}{E \text{ (incident field)}}$

Parameters ~~of~~ antenna are
dependent to distance → in near field / freshnel region
independent of distance → in far field / Fraunhoffer.
Hence far field considered / preferred more.

$A_e = \left(\frac{h_e^2 Z_0}{4 R_e} \right)$ → ~~relation between~~ $P = \frac{V^2}{4R} = \frac{E^2h^2}{4R}$ = $P = SA = \frac{E^2A}{Z}$

Effective height
Ratio of ~~inci~~ induced voltage to incident field.

Return loss - Tells us how much signal is reflected back.

Scattering or S parameters are used in the case of antenna (like z parameters → in ~~high freq.~~ equivalent model ~~at~~ high frequency).

At high frequency, we cannot measure current and voltage because it takes in account / introduces inductor and capacitor (parasitic) → hence we ~~ta~~ consider

Advisable to transmit info or signals in same plane of polarization; same plane means polarization remains uniform throughout; if not uniform; there would be multiplication loss of info.

power and make our calculations in terms of power.

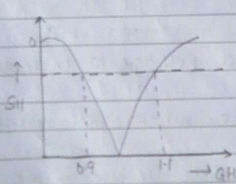
VSWR (Voltage standing wave ratio) = 2 when $S_{11} = -10dB$.
= 1 at ideal case.

Smaller the value of S_{11}; lesser the reflection, greater the radiation.

Two types of power → reactive power and radiative power

Near field - In near field; the electric and magnetic field try to dominate each other. Hence we get very random and abnormal values while measuring various parameters or verifying laws in the region of near field.

Far field → Both the fields are almost same. Hence almost uniform and data can be measured and recorded.



~~When we want to play with S11~~; When we want to improve S_{11}; we need to work upon impedance matching. Hence need to optimize the ~~feed~~ source.

In order to improve / change frequency; we need to work upon the dimensions of the antenna.

E pattern / - field pattern, - power pattern ⟹ Two types of radiation pattern.

→ Relation between Radiation resistance
- One of the factors which relates effective aperture and effective height $A_e = \frac{h^2 Z^2}{4R_r}$ $A_e = \frac{\cancel{h^2 Z^2}}{4\pi R^2}$
- two types of resistance:
  - ↳ Radiation resistance
  - ↳ Loss resistance

Radiation resistance → tells us how effectively radiation is occuring

Higher the radiation resistance; greater is the radiation.
~~Lower~~ Higher the loss resistance; greater are the losses.

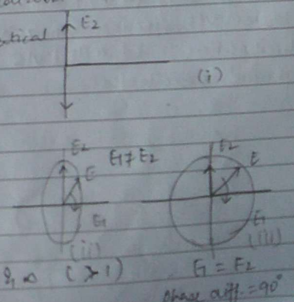
$K = \frac{\text{Radiation Resistance}}{\text{Total resistance}} = \frac{R_r}{R_L + R_r}$

Polarization occurs for both antenna and EM field

→ Polarization → Contour traced by the tip of the electric vector is known as polarization.
- Orientation of E-field vector of a wave ⇒ defi.

Polarization : only E-field vector orientation.

Three types:
i) Linear polarization — Horizontal (conductor along the ...) / Vertical
ii) Elliptical polarization
iii) Circular polarization — special case of elliptical polarization



Axial ratio = $\frac{\text{Major axis}}{\text{Minor axis}}$

(the direction of field orientation.

AR: Linear : ∞ ($\because \frac{1}{0}$)
Elliptical : in between 1 & ∞ (> 1)
Circular = $\frac{1}{1} = 1$

→ Signal to noise ratio – Ratio of signal fed to the network to the noise.

→ Antenna Temperature – Fictitious temperature at the input of an antenna which would account for noise ΔN at the output.

→ Front to back ratio – Ratio of energy in main lobe to that in block lobe.

Front to back ratio → associated only with directional antenna.

Dipole antenna → not a directional antenna. Hence front to back ratio cannot be defined for it.

Back lobes → only present for directional antenna.

Power pattern: normalised power v/s spherical coo. position
Field pattern – |E| or |H| v/s spherical coordinates

Radiation patterns:
- Omnidirectional
- Pencil Beam
- Fan Beam
- Shaped Beam

Chapter – Point Sources and their Array

→ Axial ratio is the ratio of major to minor axis of polarization of ellipse.

→ Any antenna can be considered as a point source.

→ Two types of radiation pattern:
i) field ii) power

→ Power theorem $P = \oiint S \, ds$ $(ds - r^2 \sin\theta \, d\theta \, d\phi)$
$S_r \to$ radical component of poynting vector. It is not equal to zero, however S_θ and $S_\phi = 0$

→ When we change θ and ϕ in isotropic field → no change as forms equipotential surface. However on changing r; S_r varies.

$P_r = S_r \oiint ds = S_r \cdot 4\pi r^2$ $\quad S_r = \frac{P}{4\pi r^2}$

From this we can say that magnitude of poynting vector is inversely proportional to square of distance.

$U = \frac{P}{4\pi}$ → radiation intensity → W/Sr

Q. A source has cosine radiation intensity pattern
$U = U_m \cos\theta$
$0 \le \theta \le \frac{\pi}{2}$; $0 \le \phi \le 2\pi$ Find the directivity.

$$P = \int_0^{\pi/2}\int_0^{2\pi} U_m \cos\theta \sin\theta \, d\theta \cdot d\phi$$

$$= \frac{U_m}{2}\int_0^{\pi/2}\int_0^{2\pi} 2\sin\theta\cos\theta \, d\theta \cdot d\phi$$

$$P = \pi U_m$$

$$1\,sr = (1\,rad)^2 = \left(\frac{180}{\pi}\right)^2 (deg.)^2$$

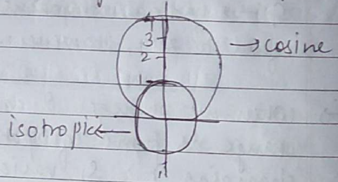
$P = \pi U_m$ $\qquad$ $r = 1$ considered.

$$D = \frac{U_m}{U_0}$$

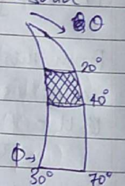
$\pi U_m = 4\pi U_0 \qquad \therefore D = \frac{U_m}{U_0} = 4$

$4\pi U_0 \rightarrow$ power radiated by isotropic point source

~~Let~~ * We are feeding the same power..
Directictivity given in terms of isotropic
Hence ~~comparision~~ in comparing it with isotropic



Q Find the number of square degrees in the solid angle Ω on a spherical surface.



$$\Omega = \iint \sin\theta \, d\theta \, d\phi$$

$$= \int_{20}^{40}\int_{30}^{70} \sin\theta \, d\theta \, d\phi$$

$$= \int_{20}^{40}\int_{30}^{70} \cancel{\cos}\sin\theta \, d\theta \, (70 - 30)$$

$$= \int_{20}^{40} [\sin\theta \cdot d\theta \cdot 40]$$

$$= 40 \cdot \cos\theta \Big|_{20}^{70} = 40[-\cos(40) - \cos(20)]$$

$$= \cancel{\frac{40° \times 2\pi}{360}} \times \frac{40° \times 2\pi}{360} \times 0.17$$

$= 0.22\pi \times 0.17 \times 3282\,(deg)^2$ $\qquad$ (converting into sq. deg. $1\,sr = \left(\frac{180}{\pi}\right)^2 (deg)^2$)

$= 397\,(deg)^2$

Q An antenna has a field pattern given by $E(\theta) = \cos^2\theta$, $0° \le \theta \le 90°$, unidirectional. Find the beam area of this pattern.

