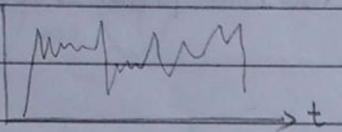
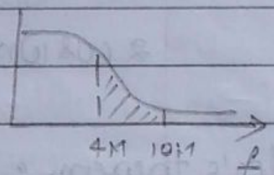
TRANFORM :

time domain Signal $\longrightarrow$ $\boxed{\text{Fourier transform}}$ $\longrightarrow$ Spectrum (freq. domain)

$$\Downarrow$$

$$\int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$$

Filtering Concept :

g(t)  g(f) 

if we have to need only till 4M spectrum, then we need Low Passfilter for that we have designed such a filter that is

$$g(t) \xleftrightarrow{IFT} \int_{0}^{2\pi f} g(\omega)\, e^{j\omega t}\, d\omega \quad \because f = 4M$$

$\Rightarrow$

Time Domain	Freq. Domain
CT	AP
P	DT
DT	P
AP	CP

DTFT $\longrightarrow$ FT $\longrightarrow$ FS

$$x(K) = \int_{-T/2}^{T/2} x(t)\, e^{-j\omega t k}\, dt$$

$\Downarrow$ $K = -\infty$ to ∞ $\longrightarrow$ Kernel Basis func

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

Analog [max. freq. $= \infty$; $t = 0 \rightarrow dc$

discrete [max. freq. $= \pi$

Discrete sinusoid concept :

(Analog) $\cos \Omega t$

↓

(discrete) $\cos(\Omega n T_s)$ put $t = nT_s$

$= \cos \omega n$

$\omega \triangleq \Omega T_s$

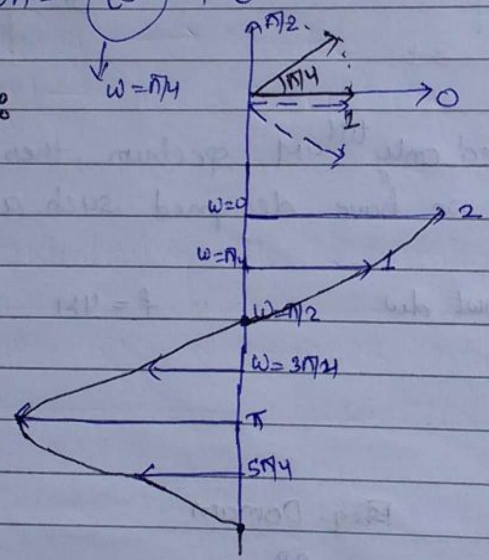
$\omega \rightarrow$ discrete freq. in rad. / sample

(how much phaser is covered rad. in per sample)

$2 \cos \omega n = e^{j\omega n} + e^{-j\omega n}$

$\omega = \pi/4$

Euler's theorem :



$\omega = \Omega \times \frac{1}{f_s}$; $T_s = 1/f_s$

discrete $\leftarrow$ $\boxed{\omega = \frac{2\pi F}{f_s}}$ $F \rightarrow$ continous signal freq. $f_s \rightarrow$ sampled freq.

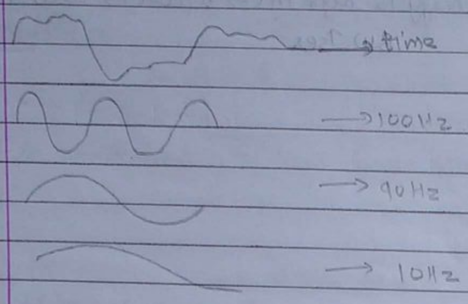
for max. discrete freq. $\omega_{max} = \pi$ $\therefore f_s = 2F$

$2\pi f = \frac{2\pi F}{F_s}$

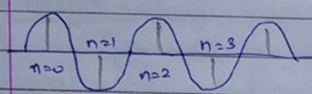
$$\boxed{f = \frac{F}{F_s}}$$

max. freq. is always π, no matter whatever be the max. F of a signal.

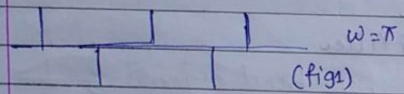
$F_{max} = 100Hz$



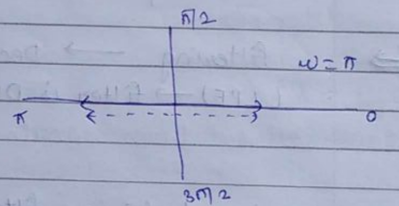
all are sampled with same frequency.

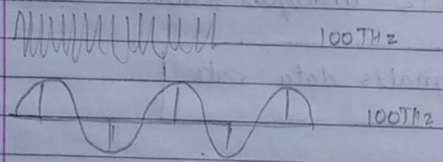


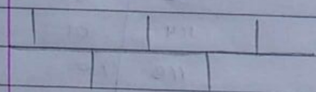
100Hz is sampled with 200Hz



(fig1)



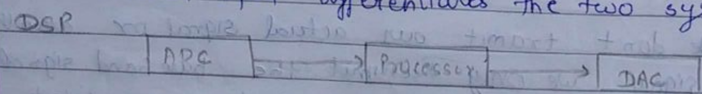




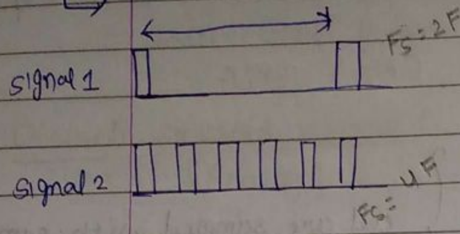
(fig2)

(fig1) & (fig2) are same for both signals 100Hz and 100THz but one thing that differs them is T_s value.

- Basic component of digital s/s is clock
  - ⇒ clock speed differentiates the two systems.

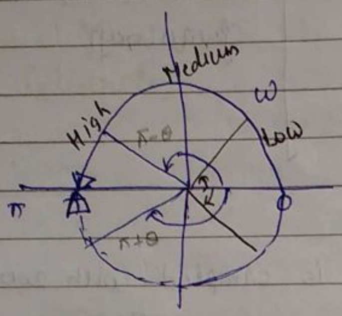


Date
Page



energy is high in (2) & cost is also high
BW ↑ses

⇒

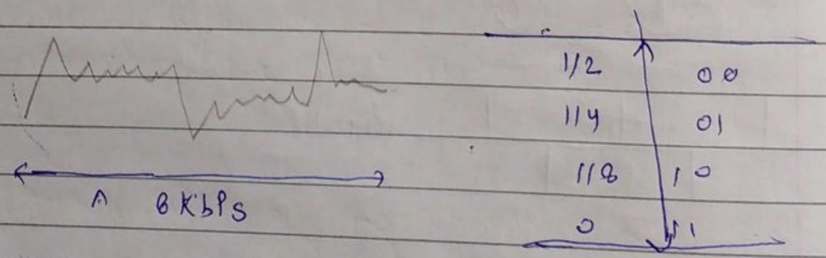


filtering ⟶ Decoder
(LPF) ⟶ filter is Digital filter
all real times signals are base band signal

and antialiasing filter is analogous filter. ⟶ sampling (CT)

$$\boxed{\text{Signal's B.W.} \propto \text{Signal's data rate}}$$

signal A:



- Signal spectrum is changed by the filter.
  minimize the bps ⟶ BW ↓es ⟶ cost ↓es

- We don't tramit our actual signal or another form of signal. we only transmit the base band signal with the help of filters. and also we detect our data back with the help of filter (base band filter). we only transmit the parameters of filters.

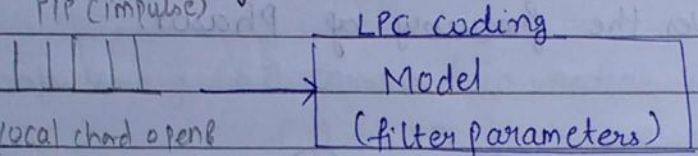
- In cellphones, for transmit the data, we don't transmit the any form of data but we only transmit the parmeters of filters.

Baseband

*

dc signal (constant) → S/W → impulse $\xrightarrow{FT}$ having all freq. compo
air
↓
required data ← By filters
(mouth cavity)

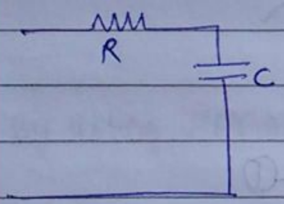
Linear Prediction Coding :



↳ Pitch (vocal chord open & close)

data rate 3 to 4 KbPS in this by filters parameters.

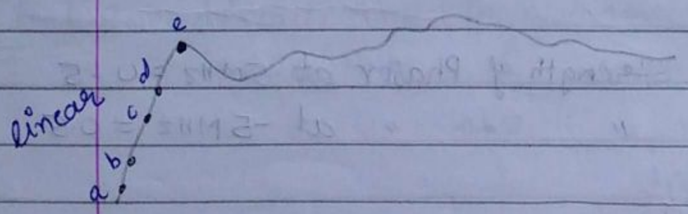
analog filter (Low Pass filter)



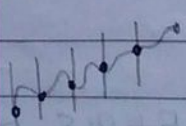
parameters are R & C

- But when we doing sampling, Quantization, encoder - - - then data rate is 64 KbPs which is very high as compare to (model method) Linear prediction coding.

- we predict the filter parameter in LPC.



$e = K_1 a + K_2 b + K_3 c + K_4 d$
(linear prediction)



By sampling we can predicted it.

- antialiasing filter is analog filter & LPF, HPF, BPF, are digital filters.

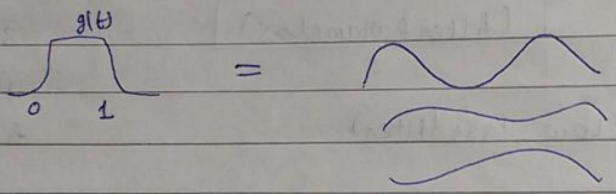
Transforms:

when we talk about transmission or signal processing then spectrum we talk about the spectrum.
We only consider on the spectrum

fourier transform: $G(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$

$G(\omega)$; its gives the frequency of Phasor.

as example:



$$g(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$$

Pos. Phase: $g(2\pi\times 10) = \int_{0}^{1} 1 \cdot e^{-j2\pi\times10t}\, dt$ —— ①

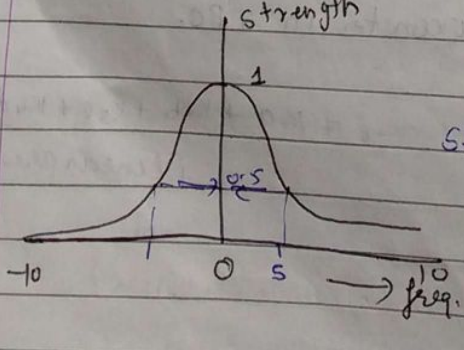
its gives the strength of Phasor at $f = 10Hz$

because $2\cos(2\pi\times10)t = e^{j(2\pi\times10)t} + e^{-j(2\pi\times10)t}$

nego Phase: $G(-2\times10\times\pi) = \int_{0}^{1} 1 \cdot e^{j2\pi\times10t}\, dt$ —— ②

its gives the strength of phasor at $f = -10$

$G(\omega) + G(-\omega) \rightarrow$ gives the strength of sinusoidal signal.

