

## chapter-1

### Fundamentals of Measurements & Instrumentation

Measurement: trying to use an instrument to determine the quantity of a variable.

Instrument: A device which we have to measure for find out its quantity.

Accuracy: Its closeness to which an instrument reading approaches to value of variable being measured.

Precision: measure of reproducibility of measurements that is given a fixed value of variable.

Precision is measure of degree to which successive measurements differ from one another.

Sensitivity: the ratio of o/p signal or response of the instrument to a change of i/p for measured variable is sensitivity.

Resolution: The smallest change in measured value to which the instrument will respond.

Error: Deviation of measured value to true value, that is error.

#### Types of Errors:

- ① Gross
- ② Systematic
- ③ Random

### ① Gross error:

- human error
- wrong connection of circuit
- not using proper instrument

### ② Systematic error:

#### Instrumental

- instrument error

#### Environment

- temp
- humidity (cleanliness, poor zoom)
- electrostatic discharge can damage our circuit  
(By antibiotic use)
- electrical & magnetic field  
(By shielding)

### ③ Random errors

- By taking no. of readings
- By taking another different instrument

### Limiting Error

Manufacture gives us knowledge or guarantee the error that is limited as example resistor having tolerance of  $\pm 10\%$  to  $\pm 20\%$  (that is already limited).

Example 8: The voltage generated by a circuit is equally dependent on the value of resistors and is

given by following equation:

$$V_{out} = \frac{R_1 R_2}{R_3}$$

If tolerance of each resistor is  $0.1\%$ . What is the max. error of generated voltage.

Sol:

$V_{out} \rightarrow \text{max} \Rightarrow R_1 \& R_2 \text{ is max. \& } R_3 \text{ is min.}$

$$R_1 + 0.001 R_1$$

$$R_2 + 0.001 R_2$$

$$R_3 - 0.001 R_3$$

$$\text{So, } V_{out} = \frac{R_1 (1+0.001) R_2 (1.001)}{R_3 (0.999)}$$

$$= 1.003 R_1 R_2$$

$R_3$

$$\text{So max. error} = 0.003 = 0.3\%$$

$$\text{total } V_{out} = 0.003 V$$

Example: The current passing through a resistor of  $100 \pm 2\Omega$  is  $2 \pm 0.01 A$ ; using  $P = I^2 R$ . Calculate the limiting error's computed value of power dissipation.

Sol:

$$I = 2 \pm 0.01 A = 2 \pm 0.5\%$$

$$R = 100 \pm 2\Omega = 100 \pm 0.2\%$$

$$P = I^2 R$$

$$\text{error's } P_r = 2 E_I + E_R$$

$$= 2 \times 0.5 + 0.2$$

$$= 1.2\%$$

$$I_r = I (1 \pm 0.002)$$

$$R_r = R (1 \pm 0.005)$$

$$P_r = I_r^2 R_r$$

$$= I^2 R (1.002)^2 (1.005)$$

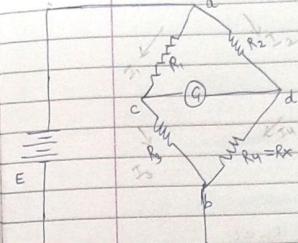
$$= (1.012) I^2 R$$

$$\text{error} = 1.2\%$$

## BRIDGES

- ↓
- ① Ac bridges
- Maxwell (for L)
- ↓
- ② Dc bridges
- ① Wheatstone (for R)
- ② Kelvin (for R)
- ③ Kelvin double (for R)
- ④ Wein (for frequency)
- ⑤ De Sauty (for capacitance)
- ⑥ Shering (for capacitor)
- ⑦ Anderson (for inductance)