Beam area ≠ ϕ_{HP} / θ_{HP}

$\Omega = \iint P_{an}(\theta, \phi) \sin\theta \, d\theta \, d\phi \rightarrow$ this is for power pattern

$E(\theta) = \cos^2\theta \rightarrow$ for electric pattern

Relation between E-pattern and power pattern:

$$P_a(\theta, \phi) = \frac{1}{2} \frac{|E(\theta, \phi)|^2}{\eta_0} \qquad \eta_0 \approx 120\pi\,\Omega \text{ / } 377\,\Omega$$

$$\therefore P_a(\theta, \phi) \propto |E(\theta, \phi)|^2$$

$$\therefore \Omega = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} \cos^4\theta \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi/2} \cos^2\theta \cdot \cos^2\theta \cdot \sin\theta \, d\theta \, d\phi$$

$$= \cancel{\int_0^{2\pi}\int_0^{\pi/2} (1 - \sin^2\theta)(1 - \sin^2\theta) \sin\theta \, d\theta \, d\phi}$$

let $\cos\theta = t$ $\qquad$ $\cos(0) = 1 = t$ $\qquad$ $\cos(\pi/2) = 1$

$-\sin\theta \, d\theta = dt$

$$= -\int_0^{2\pi}\int_1^{0} t^4 \, dt \, d\phi$$

$$= -\frac{t^5}{5} \cdot (2\pi - 0) = \frac{2\pi}{5}\,sr = \frac{2\pi \times 3282}{5} = 4124° = 1265r$$

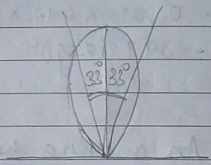
Current distribution along dipole depends upon length and it is constant along the length.
✓ Hertzian dipole.

METHOD 2.

$\cos^2\theta = 0.707$

$\theta = 33°$

∴ Total $\theta = 66°$



$\cos^2\theta = 0.707$

(HPBW)

Beam area $= \phi_{HP}\,\theta_{HP}$

$= 66 \times 66 =$

Q. The radiation efficiency of certain antenna is 95%. The maximum radiation intensity is ~~95~~ 0.5 W/sr. Calculate the directivity of antenna if
i) Input power is 0.4 W.
ii) Radiated power is 0.3 W.

$$D = \frac{U_{max}}{U_{avg.}} = \frac{U_{max}}{(P_{rad}/4\pi)} = \frac{P_d(\theta,\phi)}{P_{avg.}}$$

$$= \frac{P_d(\theta,\phi)}{(P_{rad}/4\pi r^2)}$$

$$= \frac{r^2 P_d(\theta,\phi)}{(P_{rad}/4\pi)}$$

$$= \frac{U(\theta,\phi)}{P_{rad}/4\pi}$$

$$\eta = \frac{P_{rad}}{P_{in}}$$

$$\frac{95}{100} = \frac{P_{rad}}{0.4}$$

$P_{rad} =$ ∴ $D = \dfrac{0.5}{P_{rad}/4\pi}$

Q. The loss resistance of antenna is 25 Ω. Calculate its radiation resistance if power gain is 30 and directivity is 42.

$G_{Pmax} = \eta_r \cdot G_{Dmax}$

$30 = \eta_r \cdot 25$

$30 = \dfrac{R_{rad}}{R_{rad} + r_{loss}} \times 25$ → (1)

$G = kD$

$G = k(42)$

$30 = k(42)$

$k = \dfrac{R_{rad}}{R_{rad} + r_{loss}} = \dfrac{30}{42}$ → (2)

Solve (1) and (2).

Q. Radiation intensity is $6\cos\theta$. Find directivity. Unidirectional pattern.

$D = \dfrac{U_m}{U_0} = \dfrac{6}{}$

$$P = \int_0^{\pi/2}\int_0^{2\pi} U_m \cos\theta \sin\theta \, d\theta \cdot d\phi$$

$$= \int_0^{\pi/2}\int_0^{2\pi} 6\cos\theta \sin\theta \, d\theta \cdot d\phi$$

$$= \frac{6}{2}\int_0^{\pi/2}\int_0^{2\pi} 2\sin\theta\cos\theta \, d\theta \cdot d\phi = 6\pi$$

$P = 6\pi$ $\quad D = \dfrac{U_m}{U_0}$

∴ $6\pi = 4\pi U_0$ [Because here assumption: Power radiated by both antenna - isotropic and one under consideration is same]

$6 = 4U_0$

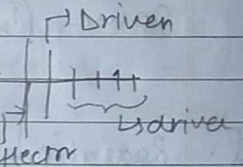
∴ $D = \dfrac{4 U_0}{U_0} = \boxed{4}$

CHAPTER - ARRAY OF ~~POINT SOURCES~~ ANTENNA.

→ Array of antenna:
A group of ~~isotro~~ isotropic radiators such that the current running through them are of different amplitudes and different phases is known as array of antenna.

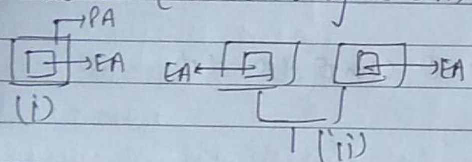
Array → the source to feed is one and with one antenna being fed; all antennas are receiving energy and radiating → this is known as array of antenna.

→ Advantages of using array of antenna:

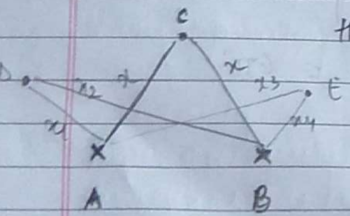


- Directivity increases:
When we use n antenna; total effective apperture increases. (Directivity ∝ E.A)

Yaagi antenna → parasitic antenna. In this; only driven gets the current and driver elements depend upon driven for functioning. Hence it is a parasitic antenna



Here overall EA increases/more than (i).



A, B → Two point sources.
At C; where distance from A and B to the point is same; there would be constructive interference.
However at points like B and E; distance from both point source to that point is not same. Hence there would be destructive interference.

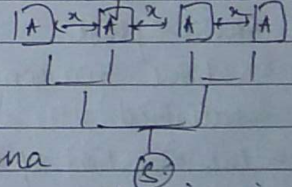
The most important advantage of array of antenna → Increase the directivity
Disadvantage - side lobes.

- Beam area reduces. - Because we try to direct the directivity at one point (As we say in the diagram before → phenomena of constructive and destructive interference).
- Beam steering - By varying the phases of current in individual antenna in an array; we can change the direction of the beam. (Overall; ~~it is~~ array of an antenna ~~a~~ acts as a directive antenna). (Transmitting sense)
- Determine direction of receiving signal (concept of beam steering → in receiving sense).

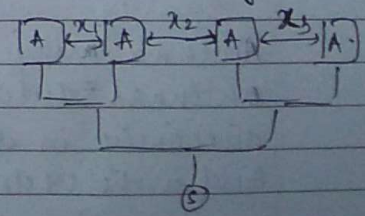
Advantages → Inc in ~~beam area~~ overall gain, increase in signal to interference plus noise ratio, beam steering, provides diversity reception.

→ CLASSIFICATION OF ARRAY OF ANTENNA.

1 Based on distance between adjacent elements:
↳ Uniform - Distance between adjacent elements is same.
same phase diff in current flowing in two adjacent antenna



↳ Non uniform - Distance between the adjacent antenna is not the same → ~~ex~~ in case wherein isolation is needed; ~~ca~~ there is non uniform arrangement.
phase diff of current is diff
Example - binomial array.



Mainly two types of array:
1) Linear - Same structures.
- Uniform
- Non uniform
2) Planar - different structures used to make an array → one antenna may be circular, one may be triangular → Ex → antenna present in car.

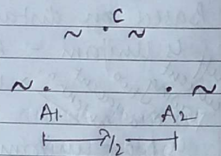
Four different types of linear array:
- Broad~~way~~side
- Parasitic
- Collinear
- End fire

In case of array - Field strength increases/reinforces in one direction → cancels out in other → hence directivity increases.

- Same phase and equal magnitude of current
- Distance between 2 adjacent antennas ~~should~~ should be $\lambda/2$

→ Conditions for reinforcement in one direction.

1 Broad~~way~~side array
- both antenna fed with same phase
- above two conditions satisfied



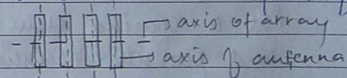
- the direction of radiation is perpendicular to the axis of array and also to axis of individual array.

Antenna array can be defined as system of similar antennas directed to get required ~~directive~~ high directivity in ~~desiree~~ desired direction. The total field is the vector sum of fields produced by individual antennas of an array.

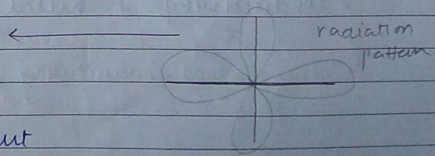
Antenna array
- Linear (Similar)
  - Uniform
    - Equal spacing
    - ~~Unco~~ Inphase current
  - Non uniform
    - Non ~~sp~~ equal spacing
    - non uniform current
    - Binomial array - Example.
- Planar ~~Non linear~~ (Non similar antenna)

→ Types:

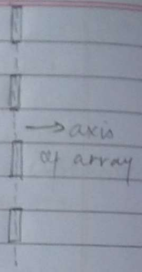
1 Broad~~way~~side
- Same elements (in array; individual antennas are known as elements of array)
- Equal spacing
- In-phase current
- Direction of maximum radiation → Normal to both axis of antenna array (x axis suppose); and axis of individual elements (y axis suppose) → (z axis → direction of maximum radiation.)



