

Applications of antennas :-

Mobile	Sensor computer	Bluetooth
Satellites	Radar	RFID
TV	Sonar	Pager
Radio	Jammers	GPS
Wi-Fi	Wireless charging	NPC
Rovers	Radio telescope	Remote control

- Antennas is an interface b/w free space and cable (guided medium)
- It is a transducer → convert electric to EM wave and vice-versa.
- It is used for impedance matching b/w free space impedance (373Ω) and output impedance of system.

Types of Antennas :-

- ① Directional: radiates energy in a single direction
- ② Non-directional:
 - (i) Isotropic → radiates equally in all direction in all plane
 - (ii) Omni directional → radiates equally in one direction while does not radiates equally in other direction. Radiation pattern is in form of ideal apple.

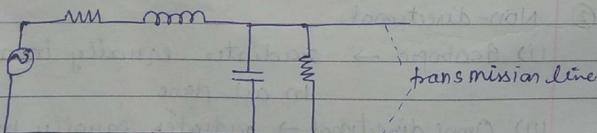
- Isotropic antenna does not exist
- Omni directional antenna exists :- ex - dipole antenna

Reciprocity theorem: Same antenna can be used as transmitting as well as receiving antenna.

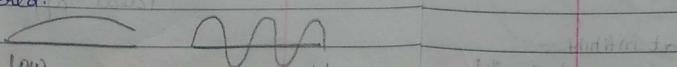
- Yagi antenna (earlier used as TV antenna \rightarrow directional) and loop antenna are mostly used for receiving but can also be used for transmission.

How does an antenna radiate ① $X_L = 2\pi f L$
 $X_C = 1/2\pi f C$

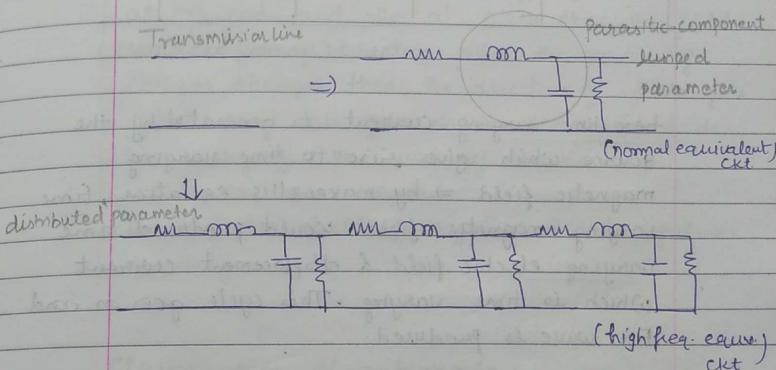
- at high frequency two parallel transmission lines will have equivalent ckt as



- this happens bcoz at high freq. X_L and X_C values are considered

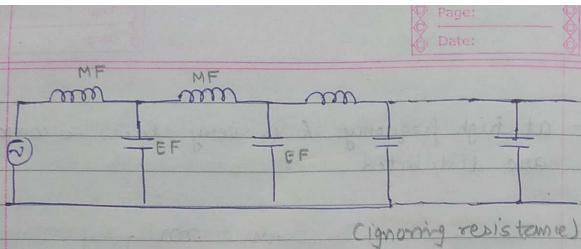


- at high frequency L is very low, so components are distributed.



The components gets distributed bcoz the phase of signal changes very rapidly at high freq. \Rightarrow the values of capacitors, inductors changes accordingly.

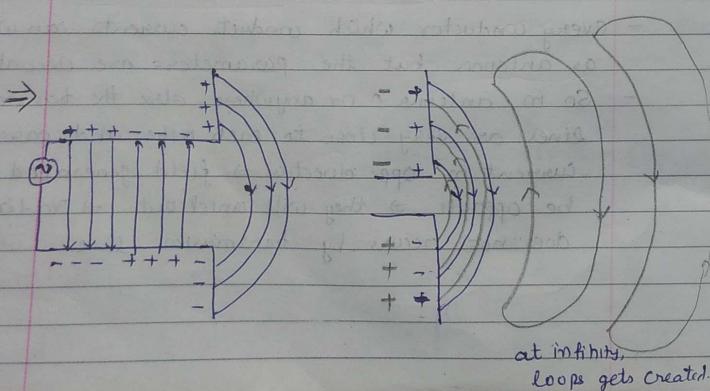
- every conductor which conducts currents can work as antenna, but the parameters are uncontrolled
- So in antenna or anywhere else the transmission lines are very close to each other and carry current in opp. direction \Rightarrow field generated will be opposite \Rightarrow they will cancel out \Rightarrow radiation does not occurs by transmission line.



here time varying current is generated by the source which gives rise to time varying magnetic field \Rightarrow by maxwell's equation, time varying magnetic field would produce time varying electric field & displacement current which is time varying. This cycle goes on and EM wave is produced.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{displacement current}$$



(Book's explanation)

In this, charges gets accumulate when the wire bends. So when positive charges gets accumulated at one end and -ve at other then the field would have a direction. When the position of charges changes then the direction of field gets opposite and this goes on as the current is time varying (produced by source)
 \Rightarrow loops are created

- The magnetic field propagates due to repulsion occurring.

Classification of antenna :-

is done on the basis of

i) frequency:

VLF, LF, HF, VHF

horn antenna: high freq. only if it's used at low freq. antennas sizes becomes very large

ii) polarization:

Polarization of antenna is in direction of electric field vector.

- Linear polarization can be in either E_x or E_y direction.

- Circular polarization is in direction of E_x and E_y direction. $E_x = E_y$

90° phase shift b/w E_x & E_y

Normalization: dividing all the values with max. value.

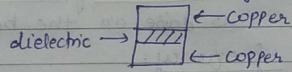
- propagation takes place in form of helical path
- types of CP → R HCP (clockwise)
LHCP (anti-clock)
- linear & circular polarization is the special case of elliptical polarization.

III) Radiations:
isotropic, omni-directional, directional
hemispherical: radiates in a hemisphere only.

IV) Aperture:
wired (monopole, dipole, loop & helical)

Reflector

Microstrip patch antenna: sandwich type.