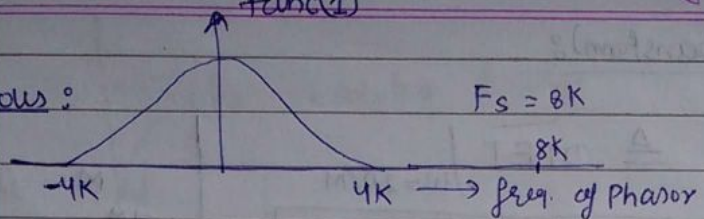


Strength of Phasor at 5MHz = 0.5
" " " at -5MHz = 0.5

Strength of sinusoidal Signal = 0.5 + 0.5 = 1.00

$\hookrightarrow$

continous :

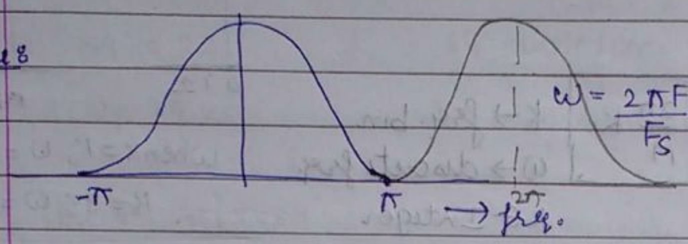


$F_s = 8K$

8K

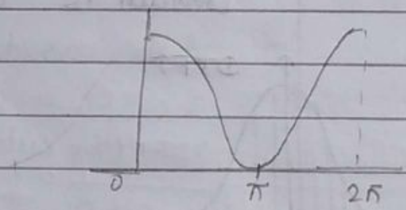
spectrum will remain same by doing sampling on the func(1). if its not then it may disorted.

discrete :



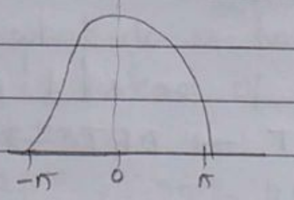
$\omega = \frac{2\pi F}{F_s}$

- By using fft command in matlab, we get fourier transform 0 to 2π
  (0 to 2π)



- By using FFT SHIFT

  (−π to π)



$* \; x(t - t_0) \rightarrow e^{j\omega t_0} X(f)$

phase change

magnitude same

$* \; x(t) \rightarrow X(f - f_0)$

- When we shifted the signal in time domain, the spectrum of phase also change / shift in freq. domain.

DTFT : discrete time fourier transform

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n}$$

discrete freq. : −π to π

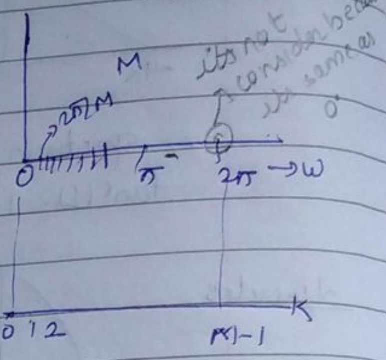
here discrete freq. are infinite. So by doing DFT, we can get finite frequencies

DFT (Discrete fourier transform):

$$DFT \triangleq DTFT \Big|_{\omega = 2\pi/M}$$

M-Point DFT ; $0 \leq K \leq M-1$

$$\boxed{x(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{M}Kn}}$$

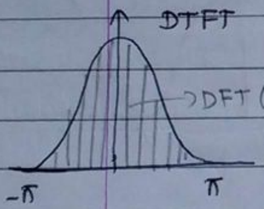


$$\boxed{\omega = \frac{2\pi}{M} K}$$

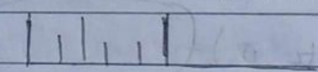
$K \rightarrow$ freq. bin
$\omega \rightarrow$ discrete freq.
$K \in$ Integer

when $K = 1$; $\omega = 2\pi/M$
$K = 2$; $\omega = \frac{2\pi}{M} \times 2$

— sampling in one domain is repeatation in another domain is occurs.



freq. domain
→ DFT (by cutting this → sampling)
inverse → in time domain we get $x(n)$; which is repeatative in time domain.



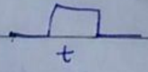
* $x(n) \rightarrow DTFT \rightarrow DFT \rightarrow x(n)$
↓ repeatation (aliasing effect)

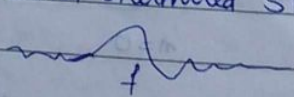
— ~~for~~ remove or overcome from the ~~Anti~~aliasing filter ;

$\boxed{M \geq N}$

how many parts between 0 to 2π ↓ in orignal signal, how much samples are there.

— time limited Signal → Band unlimited signal

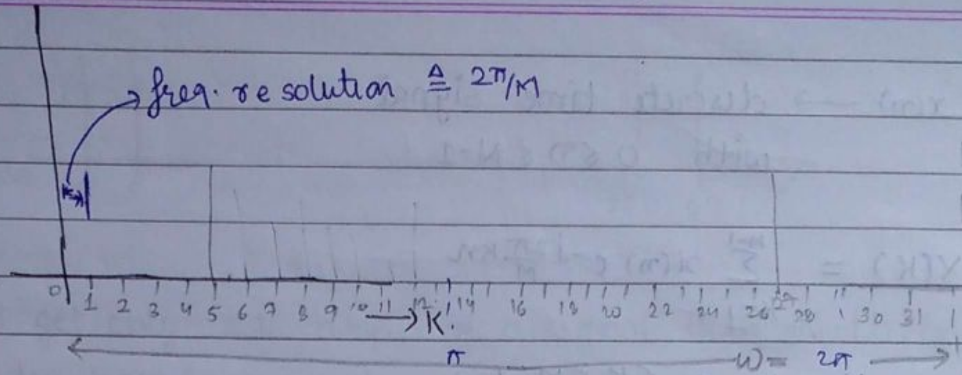




Example : Rahul samples the unknown signal at 64 sample / second. He selects 32 samples and computes 32 points DFT.

DFT is used to find the unknown frequency.

$K \rightarrow$ freq. bin



$M = 32$ $\qquad f_s = 64$ sample/sec $\qquad$ ~~For 32 sample Hz~~

$\omega_1 = \frac{2\pi}{32} \times 5 \qquad \omega_2 = \frac{2\pi}{32} \times 27$

$F_1 = \frac{2\pi \times 5 \times 64}{32 \times 2\pi} \qquad F_2 = \frac{2\pi}{32} \times 27 \times \frac{64}{2\pi}$

$= 10\,Hz \qquad\qquad = 54\,Hz$

max. discrete freq. $= \pi$; and range between 0 to π

only one ω_1 is satisfy $\omega = \frac{10\pi}{32}$;

analogous freq. $F = 10\,Hz$

Example which freq. bin, what do u expect have max. power. If the i/p was at 11 Hz instead of 10 Hz?

Sol:-

$F = 11\,Hz$

$\omega = \frac{2\pi F}{f_s} = \frac{2\pi \times 11}{64} = \frac{11\pi}{32}$

$\omega = \frac{2\pi}{M} \cdot k$

$\frac{11\pi}{32} = \frac{2\pi}{32} \cdot k$

$k = \frac{11}{2} = 5.5 \notin I$

* freq. resolution $\triangleq \frac{2\pi}{M} = \frac{2\pi}{32}$

* DFT $\longrightarrow$ 0 to 2π

Date: 26-July

$x(n) \longrightarrow$ discrete time signal
with $0 \leq n \leq N-1$

$$X(K) = \sum_{K=1}^{M-1} x(n)\, e^{-j\frac{2\pi}{M}Kn}$$

$K \rightarrow \quad 0 \leq K \leq M-1$

$N \rightarrow$ in time domain (how many samples)

$M \rightarrow$ in freq. domain (how many parts in DFT, 0 to 2π)

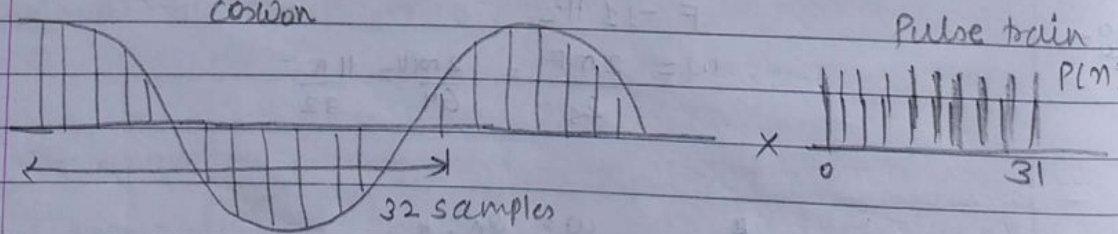
* K can never be fractional no. It should be integer always.

- DFT missed the spectrum, because DFT always takes the value of K as integer.
  the graph / spectrum acheived is 0 at all points.

Modulation property :

to get the spectrum, modulation property of fourier transform is used.

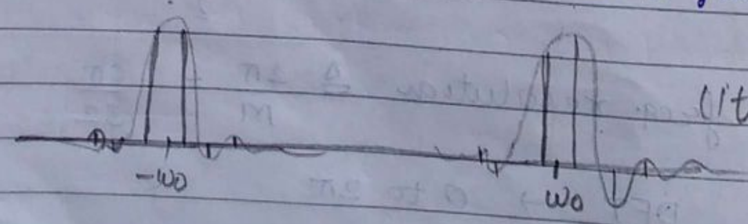
Modulation: $x(t) \cos w_c t \longleftrightarrow X(\omega + \omega_c) + X(\omega - \omega_c)$



32 samples = $[\cos W_0 n] \cdot P(n)$ = spectrum of ~~phasor~~ Pulse is shifted to at w_0 and $-w_0$ freq.