2 Endfire.
- Same element
- Equal spacing
- Out of phase current
- Direction of maximum radiation → Along the axis of antenna array. ($\phi = 0°$)

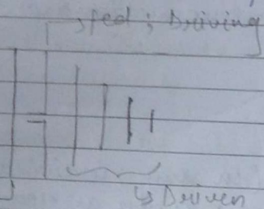


3 Collinear array
- Same elements
- Equal spacing
- Not used mostly
- Elements connected end-to-end
- Direction of maximum radiation - normal to the axis of array



4 Parasitic array
- Eg. Yaagi Uda
- Only one fed; others are dependent on it

parasitic element: lengthened → acts as reflector, shortened → director



For a spacing ~~them~~ between driven element and parasitic element is $\frac{\lambda}{4}$; and phase difference of $\pi/2$ radian; then unidirectional radiation pattern is observed.

- Density
- Conductivity

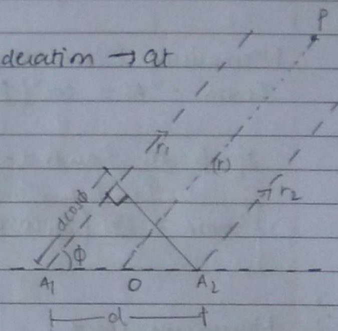
} Factors to consider for selecting metal suitable for antenna.

Silver → good conductivity; but high density.

Case 1 : Two point sources with current of equal magnitude and same phase.

Path difference.

If 'P' → point under consideration → at large distance
$r_1 = r_2 = r$



Path difference $= d\cos\phi$

Path difference in terms of $\lambda = \frac{d\cos\phi}{\lambda}$

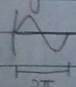
Phase change $= 2\pi \times$ (Phase difference)
$= \frac{2\pi \times d\cos\phi}{\lambda}$

$\frac{d\cos\phi}{\lambda}$ → path diff. due to one wavelength.

$\beta = \frac{2\pi}{\lambda}$

Phase change $= \beta d\cos\phi$

∴ Phase changes at 2π



$2\pi \frac{\lambda d\cos\phi}{\lambda}$

Phase shift (Ψ) ∴ $\boxed{\Psi = \beta d\cos\phi}$

$E_1 = E_0 e^{-j\Psi/2}$ $\quad E_2 = E_0 e^{+j\Psi/2}$

$E = E_0 e^{-j\Psi/2} + E_0 e^{j\Psi/2}$

$= 2E_0\left[\frac{e^{-j\Psi/2} + e^{j\Psi/2}}{2}\right]$

$= 2E_0 \cos\Psi/2$

$$\boxed{E = 2E_0 \cos\left(\frac{\beta d\cos\phi}{2}\right)}$$

At point P ; maximum field = $2E$

↳ At $\phi = 90^\circ$, $E = 2E_0$.

$$\text{Array factor} = \frac{|E|}{|E_{max}|}$$

Normalising E we get,

$$E_{norm} = AF = \cos\left(\frac{\beta d \cos\phi}{2}\right)$$

If $\beta = \frac{2\pi}{\lambda}$ and $d = \frac{\lambda}{2}$

$$\boxed{E_{norm} = \cos\left(\frac{\pi}{2}\cos\phi\right)}$$

i Maximum points

$$E_{max} = \cos\left(\frac{\pi}{2}\cos\phi\right) = \pm 1$$

$$\Rightarrow \frac{\pi}{2}\cos\phi = \pm n\pi \; ; \; n = 0, 1, 2 \ldots$$

Take $n = 0$: $\frac{\pi}{2}\cos\phi = 0$ $\therefore \boxed{\cancel{\cos}\,\phi = 90^\circ, 270^\circ}$

ii Minima points

$$\cancel{E_{min}} = \cos\left(\frac{\pi}{2}\cos\phi\right) = 0$$

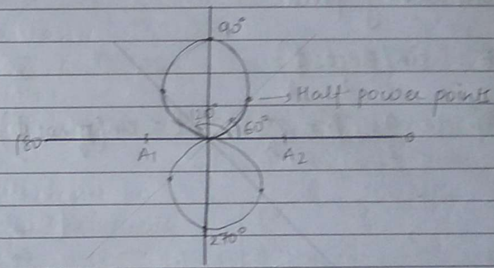
$$\cancel{\cos}\left(\frac{\pi}{2}\cos\phi\right) = \pm(2n+1)\frac{\pi}{2}$$

~~for~~ for $n = 0$; $\cancel{\cos}\left(\frac{\pi}{2}\cos\phi\right) = \pm\frac{\pi}{2}$ $\therefore \boxed{\phi = 0^\circ, 180^\circ}$

iii For Half Power points

$$\cos\left(\frac{\pi}{2}\cos\phi\right) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\phi = \pm(2n+1)\frac{\pi}{4} \quad \therefore \boxed{\phi = 60^\circ / 120^\circ}$$



Now taking A_1 as reference

$$E = E_0 e^{j\cancel{\phi}} + E_0 e^{j\psi}$$

$$= E_0 + E_0 e^{j\psi}$$

$$= 2E_0 e^{j\psi/2}\left(\frac{e^{-j\psi/2} + e^{j\psi/2}}{2}\right)$$

$$= 2E_0 e^{j\psi/2} \cos(\psi/2)$$

variation of phase of field taking ϕ

↳ variation of amplitude to EF

→ Case 2 - Two isotropic point sources of same magnitude but out of phase current.

i) $E = -E_0 e^{j\psi/2} + E_0 e^{-j\psi/2}$

$= 2jE_0 \left[\frac{e^{-j\psi/2} - e^{j\psi/2}}{2j}\right]$

$= \boxed{E = 2jE_0 \sin\left(\frac{\psi}{2}\right)}$

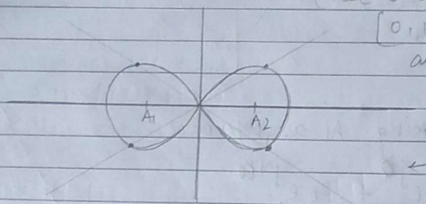
$E_0 e^{j(\psi/2+180)} = E_0 e^{j\psi/2} \cdot e^{j180} = -E_0 e^{j\psi/2}$

Normalising field

$E = \sin\left(\frac{\beta d \cos\phi}{2}\right)$

If $d = \frac{\lambda}{2}$, $\beta = \frac{2\pi}{\lambda}$ $\therefore E = \sin\left(\frac{\pi}{2}\cos\phi\right)$

(use 3 diff cases [0, 180], (90, 270) and half power to obtain this)



→ Case 3 : Two isotropic point sources of same amplitude but phase quadrature.

$\cancel{E = E_0 e^{j(\psi/2+90)} + E_0 e^{-j\psi/2}}$

$= E_0 e^{j\psi/2} e^{j90} + E_0 e^{-j\psi/2}$

$= jE_0 e^{j\psi/2} + E_0 e^{-j\psi/2}$

$e^{j90} = \cos 90 + j\sin 90$,

$E = E_0 e^{j(\psi/2+\pi/4)} + E_0 e^{-j(\psi/2+\pi/4)}$ (*)

$= 2E_0 \cos\left(\frac{\psi}{2} + \frac{\pi}{4}\right)$

(*) $E_0 e^{j(\pi/4 + \psi/2)}$ — $\pi/4$: phase shift due to source current; $\psi/2$: phase shift due to path diff.