Basic Antennas parameters:

Radiation Pattern: graphical or mathematical representation of particular coordinate system: spherical

- field pattern → pattern in terms of E or H field
- Power pattern → power vs sph. coordinate
- determined in far field region
- radiates property include
 - $0 < \theta < \infty$
 - $0 < \phi < \pi$
 - $0 < \psi < 2\pi$ (with x axis)

Azimuth & elevation plane:

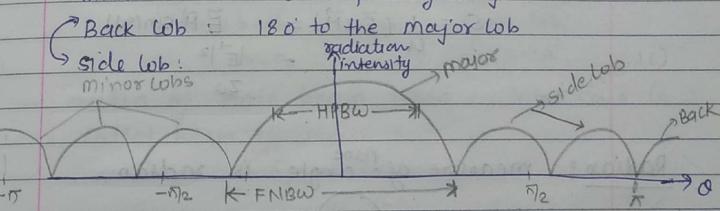
azimuth: horizontal (xz plane, $\theta = 90^\circ$)
elevation: vertical (yz plane, $\phi = 90^\circ$)

Principal pattern:

[E-plane
H-plane]

Radiation pattern lobes:

major lobes: which is in main (desired) direction
minor lobes: lob excepting major lob.



HPBW → half power beam width

- Unit is angle (degree or radian)
* $E_\theta(\theta, \phi)_m = 0.707$ (3dB down)

FNBW → first null beam width

- normalized field pattern:

$$E_\theta(\theta, \phi)_n = \frac{E_\theta(\theta, \phi)}{E_\theta(\theta, \phi)_{max}}$$

- pointing vector → power at a particular point per unit area

$$S = \frac{1}{4\pi} (\vec{E} \times \vec{H})$$

for $E(\theta)$ at half bw = 0.707 $HP_{BW} = 20 = 60 \text{ poynt}$
 $\cos\theta = 0.707$ for FNBW $E(\theta) = 0$ 6-Jan-2017

$$S(\theta, \phi) = \text{Poynting vector} \\ = E_0^2(\theta, \phi) + E_0^2(\theta, \psi) / Z_0$$

$Z_0 \rightarrow \text{intrinsic Impedance of space} = 376.7 \Omega$

power per unit area : $P_n(\theta, \phi) = \frac{S(\theta, \phi)}{S(\theta, \phi)_{\max}}$

Poynting vector gives the power per unit area and gives the direction of EM wave propagate

$$S = \frac{1}{\mu} (\vec{E} \times \vec{H}) = \vec{E} \cdot \vec{H} \sin(\theta) \\ = \frac{|\vec{E}|^2}{Z_0}$$

Radian : measure of plane angle is radian.



in circle 2π radians.

Steradian : in sphere 4π steradians.

Solid angle : $d\Omega = \frac{dA}{r^2} = \sin\theta d\phi d\theta$ (sr)

beam area : $\Omega_A = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} P_n(\theta, \phi) \sin\theta d\phi d\theta$

is the solid angle through which all of power radiated by the antenna.

By changing the size of antenna, side lobes will merge (dipole)

Date:

$$\Omega_A = \Theta HP \Phi_{HP}$$

radiation intensity : power radiated by antenna per unit solid angle called as rad. intensity.

normalised power ($\text{Unit} \rightarrow \text{Power/Sr}$)

$$P_n(\theta, \phi) = \frac{U(\theta, \phi)}{U(\theta, \phi)_{\max}}$$

Beam efficiency :

$$\Omega_A = \Omega_m + \Omega_m \\ \downarrow \quad \downarrow \\ \text{total beam area} \quad \text{major lobe area}$$

- ratio of main beam area to total area is called beam efficiency.

$$\epsilon_m = \frac{\Omega_m}{\Omega_A}$$

- ratio of minor beam area to total area is called stray factor.

$$\epsilon_m = \frac{\Omega_m}{\Omega_A}$$

$$\epsilon_m + \epsilon_m = 1$$

Directivity(D) : direction property of antenna
ratio of the max. power intensity to its avg. value over a sphere as observed in the far field of an antenna.

$$D = \frac{P(\theta, \phi)_{\max}}{P(\theta, \phi)_{\text{avg}}}$$

directivity is dimensionless ratio ≥ 1

$$\text{avg. power} = \frac{1}{4\pi} \int \int p(\theta, \psi) \sin\phi d\phi d\psi$$

$$D = \frac{4\pi}{\Omega_A} \rightarrow \text{beam area}$$

$$\therefore D = \frac{P_{\max}}{P_{\text{avg}}} = \frac{4\pi}{\int \int p(\theta, \psi) / P_{\max} d\Omega} = \frac{4\pi}{\Omega_A}$$

Normalized Power

- directivity is inversely proportional to the beam area.
- beam area ↓ so: ③ < ② < ①
- directivity is ↑ so: ③ > ② > ①



- pencil beam antennas whereas beam area is very less and directivity is very high.
- for isotropic antennas $\Omega_A = 4\pi S_r$; so $D = 1 \rightarrow$ lowest possible directivity
- Unit of D is dBi (decibels over isotropic i.e. wrt isotropic antenna)

Gain: Gain of an antenna is a quantity which is less than the D due to ohmic losses in the antenna.

- mismatch in feeding also reduces gain
antenna efficiency → ratio of gain to directivity is called antenna efficiency factor.
 $G_t = K D$ $K \rightarrow$ efficiency factor

Generator

VSWR

$\leq Z_L$

(Voltage

standing wave ratio) = 1

- Highest the VSWR \rightarrow reflected wave is there, some losses is there. it will occur due to mismatch of the impedance b/w transmission line and load.

antenna under test

$$\text{Gain} = \frac{P_{\max} (\text{AUT})}{P_{\max} (\text{ref.ant})} \times G (\text{ref.ant})$$

Antenna Apertures:

aperture of antenna is area that captures energy from a passing radio wave.

- physical aperture complete antenna
- effective aperture part of antenna which radiates

$$p = S A (\omega)$$

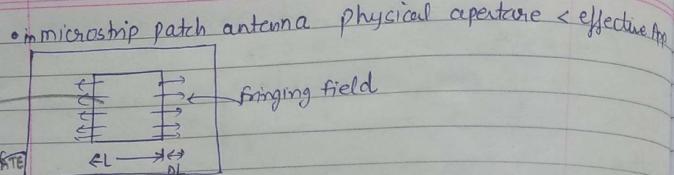
S pointing vector

effective aperture is not always equal to the physical aperture. Example is

$$\text{Aperture efficiency} = \frac{A_e}{A_p} \rightarrow \text{effective}$$

$A_p \rightarrow$ physical

reflector antenna and horn antenna



$$\text{Size of antenna} = L \quad (\text{physical aperture})$$

$$L + dL + dL \quad (\text{effective aperture})$$

$\Delta A \rightarrow$ beam area

$A_e \rightarrow$ effective aperture

$$P = \frac{E^2}{Z_0} A_e$$

(from slide)