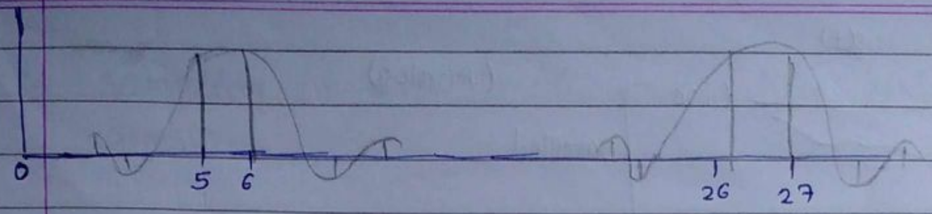
spectrum of pulse is sinc func

=



(It's continuous) but

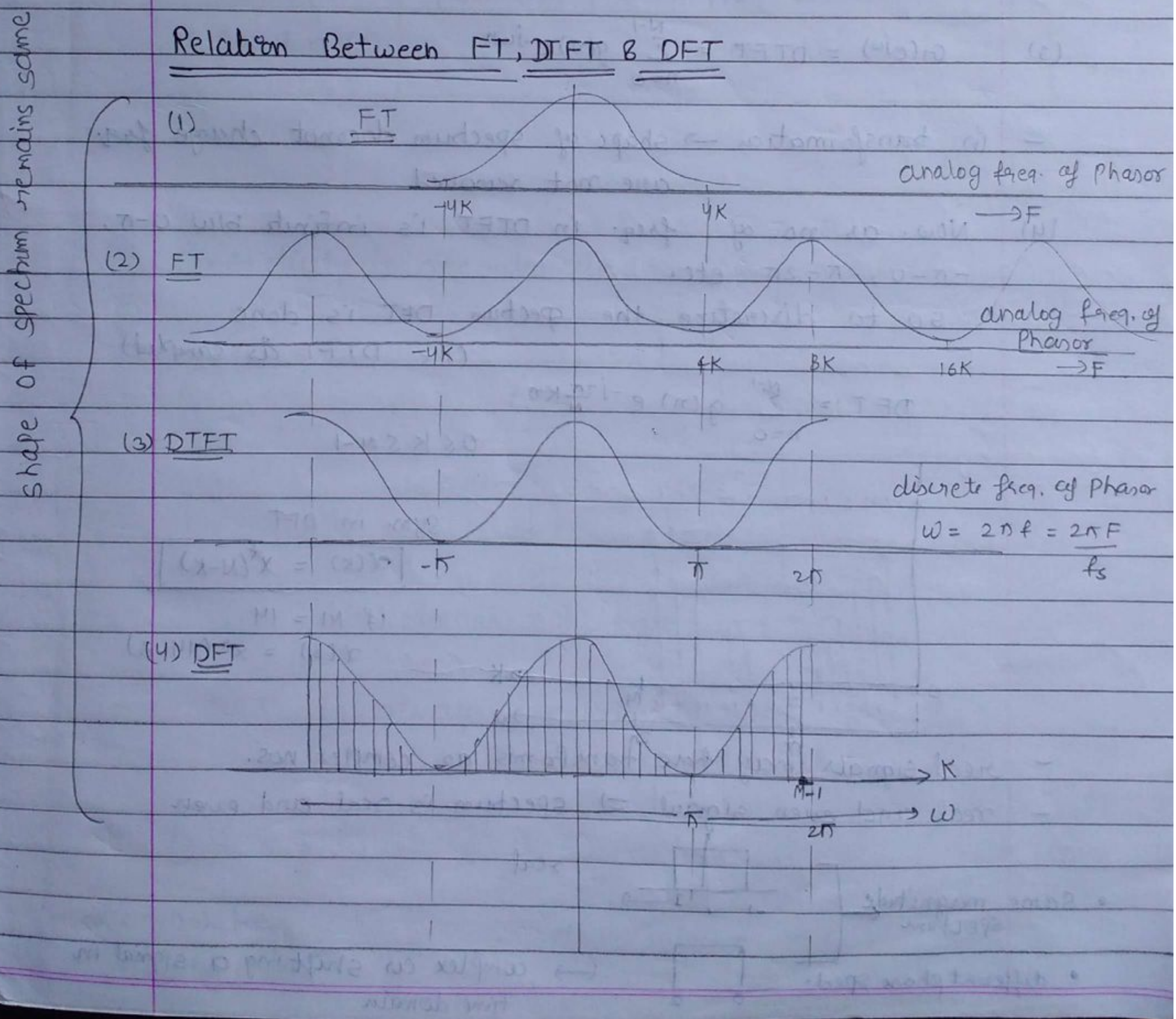
Date 27-july
Page



to get only 32 samples of a signal then multiply it with pulse train (32 samples 0 to 31)
⇒ 32 samples of signal is acheived.

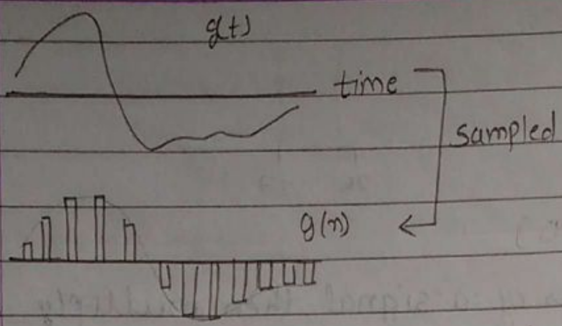
- The DFT of impulse train is discrete sinc. as we get max. peak at K = 5 & 6

Relation Between FT, DTFT & DFT



(1)

(2)



(Analog)

$g(n)\,/\,g(t)$ is quantized $\Rightarrow g(n)$ is acheived (discrete signal)

$$FT = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$$

(3) $$G(e^{j\omega}) = DTFT = \sum_{n=0}^{N-1} g(n)\, e^{-j\omega n}$$

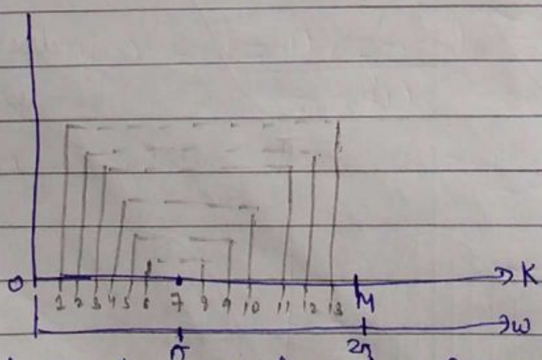
- in transformation $\rightarrow$ shape of spectrum doesnot change freq. are not removed

(4) Now, as no. of freq. in DTFT is infinite b/w $0-\pi$, $-\pi-0$, $\pi-2\pi$ etc.

so to discretize the spectrum DFT is done (i.e. DTFT is sampled)

$$DFT = \sum_{n=0}^{N-1} g(n)\, e^{-j\frac{2\pi}{M}kn}$$

$0 \leq K \leq M-1$



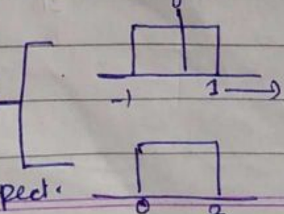
sym. in DFT

$\boxed{x(K) = X^*(N-K)}$

if $M = 14$

$x(8) = X^*(14-8)$

- real signals may have transforms as complex nos.
- real and even signal $\Rightarrow$ spectrum is real and even



real

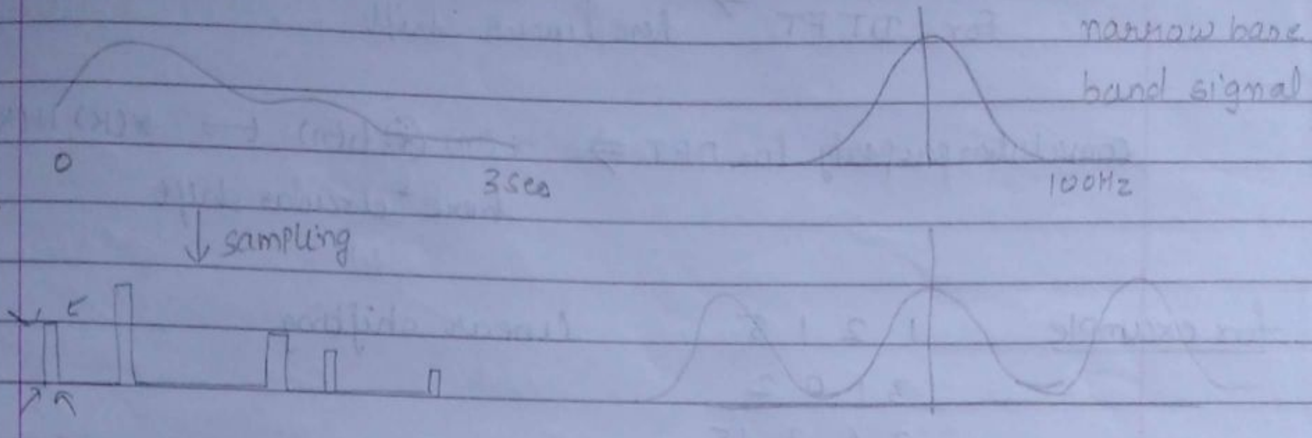
- Same magnitude spectrum
- different phase spect.

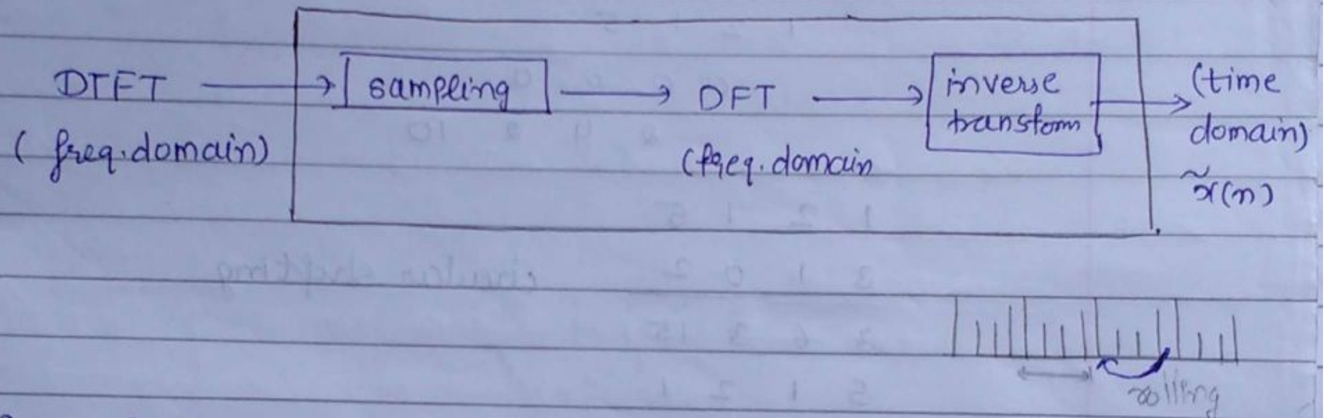
$\hookrightarrow$ complex as shifting a signal in time domain

Mo

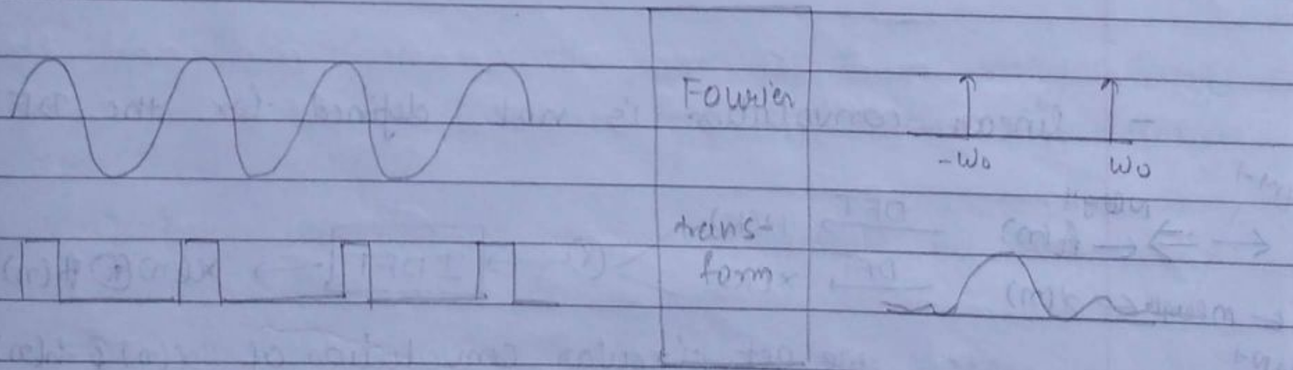
Modu

- sampling in one domain results into periodicity in other domain.



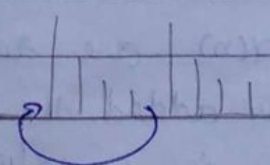


? Periodicity in one-domain results in sampling ?



its always not true.

- DFT is always periodic in both domain.
  - circular shift
  - in dFT not linear shift, it rotating shift, [which comes out that comes in]



$x(\langle n-n_0\rangle_N)$ → circular shift

$g(t-t_0) \longleftrightarrow G(\omega)\, e^{-j\omega t_0}$

$g(t-t_0)$ → linear shift; $G(\omega)\, e^{-j\omega t_0}$ → rotating / circular shift.

