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Chapter-1

VECTOR ANALYSIS

$A\cdot(B\times C) = C\cdot(A\times B) = B\cdot(C\times A)$

$A\times(B\times C) = B(A\cdot C) - C(A\cdot B)$

field: specify quantity in a particular region, that is known as field.

i) Unit Vector
ii) Distance Vector
iii) Position Vector

Example: point P & Q are located at (0, 2, 4) & (-3, 1, 5). Calculate a vector parallel to PQ with magnitude of 10.

Sol: $\vec{PQ} = -3\hat{i} - \hat{j} + \hat{k}$

(Unit vector is same for parallel vector which is to PQ)

$\hat{PQ} = \dfrac{-3\hat{i} - \hat{j} + \hat{k}}{\sqrt{11}}$

parallel vector $= \dfrac{10}{\sqrt{11}}(-3\hat{i} - \hat{j} + \hat{k})$

↳ Dot Product (scalar vector) $\Rightarrow \vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

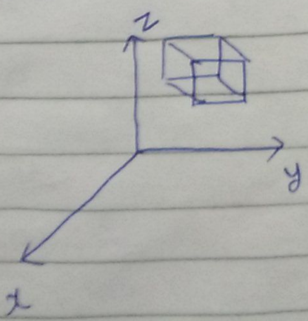
↳ Cross Product (Vector Product) $\Rightarrow \vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta$

Cordinates System:

① Rectangular CS
② cylindrical CS
③ Spherical CS

① Rectangular CS:

$-\infty \le x \le \infty$; $-\infty \le y \le \infty$; $-\infty \le z \le \infty$



Differ Displacement

$\vec{dl} = dx\,\vec{a_x} + dy\,\vec{a_y} + dz\,\vec{a_z}$

Differ Surface

$\vec{ds} = dy\,dz\,\vec{a_x}$
$dx\,dz\,\vec{a_y}$
$dx\,dy\,\vec{a_z}$

Differ volume

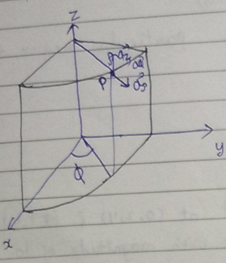
$dV = dx\,dy\,dz$

② Cylindrical CS :

there are three axis — ~~xxx~~ , ρ , z , ϕ

$\rho \downarrow$ radius

$\phi \downarrow$ angle with the x-axis



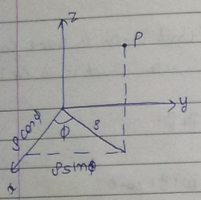
$0 \le \rho \le \infty$

$0 \le \phi \le 360^\circ$

$-\infty \le z \le \infty$

$$\vec{A} = A_\rho \vec{a_\rho} + A_\phi \vec{a_\phi} + A_z \vec{a_z}$$

$$|\vec{A}| = \sqrt{A_\rho^2 + A_\phi^2 + A_z^2}$$



$x = \rho \cos\phi$

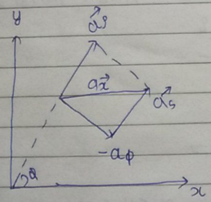
$y = \rho \sin\phi$

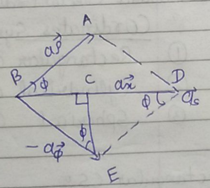
$z = z$

$\rho = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1}(y/x)$

relation between a_ρ & a_ϕ :





ΔBCE , $\sin\phi = \dfrac{BC}{BE}$

$BC = \sin\phi \, BE$

$= -\vec{a_\phi} \sin\phi$

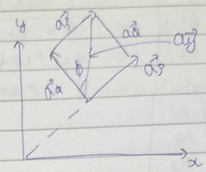
ΔCDE, $\cos\phi = CD / DE$

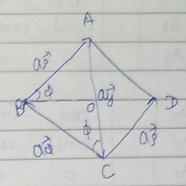
$CD = DE \cos\phi$

$= a_\rho \cos\phi$

$$BD = \boxed{\vec{a_x} = \cos\phi \, \vec{a_\rho} - \sin\phi \, \vec{a_\phi}} \quad \text{— ①}$$

Similarly;