and $D = \frac{4\pi}{\lambda^2} A_e$ (this constant, for a particular freq, antenna is designed so that λ is fix)

so directivity \propto effective aperture

Directivity of horn antenna) $>$ D of Patch antenna.)
(6170 is bigger than the patch) (size is less)

$$\left. \begin{aligned} D &= P(\theta, \phi)_{\max} \\ &\quad P(\theta, \phi)_{\text{avg.}} \\ D &= \frac{4\pi}{\Delta A} \\ D &= \frac{4\pi}{\lambda^2} A_e \end{aligned} \right\}$$

region of Antennas

$$\frac{2D^2}{\lambda} \rightarrow \text{largest dimension of an antenna}$$

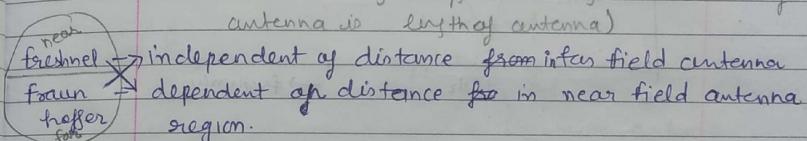
Effective Height $A_e = \frac{\hbar^2 Z_0}{4 R_s}$

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near and far field region is differentiated or decided by the $\left(\frac{2d^2}{\lambda} \right)$ mean field

where $d \rightarrow$ largest dimension of antenna
 $\lambda \rightarrow$ wavelength

if dipole antenna is there length or radius, which is large, of course largest dimension of antenna is length of antenna


 independent of distance from near field antenna dependent on distance for in near field antenna region.

So we have taken in far field region.

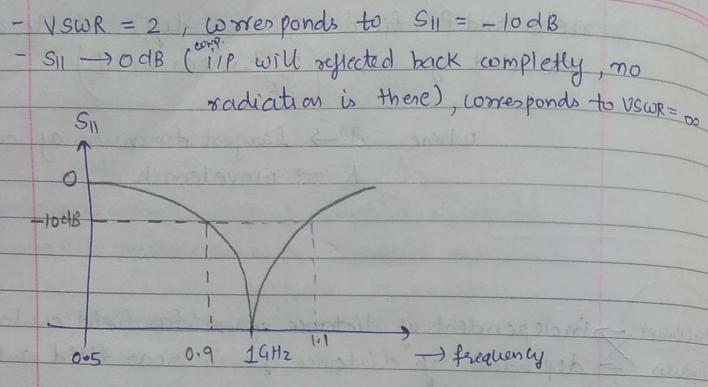
Return loss $S_{11} = -25 \text{ dB}, -5 \text{ dB}$

Scatter parameter is there and return loss is less reflection more reflection there loss is less as compare to -5 dB

at high freq. : when SC \rightarrow inductor when OC \rightarrow capacitor

In scatter parameter, we don't talk about the current & voltage. we only talk about the in term of power. \rightarrow so we used S-parameter.

- in antenna only $S_{11} \rightarrow$ parameter only there because only one part is there and $S_{11} \rightarrow$ represents the $1 \rightarrow 1$ i.e. 1 \rightarrow reflection.



- for example in horn antenna, $S_{11} = -20 \text{ dB}$, so reflection is less, so we are not allowed to reflection.
- Smaller the value of $S_{11} \rightarrow$ less reflection is occurs.
 - By ~~for~~ impedance matching, S_{11} will improve, for doing that antennas dimensions ^{will} changes.
 - as if we have designed the antenna for 1GHz but it radiates at 2GHz, so we have to decrease the frequency and wavelength will increases \rightarrow means distance between two antennas \uparrow .

Relation between effective height and A_e :

$$A_e = \frac{\pi r^2 Z_0}{4 R_r}$$

$R_r \rightarrow$ radiation resistance

(tells about the how our antenna radiates)

- Loss resistance \rightarrow how much losses occur in antennas.
- Higher the R_r , radiation by antenna is more higher the R_L , losses due to impedance mismatch, losses is higher in antenna.

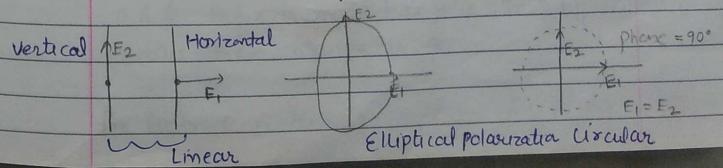
$$\text{efficiency} = \frac{R_r}{R_r + R_L}$$

free-space equations:

$$P_r = \frac{G_r G_t P_t}{4\pi d^2}$$

polarization:

Orientation of E-field vector of a wave



axial ratio = Major axis of polarization ellipse / minor

Polarization	AR	
Linear	∞	(bcz minor = 0)
Circular	1	major = minor
Elliptical	>1	major > minor

Signal to noise ratio \rightarrow ratio of signal fed to the m/w to the noise.

ratio of energy in main lobe to that in back lobe

Front to back ratio \rightarrow main lobe/major lobe
back lobe

* is the parameter of the only directional antenna only.

antenna temp \rightarrow fictitious temp at the r/r of an antenna which would account for noise AN at the MIP.

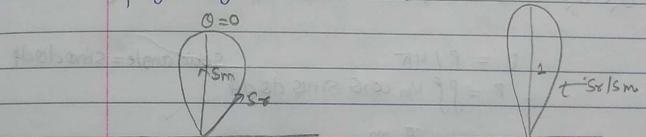
- We don't define the front to back ratio for the dipole antenna.

? numerical: JD Krauss — Online solution

= polarization of 2 antennas & their axial ratio questions.

chapter - point source and their array

- any antenna can be considered as point source, when b_{xz} field is very far we are taken.
- we fixed the observation point and antenna is moving or rotating (and vice-versa)
 - two types of
 - field
 - Power
 - Poynting vector (W/m^2)



making equals to 1, we do this $Sr/Smax \rightarrow$ normalization

Power theorem

$$P = \iint s ds$$

$$ds = r^2 \sin\theta d\theta d\phi$$

$$P = \iint s r ds$$

- $Sr \neq 0$ while $S\theta$ & $S\phi$ is equal to zero bcz it acts as equipotential surface so by changing θ & ϕ there is no change in Sr ; but by changing radius there change is Sr .

for isotropic antenna: $P = Sr \oint ds$

$$P = S_r (4\pi r^2)$$

$$S_r = \frac{P}{4\pi r^2}$$

$S_r \rightarrow$ radial charges
in Poynting vector

$$\text{radiation intensity } U = \frac{P}{4\pi} (\omega / S_r)$$

Example: A source has a cosine radiation intensity $U = U_m \cos \theta$, $0 < \theta < \pi/2$; $0 < \phi < 2\pi$. Find the Directivity?

