Analog filter

Reference book :- Analog filter design by M.E.

passive (R, L & C)

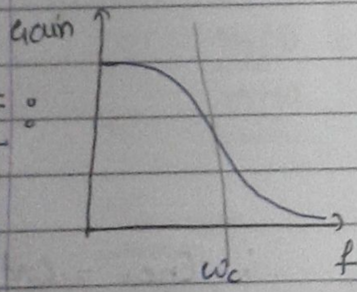
- if we design the filter by using op-amp, transistor → "Active filter"
- if we design the filter by using passive components as R, L & C → "Passive filter"

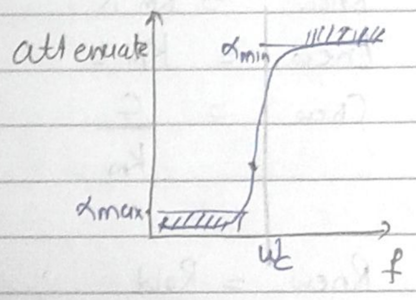
Advantage of Active over Passive filters :

- After the cut off freq. signal is attenuate in passive filters
- Gain is change as per our requirement in Active.
- Gain & attenuation are inversly Propostional.

in ~~inverting~~ ampr inverting ampr configuration is used because gain is change as per requirement. Gain > 1 & G < 1.

LPF :





$\alpha > \alpha_{max}$ → stopband or ↓ (Gain low att. High)

(i) $\alpha > \alpha_{min}$

(ii) $\alpha \le \alpha_{min}$

$\alpha > \alpha_{min}$ → Passband ↓ (Gain High att. low)

⇒ $\alpha_{stop} > \alpha_{min}$ or $\alpha_{stop} < \alpha_{m}$

stopband Gain = 0.5

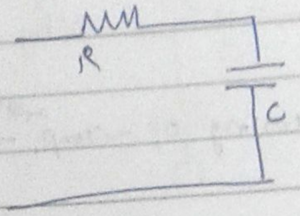
$\alpha_{stop} = 1/0.5 = 2$

Ex: Design the LPF, for Design specification   channel

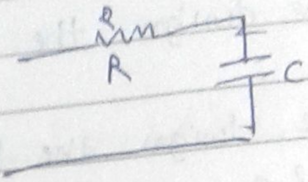
Sol: street

change specification
Design

specification $\longrightarrow$



ω_z'



$\Rightarrow R(new) = K_m R \quad \left| \frac{X_L}{\omega L} = X_L / \omega C \right.$

$L(new) = K_m . L$

$C_{new} = C / K_m$

Magnitude Scaling R, L & C :

$K_m \longrightarrow$ Scaling factor

$|X_L| = \omega L$

$|X_C| = \frac{1}{\omega C}$

- $R_{new} = K_m R$

$L_{new} = K_m L$

$C_{new} = \frac{C}{K_m}$

- $R_{new} = R_{old}$; $L_{new} = \frac{L_{old}}{K_f}$; $C_{new} = \frac{C_{old}}{K_f}$

$X_L = \omega L$

$= \omega \left(K_m \frac{L}{K_f} \right)$

$X_C = \frac{1}{\omega K_f (C/K_f)}$

- $R = K_m R_{old}$; $C = \frac{C_{old}}{K_m K_f}$; $L = \frac{K_m}{K_f} L_{old}$

When we designed filter then we take the values of R, L, C which are not practically available, so By using scaling we can used that filter.

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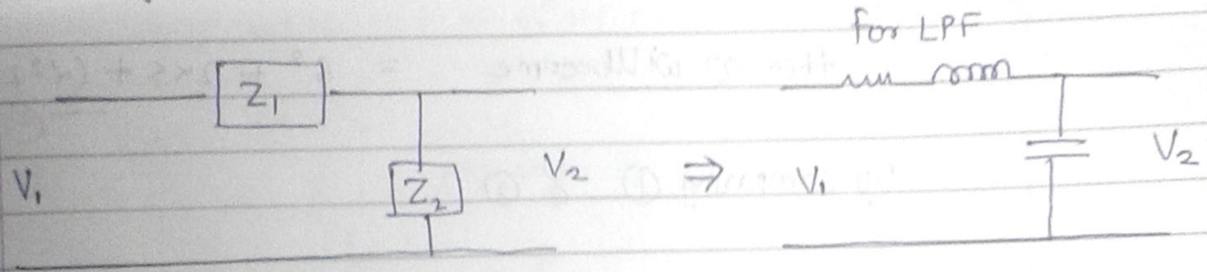
- and also by using the frequency scaling we can chose the pass band & stop band frequencies as per the already designed of LPF filter.

filter Design :

① Design Parameter Q & ω_0



when High Freq. then cap. passes the signal.

$$\frac{V_2(s)}{V_1(s)} = \frac{Z_2}{Z_1 + Z_2}$$

$$= \frac{1/\omega C}{1/\omega C + R + LC} = \frac{1/Cs}{1/Cs + R + Ls}$$

$$= \frac{1}{1 + RCs + LCs^2}$$

$$T(s) = \frac{1/LC}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}}$$

if $R = 0$ → that means loss less coil is there

$$s^2 + \frac{1}{LC} = 0$$

$$\boxed{\text{resonance freq. } \omega_0 = \frac{1}{\sqrt{LC}}}$$; for resonance freq. $X_L = X_C$

Quality factor $Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

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$$T(s) = \frac{\omega_0^2}{\underbrace{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}} \quad \text{---} \ (1)$$

If this equation having two complex conjugate pairs of roots then

$-\alpha \pm j\beta$

then eq^n will become $= s^2 + 2\alpha s + (\alpha^2 + \beta^2)$ — (2)

By comparing (1) & (2)