$= 2E_0 \cos\left(\frac{\pi}{4} + \frac{\beta d\cos\phi}{2}\right)$

$$\boxed{E_{norm} = \cos\left(\frac{\pi}{4} + \frac{\pi}{2}\cos\phi\right)}$$

i Maximum

$E_{norm} = \cos\left(\frac{\pi}{4} + \frac{\pi}{2}\cos\phi\right) = \pm 1$

$\frac{\pi}{4} + \frac{\pi}{2}\cos\phi = \pm n\pi$

for $n = 0$; $\frac{\pi}{2}\cos\phi = -\frac{\pi}{4}$ $\therefore \boxed{\phi = 120, 240}$ ($\cancel{300}$)

$\frac{\pi}{4} + \frac{\pi}{2}\cos\phi = -\pi$
$\pi/2 \cos\phi = -\frac{5\pi}{4}$
$\cos\phi = -\frac{5}{2}$

ii ~~Minimum~~ Minimum

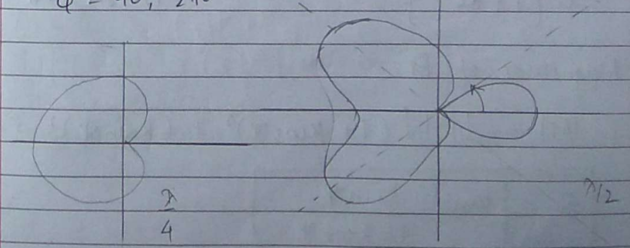
$E_{norm} = \cos\left(\frac{\pi}{4} + \frac{\pi}{2}\cos\phi\right) = 0$

$\frac{\pi}{4} + \frac{\pi}{2}\cos\phi = \pm(2n+1)\frac{\pi}{2}$

for $n = 0$; $\cos\phi = \frac{1}{2}$ $\boxed{\phi = 60, 300}$ ($\cancel{240}$)

iii Half Power points

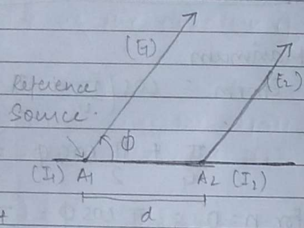
$\phi = 90, 270$



→ Case 4 : General case of 2 isotropic point sources of unequal amplitude and at any phase.

$|E_1| > |E_2|$

$\frac{E_2}{E_1} = K \quad \boxed{K<1}$



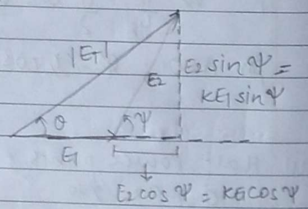
$\Psi = \beta d\cos\phi + \alpha$

Total phase shift ↓ Phase shift due to phase difference ↓ Phase shift due to applied source ↓

- $|E_1| > |E_2|$
- $\frac{E_2}{E_1} = K$
- $K<1$

* $\alpha = 0 \rightarrow$ Broadside array
* $\alpha = 180^\circ \rightarrow$ Endfire array
$0 \le \alpha \le 180^\circ \rightarrow$ range of α

$E_T = E_1 e^{j0} + E_2 e^{j\psi}$
$= E_1 + E_2 e^{j\psi}$
$= E_1\left[1 + \frac{E_2}{E_1} e^{j\psi}\right]$
$= E_1\left[1 + Ke^{j\psi}\right]$



$E_2 \sin\psi = KE_1 \sin\psi$

$E_2\cos\psi = KE_1\cos\psi$

Magnitude of E_T =

$$|E_T| = |E_1|\sqrt{(1 + K\cos\psi)^2 + (K\sin\psi)^2}$$

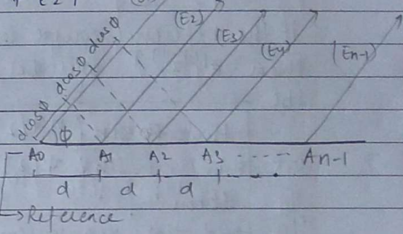
$$\boxed{\theta = \tan^{-1}\frac{K\sin\psi}{1 + K\cos\psi}}$$

→ Case 5 : n-Elements uniform linear array.

$\psi = \beta d\cos\phi \rightarrow$ for each ~~one~~ radiation

Fields, distance → uniform ∵ linear array

Total field = $E_T = E_0 + E_1 + E_2 + E_3 + \dots E_{n-1}$



Path diff. for each would be diff.
∵ A0 → ref.

∴ for A1 ; $d\cos\phi$
A2 ; $2d\cos\phi$, A3 = $3d\cos\phi$ An-1 = $(n-1)d\cos\phi$

$E_T = E_0 e^{j0} + E_1 e^{j\psi} + E_2 e^{2j\psi} + E_3 e^{3j\psi} + \dots E_{n-1} e^{(n-1)j\psi}$
∵ ($\psi = \beta d\cos\phi$).

(GP series)

$E_T = E_0\left[1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} \dots + e^{j(n-1)\psi}\right] \rightarrow ①$

$e^{j\psi} E_T = E_0\left[e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots e^{jn\psi}\right] \rightarrow ②$

(multiplied $e^{j\psi}$ on both sides)

① − ②

$E_T\left[1 - e^{j\psi}\right] = E_0\left[1 - e^{jn\psi}\right]$

$E_T = \frac{E_0\left[1 - e^{jn\psi}\right]}{\left[1 - e^{j\psi}\right]}$

$E_T = \frac{E_0 e^{jn\psi/2}\left[e^{-jn\psi/2} - e^{jn\psi/2}\right]/2}{e^{j\psi/2}\left[e^{-j\psi/2} - e^{j\psi/2}\right]/2}$

$$\boxed{E_T = E_0 e^{j\frac{(n-1)}{2}\psi} \left[\frac{\sin(n\psi/2)}{\sin(\psi/2)}\right]}$$

If $n=2$; it will give us the equation of Broadside array.

If the reference source is midway of the array; then $e^{j\frac{(n-1)}{2}\psi}$ will be eliminated, the $|E_T|$ will be

$$\boxed{E_T = \frac{E_0 \sin(n\psi/2)}{\sin(\psi/2)}}$$

To find the maximum value; put $n=0$; But this gives us indefinite value. Hence using L'Hospitals rule

$$E_{max} = \lim_{\psi \to 0} \frac{E_0 \sin(n\psi/2)}{\sin(\psi/2)} \Rightarrow nE_0$$

$$\therefore \cancel{E_{max}}\ \boxed{E_{norm} = \frac{|E_T|}{|E_{max}|}} \longrightarrow \text{Array factor}$$

$$= \frac{1}{n} \frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

Principle of Pattern Multiplication :- in this we can

→ Condition for linear array of n-isotropic point sources with equal amplitude and phase:

↳ Broadside array

1) $\alpha = 0$, $\psi = \beta d \cos\phi + \alpha$

2) for max radiation; $\psi = 0$ $\phi = 90°, \cancel{180}\ 270°$

↳ ~~Endfi~~ Endfire array

for maximum radiation, $\psi = 0$

$\psi = \beta d \cos\phi + \alpha$ ~~$\alpha =$~~ $\psi = 0$ $\phi = 0$.

$0 = \beta d \cos\phi + \alpha$ $\therefore$ $\boxed{\alpha = -\beta d}$

$d = \frac{\lambda}{4} \Rightarrow \alpha = -\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} \Rightarrow \boxed{\alpha = -\frac{\pi}{2}}$

$d = \frac{\lambda}{2} \Rightarrow \alpha = -\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \Rightarrow \boxed{\alpha = -\pi}$

$\alpha = -\beta d \Rightarrow$ does not provide us maximum directivity.

$\alpha = -\left(\beta d + \frac{\pi}{n}\right) \Rightarrow$ provides us with maximum directivity.

This equation was given by Hazennwouldyard on experimental basis. So called as condition for maximum extended directivity or Hazennwouldyard condition for max. directivity.