## ① Wheatstone Bridge :-



$R_4 \rightarrow$  Unknown arm

$R_1, R_2 \rightarrow$  Ratio arm

$R_3 \rightarrow$  Standard arm

- bridge is balanced when current through galvanometer is zero  $I_G = 0$

$$I_1 R_1 = I_2 R_2 \quad \text{--- (1)}$$

$$I_1 = I_2 \quad \& \quad I_2 = I_4$$

$$\frac{I_1}{R_1 + R_3} = \frac{I_2}{R_2} \quad \& \quad \frac{I_2}{R_2 + R_4} = \frac{I_4}{E}$$

$$\frac{R_1}{R_1 + R_3} = \frac{R_2}{R_2 + R_4}$$

$$R_1 R_2 + R_1 R_4 = R_1 R_2 + R_2 R_3$$

$$R_1 R_4 = R_2 R_3 \quad \text{--- (4)}$$

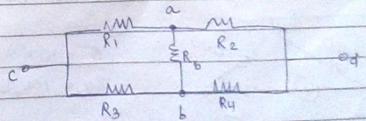
If  $R_u = R_x \rightarrow$  Unknown

$$R_x = \frac{R_2 R_3}{R_1}$$

We don't get practically exact value of  $R_x$  because  
Measurement errors :-

- Insufficient efficiency of galvanometer
- thermal emfs
- limiting errors of resistors
- $R$  of connecting leads [lead resistance is added, when resistance value is low]
- heating errors effect  $\rightarrow I^2 R$  losses

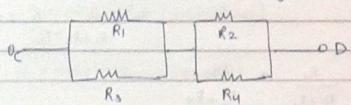
## Thevinin eq<sup>n</sup> Circuit :-



Drawback: lead resistance

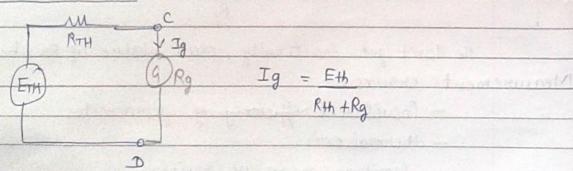
$$\begin{aligned} E_{CD} &= E_{AC} - E_{AD} \\ &= I_1 R_1 - I_2 R_2 \\ &= ER_1 - ER_2 \\ &\quad \frac{R_1 + R_3}{R_3 + R_4} \quad \frac{R_2 + R_4}{R_2 + R_4} \\ &= E \left[ \frac{R_1 R_2 + R_1 R_4 - R_2 R_2 - R_2 R_3}{(R_1 + R_3)(R_2 + R_4)} \right] \\ &= E \left[ \frac{R_1 R_2 - R_2 R_3}{(R_1 + R_3)(R_2 + R_4)} \right] \quad \text{--- (1)} \end{aligned}$$

When we assume  $R_b$  is very less  $R_b = 0$

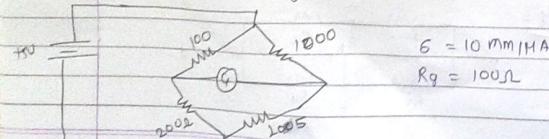


$$R_{TH} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4} \quad \text{--- (2)}$$

Equivalent circuit:



Ex: Considering the fig. the Battery voltage is 5V and  $R_b$  is negligible. The galvanometer has a current sensitivity of 10 mm/MA and  $R_g = 100 \Omega$ . Calculate the deflection of galvanometer caused by 5Ω unbalanced arm BC.



$$S = 10 \text{ mm/MA}$$

$$R_g = 100 \Omega$$

Sol:

$$\begin{aligned} V_{TH} &= S \left[ \frac{100 \times 200 - 1000 \times 200}{300 \times 300} \right] \\ &= 2.77 \times 10^{-3} \text{ V} \end{aligned}$$

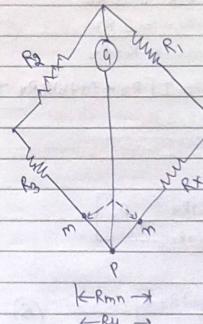
$$R_{TH} = \frac{100 \times 200}{300} + \frac{1000 \times 200}{300} = 66.67 + 666.67 = 733.89 \Omega$$

$$I_g = \frac{2.77 \times 10^{-3}}{733.89 + 100} = 3.72 \text{ mA}$$

$$\text{deflection} = 37.2 \text{ mm A}$$

Kelvin Bridge:

for low resistance value, we used Kelvin bridge.