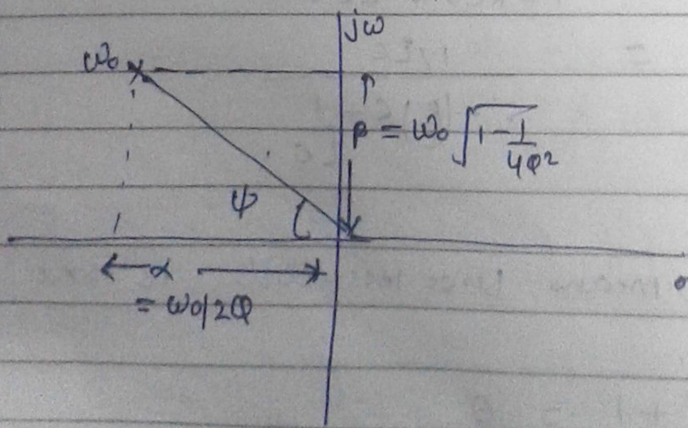
$$\alpha = \frac{\omega_0}{2Q}$$

$$\alpha^2 + \beta^2 = \omega_0^2$$

$$\beta^2 = \omega_0^2 - \frac{\omega_0^2}{4Q^2}$$

$$\beta^2 = \omega_0^2\left(1 - \frac{1}{4Q^2}\right)$$

$$\text{so}, \ \beta = \frac{\omega_0}{2Q}\sqrt{4Q^2 - 1} = \omega_0\sqrt{1 - \frac{1}{4Q^2}}$$



$$\psi = \tan^{-1}(\beta/\alpha)$$

$$= \tan^{-1}\left[\sqrt{4Q^2 - 1}\right)$$

$$\cos\phi = \frac{\omega_0/2Q}{\omega_0}$$

$$\boxed{\phi = \cos^{-1}(1/2Q)}$$

$$T(s) = \frac{\pm H\omega_0^2}{s^2 + (\omega_0/Q)s + \omega_0^2}$$

$\omega_0 = 1$; inverting ampr opamp is used so $H = -H$

$$T(s) = \frac{-H}{s^2 + s/Q + 1} = \frac{V_2(s)}{V_1(s)}$$

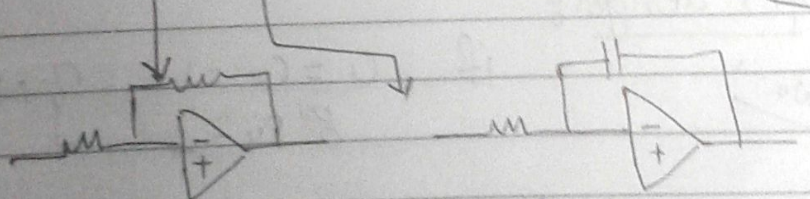
$$-V_1H = V_2\left[s\left(s+\frac{1}{Q}\right)+1\right]$$

$$V_2\left[1 + \frac{1}{s(s+1/Q)}\right] = \frac{-V_1H}{s(s+1/Q)}$$

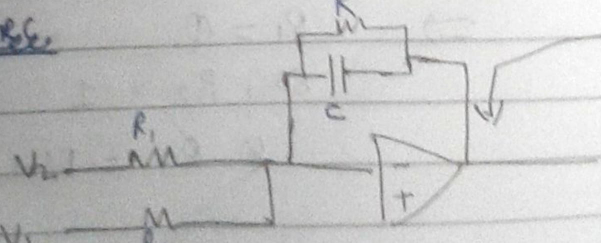
$$V_2 + \frac{V_2}{s(s+1/Q)} = \frac{-V_1H}{s(s+1/Q)}$$

$$V_2 = \frac{-HV_1}{s(s+1/Q)} - \frac{V_2}{s(s+1/Q)}$$

$$V_2 = (-1)\left(\frac{-1}{s}\right)\left[-\frac{1}{(s+1/Q)}V_2 + \frac{-H}{(s+1/Q)}V_1\right]$$



$$\frac{R\cdot 1/Cs}{1/Cs+R} = \frac{R}{RCs+1} = \frac{1/C}{s+1/RC}$$



so; $C = 1$ & $R_1 = 1$

$R = 1 \times Q$ $\quad R_2 = 1/H$

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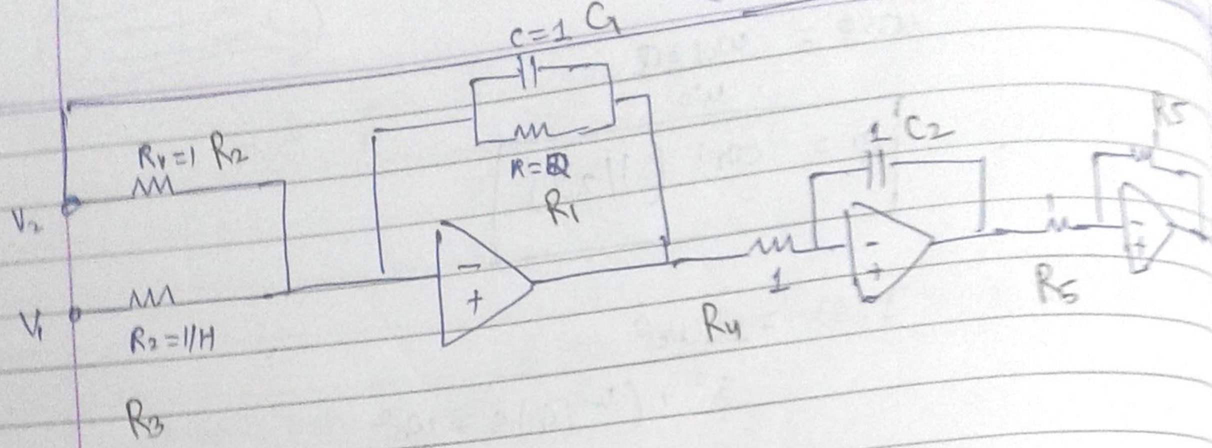

$$T(s) = \frac{-R_2/R_3 R_4 C_1 C_2 R_2}{s^2 + \left(\frac{1}{R_1 C_1}\right)s + \frac{1}{R_2 R_4 C_1 C_2}}$$

$$\omega_0 = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}$$

$$Q = \sqrt{\frac{R_1^2 C_1}{R_2 R_4 C_2}}$$

$$H = R_2 / R_3$$

for design:

$\omega_0 = 1$

if $C_1 = C_2 = 1$ & $R_4 = 1$ } Normalised values

$\Rightarrow$ $R_1 = Q \longrightarrow \omega_0$

so, $R_2 = 1$

& $R_3 = 1/H \longrightarrow$ gain

By changing the R_1, R_2 & R_3 we can change ω_0, gain & Q.

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Butterworth LPF response :

$$T(j\omega) = \left[\frac{1}{1+(\omega/\omega_0)^{2n}}\right]^{1/2}$$

butterworth pole location:

s plane $= j\omega$

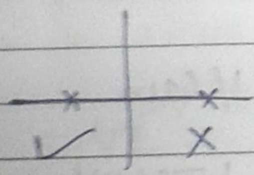
$\omega_0 = 1$

$$\text{so, } T(s) = \left[\frac{1}{1+(s/j)^{2n}}\right]^{1/2}$$

$$= \left[\frac{1}{1+s^{2n}(-1)^n}\right]^{1/2}$$

for pole location $\quad 1+s^{2n}(-1)^n = 0 = D(s)$

↳ when $n=1$; $\quad 1-s^2=0$



for stable:

$$T(s) = \frac{1}{s+1}$$

$s \to s/\omega_0$

$$T(s) = \frac{1}{1+s/\omega_0} = \frac{\omega_0}{s+\omega_0}$$

↳ when $n=2$; $\quad 1+s^4=0$

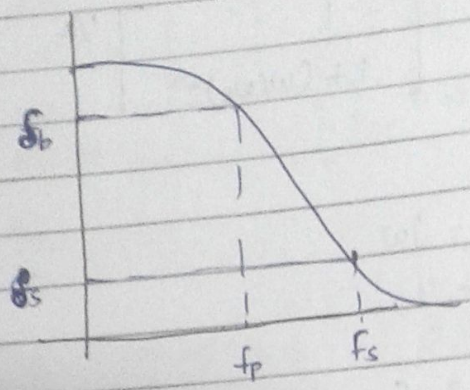
① if n is odd

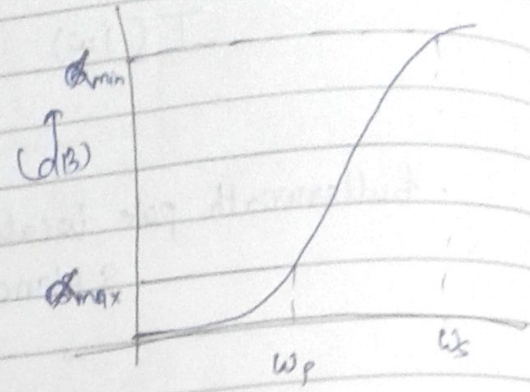
one of the pole $= 0°$

if n is even

poles at $= \pm 90°/n$

(2) All poles are separated by $= \frac{180}{n}$





$\omega_p \longrightarrow \alpha_p$ (dB) or δ_p

$\omega_s \longrightarrow \alpha_s$ (dB) or δ_s

- Gain A (dB)

$$\boxed{\alpha_{dB} = -A(dB)}$$

- attenuation = 1/ Gain

$$A = 10 \log |T^2(j\omega)|$$

$$\alpha = -A = -10 \log |T^2(j\omega)|$$

$$\alpha = -10 \log \frac{1}{1+(\omega/\omega_0)^{2n}}$$

$$\frac{\alpha}{10} = \log\left[1+(\omega/\omega_0)^{2n}\right]$$

$$10^{\alpha/10} - 1 = (\omega/\omega_0)^{2n}$$

$$\frac{\omega}{\omega_0} = \left(10^{\alpha/10} - 1\right)^{1/2n}$$

When $\omega \to \omega_p$ & $\alpha \to \alpha_p$

$$\frac{\omega_p}{\omega_0} = \left(10^{\alpha_p/10} - 1\right)^{1/2n} \quad \text{--- (1)}$$

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$$\frac{\omega_s}{\omega_0} = \left(10^{\alpha_s/10} - 1\right)^{1/2n} \quad \text{—— } (2)$$

$$\frac{\omega_p}{\omega_s} = \left(\frac{10^{\alpha_p/10} - 1}{10^{\alpha_s/10} - 1}\right)^{1/2n}$$

$$\log\left(\frac{\omega_p}{\omega_s}\right) = \frac{1}{2n} \log\left(\frac{10^{\alpha_p/10} - 1}{10^{\alpha_s/10} - 1}\right)$$

$$\boxed{n = \frac{1}{2} \cdot \frac{\log\left(\frac{10^{\alpha_p/10} - 1}{10^{\alpha_s/10} - 1}\right)}{\log(\omega_p/\omega_s)}}$$

Ex 8 -

$\omega_p = 1000$ rad/s

$\omega_s = 2000$ rad/s

$\alpha_p\,dB = 0.5$ dB

$\alpha_s\,dB = 20$ dB

Find the order of filter and ω_0 (cut off frequency)?