Solution:

$$U = P / 4\pi$$

$$P = \int \int U_m \cos \theta \sin \theta d\theta d\phi$$

$$= U_m \int_0^{\pi/2} \int_0^{2\pi} \sin 2\theta d\theta d\phi$$

$$= U_m (2\pi) \left[\frac{\cos 2\theta}{2} \right]_0^{\pi/2} = U_m (\pi) \left(\frac{1+1}{2} \right)$$

$$P = U_m \pi$$

$$\text{directivity} = \frac{U_m}{U_0}$$

Power radiated by the
(isotropic) point source

$$\pi U_m = 4\pi U_0$$

Power radiated
by point
source

Power radiated by
isotropic

Date:

Date:

$$\frac{U_m}{U_0} = 4$$

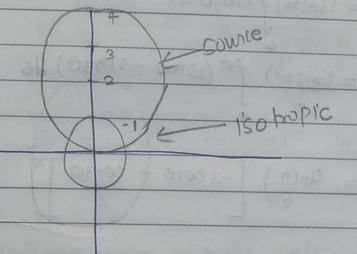
$$\text{So Directivity} = 4$$

Isotropic antenna having $D = 1$; $\theta = 0 \text{ to } \pi$

$$\phi = 0 \text{ to } 2\pi$$

but for point source $D = 4$; $\theta = 0 \text{ to } \pi/2$

$$\phi = 0 \text{ to } \pi$$



$$(i) U = U_m \sin \theta ; \quad 0 \leq \theta \leq \pi$$

$$P = \frac{U_m}{2} \int_0^{\pi} \int_0^{2\pi} \sin^2 \theta d\theta d\phi$$

$$= \frac{U_m}{2} \left[\frac{\cos 2\theta}{2} \right]_0^{\pi} (2\pi) = \frac{U_m}{2} (2\pi) \left(\frac{-1+1}{2} \right) = 0$$

$$= U_m (2\pi) \int_0^{\pi} \sin^2 \theta d\theta = \frac{U_m (2\pi)}{2} \int_0^{\pi} (1 - \cos 2\theta) d\theta$$

$$= U_m (\pi) \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\pi} = U_m \pi [\pi - 0] = U_m \pi^2$$

$$(ii) U = U_m \sin^2 \theta ; \quad 0 \leq \theta \leq \pi$$

$$0 < \phi \leq \pi$$

$$P = \frac{U_m}{2} \int_0^{\pi} \int_0^{\pi} \int_0^{\pi} \sin^2 \theta \sin \theta d\theta d\phi$$

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

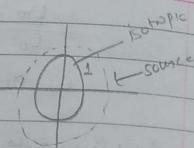
$$\sin \theta = 3 \sin \theta - \sin 3\theta$$

$$(ii) \quad \pi^2 U_m = 4\pi U_0$$

$$U_m = \frac{4}{\pi} U_0$$

$$D = \frac{4}{\pi}$$

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$$(iii) \quad P = U_m(\pi) \int_0^\pi \sin^3 \theta d\theta$$

$$= \frac{U_m(\pi)}{4} \int_0^\pi (3 \sin \theta - \sin 3\theta) d\theta$$

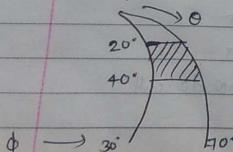
$$= U_m(\pi) \left[-3 \cos \theta + \frac{\cos 3\theta}{3} \right]_0^\pi$$

$$= U_m(\pi) \left[\frac{3-1}{3} - \left(\frac{-3+1}{3} \right) \right]$$

$$= U_m(\pi) \left[\frac{6-2}{3} \right]$$

$$= U_m(\pi) \left(\frac{16}{3 \times 2} \right)$$

Numerical: find the no. of square degree in the solid angle $d\Omega$ on the spherical surface.



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$$d\Omega = \sin \theta d\phi d\theta$$

$$\Omega = \int_{30^\circ}^{20^\circ} \int_{40^\circ}^{20^\circ} \sin \theta d\phi d\theta$$

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$$(iv) \quad (v) = [\cos \theta]_{20^\circ}^{40^\circ} (70^\circ - 30^\circ)$$

$$= 0.69 \times (0.17)$$

$$= 0.119 \text{ Sr} = 0.12 \text{ Sr}$$

$$1 \text{ Sr} = (1 \text{ rad})^2 = \left(\frac{180}{\pi} \right)^2 = 3282 (\text{deg})^2$$

$$\Omega = 393.84 (\text{deg})^2 \quad \text{Ans.}$$

Ex 8:- An antenna has a field pattern given by
 $E(\theta) = \cos^2 \theta ; 0 \leq \theta \leq 90^\circ$ find the beam area

Ans

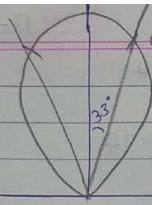
$$\Omega_A = \int \int P_n(\theta, \phi) \sin \theta d\theta d\phi$$

$$\boxed{\text{power pattern} = \frac{1}{2} \frac{(E(\theta))^2}{Z_0} \quad Z_0 = 377 \Omega \quad (\text{free space})}$$

$$= \int_{20^\circ}^{40^\circ} \int_0^{90^\circ} \cos^4 \theta \sin \theta d\phi d\theta$$

$$= 2\pi \int_{20^\circ}^{40^\circ} -t^4 dt \quad \text{let } \cos \theta = t \quad \sin \theta = -dt$$

$$= \frac{2\pi}{5} \left[\frac{t^5}{5} \right]_{20^\circ}^{40^\circ} = \frac{2\pi}{5} = 1.25 \text{ Sr} = 4102 (\text{deg})^2$$



0.707 (1,1)