$$\boxed{\frac{R_{NP}}{R_{MP}} = \frac{R_1}{R_2}} \quad \text{--- (1)}$$

$$\begin{aligned} &\text{--- balanced bridge} \\ &R_x (R_x + R_{NP}) = R_x + R_{NP} R_x \\ &R_x + R_{NP} = \frac{R_x}{R_x + R_{NP}} (R_x + R_{NP}) \quad \text{--- (2)} \end{aligned}$$

$$\text{in eqn (2)} \quad \frac{R_{NP}}{R_{NP} + R_{MP}} = \frac{R_1}{R_1 + R_2}$$

$$\Rightarrow R_{NP} = R_1 (R_2) \quad \text{--- (3)}$$

$$\frac{R_{NP} + R_{MP}}{R_{NP}} = \frac{R_1 + R_2}{R_2}$$

$$R_{NP} = \frac{R_1 (R_2)}{R_1 + R_2} \quad \text{--- (4)}$$

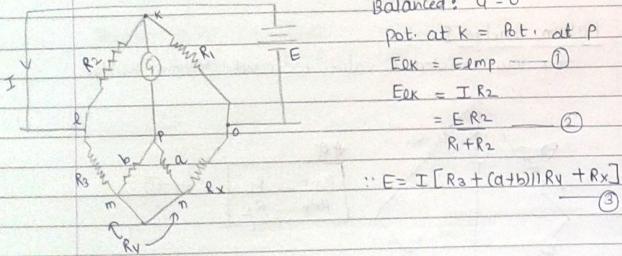
$$R_x = \frac{R_1}{R_2} \left[ \frac{R_3 + R_y R_2}{R_1 + R_2} \right] - \frac{R_y R_1}{R_1 + R_2}$$

$$= \frac{R_1 (R_1 R_3 + R_2 R_3 + R_y R_2)}{R_2 (R_1 + R_2)} - \frac{R_y R_1}{R_1 + R_2}$$

$$= \frac{R_1 (R_1 R_3 + R_2 R_3) + R_1 R_2 R_y - R_1 R_2 R_y}{(R_1 + R_2) R_2}$$

$$R_x = \frac{R_1 R_3}{R_2}$$

③ Kelvin double bridge:



$$\frac{a}{b} = \frac{R_1}{R_2}$$

Balanced:  $G = 0$

pot. at K = pot. at P

 $E_{OK} = E_{imp} \quad \text{--- (1)}$ 
 $E_{OK} = I R_2$ 
 $= E R_2$ 
 $R_1 + R_2$ 
 $\therefore E = I [R_3 + (a+b)R_y + R_x] \quad \text{--- (3)}$

$$E_{OK} = \frac{R_2 \cdot I}{R_1 + R_2} \left[ \frac{R_3 + (a+b)R_y}{a+b+R_y} + R_x \right] \quad \text{--- (4)}$$

$$E_{imp} = I \left[ R_3 + \frac{b}{a+b} \left( \frac{(a+b)R_y}{a+b+R_y} \right) \right] \quad \text{--- (5)}$$

$$(1) = (5)$$

$$\frac{R_2 R_3}{R_1 + R_2} \left[ 1 + \frac{R_2 \cdot R_y + R_2 (a+b)R_y}{R_1 + R_2 (a+b+R_y)} \right] = R_3 + \frac{b}{a+b} \frac{(a+b+R_y)}{(a+b)R_y}$$

$$\frac{R_3}{b} + \frac{R_x}{b} + \frac{(a+b)R_y}{(a+b)(a+b+R_y)} = R_3 + \frac{b(a+b+R_y)}{(a+b)^2 R_y}$$

$$\begin{aligned} & \cancel{\Rightarrow \frac{bR_3}{a+b} + \frac{bR_x}{a+b} + \frac{b(a+b)R_y}{(a+b)(a+b+R_y)} = \frac{R_3(a+b)}{(a+b)} + \frac{b(a+b+R_y)}{(a+b)^2 R_y}} \\ & \Rightarrow b [R_3 + R_x] + b(a+b)R_y = R_3(a+b) + b(a+b+R_y) \\ & \Rightarrow [bR_x + b(a+b)R_y] [(a+b)R_y] = aR_3R_y(a+b) + b(a+b+R_y) \\ & \Rightarrow abR_xR_y + ab^2R_y^2 + a^2bR_y^2 + ab^2R_y^2 + b^3R_y^2 \\ & = \end{aligned}$$