$$\cancel{n = \frac{1}{2} \log\left(10^{\alpha}\right.}$$

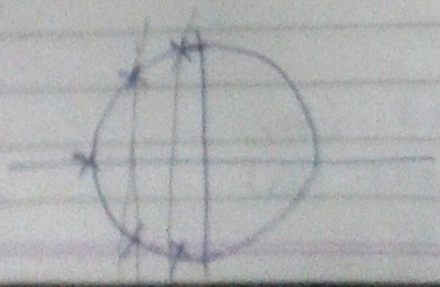
Sol:

$$n = \frac{1}{2} \cdot \frac{\log\left(\frac{10^{\alpha_s/10} - 1}{10^{\alpha_p/10} - 1}\right)}{\log(\omega_s/\omega_p)} \cong 4.8 = 5$$

$$\frac{\omega_s}{\omega_0} = \left(10^{\alpha_s/10} - 1\right)^{1/2n}$$

$$\frac{\omega_s}{\omega_0} = (99)^{1/2\times 5}$$

$\omega_0 = 1263.18$ rad/s $\rightarrow \omega_s$



Order is odd then $\rightarrow 0^\circ$

$180/5 = 36^\circ$

$\Rightarrow \quad \psi = 0^\circ, \pm 36^\circ, \pm 72^\circ$

2nd order: $\boxed{T(s) = \dfrac{\omega_0^2}{s^2 + \left(\dfrac{\omega_0}{Q}\right)s + \omega_0^2}}$

when $\omega_0 = 1$

$$T(s) = \frac{1}{s^2 + s/Q + 1}$$

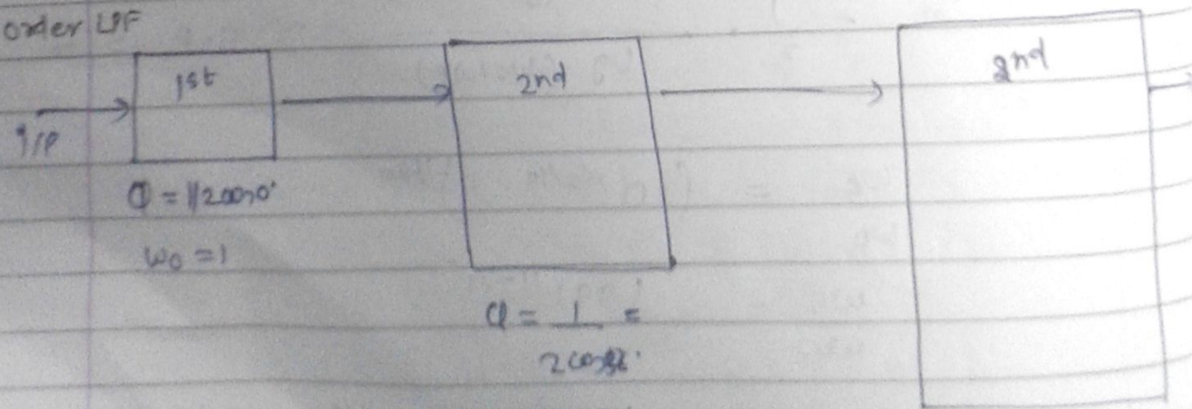
$$\psi = \cos^{-1}\left(\frac{1}{2Q}\right) \quad \& \quad Q = \frac{1}{2\cos\psi}$$

$$T(s) = \frac{1}{(s+1)(s^2 + s/Q + 1)(s^2 + s/Q + 1)}$$

$$T(s) = \frac{1}{(s+1)(s^2 + \underset{\substack{\downarrow \\ 36^\circ}}{2\cos\psi}\, s + 1)(s^2 + \underset{\substack{\downarrow \\ 72^\circ}}{2\cos\phi}\, s + 1)}$$

$$= \frac{1}{(s+1)(s^2 + 2\cos 36\, s + 1)(s^2 + 2\cos 72^\circ s + 1)}$$

5th order LPF



$Q = 1/2\cos 0^\circ$

$\omega_0 = 1$

$Q = \dfrac{1}{2\cos 36^\circ} =$

$\omega_0 = 1$

$Q = 1/2\cos 72^\circ$

$\omega_0 = 1$

$\omega_0 = 1263 \quad (\omega_p)$

$\omega_0 = 1221 \quad (\omega_s)$

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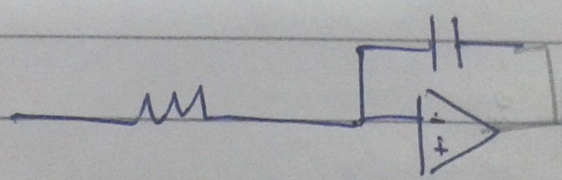
Byquist circuit:

1st



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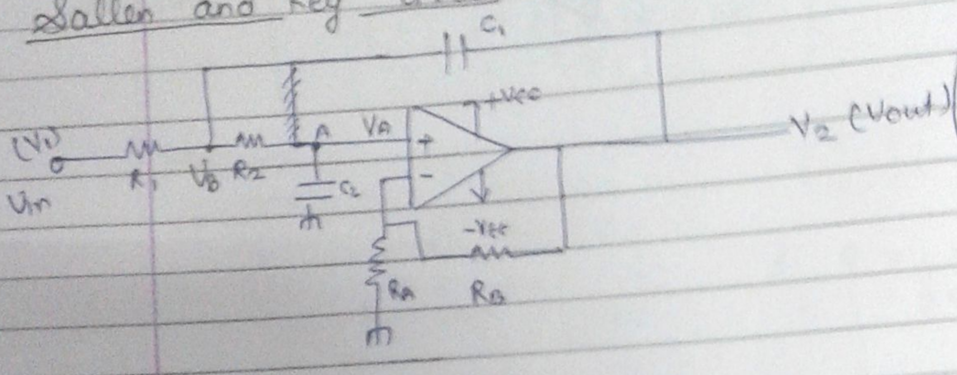
Analog filter Design:

Sallen and Key circuit:



Design-2

$R_1 = 1$

$R_2 = 1$

$C_1 = 2Q$

$C_2 = 1/2Q$

$$T(s) = \frac{V_{out}}{V_{in}} = \frac{V_2}{V_1}$$

$$\frac{V_2}{V_A} = \boxed{1 + \frac{R_B}{R_A} = K} \quad \text{(non inverting)}$$

$$V_A = \frac{V_2}{K}$$

KCL at node A: $\dfrac{V_A - V_B}{R_2} + V_A \cdot (sC_2) = 0$ — (1)

node B: $\dfrac{V_B - V_A}{R_2} + \dfrac{V_B - V_1}{R_1} + (V_B - V_2) \cdot C_1 s = 0$ — (2)

in eq^n (1) $V_A = V_2/K$

$$\frac{V_2}{K} - V_B = -\frac{V_2}{K}(sC_2) \cdot R_2$$

$$V_B = \frac{V_2}{K}\left(1 + C_2 s R_2\right)$$

in eq^n (2)

$$\frac{1}{R_2}\left[\frac{V_2}{K}\left(1 + C_2 s R_2\right) - \frac{V_2}{K}\right] + \frac{V_2}{K R_1}\left(1 + C_2 s R_2\right) - \frac{V_1}{R_1}$$

$$+ \frac{V_2}{K}\left(1 + C_2 s R_2\right) C_1 s - V_2 C_1 s = 0$$

$$\frac{V_2}{K}\left[\frac{1}{R_2} + C_2 s - \frac{1}{R_2} + \frac{1}{R_1} + \frac{C_2 s R_2 + C_1 s + C_1 C_2 s^2 R_2 - K C_1 s}{R_1}\right] = \frac{V_1}{R_1}$$

$$\boxed{\frac{V_2}{V_1} = \frac{K \cdot 1/R_1R_2C_1C_2}{s^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} - \frac{K}{R_2C_2}\right)s + \frac{1}{R_1R_2C_1C_2}}} \quad \text{— (A)}$$

$$T(s) = \frac{K\omega_0^2}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2} \quad \text{— (B)}$$

(second order LPF eqn)

By comparing (A) & (B)

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$\frac{\omega_0}{Q} = \frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} - \frac{K}{R_2C_2}$$

for designing the filter:

Design-1:

$\omega_0 = 1$

& if $R_1 = R_2 = 1$

& $C_1 = C_2 = 1$

Then eqn (A)

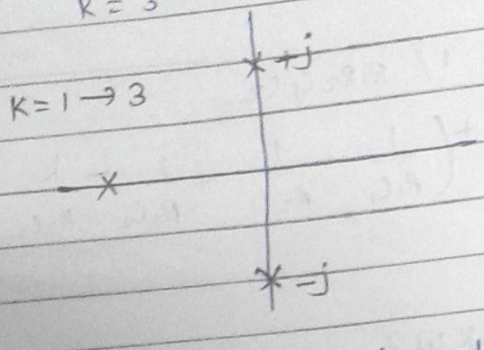
transfer func : $T(s) = \dfrac{K}{s^2 + (3-K)s + 1}$

so; $Q = 1/3-K$

for the value of Q as per our choice, we have to only change in K.

When $K = 1 \rightarrow$ real axis (Poles)

$K = 3 \rightarrow$ imag. axis (Poles)



By changing the value of K, Poles locus is also change

$$K = 1 + \frac{R_B}{R_A}$$

$$Q = \frac{1}{3-K}$$

$$3-K = \frac{1}{Q}$$

$$K = -1/Q + 3$$

$$\frac{-1}{Q} + 1 + 2 = 1 + \frac{R_B}{R_A}$$

$$-\frac{1}{Q} = \frac{R_B}{R_A} - 2$$

if $R_A = 1$ (let we assume)

$$\frac{-1}{Q} = R_B - 2$$

$$R_B = 2 - 1/Q$$

Design-2 :

let assume $K = 1$

ω_0 & Q are given

if $R_1 = R_2 = 1$ & $\omega_0 = 1$

$C_1 = ?$

$C_2 = ?$

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$C_1 \cdot C_2 = 1$ (By ③ eqn)

$$\frac{1}{Q} = \frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{C_2} - \frac{1}{C_2}$$ (By ④)

$$\frac{2}{C_1} = \frac{1}{Q}$$

$$C_1 = 2Q$$

So. $C_2 = 1/2Q$

Design-3 :

if $R_A = R_B$; $K = 2$

& $\omega_0 = 1$

let $C_1 = 1$ & $R_1 C_1 = R_2 C_2$

$R_1 = ?$, $R_2 = ?$; $C_1 = ?$; $C_2 = ?$

$$1 = \frac{1}{R_1 C_1} \Rightarrow R_1 C_1 = 1$$ (By ③)

so', $\boxed{R_1 = 1}$

$$\frac{1}{Q} = \frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_1 C_1} - \frac{2}{R_1 C_1}$$ (By ⑤)