Date: P = E²

$$\cos \theta = 0.707$$

$$\theta = 33^\circ$$

$$\Delta \approx \Theta_{HPBW}^{\circ} \times \Theta_{HPBW}^{\circ}$$

(E) (H)

$$P_d(\theta, \phi) \propto |E(\theta, \phi)|^2$$

$$= \frac{g^2 P_d(\theta, \phi)}{(P_{rad}/4\pi)}$$

$$= \frac{U(\theta, \phi)}{(P_{rad}/4\pi)}$$

$$\eta \% = \frac{P_{rad}}{P_{in}} \times 100$$

$$i) D = \frac{U_{max}}{P_{rad}/4\pi} = \frac{0.5 \text{ W/Sr}}{0.34/4\pi} = 18.47$$

$$ii) P_{rad} = 0.5 \times 0.4 = 0.38$$

$$D = \frac{0.5 \times 4\pi}{0.38} = 16.53$$

Ex8- The loss resistance of antenna is 25Ω . Calculate its radiation resistance Power gain = 30, D = 42

$$G = K D \Rightarrow K = \frac{30}{42} = 0.712$$

$$K = \frac{R_{rad.}}{R_{rad} + R_{loss}}$$

$$0.712 = \frac{R_{rad.}}{R_{rad} + 25}$$

$$17.85 = R_{rad} (1 - 0.712)$$

$$R_{rad.} = 61.80 \Omega$$

Ex8- Radiation intensity = $6 \cos \theta$, find D? Pattern is Unidirectional

- Current distribution along the antenna is ~~not~~ constant then antennas do not radiate.

* if Hertzian dipole antenna is then

$$P_d(\theta, \phi) \approx E^2(\theta, \phi)$$

(bcz current is constant)

But other than this antenna only in Linear dipole

$$P_d(\theta, \phi) \approx E^2(\theta, \phi)$$

\rightarrow We have to consider this

Ex8- Radiation efficiency for certain antenna is 95%. The max. radiation intensity 0.5 W/Sr .

Calculate the Directivity of antenna if

$$i) \text{ I/P Power} = 0.4 \text{ W}$$

$$ii) \text{ Radiated Power} = 0.34 \text{ W}$$

Sol8

$$D = \frac{P_d(\theta, \phi)}{P_{avg.}} = \frac{U_{max.}}{U_{avg.}} = \frac{U_{max.}}{P_{rad}/4\pi}$$

$$= \frac{P_d(\theta, \phi)}{(P_{rad}/4\pi r^2)}$$

$$U = \frac{P}{4\pi} \quad \text{if } U_m = 6$$

$$\begin{aligned} P &= \int \int U_m \cos \theta \sin \phi d\theta d\phi \\ &= \int \int 6 \cos \theta \sin \phi d\theta d\phi \\ &= 3 \int_0^{\pi} \int_0^{2\pi} \sin(2\theta) d\theta d\phi \\ &= 3(2\pi) [(-\cos 2\theta)]_0^{\pi} \\ &= 3\pi [+1 +1] = 6\pi \\ P &= 6\pi \approx U_m \times \pi \end{aligned}$$

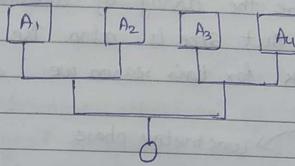
$$\begin{aligned} 6\pi U_m &= 4\pi y_0 \\ U_m &= y_0 \\ y_0 &= \frac{D}{3} \end{aligned}$$

$$U_m \times \pi = 4\pi y_0$$

$$D = 4$$

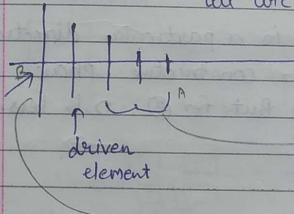
ARRAY OF ANTENNA :

group of antennas which are having different amplitude and phase → array of antenna



- all antennas are in physical contact.
- all antennas are fed by only single source then it called array.

- ~~yagi~~ antenna is parasitic antenna. In this all are not in physical contact.



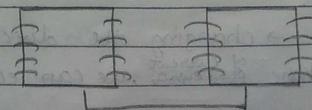
(it depends on the host means ① depends on only ②)

- array of antenna get the excitation through a single source.

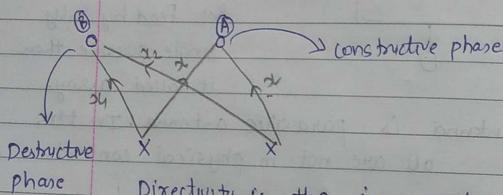
- disadvantage of array is sidelobes.

Advantages:

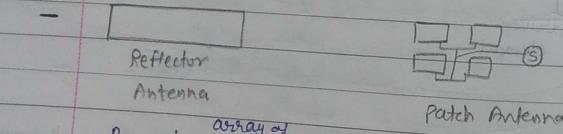
- ① D or A_e (effective aperture)
for example: patch antenna



In patch antenna as effective aperture is increases then directivity is increases. So for increasing the directivity, we used array of antenna. For reflector ^{antenna} we know that AE is higher, so we don't require the array of antenna for increasing the directivity, But size is also Tse that is the draw back for this reason we don't used it.



Directivity is the in a particular direction max power is radiated. For constructive phase, D is T_{Ser} in a direction of (A). But for (B), D is less.



By using ^{array of} patch antennas D is T_{Ser} and area is also less as compare to single reflector antenna. D is increases, beam area is decreases.

(2) D is T_{Ser} then gain is also T_{Ser}
 $G = K D$

(3) Beam steering \rightarrow changing the direction of beam. By changing the phase of current, we can easily

change the beam direction.

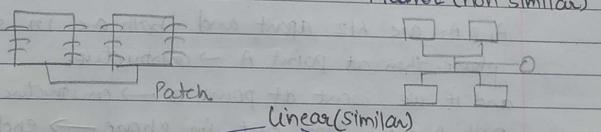
Phase Diff = 0

Phase Diff $\neq 0$

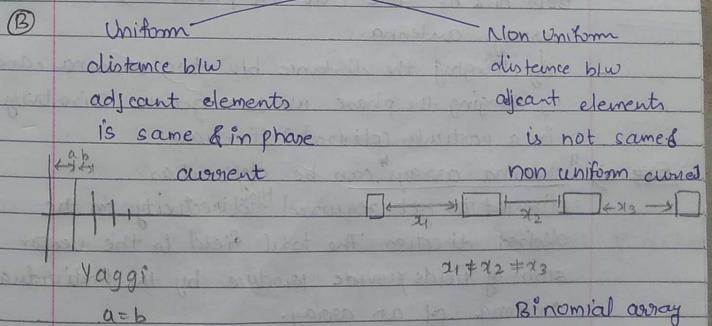
- (4) Determine direction of receiving signal.
- (5) Signal interference and noise ratio (SNR T_{Ser})

Classification of array of antennas:

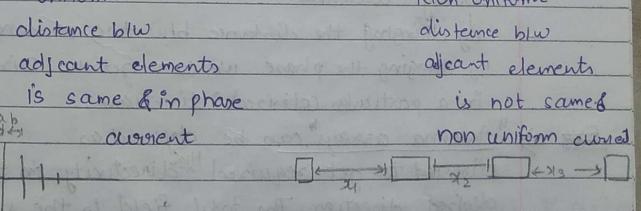
(A) linear



Planar (non similar)

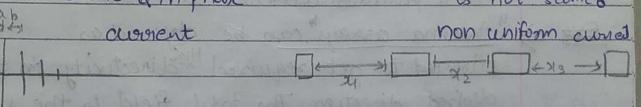


(B) Uniform



Non Uniform
distance b/w adjacent elements

adjacent elements is same & in phase



Yaggi
 $x_1 + x_2 + x_3$
 $a=b$

Binomial array

In dipole antenna can have the sidelobes, when size of antenna we are changing.

linear array

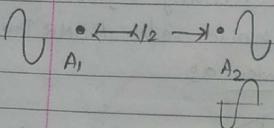
Broad side End fire Collinear Parasitic

Broadside:

A
constructive

D

destructive



A_1 & A_2 are $\lambda/2$ apart and both are in same phase. Then at point A \rightarrow constructive. and if we went at point B \rightarrow constructive then Both A_1 & A_2 are out of phase \rightarrow end fire array antenna.

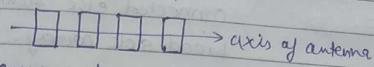
By changing the distance b/w two antenna, and By changing the phase, we can get the Directivity in a particular desired direction.

"Antenna arrays" can be defined as directed to get required directivity in the desired direction. The total field is the vector sum of fields produced by the individual antenna of an array.

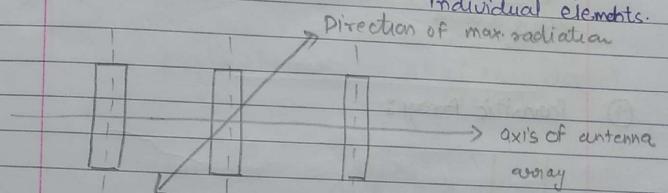
for ex: if phase shift is $\pi/3$ then what is distance b/w sources: $\frac{\pi}{\lambda/2} = h/2$
 $\frac{\pi}{\lambda/2} \times \frac{\pi}{3} = h/6$

Types:

(1) Broadside:



- same elements
- equal spacing
- in-phase current
- direction of max. radiation = normal to both axis of antenna array & axis of individual elements.



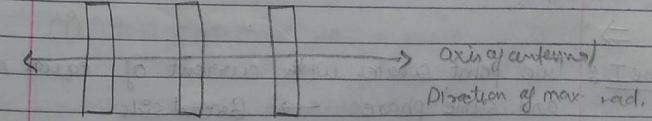
(2) End fire:

- same elements

- equal spacing

- out of phase current

- Direction of max. rad. is along the axis of antenna arrays.

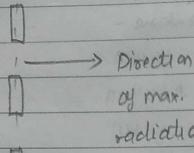


material for antenna is copper
 Density & conductivity → both factors are we have to consider

③ Colinear arrays

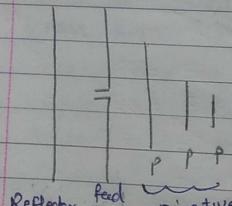


- Antennas are end-to-end connected, else same as broad side array.
- not used mostly.



axis of antenna

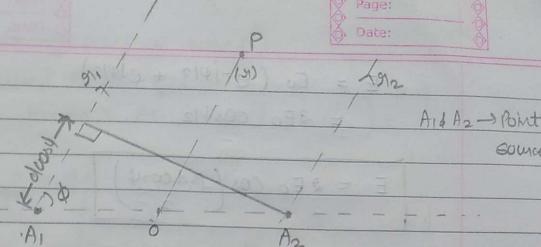
④ Parasitic Array:



Reflector feed
 for a spacing b/w driven element & parasitic elements
 If it is $\lambda/4$ and phase diff by $\pi/2$ rad then
 Unidirectional radiation pattern is achieved.



Case I: Two point sources with current of equal mag and same phase → Broad side



if 'p' is at large distance $\delta_1 = \delta_2 = \delta$

$$\text{path difference} = d \cos\phi$$

$$\text{path difference in terms of } h = \frac{d \cos\phi}{h}$$

$$\text{phase change} = 2\pi \times (\text{phase change})$$

$$= \frac{2\pi (d \cos\phi)}{h}$$

$$\beta = 2\pi/h \rightarrow \text{wave no.}$$

$$\text{phase change}/\text{shift } \psi = \beta d \cos\phi$$

ψ is the phase change b/w point source A1 & A2 and Point P.

E_1 → is the electric field at point P due to point source A1

E_2 → due to A2

(i) taking O point as reference

$$E_1 = E_0 e^{-j\psi/2}$$

$$E_2 = E_0 e^{+j\psi/2}$$

$$E = E_0 (e^{-j\psi/2} + e^{j\psi/2})$$

$$= 2E_0 \cos(\frac{\psi}{2})$$

$$E = 2E_0 \cos\left(\frac{\beta d \cos\phi}{2}\right)$$

- When $\phi = 90^\circ$, $E = 2E_0 \rightarrow$ max. field at point P
- When $\phi = 0^\circ$, $d = \lambda/2$, $E = 0 \rightarrow$ min.

$$\text{Array factor} = \frac{E}{|E_{\max}|}$$

Case (ii) taking A₁ point as reference

$$E_1 = E_0$$

$$E_2 = E_0 e^{j\psi}$$

$$E = E_0 (1 + e^{j\psi})$$

$$= E_0 (\cos\phi + j\sin\phi)$$

$$= 2E_0 \cos$$

$$= E_0 e^{j\psi/2} [e^{-j\psi/2} + e^{j\psi/2}]$$

$$E = 2E_0 e^{j\psi/2} \cos(\psi/2)$$

Variation of amplitude of electric field

Variation of phase if reference is A₁

Case I: (it is the part of case I)