$$\frac{R_2}{R_1 + R_2} \left[ R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} \right] = R_3 + \frac{b(a+b)R_y}{a+b+R_y}$$

$$\frac{R_2 R_3 + R_2 R_x + (a+b)R_y R_2}{a+b+R_y} = R_3 R_1 + R_3 R_2 + b R_y (R_1 + R_2) \quad (a+b+R_y)$$

$$\frac{R_2 R_x + a R_2 R_y - b R_y R_1}{a+b+R_y} = R_1 R_3$$

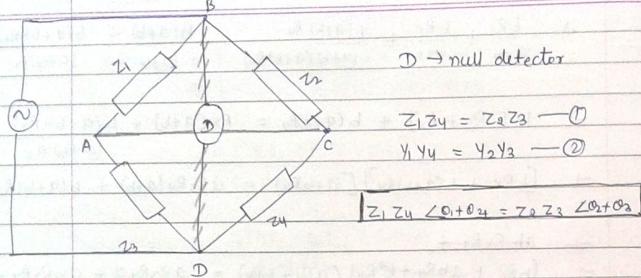
$$R_x = \frac{R_1 R_3}{R_2}$$

Assumption: (1)  $\frac{a}{b} = \frac{R_1}{R_2}$

(2) current through both branches is same.

AC BRIDGES:

- for DC, we used galvanometer, but we can't use this for AC.  
Because galvanometer calculates the avg. value:  $I_{avg} = 0$



-  $Z_1 Z_4 = Z_2 Z_3$   
 $\angle \theta_1 + \theta_4 = \angle \theta_2 + \theta_3$

Example: the impedances of basic AC bridge are as follows:

$$Z_1 = 100\Omega \angle 0^\circ \rightarrow R$$

$$Z_2 = 250\Omega \angle 0^\circ \rightarrow R$$

$$Z_3 = 400\Omega \angle 30^\circ \rightarrow L \text{ find } Z_4 = ?$$

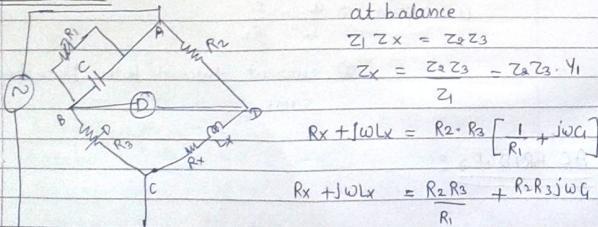
Soln:  $Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{250 \times 400}{100} = 1000\Omega$

$$\theta_2 + \theta_4 = 30^\circ$$

$$\theta_4 = -50^\circ$$

$$\text{so, } Z_4 = 1000\Omega \angle -50^\circ \rightarrow \text{capacitor}$$

### ① Maxwell Bridge



$$\begin{aligned} 0 \rightarrow +ve &\rightarrow L \\ 0 \rightarrow 0^\circ &\rightarrow R \\ 0 \rightarrow -ve &\rightarrow C \end{aligned}$$

$$\begin{aligned} L &\rightarrow jwLx \\ C &\rightarrow 1/jwCx \end{aligned}$$

Date - 28-July  
Page - 1

$$Rx = \frac{R_2 R_3}{R_1}; \quad Lx = R_2 R_3 C_1$$

- maxwell:  $1 < Q < 10$ , medium & coil (inductor)

Example: A Maxwell bridge is used to measure an inductive impedance. The bridge constant are balanced  $C_1 = 0.01\text{HF}$ ;  $R_1 = 470\text{k}\Omega$ ;  $R_2 = 5.1\text{k}\Omega$  &  $R_3 = 100\text{k}\Omega$ .

Soln:

$$Rx = \frac{R_2 R_3}{R_1} = \frac{5.1 \times 100}{470} = 1.08\text{k}\Omega$$

$$Lx = R_2 R_3 C_1 = 5.1 \times 100 \times 0.01\text{HF} = 5.1\text{H}$$

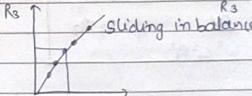
② Why Maxwell bridge is not used for finding the low-Q and high-Q?