– $x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n) = x(n) * h(n)$

convolution property for DTFT $\Rightarrow$ $x(n) * h(n) \longleftrightarrow X(e^{j\omega})\, H(e^{j\omega})$
here linear shift

convolution property for DFT $\Rightarrow$ $x(n) \circledast h(n) \longleftrightarrow X(K)\, H(K)$
here circular shift

for example

1	2	1	5			
3	1	0	2			
3	6	3	15			
	1	2	1	5		
		0	0	0	0	
			2	4	2	10

linear shifting

1	2	1	5
3	1	0	2
3	6	3	15
5	1	2	1
0	0	0	0
10	2	4	2

circular shifting

– linear convolution is not defined for the DFT.

N+M-1

N length $\Rightarrow$ $h(n) \xrightarrow{DFT} H(K)$

M length $x(n) \xrightarrow{DFT} X(K)$ $\longrightarrow \otimes \longrightarrow \boxed{IDFT} \longrightarrow x(n) \circledast h(n) = y(n) = ?$

we get circular convolution of $x(n)$ & $h(n)$

but as we know that we always have linear convolution. so we have to change this circular to linear

M+N-1

$X(K)$ $H(K)$ $\rightarrow \otimes \rightarrow \boxed{IDFT} \rightarrow y(n)$

– same length of $h(n)$ & $x(n)$ are required for linear convolution. so we added the $n+m-1$

– O/P of any filter $(h(n))$ is convolution.

$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n) = x(n) * h(n)$ $\quad \Sigma\, x(K)\, h(n-K)$

Date 2-Aug.
Page

- By doing $\Sigma\, x(k)\, h(n-k)$, we also get the linear convolution but its requirment of multiplication and division is more as compare to DFT. So processor work will more in $\Sigma\, x(K)\, h(n-k)$.

Notation of Sanjit Mitra :

$$\omega_N = e^{-j2\pi/N}$$
$$\omega_N^4 = e^{-2\pi/N \cdot 4}$$

can be written in matrix form

DFT $\Rightarrow$ $x(K) = \sum_{n=0}^{M-1} x(n)\, e^{-j\frac{2\pi}{M}Kn}$ $\quad 0 \le K \le M-1$

IDFT $\Rightarrow$ $x(n) = \frac{1}{N} \sum_{K=0}^{M-1} x(K)\, e^{j\frac{2\pi}{M}Kn}$

$X = x \cdot D \Rightarrow D = \begin{bmatrix} \cdots \\ \omega_n \\ \omega_n^1 \\ \cdots \end{bmatrix}$

Inverse of processing speed of processor :

- in reality no. of points for DFT is very large so no. of operations of multiplication & addition etc. ↑ses
  - ⇒ time taken and power consumed by processor was more.
  - ⇒ we need to ↓se the time taken.

- $X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{M}Kn}$

for $N = 2$; $\boxed{M = 2}$ → min. M for N = 2 as $M \ge N$

$$X(K) = x(0)\, e^{-j\frac{2\pi}{M}K(0)} + x(1)\, e^{-j\frac{2\pi}{M}K(1)}$$

$$X(0) = x(0) + x(1)$$
$$X(1) = x(0) + x(1)\, e^{-j\pi}$$

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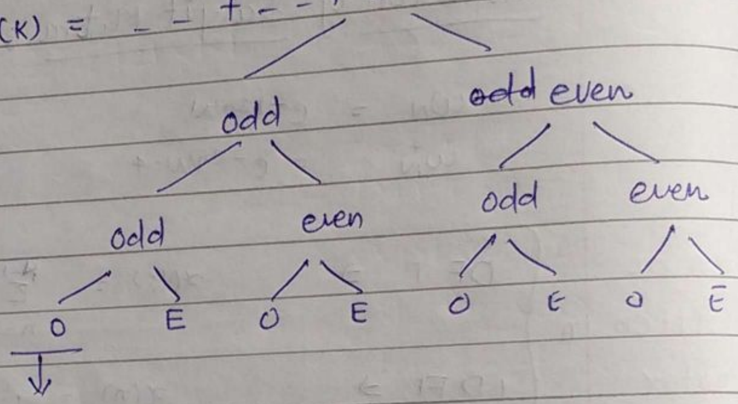
- DFT of 2 points is most simple. So the basic idea to decrease time was to convert 1000 samples to 2 samples and then computing.

1000 samples $\longrightarrow X(K) = \_ \_ + \_ \_ + \_ \_ + \_ \_ + \_ \_$



Here $X(K)$ is divided into many 2 samples pair (even & odd)

<u>for N-Points</u>

$$\left.\begin{array}{l} X(K) \longrightarrow 2N^2(\times) \longrightarrow N/2 \log_2 N \\ \qquad\quad\;\; 2N^2(+) \longrightarrow N \log_2 N \end{array}\right] - FFT$$

and if no. of samples are odd then a zero valued sample is added $\longrightarrow$ called as zero padding.

DTFT : $X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n}$

Z- transform

Date 3-Aug

* ⇒ multiplication of spectrum by $e^{-j\omega t_0}$ in freq. domain, this cause real spectrum will change into conjugate spectrum

***Ex:-** DTFT $X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n}$; what is the condition for $X(e^{j\omega})$ DTFT to exist ?

Sol: i.e. magnitude is finite for existence of DTFT

$$X(e^{j\omega}) < \infty$$

$$X(e^{j\omega}) = x(0)e^{-j\omega 0} + x(1)\, e^{-j\omega(1)} + x(2)\, e^{-j\omega(2)} + x(3)\, e^{-j\omega 3} \ldots$$

magnitude: $|X(e^{j\omega})| = |x(0)| + |x(1)| + |x(2)| + |x(3)| + \ldots$

So; $\boxed{\sum_{n=0}^{N-1} |x(n)| < \infty}$ — ① for existence of DTFT

$X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n}$ → how to make this convergence or absolute summable for existence of DTFT

- if we don't know about the s/s then how can we find out its response. By giving know i/P & see the o/P we can evaluate the s/s response.
- i/P signals :- Unit step , ramp

  but Unitstep signal is, $\sum_{n=0}^{N-1} |x(n)| < \infty$ not satisfy this

By doing z-transform, we can satisfy this eqn ①

z-transform:

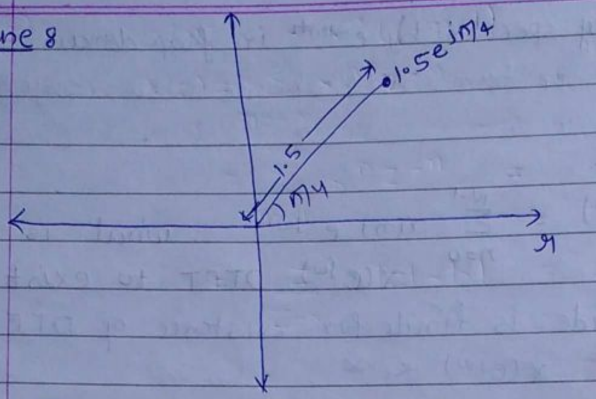
$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n} \longrightarrow \sum_{n=0}^{N-1} r^{-n}\, e^{-j\omega n}\, x(n)$$

added one series when $n \to \infty$, $r^{-n} \to 0$

$$= \sum_{n=0}^{N-1} x(n)\, (r e^{j\omega})^{-n}$$

$$\boxed{X(z) = \sum_{n=0}^{N-1} x(n)\, z^{-n}}$$ "z-transform" $z = r e^{j\omega}$

z-plane:



$z = r e^{j\omega}$

$= 1.5 e^{j\pi/4}$

To analyze, implementation the digital signal, z-transform is used.

* In laplace $s = \sigma + j\omega$

$\sigma = ?$

* By putting $r = 1$

z-transform = DTFT

ROC (Region of Convergence):

for what value of z, the z-transform exist?
for which value of z, transform converge is called ROC.

Example:

$$H(z) = \frac{(z-1)(z-0.5)}{z(z+1)} \begin{array}{l} \rightarrow \text{zeros} \\ \rightarrow \text{poles} \end{array}$$

H(z) is the s/s response of z-plane.

Date 5-Aug

concept of zero — sensors + 50Hz ($\omega = 2\pi F/f_s$) → for suppressed the 50Hz freq. → for this at this freq. $H(z) = 0$; zeros →
$z_1 = r e^{j\omega}$ → converted into z-plane →
$H(z) = (z - z_1)$

- Digital s/s design : if we want to suppress any freq. we find the 'z' of the freq., we keep at zero
The nearby surrounding freq. which is unwantedly suppressed is lifted by assigning poles at those frequencies.
- signals which are not absolutly summable, than we used z-transform.
- digital filter are designed, expressed, applied and represented in terms of z-transform.

Example (1) $x(n) = \alpha^n u(n)$

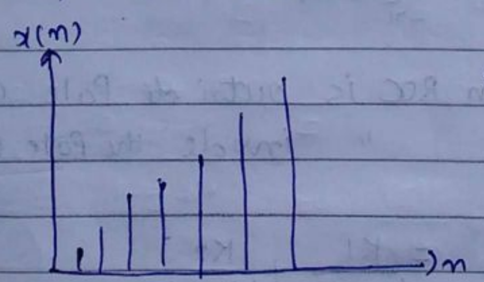
z-transform: $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$= \sum_{n=0}^{\infty} \alpha^n z^{-n} = \sum_{n=0}^{\infty} (\alpha z^{-1})^n = \frac{1}{1-\alpha z^{-1}}$

provided $|\alpha z^{-1}| < 1$

$\boxed{X(z) = \frac{z}{z-\alpha} \; ; \text{ if } |z| > |\alpha|}$ → ROC

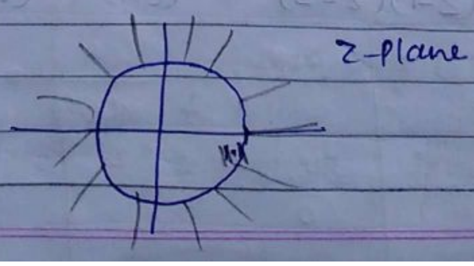
if $\alpha = 1.1$, $x(n) = (1.1)^n u(n)$



$X(z) = \frac{z}{z - 1.1}$; $|z| > 1.1$

ROC

ROC on z-plane



Ex:- ②

Sol:

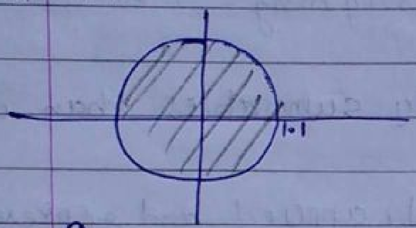
$$x(n) = -\alpha^n u(-n-1)$$

$$X(z) = -\sum_{n=-\infty}^{-1} \alpha^n z^{-n}$$

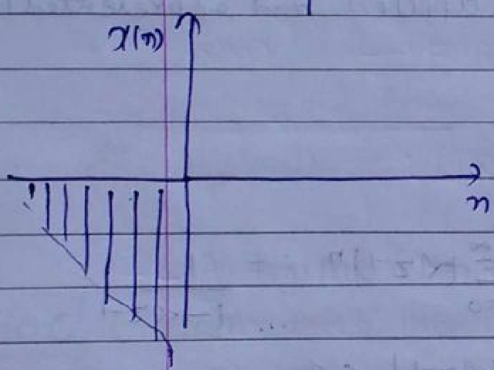
$$= \sum_{n=-1}^{\infty} \alpha^n z^{-n} =$$

$$= \frac{z}{z-\alpha} \; ; \; |z| < |\alpha|$$

if $\alpha = 1.1$



- DTFT of a sequence exists, if and only if the ROC includeds the Unit circle.
- existence of DTFT is not a guarantee for the existence of the z-transfom.