Q Two vertically oriented half wave dipoles are spaced 1.5λ apart to form an array. Calculate the half power ~~band~~ beam width of the major lobes of the array in horizontal planes for the following 2 cases; when the dipoles are fed with:

i Equal and in phase current

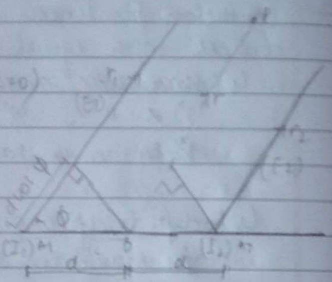
ii Equal current but the phase diff. of 540°. Between 2 currents

Path difference $= d\cos\phi$

$\psi = \beta d\cos\phi + \alpha$

(But as currents in phase, $\alpha = 0$)

$\psi = \beta d\cos\phi$



$r_1 = r + \frac{d}{2}\cos\phi$

$r_2 = r - \frac{d}{2}\cos\phi$

$E_T = E_1 e^{j\beta(r + \frac{d}{2}\cos\phi)} + E_2 e^{j\beta(r - \frac{d}{2}\cos\phi)}$

$d << r$

$E_1 = E_2 = E_0$

$E_T = 2E_0 e^{j\beta r}\left[\frac{e^{j\beta d\cos\phi} + e^{-j\beta d\cos\phi}}{2}\right]$

$E_T = 2E_0 e^{j\beta r}\cos(\beta d\cos\phi)$

For half power beam width,

$\cos(\beta d\cos\phi) = \frac{\pm 1}{\sqrt{2}}$ $\qquad 0.75\lambda$, $2d = 1.5\lambda$

$\beta d\cos\phi = \pm\pi/4$

$\cos\phi = \frac{\pi}{4\beta d} = \frac{\pi}{4 \times \frac{2\pi}{\lambda} \times 0.75\lambda}$ $\qquad \boxed{\phi = 80.40^\circ}$

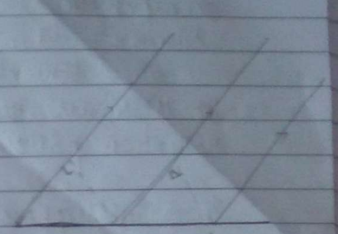
$\beta d\cos\phi = 54°$ and solve as above.

In a radiation ~~part~~ pattern of a 3 element array of isotropic radiators equally spaced at a distance of $\lambda/4$. It is required to place a null ~~of~~ at an angle 33.56° of the $n\pi$ direction. Calculate the progressive phase shift to be applied to the element. Also calculate the angle at which the main beam is placed for this phase distribution.

$\psi = \beta d\cos\phi + \alpha$

$= \frac{2\pi}{\lambda} \times \frac{\lambda}{4}\cos\phi + \alpha$

$= \left(\frac{\pi}{2}\cos\phi\right) + \alpha$



$E_T = E_0\left[e^{-j\left(\frac{\pi}{2}\cos\phi + \alpha\right)}\right] + 1 + E_0\left[e^{j\left(\frac{\pi}{2}\cos\phi + \alpha\right)}\right]$

lag (current 1) wrt 2 $\qquad$ lead (current 3)

$= E_0\left[1 + 2\cos\left(\frac{\pi}{2}\cos\phi + \alpha\right)\right]$

<u>Null</u> $\therefore\ 0 = E_0\left[1 + 2\cos\left(\frac{\pi}{2}\cos\phi + \alpha\right)\right]$

$0 = E_0\left[1 + 2\cos\left(\frac{\pi}{2}\cos 33.56 + \alpha\right)\right]$

$-\frac{1}{2} = \cos\left(\frac{\pi}{2}\cos 33.56 + \alpha\right)$

$120 - \frac{180}{2}\cos 33.56 = \alpha$ $\qquad \boxed{\alpha = 45^\circ}$

→ The total field pattern as a non isotropic but SIMILAR SOURCES is the multiplication of individual source pattern and the pattern of an array of isotropic point sources each located at the phase centre of individual source and having relative amplitude and phase whereas the total phase pattern ~~of individual~~ is the addition of the phase pattern of the individual sources and that of the array of isotropic point sources → Principle of pattern multiplication.

→ Significance of array factor / Information given by array factor:

$E_{norm} = \frac{|E_T|}{|E_{max}|}$ ⇒ ~~po~~ normalised power.

It tells us ~~the~~ as to what would be the radiation pattern of the antenna considered

Ψ → Total phase shift ⇒ max. field ~~in the directi~~ when $\Psi = 0$.

Field from any array would be maximum in any direction ϕ for which $\Psi = 0$

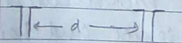
This can be again understood in terms of receiver as the ~~ma~~ maximum field received from any direction ϕ when $\Psi = 0$.

→ Principle of pattern multiplication.

Just by simple multiplication, we can get to know the radiation pattern of a field.

Non isotropic but similar point sources → field pattern has same shape and orientation.

Not necessary that amplitude is same of both the patterns.



(Two dipole antenna considered)

↓

Broadside array case

$E_0 = E_1 \sin\phi$

$E = 2E_0 \cos\Psi/2$

$\Psi = \beta d \cos\phi$

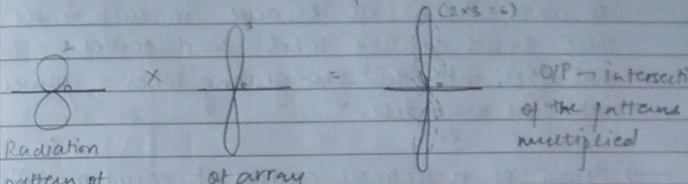
$E = 2E_1 \sin\phi \cos\Psi/2$

$E = (\sin\phi)(2E \cos\Psi/2)$

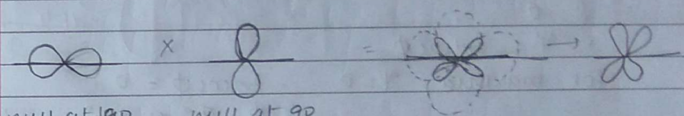
$E = \{\underbrace{E_i(\theta,\phi) \times E_a(\theta,\phi)}_{\text{Product of field pattern}}\}\{\underbrace{E_{pi}(\theta,\phi) + E_{pa}(\theta,\phi)}_{\text{Addition of phase pattern}}\}$

E_i → individual E_a → array parameter

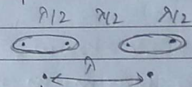
1



2



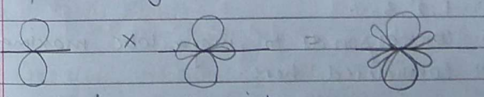
→ 4 point sources



→ ~~Two~~ Four point sources separated by distance of λ/2 each (we have grouped 2)

Phase centre → here we take the centre of → between 2 adjacent antenna

∴ Now phase centre case → 2 point sources of separated by λ distance



Distance between 2 elements in an array is never λ, 2λ, etc because in such cases there exists grating lobes. and power that goes into this ~~grad~~ grating lobes = power into the major lobe.

→ Array of n-isotropic sources of equal amplitude and spacing (Broadside)

$\psi = \beta d\cos\phi + \delta$

$\delta = 0$

$\therefore \psi = \beta d\cos\phi$

For maxima, $\psi = 0$ $\therefore \beta d\cos\phi = 0$

$\phi = 90°$ or $270°$

→ Pattern maxima

$$E = \frac{1}{n}\frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

Because we want to find the maxima;

$\sin(n\psi/2) = 1$

$n\psi/2 = \pm(2N+1)\frac{\pi}{2}$

$n = 1, 2, 3 \ldots$

$n = 0$ → corresponds to major lobe maxima Hence not considered here.

$\therefore \sin(n\psi/2) = 1$

$n\psi/2 = \pm(2N+1)\frac{\pi}{2}$

$N = 1, 2, 3 \ldots$

$$\psi/2 = \pm\frac{(2N+1)\pi}{2n}$$

$$\psi = \pm\frac{(2N+1)\pi}{n}$$

$$\beta d\cos\phi_{max} = \pm\frac{(2N+1)\pi}{n}$$

$$\phi_{max} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{(2N+1)\pi}{n}\right\}\right]$$

$$\boxed{\phi_{max} = \cos^{-1}\left[\pm\frac{(2N+1)\lambda}{2nd}\right]}$$

$\beta = \frac{2\pi}{\lambda}$

n = no. of elements

~~Putting~~

Q $n = 4$ $d = \lambda/2$ $N = 1$

$$\phi_{max} = \cos^{-1}\left[\pm\frac{(2(1)+1)\cdot\lambda}{2(4)(\lambda/2)}\right]$$

$$= \cos^{-1}\left[\pm\frac{(2+1)\lambda}{(4\lambda)}\right]$$

$$= \cos^{-1}\left[\pm(3/4)\right]$$

$$\boxed{\phi_{max} = \pm 41.4°, \pm 138.6°}$$

~~Pattern maxima~~.