Low-Q

$Lx \rightarrow \text{low } Q$

$R_3 \leftrightarrow R_1 \leftrightarrow R_2 \leftrightarrow R_1$

$\downarrow$



High-Q

$$\angle \theta_1 + \angle \theta_4 = \angle \theta_2 + \angle \theta_3$$

$= 0$  (because of Resistors)

$$\angle \theta_1 = -\angle \theta_4$$

$$\approx 90^\circ$$

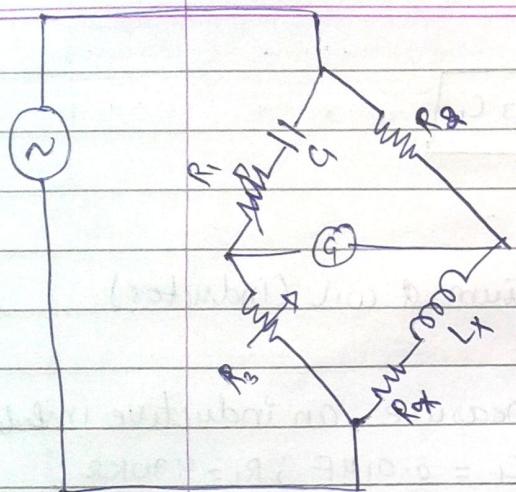
for getting the  $-90^\circ$  circle we have to be the value of  $x_c$ , so we have to increase the value of  $R_1$  ( $M_2$ ) so it's not practically possible.

②

### Hay's Bridge

- for High Q measurement

15-3-2020  
Date \_\_\_\_\_  
Page \_\_\_\_\_



$$Z_1 = R_1 - \frac{1}{j\omega C_1}$$

$$Z_4 = R_4 + j\omega L x_1$$

at balance

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left( R_1 - \frac{1}{j\omega C_1} \right) (R_4 + j\omega L x_1) = R_2 R_3$$

$$R_1 R_4 + j\omega L x_1 - \frac{R_4}{j\omega C_1} - \frac{j\omega L x_1}{C_1} = R_2 R_3$$

$$j\omega L x_1 + R_4 j = 0$$

$\omega Q$

$$\omega R_1 L x_1 = \frac{R_4}{\omega Q} \quad \textcircled{1}$$

$$R_1 R_4 - L x_1 = R_2 R_3$$

$$L x_1 = \frac{R_4}{\omega^2 C_1 R_1}$$

$$C_1 (R_1 R_4 - R_2 R_3) = L x_1$$

$$C_1 (R_1 R_4 - R_2 R_3) = L x_1 \quad \textcircled{2}$$

putting the value from  $\textcircled{1}$  in  $\textcircled{2}$

$$C_1 (R_1 R_4 - R_2 R_3) = R_4$$

$$\left( \frac{R_1 - 1}{j\omega C_1} \right) (R_X + j\omega L_X) = R_2 R_3$$

$$\frac{R_1 R_X - L_X}{C_1} - \frac{R_X}{j\omega C_1} + R_1 j\omega L_X = R_2 R_3$$

$$C_1 (R_1 R_X - R_2 R_3) = L_X \quad \textcircled{1} \quad \text{and} \quad L_X = \frac{-R_X}{\omega^2 R_1 C_1} \quad \textcircled{2}$$

$$\textcircled{1} = \textcircled{2} \quad C_1 R_1 R_X - C_1 R_2 R_3 = \frac{-R_X}{\omega^2 R_1 C_1}$$

$$R_X \left( C_1 R_1 + \frac{1}{\omega^2 R_1 C_1} \right) = C_1 R_2 R_3 \Rightarrow \boxed{R_X = \frac{\omega^2 C_1^2 R_2 R_3 R_1}{1 + \omega^2 R_1^2 C_1^2}}$$

$$\boxed{L_X = \frac{C_1 R_2 R_3}{1 + \omega^2 R_1^2 C_1^2}}$$

II. balanced condition :

$$\tan \theta_L = \frac{x_L}{R} = \frac{\omega L_X}{R_X} \quad \textcircled{7}$$

$$\tan \theta_C = \frac{x_C}{R} = \frac{1}{\omega C_1 R_1} \quad \textcircled{8}$$

$$\frac{\omega L_X}{R_X} = \boxed{\frac{1}{\omega C_1 R_1} = Q} \quad \textcircled{9}$$

$$\text{so, } R_X = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1 + (1/Q)^2} \approx \omega^2 C_1^2 R_1 R_2 R_3$$

$$L_X = \frac{R_2 R_3 C_1}{1 + (1/Q)^2} \approx R_2 R_3 C_1$$

$$\text{Hay's } Q > 10 \Rightarrow \left(\frac{1}{Q}\right)^2 < \left(\frac{1}{10}\right)^2$$