(Right to infinite Point)

- if the sequence is Right sided, then ROC is outside Pole circle. (casual)
- if the sequence is left sided, then ROC is inside of pole circle (Anti causal)

- if signal is causal, then ROC is outside Pole circle. ~~vice versa~~ " " anticasual, " " inside the Pole circle.

Ex:- $x(z) = \frac{1}{(z-1)(z-2)} = \frac{K_1}{(z-1)} + \frac{K_2}{(z-2)}$

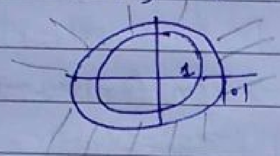
①	②	ROC
causal	causal	outside 2
causal	anticausal	between 1 & 2
anticausal	causal	0
anti ca..	anti causa..	inside 1

- for finite sequence $x(n) = [1, 2, 1.5, -1, 2, 0]$
  (_ _ _) z-transform is always convergence and z-transform is exist.

- for infinite sequence: $(-\infty, \infty)$ there is possibility that summision of all value will infinite, the z-transform is not exist.

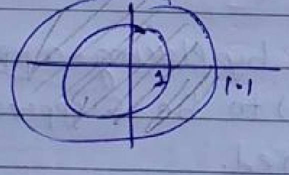
- $X(z)$ = we are having
  then if we have to find $DTFT = X(z)\big|_{z=e^{j\omega}}$ is finite then DTFT exist.

i) Unit circle $r = 1$; $z = e^{j\omega}$; $X(z) = \dfrac{z}{z-1.1}$; $|z| > 1.1$



DTFT is not exist

ii) $r = 1$; $z = e^{j\omega}$; $X(z) = \dfrac{z}{z-1.1}$; $|z| < 1.1$



DTFT is exist

DTFT ⟶ z-transform ⟶ ROC ⟶ Poles, zeros

⇒

$h(n)$, $H(\omega)$ ⟶ [LTI] ⟶

- $\sum |h(n)| < \infty$; for BIBO stability

  $H(\omega) = \sum h(n) e^{-j\omega n}$

* Unstable s/s (without giving any I/P, we get O/P that's Oscillator)

Date 16-Aug

- if we want that s/s is stable
  if $h(n)$ is stable $\longrightarrow$ impulse response summable

$\downarrow$

$H(\omega)$ exists

$\downarrow$

DTFT exists

$\downarrow$

Poles $\longleftarrow$ ROC must include unit circle

$h(n) \longrightarrow$ FIR (finite) $\longrightarrow x(n)$ finite $\rightarrow x(\omega)$ exists
$\longrightarrow X(z)$ exist

$\longrightarrow$ IIR (infinite)

- A digital filter is designed by placing appropriate no. of zeros at the freq. (z-value) to be suppressed, and poles at the freq. to be amplified.

Concept of Poles & Zeros :

$x(n) \longrightarrow \boxed{LTI} \longrightarrow y(n) = \frac{1}{2}\left(x(n) + x(n-1)\right)$

① in z-plane; $y(z) = \frac{1}{2}x(z) + \frac{z^{-1}}{2}x(z)$

$$2\frac{y(z)}{x(z)} = 1+z^{-1} = \frac{z+1}{z} \quad \text{—①}$$

(zeros = -1 & Poles at ori

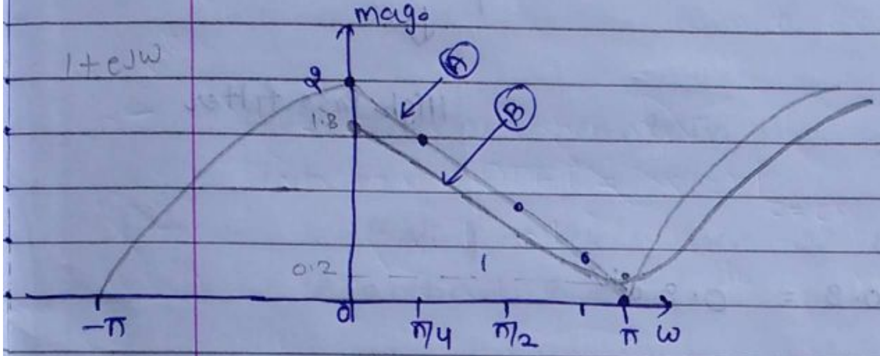
DTFT from ①; $\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{e^{j\omega}+1}{e^{j\omega}}$ —②

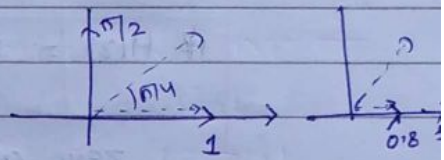
by taking magnitude;

Date ______
Page ______

$$\left|\frac{Y(e^{j\omega})}{X(e^{j\omega})}\right| = \left|\frac{1+e^{j\omega}}{e^{j\omega}}\right|$$

$e^{j\omega}$ — poles at origin does not contribute anything to magnitude response, i.e, magnitude = 1 ; $\because$ $|e^{j\omega}| = 1$, for all frequencies



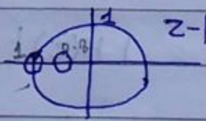


$\Downarrow$

its low pass filter (moving average filter)
at $\omega = \pi$; magnitude = 0 ; $\Rightarrow$ zeros at $z=-1$; so we get it at $\omega = \pi$

(i) zero is at $z = -1 = 1 \cdot e^{j\pi}$

$1+e^{j\omega}$ $\quad H(z) = \dfrac{z+1}{z}$ — (A)



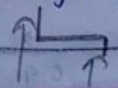
$0.8+e^{j\omega}$ (ii) if $H(z) = \dfrac{z+0.8}{z}$ $\xrightarrow{}$ (B) zero is at $z=-0.8 = 0.8\, e^{j\pi}$

here $r = 0.8$
$\omega = \pi$

To remove the value at a certain freq. completly then keep a zero at that freq. and on radius of circle = 1 (Unit circle) because as seen at $r = 0.8$ $\Rightarrow$ value at π does not become zero infact has 0.2 value left at that freq.

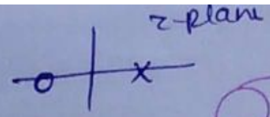
(2) $\quad y(n) = x(n) - x(n-1)$ } $\rightarrow$ High pass filter

$$\frac{Y(z)}{H(z)} = \frac{z-1}{z}$$



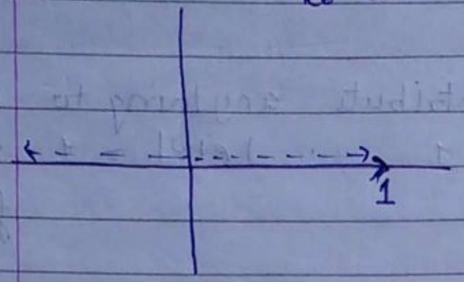
$\times \to$ poles

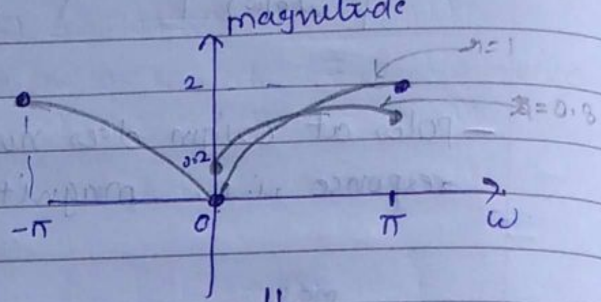
$O \to$ zeros



zero is at $z = 1 = 1 \cdot e^{j0}$

$e^{j\omega} - 1$





$\Downarrow$

High Pass filter

if $H(z) = \frac{z - 0.8}{z}$

zero at $z = 0.8 = 0.8\, e^{j0}$

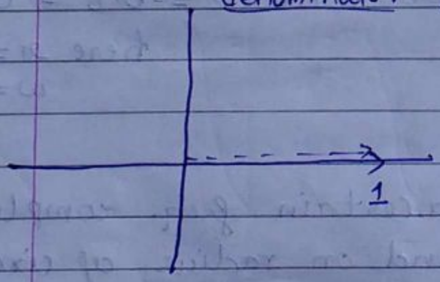
$\Rightarrow$ <u>concept of Poles:</u>

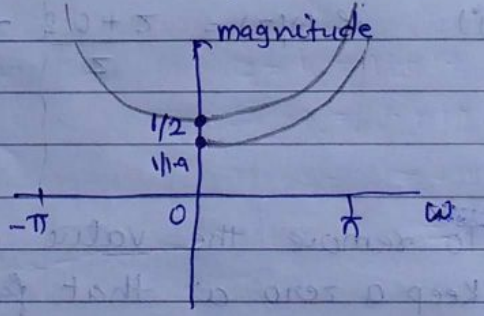
$$H(z) = \frac{z}{z+1} = \frac{e^{j\omega}}{e^{j\omega}+1}$$

- zero at origin does not contribute anything to magnitude response.

$$|H(z)| = \frac{|e^{j\omega}|}{|1+e^{j\omega}|} \longrightarrow \text{pole at } z = -1 = 1 \cdot e^{j\pi}$$

<u>denominator</u> $1 + e^{j\omega}$

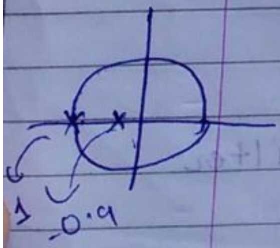




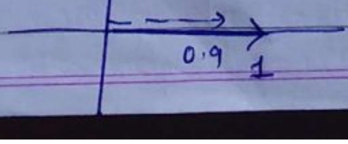
$$H(z) = \frac{e^{j\omega}}{1+e^{j\omega}}$$

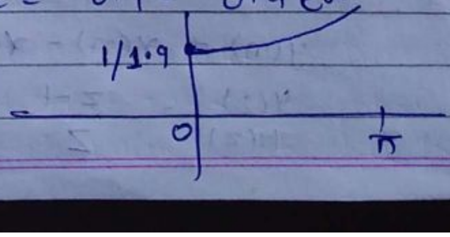
S/S is unstable because $\omega = \pi$; magnitude $= \infty$

if $H(z) = \frac{z}{z+0.9}$



poles at $z = -0.9 = 0.9\, e^{j\omega}$



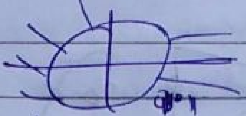


- Power of zeros ≤ powers of Poles
- Pole must be inside the Unit circle, for stability
zeros

$w = \pi$; $0.9e^{j\pi} = -0.9$ then $|H(z)| = 1/0.1 \Rightarrow$ s/s stable.

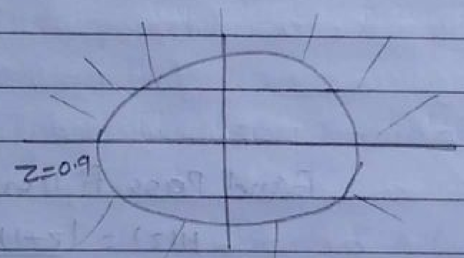
- if we want to inhance the freq., placed the pole at that freq. and $r < 1$ (nearer to pole circle)

- all the real time signals are causal, so if we taken $z = 1.1$, then ROC is outside the circle



then its not contain the Unit pole circle, DTFT not exist.