→ Pattern minima.

$$E = \frac{1}{n}\frac{\sin n\psi/2}{\sin\psi/2}$$

Because minima; $\sin n\psi/2 = 0$

$n\psi/2 = \pm N\pi$

$$\psi = \pm\frac{2N\pi}{n}$$

$$\beta d\cos\phi_{min} = \pm\frac{2N\pi}{n}$$

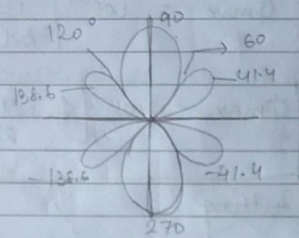
$$\cos\phi_{min} = \frac{1}{\beta d}\left(\pm\frac{2N\pi}{n}\right)$$

$$\phi_{min} = \cos^{-1}\left[\pm\frac{N\lambda}{nd}\right]$$

$n = 4 \quad d = \lambda/2 \quad N = 1$

$\phi_{min} = \cancel{0^\circ},\ 60^\circ, 120^\circ$

For $N = 2$; $\phi_{min} = 0^\circ, 180^\circ$



→ Array of n-isotropic sources of equal amplitude ~~an~~ but out of phase by 180° → opposite direction (Endfire)

→ ~~Pattern maxima~~.

$\psi = \beta d \cos\phi + \delta$

$\psi = \beta d \cos\phi + (-\beta d)$

$\psi = \beta d \cos\phi - \beta d$

We know;
for maximum radiation

$\psi = 0 \quad \phi = 0$

$\psi = \beta d \cos\phi + \alpha$

$0 = \beta d \cos 0 + \alpha$

$\alpha = -\beta d$.

For maxima, $\psi = 0$

$0 = \beta d \cos\phi - \beta d$

$\beta d = \beta d \cos\phi$ $\therefore$ $\boxed{\phi_{max} = 0^\circ, 180^\circ}$

→ ~~Pattern minima~~

~~$\psi = \beta d \cos\phi + \delta$~~

~~$\psi = \beta d \cos\phi + (-\beta d)$~~

→ Pattern maxima.

$$E = \frac{1}{n}\frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

$$\sin(n\psi/2) = 1$$

$$n\psi/2 = \pm(2N+1)\frac{\pi}{2}, \quad n = 1, 2, 3 \ldots$$

$$\sin(n\psi/2) = 1 \quad \therefore \psi = \frac{(2N+1)\pi}{n}$$

$$\beta d \cos\phi_{max} - \beta d = \frac{(2N+1)\pi}{n}$$

$$\beta d \cos\phi_{max} = \frac{(2N+1)\pi}{n} + \beta d$$

$$\cos\phi_{max} = \frac{1}{\beta d}\left[\frac{(2N+1)}{n}\pi + \beta d\right]$$

$$\phi_{max} = \cos^{-1}\left[\frac{1}{\beta d}\left(\frac{(2N+1)\pi}{n} + \beta d\right)\right]$$

$$= \cos^{-1}\left[\frac{1}{\beta d n}\left((2n+1)\pi + n\beta d\right)\right]$$

$$= \cos^{-1}\left[\frac{\lambda}{2\pi d n}\left((2n+1)\pi + n\frac{2\pi d}{\lambda}\right)\right]$$

$$= \cos^{-1}\left[\frac{\lambda}{2\pi d n}\left[(2n+1)\pi + \frac{2\pi n d}{\lambda}\right]\right]$$

$$\boxed{\phi_{max} = \cos^{-1}\left[\frac{\lambda}{2nd}(2n+1) + 1\right]}$$

→ Pattern minima

$$E = \frac{1}{n}\frac{\sin n\psi/2}{\sin \psi/2} \longrightarrow \text{array factor}$$

$$\sin n\psi/2 = 0$$

$$\frac{n\psi}{2} = \pm N\pi$$

$$Bd\cos\phi - Bd = \pm\frac{2N\pi}{n}$$

$$\therefore \boxed{\phi_{min} = \cos^{-1}\left[\pm\frac{N\lambda}{nd} + 1\right]}$$

→ NULL DIRECTION FOR N ISOTROPIC POINT SOURCE ARRAY WITH SAME AMPLITUDE AND SPACING.

From the derivation of E for 'n' elements, we can say;

$$E = \frac{1-e^{jn\psi}}{1-e^{j\psi}}$$

if $E = 0 \Longrightarrow$ because we want to find null.

$$1 - e^{jn\psi} = 0 \quad \therefore e^{jn\psi} = 1$$

$$\therefore n\psi = \pm 2k\pi \; ; \; k = 1, 2, 3 \ldots$$

$$\psi = Bd\cos\phi + \delta$$

$$Bd\cos\phi + \delta = \pm\frac{2k\pi}{n}$$

$$\phi_0 = \cos^{-1}\left[\left(\pm\frac{2k\pi}{n} - \delta\right)\frac{1}{Bd}\right]$$

For Broadside $\delta = 0$

$$\phi_0 = \cos^{-1}\left(\pm\frac{2k\pi}{nBd}\right)$$

$$B = \frac{2\pi}{\lambda}$$

$$\phi_0 = \cos^{-1}\left(\pm\frac{k\lambda}{nd}\right)$$

- if $n = 4$ $d = \frac{\lambda}{2}$ $\therefore \phi_0 = \cos^{-1}\left(\pm\frac{k}{2}\right)$
- for $k = 1$, $\phi_0 = \pm 60°, \pm 120°$ ⇒ symmetric patterns hence ±.
  $k = 2$ $\phi_0 = 0°, \pm 180°$

Thus there are total 4 nulls in 4 point source array at $d = \frac{\lambda}{2}$.

Complementary angle of $\phi_0 = \cos^{-1}\left(\pm\frac{k\lambda}{nd}\right) =$

$$= \gamma_0 = \sin^{-1}\left(\pm\frac{k\lambda}{nd}\right)$$

If the array is huge → has large number of elements and distance between 2 elements is large; $nd \gg k\lambda$. In such cases;

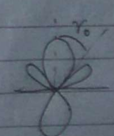
$$\gamma_0 \simeq \pm\frac{k\lambda}{nd}, \quad nd \gg k\lambda$$

First null at either side of maxima;

$$k = 1$$

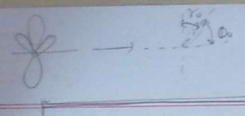
$$\gamma_{01} \simeq \frac{k\lambda}{nd} \longrightarrow \text{first null.}$$

$$\gamma_{01} \simeq \frac{\lambda}{nd}$$



Total Beam width first Null Beam width $= 2\gamma_{01} \simeq \frac{2\lambda}{nd}$

Scanned by CamScanner



$\phi_0 = 90 - \gamma_0$

$\gamma_0 = 90 - \phi_0$

$\cancel{2\gamma_0}$ $\boxed{2\gamma_{01} \simeq \frac{2\lambda}{nd} = \frac{2}{L/\lambda} \text{ rad.} = \frac{2 \times 57.3}{L/\lambda} \text{ deg.} = \frac{114.6}{L/\lambda}}$

$\frac{L}{\lambda} = L\lambda \rightarrow$ means L in terms of λ

$\boxed{\text{Half Power Band Width (HPBW)} = \frac{FNBW}{2} = \frac{57.3^\circ}{L/\lambda}}$

Q → ϕ_0 for Endfire array.

$\delta = -\beta d$

$\cancel{\phi_0}$ $\psi = \beta d \cos \phi_0 + \delta$

$\beta d \cos \phi_0 + \delta = \pm \frac{2k\pi}{n}$

$\phi_0 = \cos^{-1}\left[\left(\pm \frac{2k\pi}{n} - \delta\right)\frac{1}{\beta d}\right]$

$\phi_0 = \cos^{-1}\left[\left(\pm \frac{2k\pi}{n} + \beta d\right)\frac{1}{\beta d}\right]$

$= \cos^{-1}\left[\pm \frac{2k\pi}{n\beta d} + 1\right]$

$= \cos^{-1}\left[\cancel{\cot} \pm \frac{k\lambda}{nd} + 1\right]$

$\psi = \beta d \cos \phi_0 - \beta d = \pm \frac{2k\pi}{n}$

$\cos \phi_0 - 1 = \pm \frac{2\pi k}{n\beta d}$

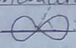
$-2\sin^2 \frac{\phi_0}{2} = \pm \frac{2k\pi}{n\beta d}$

$$\boxed{\phi_0 = 2\sin^{-1}\left(\pm\sqrt{\frac{k\pi}{n\beta d}}\right)}$$

$n = 4$ $d = \lambda/2$
FNBW, HPBW

2) Repeat for endfire with incr. directivity; HPBW, FNBW

$k=1$ $\cancel{\phi_0 = \pm 6}$ $\phi_0 = \pm 60^\circ$,
$k=2$, $\phi_0 = \pm 90^\circ$
$k=3$, $\phi_0 = \pm 120^\circ$
} for $n=2$, $d = \frac{\lambda}{2}$

Here we do not need to find complementary because orientation of field here: 

if array is huge ~~then~~ then;

$\phi_0 = 2\sin^{-1}\left(\pm\sqrt{\frac{k\pi}{n\frac{2\pi}{\lambda}d}}\right) = 2\sin^{-1} \pm \sqrt{\frac{k\lambda}{2nd}}$

$nd >> k\lambda$

$\frac{\phi_0}{2} = \pm\sqrt{\frac{k\lambda}{2nd}}$. $\boxed{\phi_0 = \pm\sqrt{\frac{2k\lambda}{nd}}}$ HPBW

For $k=1$, $\phi_0 \simeq \pm\sqrt{\frac{2\lambda}{nd}}$.