Example find series equivalent  $L_x$  &  $R_x$  of a NW that ~~posses~~ opp.

$$\omega = 3000 \text{ rad/s}$$

$$R_2 = 10 \text{ k}\Omega$$

$$R_1 = 2 \text{ k}\Omega$$

$$C_1 = 1 \text{ MF}$$

$$R_3 = 1 \text{ k}\Omega$$

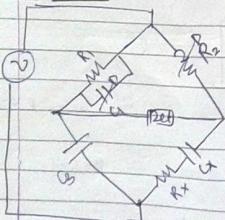
Sol:-

$$\varphi = \frac{1}{\omega R_1 C_1} = \frac{1}{3000 \times 2 \times 10^3 \times 1 \times 10^6} = 0.16 \times 10^{-12}$$

$$\begin{aligned} R_x &= \omega^2 G^2 R_1 R_2 R_3 \\ &= \frac{(3000)^2 (1 \text{ M})^2 (10 \times 2 \times 1 \times 10^6)}{1 + (\omega^2 G^2 R_2^2)} \\ &= \frac{180 \times 10^{21}}{1 + 36 \times 10^{24}} = 5 \text{ k}\Omega \end{aligned}$$

$$L_x = \frac{R_2 R_3 G}{1 + \omega^2 G^2 R_1^2} = \frac{10 \times 1 \times 2 \times 10^6 \times 10^6}{36 \times 10^{24}} = 0.027 \times 10^{-11} \text{ H}$$

(3) Schering Bridge :-



$$Y_1 = \frac{1}{R_1} + j\omega C_1$$

$$Z_2 = R_2$$

$$Z_3 = -j \frac{1}{\omega C_3}$$

$$Z_X = R_x - j \frac{1}{\omega C_x}$$

Sol:-

Ex:- An ac bridge has following constants ;  $f = 1 \text{ kHz}$   
Arm AB :  $C = 0.5 \text{ nF}$  ||  $R = 2 \text{ k}\Omega$

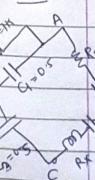
$$\text{Arm AD : } R = 2 \text{ k}\Omega$$

$$\text{Arm BC : } C = 0.5 \text{ nF}$$

CD :  $C_x$  &  $R_x$  in series with other  
determine  $R_x$  &  $C_x$  and dissipation factor  $D = ?$

$$R_x = R_2 \cdot C_1 = \frac{2 \times 10^3 \times 0.5 \times 10^{-9}}{0.5 \times 10^{-6}} = 2 \text{ k}\Omega$$

$$C_x = \frac{R_1 \cdot C_3}{R_2} = \frac{1 \times 10^3 \times 0.5}{2 \times 10^3} = 0.25 \text{ nF}$$



$$D = \omega C_1 R_1 = \frac{2\pi f (0.5 \times 10^{-9}) (10^3)}{10^6} = \frac{10^{-3} (0.5)}{10^6 (2\pi)} = \frac{0.5}{2\pi \times 10^9} = 3.14 \text{ Ans.}$$

at balance

$$Z_1 Z_X = Z_2 Z_3 \quad \text{--- (1)}$$

$$\begin{aligned} Z_X &= Z_2 Z_3 Y_1 \\ \left( R_x - \frac{j}{\omega C_x} \right) &= -R_2 j \left( \frac{1}{R_1} + j\omega C_1 \right) \\ &= -j \frac{R_2 R_1}{\omega C_3 R_1} + \frac{R_2 j^2 C_1}{C_3} \end{aligned}$$

$$\left( R_x - \frac{j}{\omega C_x} \right) = -j \left( \frac{R_2}{R_1 \omega C_3} \right) + \frac{R_2 C_1}{C_3}$$

$$R_x = \frac{R_2 C_1}{C_3} \quad \text{--- (2)}$$

$$C_x = \frac{R_1 C_3}{R_2} \quad \text{--- (3)}$$

dissipation factor  $\delta$  : (D)