$$\frac{1}{Q} = \frac{1}{R_2 C_1}$$

~~$R_2 C_2 = Q$~~

$R_2 = Q$

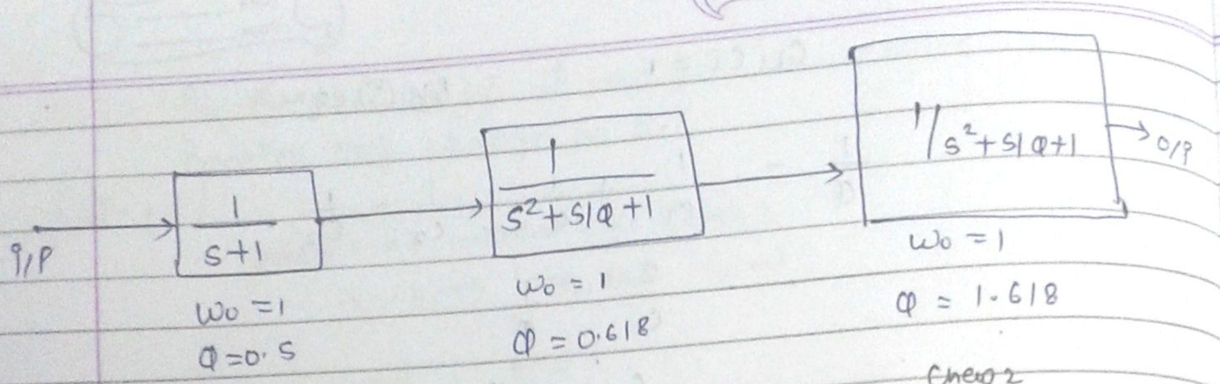
$C_2 = 1/Q$

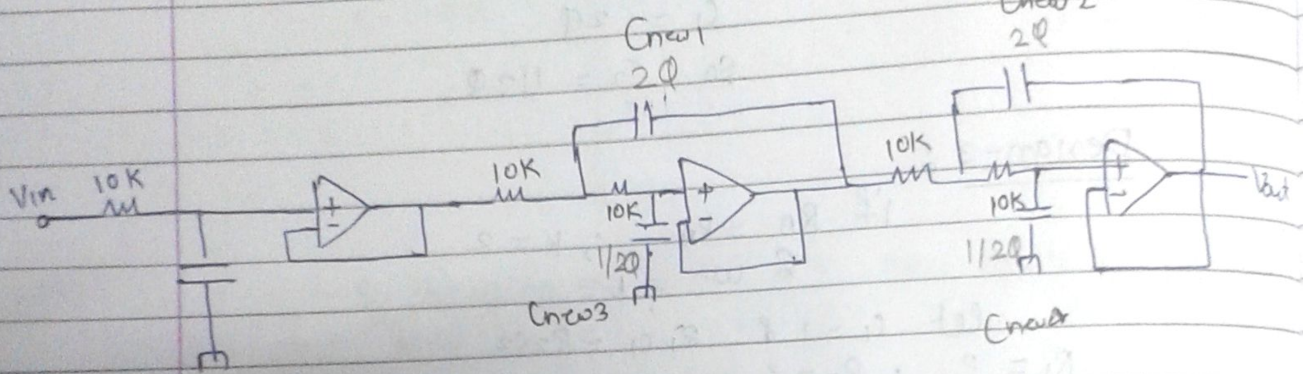
$$T(s) = \frac{1}{(s+1)(s^2 + s/Q + 1)(s^2 + s/Q + 1)}$$

0.5 0.618 1.618

$n = 5$; $\phi = 0, \pm 36, \pm 72$

$Q = 1/2\cos\phi$





specifications are given : $\alpha_p = 0.5\,dB$

$\alpha_s = 20\,dB$

$\omega_p = 1000$ rad/s

$\omega_s = 2000$ rad/s

$\omega_0 = 1263$

freq. selective $K_f = \frac{\omega_0\,new}{\omega_0\,old} = \frac{1263}{1} = 1263$

mag. $R_{new} = K_m R_{old}$

$10K = K_m \times 1$ if $R_{new} = 10K\Omega$

$K_m = 10K$

$C_{new} = \frac{1}{K_m K_f} C_{old}$

$C_{new1} = \frac{1}{1000 \times 10 \times 1263} \times 2 \times 0.618$

$C_{new2} = \frac{1}{10^4 \times 1263} \times 2 \times 1.618$

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$$C_{new_1} = \frac{1}{10^4 \times 1263 \times} \times \frac{1}{2 \times 1.618}$$

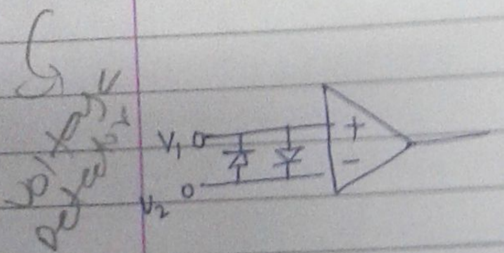
$$C_{new_2} = \frac{1}{10^4 \times 1263} \times \left(\frac{1}{2 \times 0.618}\right)$$

Non linear application of op-amp :

Topics :

- Comparator (non linear application)
- Schmitt trigger
- Square wave gen^n
- traingular wave gen^n
- wave shaping: Clipper - Rectifier
- Clamper
- Sample & hold
- Peak detector
- log and antilog are non linear

Comparator :



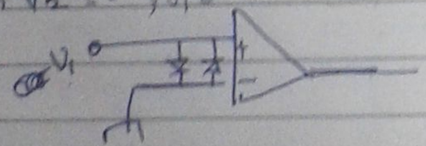
$V_1 > V_2$ $+V_{sat}$

$V_1 < V_2$ $-V_{sat}$

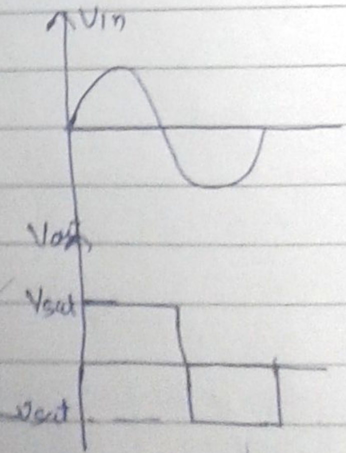
$V_{id} = V_1 - V_2$

- to protect from the i/p high voltage ~~from~~ we used diode.
- O/P of comparator is square wave.