But for existence of DTFT it should be less than $z < 1$. (for poles and zeros both)



$\Rightarrow$ then its contain Unit pole circle $\Rightarrow$ DTFT exist $\Rightarrow$ causal s/s

- if we inhance the freq. $\pi/3$

then $= \frac{z}{z - 0.9e^{j\pi/3}} \rightarrow$ poles

$= \frac{z - 0.9e^{j\pi/3}}{z} \rightarrow$ zeros

Ques. Draw magnitude response for T.F with following Pole zero plots:-

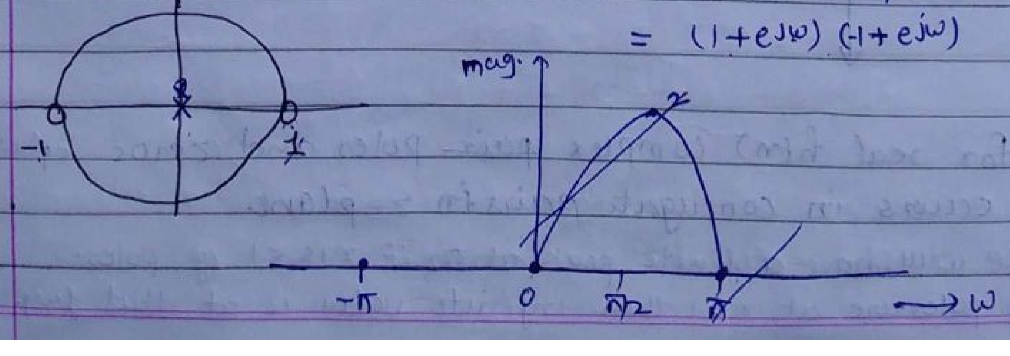
1. $H(z) = (z+1)(z-1) / z^2$

$= (1+e^{jw})(-1+e^{jw})$

$z = re^{jw}$

$z = 1 = e^{j0}$

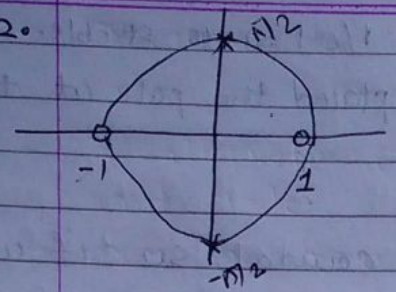
$z = -1 = 1 \cdot e^{j\pi}$



- $h(n)$ is real, in z-plane poles are zeros in conjugate pairs. (complex)

Pole ↑, zeros ↓

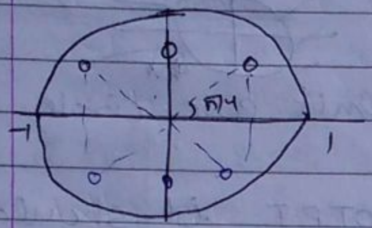
2.



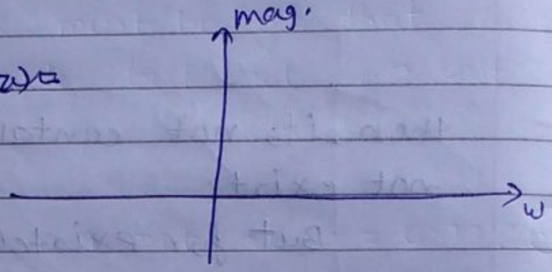
$$H(z) = \cancel{\quad} \frac{(z-1)(z+1)}{(z-\pi/2)(z+\pi/2)}$$

$z = 1 = 1 \cdot e^{j0}$; $z = -1 = 1 \cdot e^{j\pi}$

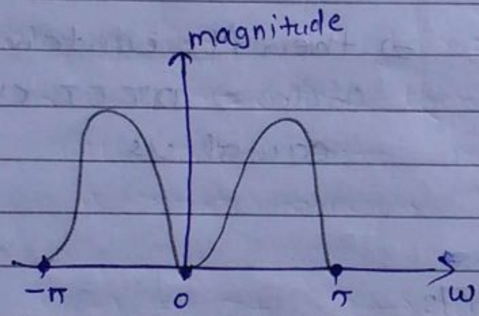
3.



$\cancel{H(z) =}$



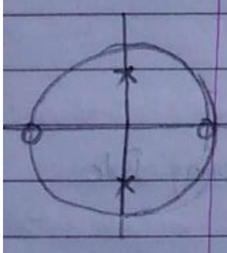
<u>Sol (i)</u>

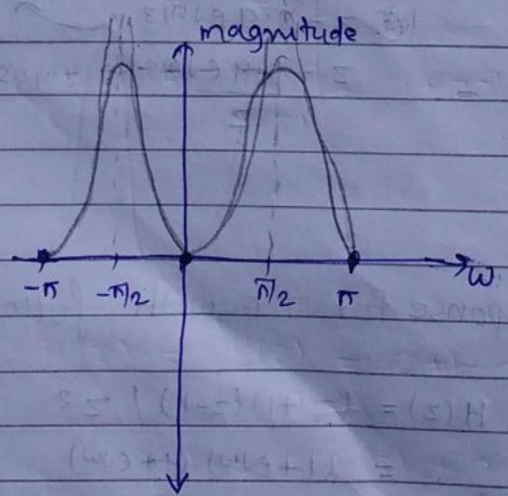


Band Pass filter

$$H(z) = \frac{(z+1)(z-1)}{z^2}$$

(ii)





$z = 1 = 1 \cdot e^{j0}$

$z = -1 = 1 \cdot e^{j\pi}$

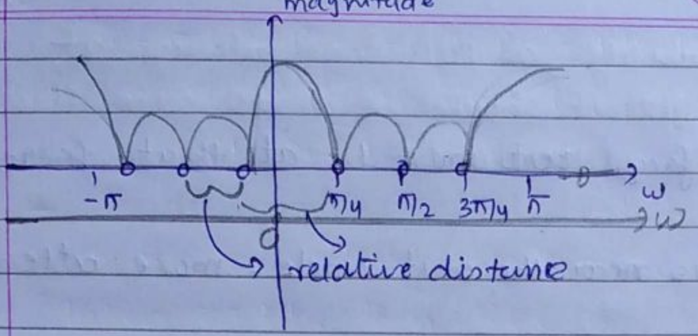
$$H(z) = \frac{(z+1)(z-1)}{z(}$$

- poles are enhancing

- for real $h(n)$ complex ~~pairs~~ poles and zeros always occurs in conjugate pairs in z-plane.
- we will have a finite peak at $\pi/2$ if r is < 1 of poles.

If poles are at $r = 1$ then infinite value is at that freq.

(iii)



zeros are in conjugate pair
poles are 6

as the relative distance b/w $\pi/4$ & $\pi/2$ is less compared to distance b/w $-\pi/4$ & $\pi/4$ as there is no zero at $\omega = 0$.

$\Rightarrow$ so, the peak between of $-\pi/4$ & $\pi/4$ is higher than peak between $\omega = \pi/4$ & $\pi/2$.

— in (i) the peaks are lower than (2) bcoz in (2) there are poles present at $\pi/2$ & $-\pi/2$ frequencies.

19-Aug.

given is stable causal s/s

$\Downarrow$

i/p is bounded & o/p is bounded

$\Downarrow$

$$\underset{\text{Bounded}}{y(n)} = \underbrace{x(n) * h(n)}_{\text{bounded}}$$

Now, if $x(n)$ [i/p] is bounded then $h(n)$ [impulse response of system] should also be bounded / completely summable/ covergence

$\Downarrow$

$$\sum |h(n)| < \infty$$

$\Downarrow$

$$\sum h(n)\, e^{j\omega} < \infty \qquad \text{bcoz } |e^{j\omega}| = 1$$

$\Downarrow$

DTFT exists $\quad$ DTFT $= z|_{z=e^{j\omega};\, r=1}$

$\Downarrow$

z - transform exists

$\Downarrow$

As DTFT exists, ROC contains unit circle

ROC of causal system > largest pole

[opp. of anti causal] $\Rightarrow$ largest pole should be within unit circle for stability of causal s/s.

- To attenuate a freq, zero must be at that freq. and on unit circle.
- more the zero is near the unit circle; more attenuation takes place.
- To enhance a freq., pole must be at that freq. and as close as unit circle but not on unit circle.

- Why pole should be at 0.9 not at $r = 0.99$? ⓢ

 Bcoz as the parameters of filters are not exact to calculated parameters (in reality). so, a small change in parameter if $r = 0.99$, then pole gets placed at $r = 1$ → not required.

 so, if $r = 0.9$, then a small change in parameter will not bring pole at $r = 1$

 ∴ error margin ↑ses if $r = 0.9$

Ex: $H(z) = \frac{z+1}{z}$ → causal s/s because zero at $z = -1$

$$H(z) = \frac{Y(z)}{X(z)} = 1 + z^{-1} \Rightarrow Y(z) = X(z) + X(z) z^{-1}$$

$$y(n) = x(n) + x(n-1)$$

causal s/s

$$H'(z) = \frac{Y(z)}{X(z)} = z+1$$

$$Y(z) = z X(z) + X(z)$$

$$y(n) = x(n+1) + x(n) \quad \leftarrow \text{anticausal s/s}$$

∴ Order of denominator ≥ order of numerator for causal systems.

FIR & IIR filters :

$$y(n) = x(n) + x(n-1)$$

$$H(z) = \frac{y(z)}{x(z)} = 1 + z^{-1}$$

$$h(n) = \delta(n) + \delta(n-1)$$

$$h(n) = [\underset{\uparrow}{1} \quad 1]$$

$H(z) = \frac{z+1}{z^0}$ → shifting pole from origin to somewhere else then s/s becomes

Pole at origin IIR

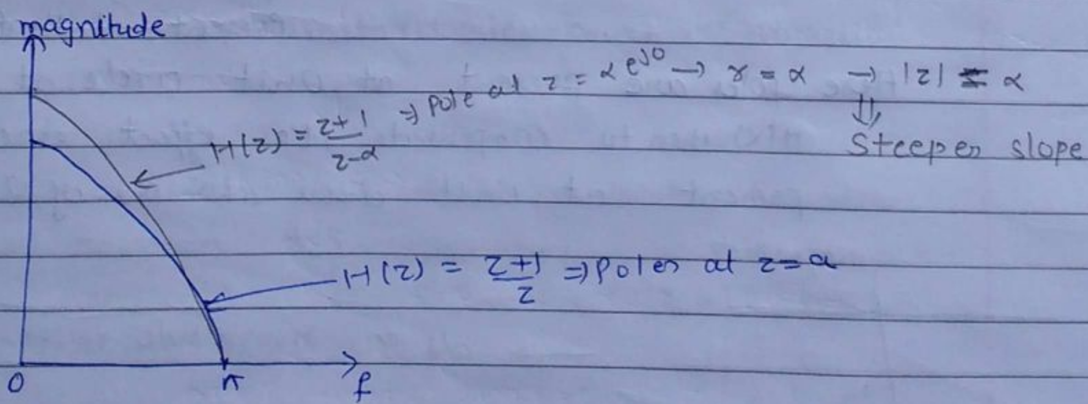
⇓ because

$H(z) = \frac{z+1}{z-\alpha}$ → Pole somewhere else

$$\frac{y(z)}{x(z)} = \frac{1+z^{-1}}{1-\alpha z^{-1}}$$

$$= \frac{1}{1-\alpha z^{-1}} + \frac{z^{-1}}{1-\alpha z^{-1}}$$

$h(n) = \alpha^n u(n) + \ldots$ → infinite sequence



for IIR s/s, α should be < 1 so that s/s is stable
⇒ ROC would contain unit circle.