For FNBW:

$2\phi_{01} \simeq 2\sqrt{\frac{2\lambda}{nd}}$

$= 57.3 \times 2\sqrt{\frac{2}{L/\lambda}} \simeq \pm 114.6\sqrt{\frac{2}{L/\lambda}}$ (deg)

→ Endfire array with increased directivity.

$\delta = -\left(\beta d + \frac{\pi}{n}\right)$ ~~(give)~~

$\psi = \beta d \cos \psi_0 + \delta$

$= \beta d \cos \psi_0 - \left(\beta d + \frac{\pi}{n}\right)$

$\pm \frac{2k\pi}{n} = \beta d \cos \phi_0 - \beta d + \frac{\pi}{n}$.

$\cancel{\beta d e}\ \beta d \cos\phi_0 = \pm \frac{2k\pi}{n} + \left(\beta d + \frac{\pi}{n}\right)$

$\phi_0 = \cos^{-1}\left(\pm\frac{2k\pi}{n\beta d} + 1 + \frac{\pi}{n\beta d}\right)$

$\pm\frac{2k\pi}{n} + \frac{\pi}{n} = \beta d(\cos\phi - 1)$

$2\sin^2\phi/2 = \pm\frac{2k\pi}{n\beta d} + \frac{\pi}{n\beta d}$

$\sin^2\phi/2 = \pm\frac{k\pi}{n\beta d} + \frac{\pi}{2n\beta d}$

$\cancel{\phi_0 = 2\sin^{-1}\left(\pm\sqrt{\frac{k\pi}{n\beta d} + \frac{\pi}{2n\beta d}}\right)}$

$\cancel{\left(\because \beta = \frac{2\pi}{\lambda}\right)}$

$\cancel{\phi_0 = 2\sin^{-1}\left(\pm\sqrt{\frac{\pi}{2n\beta d}(2k+1)}\right)}$

$\sin^2\phi/2 = \frac{\pi}{2n\beta d}(\pm 2k + 1)$

$\because$ [$(\cos\phi - 1)$ will always be negative
$\therefore\ \pm 2k \rightarrow$ omitted as we need RHS side to be negative]

$-\sin^2\phi/2 = \frac{\pi}{2n\beta d}(-2k+1)$

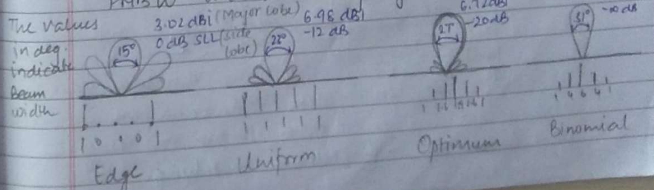
$\phi_0 = \sin^{-1}\sqrt{\left(\frac{\pi}{2n\beta d}(2k-1)\right)}$

$$\boxed{\phi_0 = \sin^{-1}\sqrt{\frac{\pi}{4nd}(2k-1)}}$$

→ Orientation of maxima
→ feeding

Parameters	Broadside	Endfire	Endfire with increased directivity
Null direction (any length)	$r_0 = \sin^{-1}\left(\pm\frac{k\lambda}{nd}\right)$	$\phi_0 = 2\sin^{-1}\left(\pm\sqrt{\frac{k\lambda}{nd}}\right)$	$\phi_0 = 2\sin^{-1}\left[\pm\sqrt{\frac{\lambda(2k-1)}{4nd}}\right]$
Null direction (long array)	$r_0 \simeq \frac{k\lambda}{nd}$	$\phi_0 \simeq \pm\sqrt{\frac{2k\lambda}{nd}}$	$\phi_0 \simeq \pm\sqrt{\frac{\lambda(2k-1)}{nd}}$
FNBW	$2r_{01} \simeq \frac{2\lambda}{nd} \simeq \frac{114.6^\circ}{L/\lambda}$	$2\phi_{01} \simeq 2\sqrt{\frac{2\lambda}{nd}}$ $\simeq 114.6\sqrt{\frac{2}{L/\lambda}}$	$2\phi_{01} \simeq 2\sqrt{\frac{\lambda}{nd}}$ $\simeq 114.6\sqrt{\frac{1}{(L/\lambda)}}$ (deg)
Directivity	$\frac{2nd}{\lambda} = \frac{2L}{\lambda}$	$\frac{4nd}{\lambda} = \frac{4L}{\lambda}$	$1.789\left(\frac{4nd}{\lambda}\right) \simeq 1.789\left(\frac{4L}{\lambda}\right)$

→ Non Uniform Amplitude Distribution
Endfire: Bigger major lobe than Broadside for a given length.
With increase in directivity; beam width decreases. Endfire is an exception. → compared to broadside; Endfire has both → FNBW and directivity greater than Broadside.



Ques For the following scanarios; which case to be considered

~~Uniform~~ : ~~Adv~~:

→ Edge - Sidelobes → large ⇒ disadvantage

~~The first sidelobe~~

→ Antenna Synthesis. / Pattern Synthesis (J.D. Krauss)

Process of finding the source or array of sources that produces ~~the~~ a desired pattern as specified

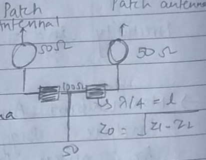
Primary pattern × Secondary pattern = Specified pattern.

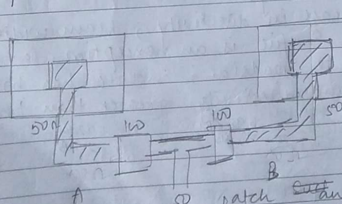
~~to~~ Quarter wave transformer

→ Microchip pattern antenna.

Quarter wave transformer.

Basically → array of antenna (patch form)





This way various patch ~~ant~~ antenna are connected together to form an array

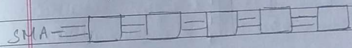
Now A and B → impedance matching. Because

50 (C) is divided (parallely) 100 on each side.

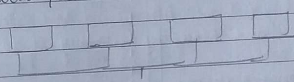
Quarter wave transformer → used for impedance matching.

→ ~~Series feed~~ Feeding Techniques

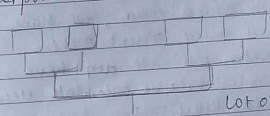
1. Series feed.



2. Parallel feed



3. Corporate

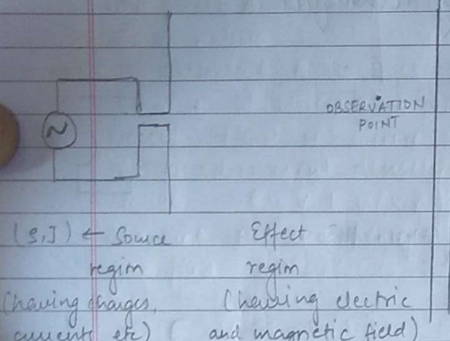


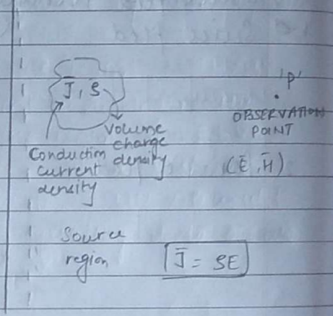
In this; the wave has to travel:

Lot of losses due to internal reflections → TIR, etc. less Gain. Therefore Shempheing technique used which converts 90° into: →

CHAPTER - THIN LINEAR ANTENNA AND RADIATION CHARACTERISTICS

Reference - 1) NPTEL lecture - 43 (Transmission line and EM waves)
2) K.D. Prasad.





We can calculate electric field in source regim because in source region; we have ~~current density~~ and source present there.
However at observation point we don't have anything which could help us calculate electric and magnetic field. Hence:
MOTIVE: We are required to establish a relation between parameters of source region and source free regim

Observation point regim → source free region → unbounded free space regim → no current and charges present.