Normalized E we get

$$E_{\text{norm}} = AF = \cos\left(\frac{\beta d \cos\phi}{2}\right)$$

$$\text{if } \beta = 2\pi; \beta d = \lambda/2$$

$$E_{\text{norm}} = \cos\left(\frac{\pi \cos\phi}{2}\right)$$

i) max. points

$$E_{\max} = \cos\left(\frac{\pi}{2} \cos\phi\right) = \pm 1$$

$$\frac{\pi}{2} \cos\phi = +n\pi$$

$$\therefore n = 0, 1, 2, \dots$$

$$\text{take } n=0; \frac{\pi}{2} \cos\phi = 0$$

$$\therefore \phi = 90^\circ, 270^\circ$$

ii) min. points

$$E_{\min} = \cos\left(\frac{\pi}{2} \cos\phi\right) = 0$$

$$\left. \begin{array}{l} \frac{\pi}{2} \cos\phi = \frac{n\pi}{2} \\ \text{take } n=1; \cos\phi = 0 \end{array} \right\} \begin{array}{l} m=1, 3, 5, \dots \\ \therefore \phi = 0, \pi \end{array}$$

$$\frac{\pi}{2} \cos\phi = \pm(2n+1)\pi$$

$$\text{for } n=0; \cos\phi = \pm 1$$

$$\therefore \phi = 0, \pi$$

(vii) for half power points:

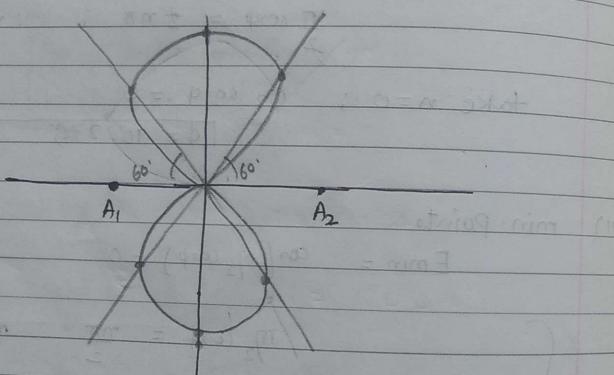
$$E_{half} = \pm \frac{E}{\sqrt{2}}$$

$$\cos\left(\frac{\pi}{2}\cos\phi\right) = \pm \frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\phi = \pm \frac{(2n+1)\pi}{4}$$

$$\text{take } n=0 \quad \frac{\pi}{2}\cos\phi = \pm \frac{\pi}{4}$$

$$\phi = 60^\circ / 120^\circ$$



Case II: Two point sources with current of same magnitude and opposite phase. (out of phase)

$$E = -E_0 e^{j\psi/2} + E_0 e^{-j\psi/2}$$

$$= E_0 [-e^{j\psi/2} + e^{-j\psi/2}]$$

$$= -2E_0 j \sin(\psi/2)$$

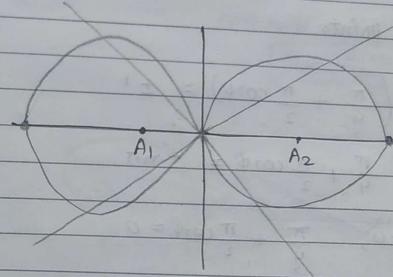
Ignored

normalised field

$$E = \sin\left(\frac{\beta d \cos\phi}{2}\right)$$

$$\text{if } d = h/2; \beta = 2\pi/h$$

$$E = \sin\left(\frac{\pi}{2}\cos\phi\right)$$



Case III: two isotropic point sources of same amplitude and in phase Quadrature

$$E = E_0 e^{j[\psi/2 + \pi/4]} + E_0 e^{-j[\psi/2 + \pi/4]}$$

Phase shift introduced by the point sources

Phase shift due to path diff.

dist. all = 100

angle 100 = 100

$$E = 2E_0 \cos\left(\frac{\psi}{2} + \frac{\pi}{4}\right)$$

$$E = 2E_0 \cos\left(\frac{\pi}{4} + \frac{\beta d \cos\phi}{2}\right)$$

$$\boxed{E_{\text{num}} = \cos\left(\frac{\pi}{4} + \frac{\pi}{2} \cos\phi\right)}$$

i) Max. points

$$\cos\left(\frac{\pi}{4} + \frac{\pi}{2} \cos\phi\right) = \pm 1$$

$$\frac{\pi}{4} + \frac{\pi}{2} \cos\phi = \pm n\pi$$

$$\text{for } n=0; \quad \frac{\pi}{4} + \frac{\pi}{2} \cos\phi = 0$$

$$\cos\phi = -1/2$$

$$\boxed{\phi = 120^\circ / 240^\circ}$$

ii) min points

$$\cos\left(\pi_4 + \pi_2 \cos\phi\right) = 0$$

$$\pi_4 + \pi_2 \cos\phi = \pm (2n+1)\pi_2$$

$$\text{for } n=0; \quad \pi_4 + \pi_2 \cos\phi = \pm \pi_2$$

$$1/2 + \cos\phi = \pm 1$$

$$\begin{cases} \cos\phi = 1/2, \\ \phi = 60^\circ / 300^\circ \end{cases} \rightarrow$$

iii)

Half power points

$$\cos(\pi_4 + \pi_2 \cos\phi) = \pm \frac{1}{\sqrt{2}}$$

$$\pi_4 + \pi_2 \cos\phi = \pm (2n+1)\pi_4$$

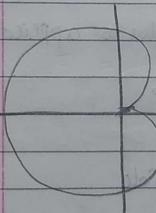
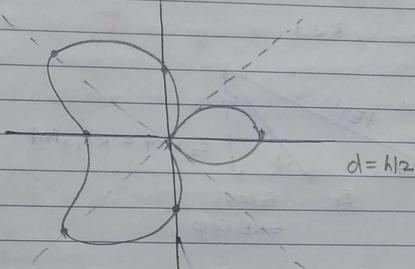
for $n=0$;