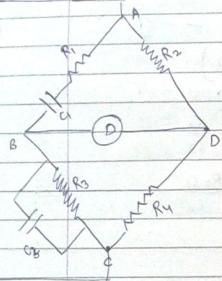
$$\delta = \cot(\delta_c) = \omega C_x R_x \quad \text{--- (4)}$$

put (2) & (3) in (4)

$$\delta = \omega C_1 R_1 \quad \text{--- (5)}$$

Date 2-Aug  
Page

#### ④ Wein Bridge: (for frequency)



$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_2 = Z_1 Z_4 \gamma_3$$

$$R_2 = \left( R_1 - j \frac{1}{\omega C_1} \right) R_4 \left( \frac{1}{R_3} + j \omega C_3 \right)$$

$$R_2 = \frac{R_1 R_4 + R_1 R_4 j \omega C_3 - j R_4 + \epsilon_3 R_4}{\omega C_1 R_3 C_3}$$

Equate real terms

$$R_2 = \frac{R_1 R_4 + C_3 R_4}{R_3 C_1}$$

$$R_2 = \frac{R_1 + C_3}{R_3 C_1} \quad \text{--- (1)}$$

Equate Imag. terms

$$R_1 R_4 j \omega C_3 = \frac{R_4}{\omega C_1 R_3}$$

$$\omega^2 = \frac{1}{C_1 C_3 R_1 R_3}$$

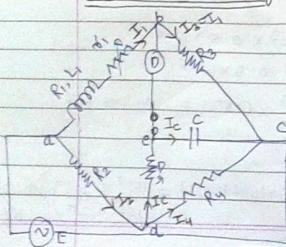
$$f = \frac{1}{2\pi \sqrt{C_1 C_3 R_1 R_3}} \quad \text{--- (2)}$$

$$C_1 = C_3 = C \quad \& \quad R_1 = R_3 = R$$

$$\text{So freq. } f = \frac{1}{2\pi R C}$$

#### ⑤ Anderson's Bridge: (measure the self inductance of inductor)

at balance



$$I_1 = I_3$$

$$I_2 = I_4 + I_3$$

$$I_4 = I_2 - I_3$$

Potential at b = Potential at e

$$E_{bc} = E_{ec}$$

$$I_1 R_3 = I_c \left( \frac{1}{j\omega c} \right) \quad \text{--- (1)}$$

$$\Rightarrow I_c = I_1 (R_3 j \omega c) \quad \text{--- (2)}$$

$$E_{ab} = E_{ad}$$

$$I_1 (\gamma_1 + R_1 + j \omega L_1) = I_2 R_2 + I_c \gamma \quad \text{--- (3)}$$

$$E_{dc} = E_{dec}$$

$$I_4 R_4 = I_c \gamma + I_c \left( \frac{1}{j\omega c} \right)$$

$$\therefore I_4 = I_2 - I_c$$

$$(I_2 - I_c) R_4 = I_c \left( \gamma + \frac{1}{j\omega c} \right) \quad \text{--- (4)}$$

put (2) in (3)

$$I_1 (\gamma_1 + R_1 + j \omega L_1) = I_2 R_2 + I_1 (\gamma R_2 j \omega c)$$

$$I_2 R_2 = I_1 [\gamma_1 + R_1 + j \omega L_1 - \gamma R_2 j \omega c]$$

$$I_2 = \frac{I_1}{R_2} [\gamma_1 + R_1 + j \omega L_1 - \gamma R_2 j \omega c] \quad \text{--- (5)}$$

put (2) in (4)

$$I_2 R_4 - R_4 I_2 R_3 j \omega c = I_1 (R_3 j \omega c) \left( \gamma + \frac{1}{j\omega c} \right)$$

$$I_2 R_4 = I_1 R_3 j \omega c \left[ \gamma + \frac{1}{j\omega c} + R_4 \right]$$

$$I_2 = \frac{I_1 R_3 j \omega c}{R_4} \left( \gamma + R_4 + \frac{1}{j\omega c} \right) \quad \text{--- (6)}$$