- Response of IIR is steeper than FIR due to presence of pole.

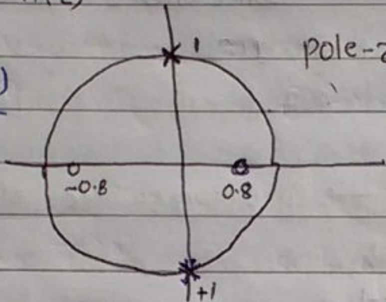
Date 23-Aug.
Page

- FIR is always stability as poles are always at origin while in IIR it should be made sure that poles are within unit circle.

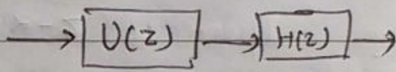
Pole-zero Compensation:

Ex- When s/s $H(z)$ is connected in series with unknown s/s $U(z)$, gives overall response allpass. find out $U(z)$. where $H(z)$

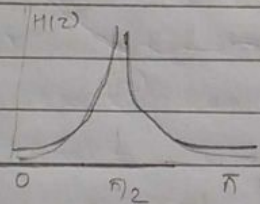
$$H(z) = \frac{(z^2+0.8)(z-0.8)}{z(z-j)(z+j)}$$



pole-zero of $H(z)$

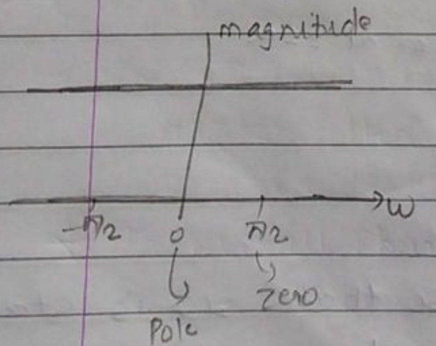


Sol:

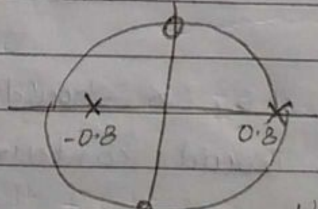


pole-zero compensation concept is used i.e., if pole is present at a freq. then to compensate it's effect zero should be present at that freq.

= Here poles are present at unit circle at $\pi/2$ of $H(x)$ then to compensate it's effect zero is present at unit circle at $\pi/2$ of $U(x)$



→ all pass magnitude response at o/p



Pole zero of $U(z)$

$$U(z) = \frac{(z-e^{j\pi/2})(z-e^{-j\pi/2})}{(z+0.8)(z-0.8)}$$

Scanned by CamScanner

– Order of filter is defined by the max. power of denominator

$$H(z) = \frac{(z-1)}{z^2-1}$$ the order of filter = 2

Application eg of IIR : (for very steep response)

Multistation channel (Base Station)

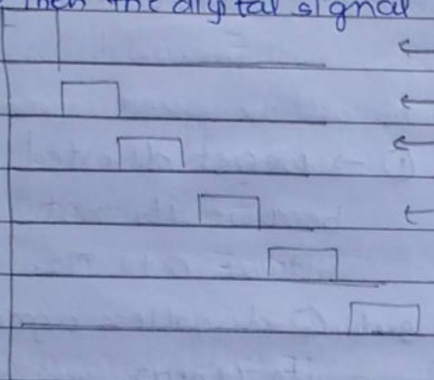
IIR is used when 10 signal are Rxed at base station then instead of having 10 different receivers.

→ 10 signals are allowed to pass through with wide-spectrum filter (removes noise)

→ then the signal is converted (A to D takes place)

(used to separate the diff. channels in multiple channel type of commⁿ)

→ then the digital signal is placed through IIR filter.



← for 1st signal

← " 2nd "

← " 3rd "

← " 4th

IIR filters have steeper response at certain frequencies

– FIR → used for linear prediction, equalizer

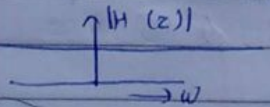
⇒ saves BW

eg: when speech signal is produced, the FIR of mobile predict the parameter (pole-zeros) of signals.

Analogy :- when you talk, the impulse response remains same but due to your mouth structure ⇒ words are produced ⇒ we only need to Tx the freq. of impulse response & FIR filter parameters.

⇒ **Condition** for filter to be disortion-less in the pass band,

(i) $|H(z)| = |H(e^{j\omega})| = K = \text{constant}$ within freq. band of interest



(ii) $\angle H(e^{j\omega}) \rightarrow$ must be linear w.r.t. freq.

i.e. func of ω within passband

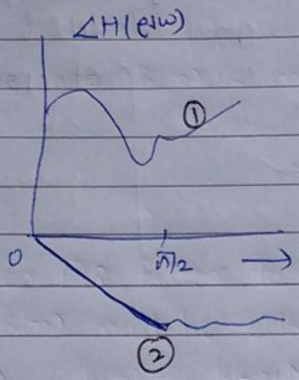
time delay in filter for all freq. of band of interest has to be same & constant. $\Rightarrow$ group delay = constant

$$A\cos(\omega t \xrightarrow{} \text{time delay})$$

A $\downarrow$ amplitude

ωt $\rightarrow$ Phase

$$\frac{d}{d\omega}(\text{Phase}) = \text{time} \rightarrow \text{we want this to be constant.}$$



① → we get distorted signal because its not linear in between 0 to π/2.

But ② distortless signal, because its linear.

Ex: $H(z) \Rightarrow H(e^{j\omega}) = \text{magnitude} \angle$

$$H(e^{j\omega}) = \underset{\text{magnitude}}{\cos\omega}\; e^{j\underset{\text{Phase}}{\omega^2}}$$

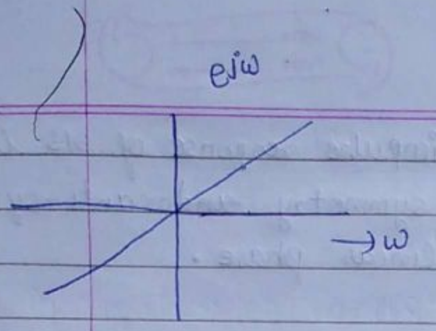
$\because$ Phase response is ω^2

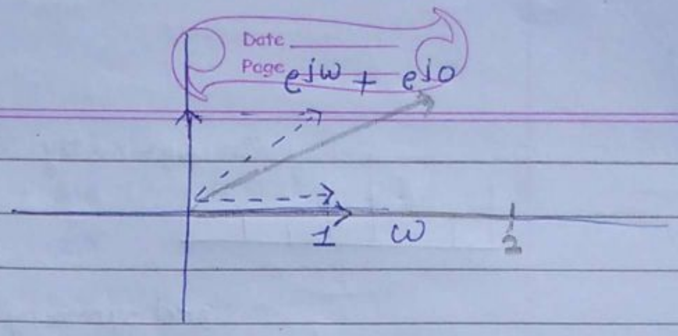
So here we can say that our signal is distorted because $\cos\omega$ (magnitude) is not constant and phase response ω^2 is not linear.

$\Rightarrow$ filter is distorted.

* the overall phase response = submission of all phase of zeros - poles

Ex:
$$H(z) = \frac{z+1}{z} \approx \frac{e^{j\omega} + e^{j0}}{e^{j\omega}}$$





ω	For num	For den.	total Phase
at = 0	$0^\circ + 0^\circ = 0^\circ$	0°	0
at = $\pi/4$	$0 + \pi/4 = \pi/8$	$\pi/4$	$-\pi/8$
$\pi/2$	$0 + \pi/2 = \pi/4$	$\pi/2$	$-\pi/4$

Phase of resultant vector

Linear Phase FIR filter :

Ex-: $h(n) = [\, h(0) \quad h(1) \quad h(2) \,]$

$$H(z) = \sum_{n=0}^{2} h(n) z^{-n}$$

$$H(z) = h(0) + h(1) z^{-1} + h(2) z^{-2}$$

$$H(e^{j\omega}) = h(0) + h(1) e^{-j\omega} + h(2) e^{-2j\omega} = |\quad| e^{j\cdot\cdot}$$

$$= e^{-j\omega} [\, h(0) e^{j\omega} + h(1) + h(2) e^{-j\omega} \,]$$

if $h(0) = h(2)$

$$= \underbrace{e^{-j\omega}}_{\text{linear Phase}} [\, \underbrace{2\cos\omega\, h(0) + h(1)}_{\text{real magnitude}} \,] \rightarrow \text{even sym.}$$

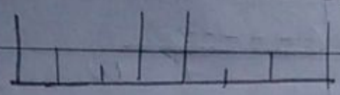
↳ if $h(0) = -h(2)$ & $h(1) = 0$

then

$$H(e^{j\omega}) = \underbrace{j}_{\text{odd sym.}} e^{-j\omega} [\, 2\cos\omega . h(0) \,]$$

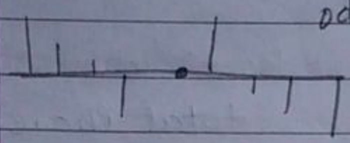
Date ______
Page ______

even symmetry



- impulse response of s/s is must symmetry ~~and~~ or anti-symm for linear phase.

odd symmetry



- FIR filter is always linear phase when H(z) func having even symmetry (with even no.)

- IIR filter can never get linear phase because h(n) is infinite ⇒ no odd or even symmetry is there.

-

<u>24-Aug</u>

for FIR filter to be of linear phase,
h(n) must be either symmetric or anti symmetric
↳ $h(n) = h(-n+N)$ ⟶ symmetric $h(n) = h(M-n)$
$h(n) = -h(n+N)$ ⟶ anti symmetric $h(n) = -h(M-n)$

- the length of filter is defined as M i.e. length of h(n) and order of filter is defined as M−1 i.e. order of denomenator of H(z).

<u>7-Sep</u>

- Order of Polynomial of T.F. H(z) → M
- length of array h(n) ⟶ L

$$h(n) = \pm h(M-n)$$

where M is the order of filter
(M+1) is the length of filter

Ex: $h(n) = \{ \underset{\uparrow}{1.2} \quad -1 \quad -1.1 \quad 0.5 \quad 0.8 \quad 1.2 \}$

FIR filter

length = 6

$$H(z) = 1.2 + (-1)z^{-1} + (-1.1)z^{-2} + 0.5(z^{-3}) + 0.8\,z^{-4} + 1.2\,z^{-5}$$

$$H(z) = \frac{1.2 z^5 - z^4 + 0.5 z^3 + 0.5 z^2 + 0.8 z + 1.2}{z^5}$$

Order = 5

Change of variable: $h(n) = \pm h(M-n)$ —— ①

$H(z) = \sum \pm h(M-n) z^{-n}$

↓ change of variable

$\boxed{H(z) = z^{-M} H(z^{-1})}$ —— ②

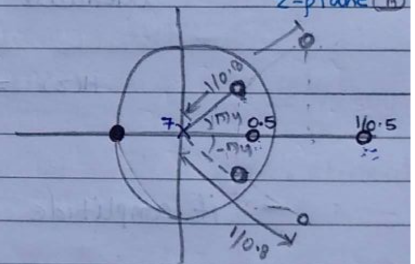
① & ② are char^c eqn of linear phase FIR filters.