$$1 + \cos\phi = \pm 1$$

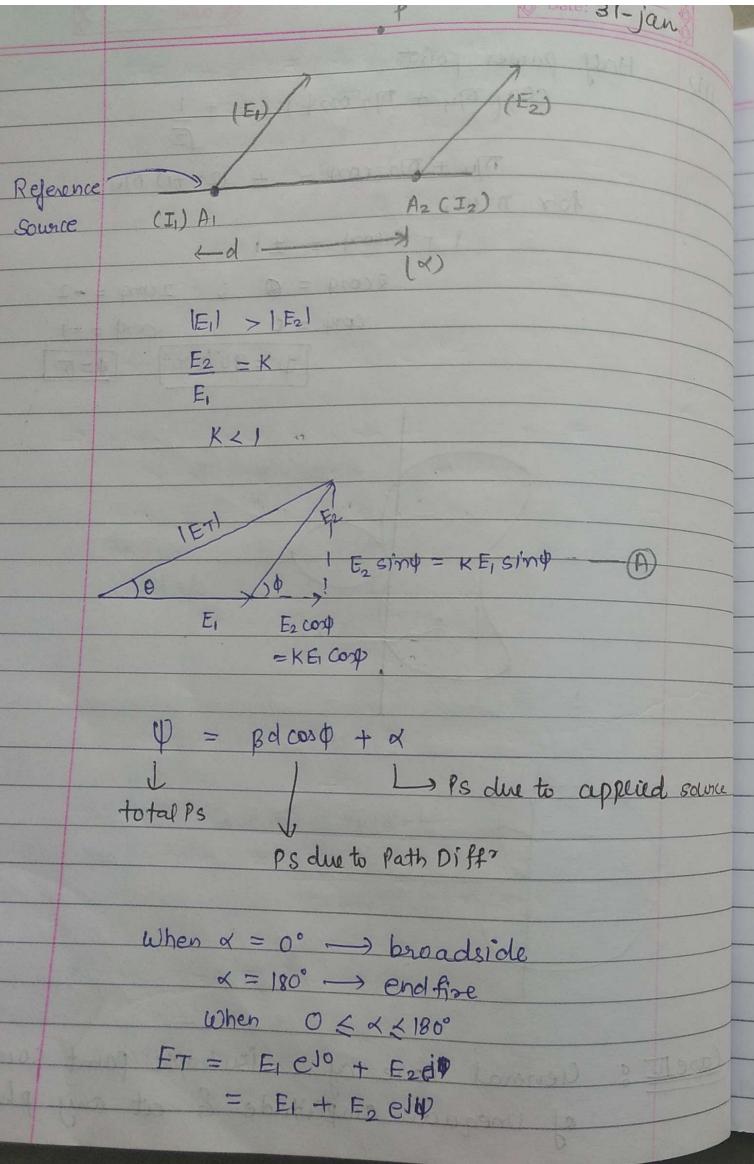
$$2\cos\phi = 0; \quad 2\cos\phi = -2$$

$$\cos\phi = 0; \quad \cos\phi = -1$$

$$\boxed{\phi = 90^\circ / 270^\circ} \quad \boxed{\phi = \pi}$$



Case 12: General case of 2-isotropic point sources of unequal amplitude & at any phase:



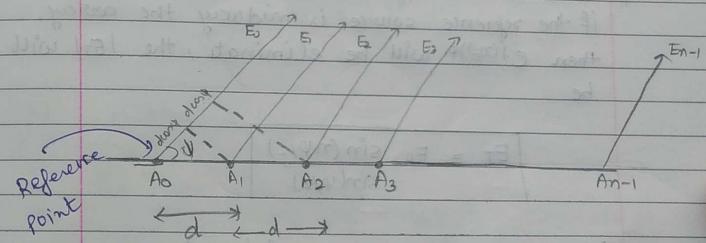
$$= E_1 \left(1 + \frac{E_2}{E_1} e^{j\psi} \right)$$

$$E_T = E_i (1 + \kappa e^{j\psi})$$

$$- \text{magnitude of } |E_T| = |E_1| \sqrt{(1 + K \cos \psi)^2 + (K \sin \psi)^2}$$

$$\text{angle } \theta = \tan^{-1} \left(\frac{K \sin \phi}{1 + K \cos \phi} \right) \quad (\text{from Dia.})$$

\Rightarrow n-elements uniform linear Array :



Uniform : $E_0 = E_1 = E_2 = \dots = E_{n-1}$

$$(c) ET = E_0 e^{j\phi} + E_1 e^{j\psi} + E_2 e^{j2\psi} + \dots$$

$$E_T = E_0 [1 + e^{j\psi} + e^{2j\psi} + e^{3j\psi} - e^{(n-1)j\psi}]$$

$\therefore \psi = \beta d \cos \phi$ (1)

$$e^{j\psi} ET = E_0 [e^{j\phi} + e^{2j\phi} + e^{3j\phi} + \dots + e^{nj\phi}]$$

$$\begin{aligned}
 E_T(1 - e^{j\psi}) &= E_0(1 - e^{jn\psi}) \\
 E_T &= E_0 \frac{(1 - e^{jn\psi})}{(1 - e^{j\psi})} \\
 &= E_0 \frac{e^{jn\psi/2}}{e^{j\psi/2}} \left[e^{-jn\psi/2} - e^{jn\psi/2} \right] \\
 E_T &= E_0 e^{j(n-1)\psi/2} \frac{\sin(n\psi/2)}{\sin(\psi/2)}
 \end{aligned}$$

When $n = 2$; $E_T = 2E_0 e^{j\psi/2} \cos(\psi/2)$ → broadside [Condition]

when we take reference at mid point if the reference source is midway the array, then $e^{j(n-1)\psi}$ will be eliminated, the $|E_T|$ will be

$$E_T = E_0 \frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

$$\text{for max value } \lim_{\psi \rightarrow 0} E_0 \frac{\sin(n\psi/2)}{\sin(\psi/2)} = nE_0$$

$$\text{array factor } \rightarrow E_{\text{max}} = \frac{|E_T|}{|E_{T\text{max}}|} = \frac{\sin(n\psi/2)}{n \cdot \sin(\psi/2)}$$

When $n=4$ (four sources)

$$\alpha = \pi/2$$

$$d = \lambda/2$$

→ condition for linear array of n -isotropic point sources with equal amplitude and phase α

a) Broad side array:

$$\alpha = 0$$

$$\begin{aligned}
 \Psi &= \beta d \cos\phi + \alpha \\
 \text{for max } \alpha &\stackrel{\text{radio}}{=} 0 \\
 \phi &= \pi/2, 3\pi/2
 \end{aligned}$$



b) end fire array:

$$\begin{aligned}
 \text{for max } \alpha &\stackrel{\text{radio}}{=} 0 \quad \text{along the axis so that } \phi = 0^\circ \\
 \beta d \cos\phi + \alpha &= 0 \quad \text{depends on spacing b/w sources} \\
 \alpha &= -\beta d \\
 \alpha &= -2\pi \times \frac{d}{\lambda} \quad d = \lambda/2; \alpha = -\pi \\
 \alpha &= -\pi/2 \quad d = \lambda/4; \alpha = -\pi/2
 \end{aligned}$$



when $d = \lambda/2$

$$\alpha = -2\pi/\lambda \times \lambda/2$$

$$\alpha = -\pi$$

Hazen woodward: end fire array with increase directivity $\alpha = -(\beta d + \pi/n)$