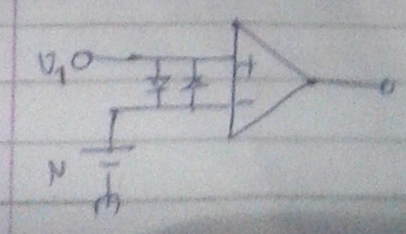
- when $V_2 = 0$, $V_1 = 0$

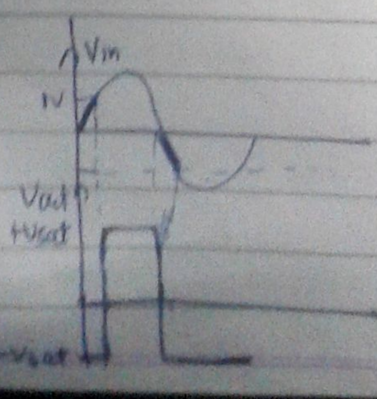


reference voltage is zero then "zero crossing detector".



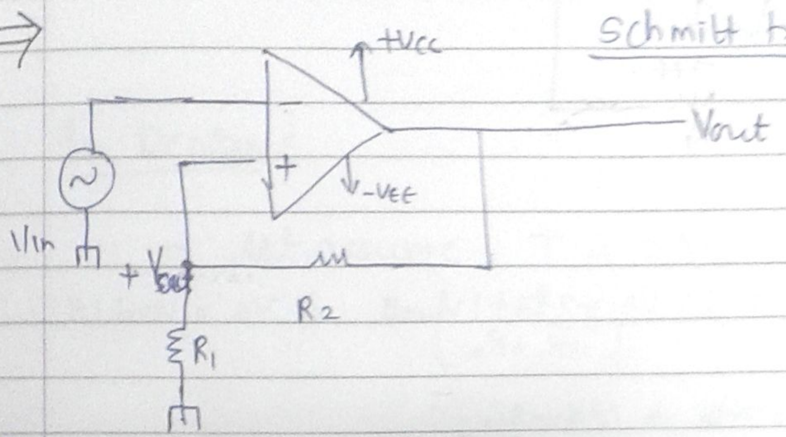
- when $V_{ref} \neq 0$





we can't connect the O/P of comparator to digital circuits because Digital IC → 5V → and if comparator O/P is 13V → then we have to clip that comparator O/P

⇒ Schmitt trigger circuit:

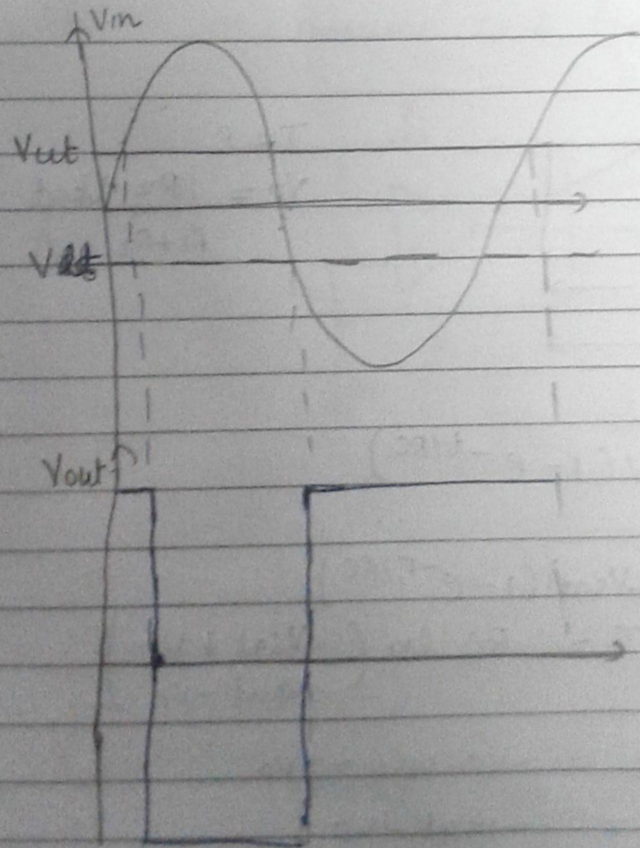


Vout = +Vsat

$V_{ut} = \frac{R_1}{R_1+R_2}(+V_{sat})$

Vout = −Vsat

$V_{Lt} = \frac{R_1}{R_1+R_2}(-V_{sat})$



hysteresis

$V_{hy} = V_{ut} - V_{lt}$

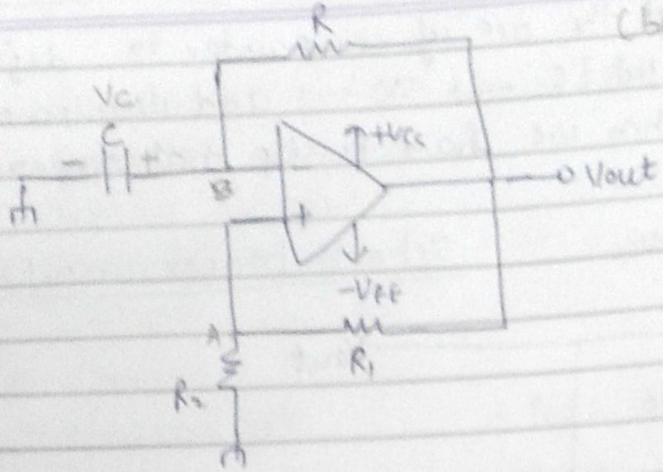
$= \frac{R_1}{R_1+R_2}\left[V_{sat} - (-V_{sat})\right]$