– if $H(z) = \frac{z^2+z+1}{z^2}$

$H(z^{-1}) = \frac{z^{-2}+z^{-1}+1}{z^{-2}} = z^2+z+1$

– Let's say z_0 is a zero of $H(z)$

e.g. $z_0 = 0.8\, e^{j\pi/4}$



$H(z_0) = 0$

$\Rightarrow z^{-m} \cdot H(z^{-1}) = 0$

$\Rightarrow H(z_0^{-1}) = 0$

$\frac{1}{0.8} e^{-j\pi/4}$

– So we can say that for linear phase FIR filter, there are (four zeros) "Quadruiple zeros"

(conjugate is there bcuz $h(n)$ is real)

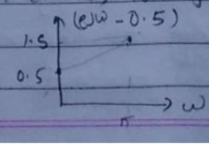
Quadruple zeros

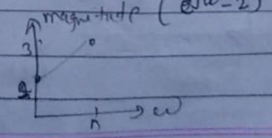
(* $h(n)$ real causal linear phase FIR)

– effect of mirror image zeros, same freq. on (same) amplitude & phase response Ⓑ

– If our s/s is phase sensing then zero must be inside the unit circle, for giving Ⓐ

(magnitude response)





Zeros $\rightarrow$ attenuate $\downarrow$

Poles $\rightarrow$ enhance $\uparrow$

Date ____
Page ____

phase Response

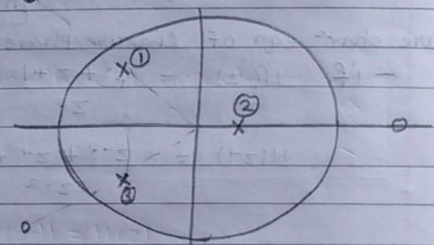
* – max. phase s/s meaning s/s having all zeros outside the unit circle
– min. phase s/s meaning s/s having all zeros inside unit circle

filters :

(1) All Pass filter :

– $h(n)$ is real and IIR response and (One pole is present there.)

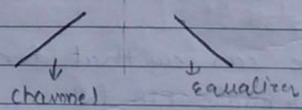
③ conjugate of ①
and all zeros are mirror images of poles.



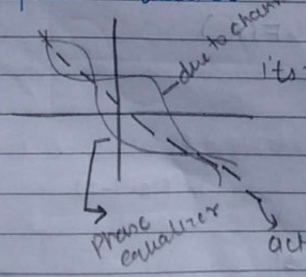
– So this filter is all pass filter. Bcuz zeros attenuate and poles enhanced at all freq. (ω).

$$H(z) = \frac{-0.2 + 0.18z^{-1} + 0.43z^{-2} + z^{-3}}{1 + 0.43z^{-1} + 0.18z^{-2} - 0.2z^{-3}}$$

– if amplitude equalizer



– phase equalizer



it's not linear. for making its linear we have to use something that's make this linear. So for this we are using all pass filter.

actual phase of signal response

IIR BPF, LPF, HPF (?) Random ...

Date 9-Sep0

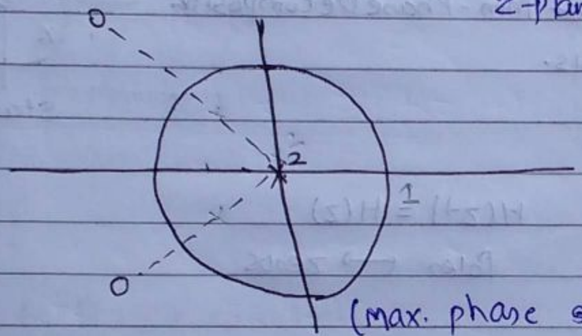
Page

- These type of all-pass filter are used to nullify the effect of channel on phase response of signal.
  Eg: used in phase equalizer

- $H(z) = \frac{1}{z^5}$ (FIR, all pass filter, but phase response is linear so we don't require any phase equalizer.)

Ex:

Design a Equalizer filter for following wireless channel that is having multipath.



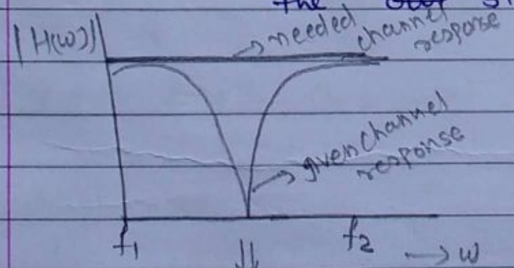
- multipath (Reflection)

- channel response of S/S is constant (non dispersive)

- wireless channel should be non-dispersive i.e., should not effect the magnitude spectrum of signal (in BW of signal)

(max. phase ~~shift~~ S/S)

Sol: Because of reflection, we can't decide which we select the ~~out~~ signal.

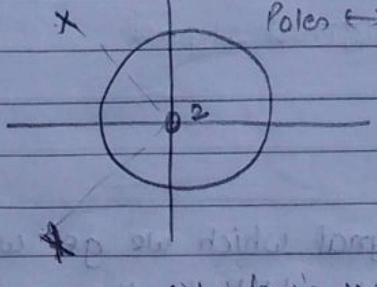


to equalize this S/S equalizer would be then

- Signal are transmitted using electrical path in wireless. electrical path depends on λ and λ depends freq. of signal

- if 10 diff^r freq. are transmitted then if f_1 is transmitted → let it takes a path - 1st, line of sight and 2nd, by some reflection But the λ for any of it don't match with λ of any of other frequencies.

Equalizer:

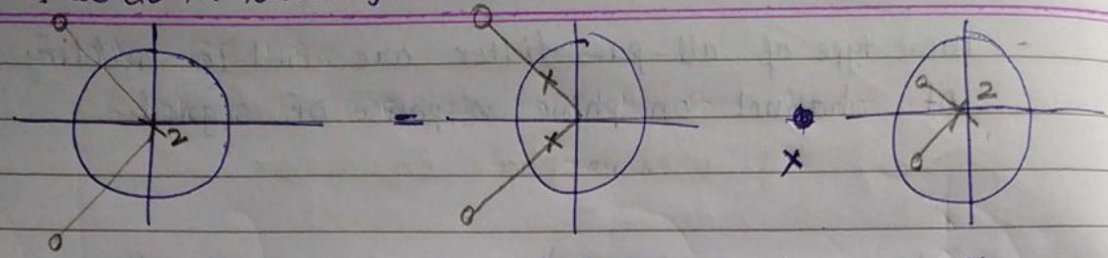


Poles ⟷ zeros

which is not stable

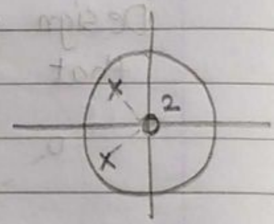
- f_4 → path 1 - delay 0° ; path 2 - delay 180°
  ⇒ total = 0, i.e f_4 freq. is not recognised

∴ we do the following:



max. phase s/s — All Pass (s/s res = 1 so no change in Equalizer) — min phase s/s

Equalizer



stable

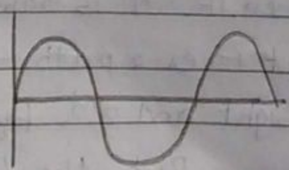
Now we required to equalize minphase and all phase not effect amplitude response.

∴ This is "All Pass- min-Phase Decomposition" of max.-phase s/s.

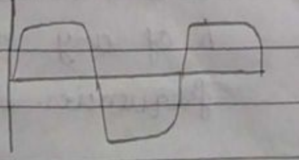
- In equalizer $H(z^{-1}) = H(z)$
  Poles $\longleftrightarrow$ zeros

- this is application of all pass filter.

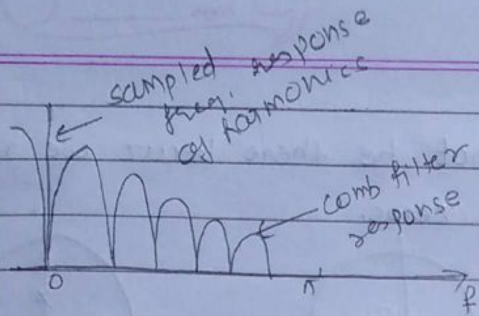
Comb filter:



← original signal

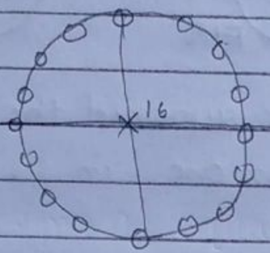


← signal which we get when harmonics gets added to the required signal by the channel

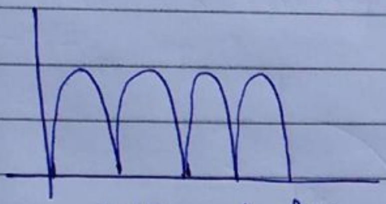


← To remove these harmonics we use comb. filter as shown.

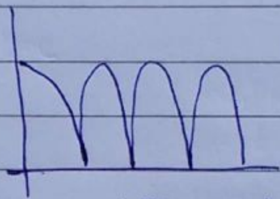
z-Plane (for comb filter)



in comb filter, the zeros are placed in harmonical manner to remove the harmonics. → ($\omega, 2\omega, 4\omega, 6\omega$ ...)



HPF comb filter



LPF comb. filter

To get comb filter equation, replace z by z^L in filter equation.

where L is no. zeros required in the filter.

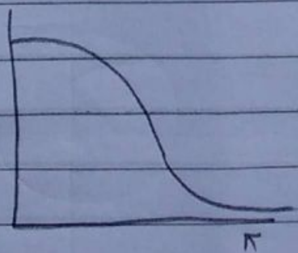
<u>14 - Sep</u>

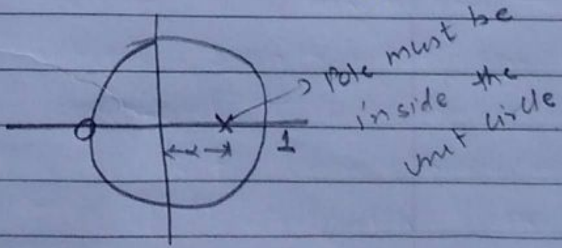
IIR filter Design:

① Simple LPF IIR:

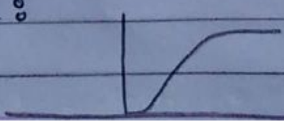
↓

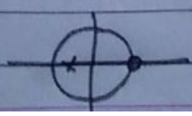
(first order → den. Power → Poles





② Simple HPF IIR:

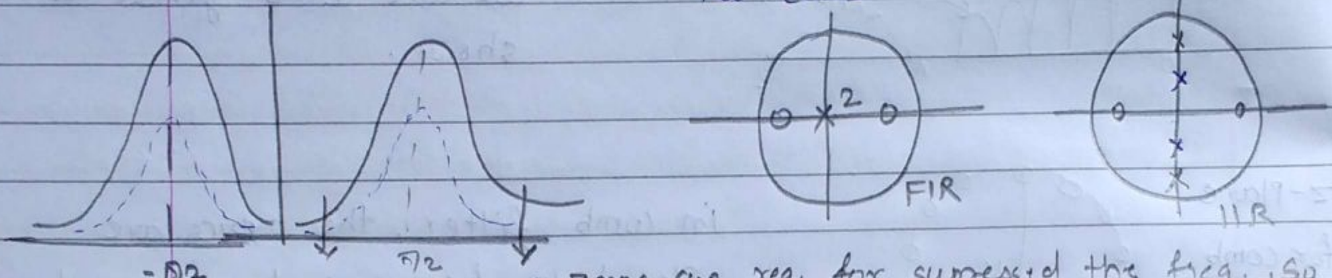




Date
Page

③ Simple BPF IIR:

Single order BPF ~~may~~ not be there becuz we required two zeros.



2 zeros are req. for supressed the freq., so there should be two or more poles are in s/s to becomes the s/s causal.

- α, β