The parameters which relate source region and source free ~~eq~~ regim's electric and magnetic field regim:
→ ~~Scalar~~ Electric scalar potential → ~~Vec~~ ~~Electric~~ Magnetic vector potential.

→ OBJECTIVES

1. Define the potentials which are consistent with the Maxwell's equation.
2. Find the solution for the potentials.
3. From potential, find the electric and magnetic fields at large distance
4. Calculate the power and the radiation parameters at observation point P due to source region.

→ MAXWELL EQUATIONS (represents inter relation between electric and magnetic field)

1) $\nabla \cdot \bar{B} = 0$

assumption ← $\bar{B} = (\nabla \times \bar{A})$ $\quad$ $\bar{A} \rightarrow$ magnetic vector potential.

$\nabla(\nabla \times \bar{A}) = 0 \longrightarrow (1)$

→ time derivative

2) $\nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t} = \dot{\bar{B}}$

$= -(\nabla \times \dot{\bar{A}})$

$\nabla \times \bar{E} = -(\nabla \times \dot{\bar{A}})$

$\nabla \times (\bar{E} + \dot{\bar{A}}) = 0$ $\quad \therefore \boxed{E + \dot{\bar{A}} = -\nabla V} \rightarrow$ Electric scalar potential.

-ve sign to satisfy and balance electrostatic property

3) $\nabla \times H = \bar{J} + \frac{\partial \bar{D}}{\partial t}$

$D = \cancel{\epsilon E}\, \epsilon E$

$\nabla \times H = \bar{J} + \epsilon \dot{\bar{E}}$

$\bar{B} = \mu \bar{H} = \nabla \times \bar{A}$

$\bar{H} = \frac{1}{\mu}(\nabla \times \bar{A}) \longrightarrow$ putting this in $\nabla \times \bar{H} = \bar{J} + \epsilon \dot{\bar{E}}$

$\therefore \nabla \times (\nabla \times \bar{A}) = \mu \bar{J} + \mu \epsilon \dot{\bar{E}}$

$$\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu \bar{J} + \mu\epsilon\left[-\nabla \dot{V} - \ddot{\bar{A}}\right]$$

$$\boxed{\nabla^2 \bar{A} - \mu\epsilon \ddot{\bar{A}} = -\mu \bar{J} + \mu\epsilon \nabla \dot{\bar{V}} + \nabla(\nabla \cdot \bar{A})}$$

$\nabla^2 \bar{A} - \mu\epsilon \ddot{\bar{A}} \longrightarrow$ represents wave equation / represent wave nature

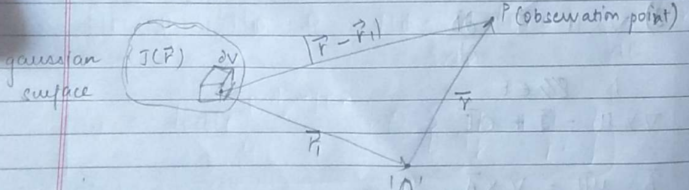
To calculate A in source free region; LHS = 0 because at observation point → nothing really present. Now in source free region; $\mu \bar{J} = 0$ ($\because J \to$ current density = 0). Thus if we make $\mu\epsilon\nabla \dot{\bar{V}} + \nabla(\nabla \cdot \bar{A}) = 0$; LHS ≠ 0 would become 0 and hence by equating it to 0; we can find A.

$$\mu\epsilon \nabla \dot{\bar{V}} + \nabla(\nabla \cdot \bar{A}) = 0$$

$$\mu\epsilon \nabla \dot{\bar{V}} = -\nabla(\nabla \cdot \bar{A})$$

$$\boxed{\mu\epsilon \dot{V} = -(\nabla \cdot \bar{A})} \rightarrow \text{Lorentz Gauge condition}$$

Gives inter relation between 2 potentials (V and A). If we know any one of the potential, other one can easily be found.



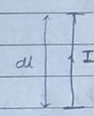
$$\bar{A} = \oint_V \mu J(\bar{r}') \frac{e^{-j\beta(\bar{r}-\bar{r}')}}{4\pi|\bar{r}-\bar{r}'|} \partial v'$$

A is found from this equation and kept in Lorentz Gauge condition to find V.

→ Analysis for small current element (Hertz Dipole)

Current moment = $I\bar{dl}$

'I' current is varying sinusoidally with respect to 't'.



$I = I_0 e^{j\omega t}$

$\therefore$ Current moment = $I_0 \bar{dl} e^{j\omega t}$

Now if we place the whole gaussian surface and the object at the origin; $\bar{r}_1 = 0$

Potential due to current element Idl at O' (origin)

$$A(r) = \frac{\mu e^{-j\beta r}}{4\pi r} \int J(r)\, dv$$

$$\int J(r)\, dv = I\bar{dl}$$

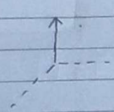
$$A(r) = \frac{\mu}{4\pi} I\bar{dl} \frac{e^{-j\beta r}}{r}$$

$$\boxed{\bar{A}(r) = \frac{\mu}{4\pi} I_0 \bar{dl} e^{j\omega t} \frac{e^{-j\beta r}}{r} \hat{z}}$$ ($\because$ direction of dl → positive z direction).

Antenna theory → Spherical coordinate system used always.

$$A_z(r) = \frac{\mu}{4\pi} I_0 dl e^{j\omega t} \frac{e^{-j\beta r}}{r} \hat{z}$$

In spherical coordinate system.



$\mu \bar{H} = \nabla \times \bar{A}$

$\bar{H} = \frac{1}{\mu} \{ \nabla \times \bar{A} \}$

$\bar{H} = \frac{1}{\mu} \left\{ \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ d/dr & d/d\theta & d/d\phi \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix} \right.$

$\frac{d}{d\phi}$, $r\sin\theta\, d\phi = 0$ $\because$ an element placed in a field; when observed from above on the ϕ plane, no change observed. Hence change with respect to $\phi = 0$.

$\bar{H} = \frac{1}{\mu} \left\{ \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ d/dr & d/d\theta & 0 \\ A_r & rA_\theta & 0 \end{vmatrix} \right.$

Solving this, we can conclude that only ϕ component exist

$\bar{H} = \frac{1}{\mu} \left\{ \frac{1}{r^2\sin\theta} \left[r\sin\theta\,\hat{\phi} \left[rA_\theta \frac{d}{dr} - A_r \frac{d}{d\theta} \right] \right] \right\}$

$H_r = 0 \quad H_\theta = 0 \quad H_\phi = \frac{d}{dr}(rA_\theta) - \frac{d}{d\theta}(A_r)$

$$\boxed{H_\phi = \frac{I_0 dl e^{j\omega t - j\beta r}}{4\pi} \sin\theta \left\{ \frac{j\beta}{r} + \frac{1}{r^2} \right\}}$$

$\nabla \times \bar{H} = j\omega\epsilon \bar{E}$

$\bar{E} = \frac{1}{j\omega\epsilon} (\nabla \times \bar{H})$

$= \frac{1}{j\omega\epsilon\,(r^2\sin\theta)} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ d/dr & d/d\theta & d/d\phi \\ 0 & 0 & H\phi \end{vmatrix}$

$E_r = \frac{I_0 dl \cos\theta\, e^{j\omega t - j\beta r}}{4\pi\omega\epsilon} \left\{ \frac{\beta}{r^2} - \frac{j}{r^3} \right\}$

$E_\theta = \frac{I_0 dl \sin\theta\, e^{j\omega t - j\beta r}}{4\pi\epsilon} \left\{ \frac{j\beta^2}{\omega r} + \frac{\beta}{\omega r^2} - \frac{j}{\omega r^3} \right\}$

E_r and E_θ $\rightarrow$ these fields are inversely proportional to r at different order:

$\propto \frac{1}{r}$ $\rightarrow$ radiation field component

$\propto \frac{1}{r^2}$ $\rightarrow$ induction field component

$\propto \frac{1}{r^3}$ $\rightarrow$ electrostatic field component.

E_r $\rightarrow$ available only in near field $\rightarrow$ available in very small area $\rightarrow$ disappears in far field.

Now $\beta = \omega\sqrt{\mu\epsilon}$

replacing $\beta = \omega\sqrt{\mu\epsilon}$ in equations:

Scanned by CamScanner