– open loop configuration, BW is very high, so noise is the (false triggering → unwanted transient)

→ when CMRR is very large it is possible to avoid false triggering

Oscillator :

(becuz i/p is not there for oscillation Pos feedback must be there)

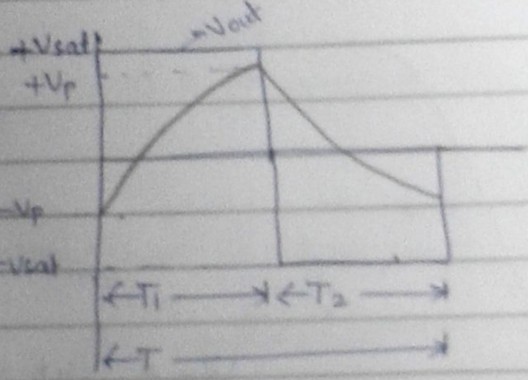


when $V_{out} = V_{sat}$; $V_A = \frac{R_2}{R_1+R_2} V_{sat}$ & $V_B = V_{out}/R$

→ then cap. starts to charging

$V_c > V_{th}$ → then $V_{out} = -V_{sat}$ & $V_A = \frac{-R_2}{R_1+R_2} V_{sat}$ &

cap. starts to discharged.



T = ?

$V_P = \frac{R_2}{R_1+R_2} V_{sat}$

$$V_c = V(1-e^{-t/RC})$$

when cap. is charging :

$$2V_P = (V_P + V_{sat})(1-e^{-T_1/RC})$$

$$T_1 = RC \ln\left(\frac{V_{sat}+V_P}{V_{sat}-V_P}\right)$$

when cap is discharging :

$$V_c = -V_P - V_P = -2V_P$$

$$V = -V_{sat} - V_P$$

$$-2V_P = (-V_{sat} - V_P)\left[1-e^{-T_2/RC}\right]$$

$$T_2 = RC \ln\left(\frac{V_{sat}+V_P}{V_{sat}-V_P}\right)$$

$$T = T_1 + T_2$$
$$= 2RC \ln\left(\frac{V_{sat} + V_P}{V_{sat} - V_P}\right)$$
$$T = 2RC \ln\left(1 + \frac{2R_2}{R_1}\right)$$

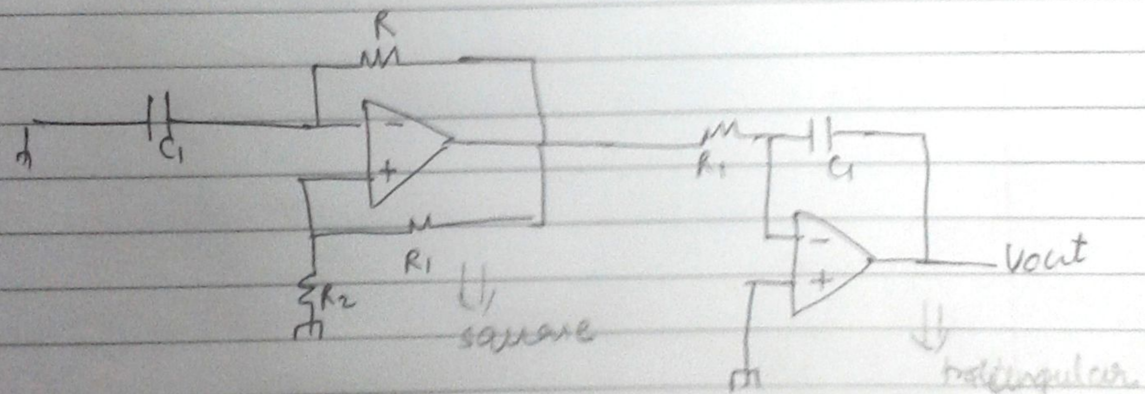
<u>for Design :</u>

- let assume $T = 2RC$

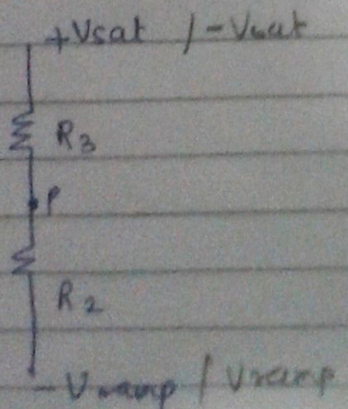
$$\ln\left(1 + \frac{2R_2}{R_1}\right) = 1$$
$$1 + \frac{2R_2}{R_1} = e^1$$
$$\frac{2R_2}{R_1} = 2.7 - 1 = 1.7 \quad \Rightarrow R_2 = R_1 (0.85)$$

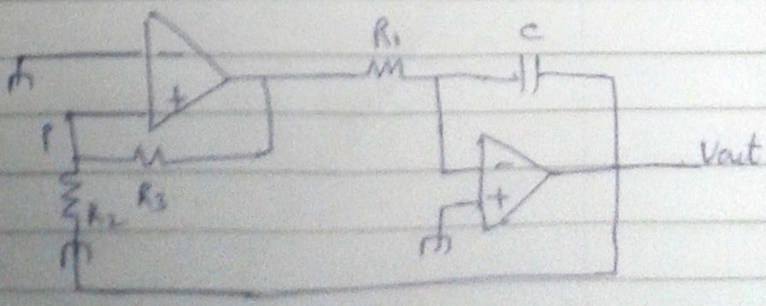
- assume C & find R

⇒



O/P of this is triangular wave





$$T = \frac{4 R_1 C_1 R_2}{R_3}$$

$$V_{O/P\,(P\text{-}P)} = 2\frac{R_2}{R_3} V_{sat}$$