$$\text{By equating } \frac{\gamma_1 + R_1 + j \omega L_1 - \gamma R_2 j \omega c}{R_2} = \frac{R_3 j \omega c \gamma + R_3 j \omega c \frac{1}{j\omega c} + R_4}{R_2} \quad \text{--- (7)}$$

$$\text{real terms: } \frac{\gamma_1 + R_1}{R_2} = \frac{R_3}{R_4} \quad \text{Imag. Part:}$$

$$\frac{R_1}{R_2} = \frac{R_3 R_2 - \gamma_1}{R_4} \quad \frac{L_1}{R_2} = \frac{R_3 C \gamma}{R_4} + \frac{R_3 C + \gamma R_3 C}{R_2}$$

$$L_1 = \frac{C R_3}{R_4} \left( \gamma + 1 + \frac{\gamma}{R_2} \right) R_2$$

$$R_1 = \frac{R_2 R_3}{R_4} - j_1$$

$$L_1 = C R_0 R_3 \left( 1 + \frac{j_1}{R_2} + \frac{j_1}{R_4} \right)$$

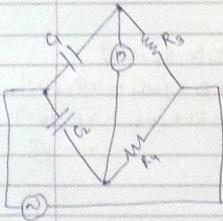
$$= \frac{C R_2 R_3}{R_2 R_4} \left[ R_2 R_4 + j_1 (R_2 + R_4) \right]$$

$$L_1 = \frac{C R_3}{R_4} \left[ j_1 (R_2 + R_3) + R_2 R_4 \right]$$

- low Q measurement (Advantage)

## ⑥ De Sauty Bridge:

- used for measuring capacitance



$C_1 \rightarrow$  Unknown

$C_2 \rightarrow$  Standard

$R_0, R_4 \rightarrow$  non-inductive resistor

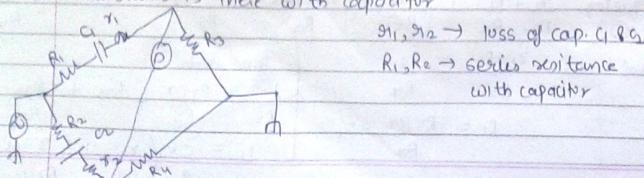
$$Z_1 Z_4 = Z_2 Z_3$$

$$\frac{1}{j\omega C_1} \times R_4 = R_2 \times \frac{1}{j\omega C_2}$$

$$\boxed{\frac{C_1}{C_2} = \frac{R_4}{R_2}} \quad \text{--- (1)}$$

eqn (1) is valid when (i.e. ideally) when cap has not dielectric losses by capacitor

- But if dielectric loss is there, then some series resistance is there with capacitor



$j_1, j_2 \rightarrow$  loss of cap. C1 & C2

$R_1, R_0 \rightarrow$  series resistance with capacitor

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left( R_1 + j_1 + \frac{1}{j\omega C_1} \right) R_4 = R_2 \left( R_2 + j_2 + \frac{1}{j\omega C_2} \right)$$

$$\text{real: } \frac{R_1 + j_1}{R_2 + j_2} = \frac{R_3}{R_4}$$

$$\text{imag: } \frac{R_4}{R_2} = \frac{C_2}{C_1} \quad \Rightarrow \quad C_1 = C_2 \frac{R_4}{R_2}$$

$$\Rightarrow \boxed{\frac{C_1}{C_2} = \frac{R_2 + j_2}{R_1 + j_1}} \quad \text{--- (2)}$$

dissipation factors

$$D_1 = \omega C_1 j_1$$

$$D_2 = \omega C_2 j_2$$

$$\text{In eqn (2)} \quad C_1 R_1 + C_1 j_1 = C_2 R_2 + C_2 j_2$$

$$\omega (C_1 R_1 - C_2 R_2) = \omega (C_2 R_2 - C_1 j_1)$$

$$\omega (C_1 R_1 - C_2 R_2) = D_2 - D_1 \quad \text{--- (3)}$$

$$\text{So, } \boxed{D_2 - D_1 = \omega C_2 \left( \frac{R_1 R_4 - R_2}{R_3} \right)}$$