ΔBOC ; $\cos\phi = \dfrac{CO}{BC}$

$\therefore CO = BC \cos\phi = \vec{a_\phi} \cos\phi$

ΔAOB ; $\sin \cancel{\cos\phi} = \dfrac{AO}{AB}$

$\therefore AO = \vec{a_\rho} \cancel{\cos\phi} \sin\phi$

So; $AC = AO + OC$

$$\boxed{\vec{a_y} = \cos\phi \, \vec{a_\phi} + \sin\phi \, \vec{a_\rho}} \quad \text{— ②}$$

multiplying eqn ① with $\cos\phi$

$$\cos\phi \, \vec{a_x} = \cos^2\phi \, \vec{a_\rho} - \cos\phi \sin\phi \, \vec{a_\phi} \quad \text{— (4)}$$

multiplying eqn ② with $\sin\phi$

$$\vec{a_y} \sin\phi = \cancel{\cos\phi} \sin^2\phi \, \vec{a_\rho} + \cos\phi \sin\phi \, \vec{a_\phi} \quad \text{— (5)}$$

adding (4) & (5)

$$\boxed{\vec{a_\rho} = \cos\phi \, \vec{a_x} + \sin\phi \, \vec{a_y}}$$

sub. (4) & (5)

$$\boxed{\vec{a_\phi} = \cos\phi \, \vec{a_y} - \sin\phi \, \vec{a_x}}$$

rectangular cs: $\vec{A} = A_x \vec{a_x} + A_y \vec{a_y} + A_z \vec{a_z}$

cylindrical cs: $\vec{A} = (\quad) \vec{a_\rho} + (\quad) \vec{a_\phi} + A_z \vec{a_z}$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$\rho = \sqrt{x^2+y^2}$

$\phi = \tan^{-1}(y/x)$; $z=z$

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Example Point $(-2, 6, 3)$ and

$\vec{A} = y\,\vec{a_x} + (x+z)\,\vec{a_y}$ express vector A in Cylindrical cordinates.

Sol:

$A_x = y$

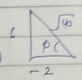
$A_y = x+z$

$A_z = 0$

$x=-2$; $y=6$; $z=3$

$\rho = \sqrt{4+36} = \sqrt{40}$

$\phi = \tan^{-1}(-6/2) = -\tan^{-1}(3)$



$\sin\phi = 6/\sqrt{40}$

$\cos\phi = -2/\sqrt{40}$

$A_\rho = y\cos\phi + (x+z)\sin\phi$

$A_\phi = -y\sin\phi + (x+z)\cos\phi$

$A_z = A_z = 0$

$A_\rho = \rho\sin\phi\cos\phi + (\rho\cos\phi\sin\phi) + z\sin\phi$

$A_\phi = -\rho\sin^2\phi + \rho\cos^2\phi + z\cos\phi$

$A_z = 0$

So; $A_\rho = \sqrt{40} \times \frac{6}{\sqrt{40}} \times \frac{-2}{\sqrt{40}} + \sqrt{40} \times \frac{6}{\sqrt{40}} \times \frac{-2}{\sqrt{40}} + 3 \times \frac{6}{\sqrt{40}}$

$= \frac{-24}{\sqrt{40}} + \frac{18}{\sqrt{40}} = \frac{-6}{\sqrt{40}} = -0.948$

$A_\phi = -\sqrt{40} \times \frac{36}{40} + \sqrt{40} \times \frac{4}{40} + 3 \times \frac{-2}{\sqrt{40}}$

$= \frac{-36+4-6}{\sqrt{40}} = \frac{-38}{\sqrt{40}} = -6.008$

③ Spherical Cordinates System: (Antena)

↳ $r, \phi, \theta \longrightarrow$ angle wrt to z-axis (Solid angle)

ϕ ↳ angle wrt to x-axis

r ↓ radius of sphere

↳ $0 \leq r \leq \infty$

$0 \leq \theta \leq \pi$

$0 \leq \phi < 2\pi$

↳ $r = \sqrt{x^2+y^2+z^2}$

$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$\phi = \tan^{-1}(y/x)$

$x = r\sin\theta\cos\phi$; $y = r\sin\theta\sin\phi$; $z = r\cos\theta$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Example:

Sol:

$A_r = \sin\theta\cos\phi\,A_x + \sin\theta\sin\phi\,A_y + \cos\theta\,A_z$

$A_\theta = \cos\theta\cos\phi\,A_x + \cos\theta\sin\phi\,A_y - \sin\theta\,A_z$

$A_\phi = -\sin\phi\,A_x + \cos\phi\,A_z$

$A_x = y$ $A_z = 0$ $A_y = (x+z)$

$A_r = r\sin^2\theta\sin\phi\cos\phi + \sin^2\theta\sin\phi\,r\cos\phi + r\cos\theta\sin\theta\sin\phi$ — (A)

$A_\theta = r\sin\theta\cos\theta\sin\phi\cos\phi + r\cos^2\theta\sin\phi + r\sin\theta\sin\phi\cos\theta\cos\phi$ — (B)

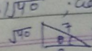
$A_\phi = -r\sin\theta\sin^2\phi$ — (C)

So; $r = \sqrt{x^2+y^2+z^2}$ $x=-2, y=6, z=3$

$= \sqrt{4+36+9} = 7$

$\tan\phi = -3$; $\sin\phi = 6/\sqrt{40}$; $\cos\phi = -2/\sqrt{40}$

$\tan\theta = \frac{\sqrt{4+36}}{3} = \frac{\sqrt{40}}{3}$



; $\sin\theta = \frac{\sqrt{40}}{7}$; $\cos\theta = \frac{3}{7}$

H.W → spherical cordinates derivation

$$A_r = 7 \times \frac{40}{49} \times \frac{(-12)}{40} \times 2 + 7 \times \frac{3 \times \sqrt{40}}{49} \times \frac{6}{\sqrt{40}}$$

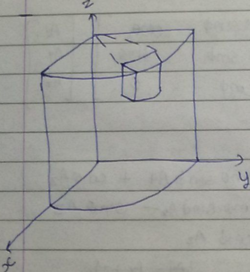
$$= \frac{-24}{7} + \frac{18}{7} = \frac{-6}{7}$$

$$A_\theta = 7 \times 2 \times \frac{(-12)}{40} \times \frac{3 \times \sqrt{40}}{49} + 7 \times \frac{9}{49} \times \frac{6}{\sqrt{40}}$$

$$= \frac{-72 + 63}{7\sqrt{40}} = \frac{-9}{7\sqrt{40}}$$

$$A_\phi = -7 \times \frac{6}{\sqrt{40}} \times \frac{40}{49} = \frac{-6\sqrt{40}}{7}$$

* cylindrical :



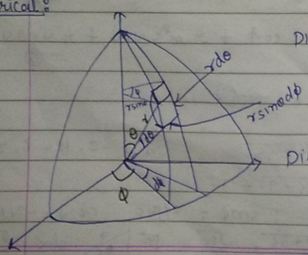
Differential Displacement :

$$\vec{dl} = d\rho\, \vec{a_\rho} + \rho\, d\phi\, \vec{a_\phi} + dz\, \vec{a_z}$$

$$\vec{ds} = \rho\, d\phi\, dz\, \vec{a_\rho}$$
$$d\rho\, dz\, \vec{a_\phi}$$
$$\rho\, d\phi\, d\rho\, \vec{a_z}$$

$$dV = \rho\, d\rho\, d\phi\, dz$$

* spherical :



Differential displacement

$$\vec{dl} = dr\, \vec{a_r} + r\, d\theta\, \vec{a_\theta} + r \sin\theta\, d\phi\, \vec{a_\phi}$$

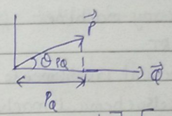
Differⁿ surface

$$\vec{ds} = r^2 \sin\theta\, d\theta\, d\phi\, \vec{a_r}$$
$$= r \sin\theta\, dr\, \vec{a_\theta}\, d\phi$$
$$= r\, dr\, d\theta\, \vec{a_\phi}$$

Differential Voltume $dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$

types of ques
- if we change radius of sphere.
- $\phi = \pi/2$, what dues it represents? → plane
- which type of cordinate s/s is used if some senario is given

* The component of P along Q = $(P \cdot a_Q) a_Q = \frac{(\vec{P} \cdot \vec{Q})\, \vec{Q}}{|Q|^2} = P_Q$



*

in spherical cordinate s/s

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Date 20-july

chapter-2

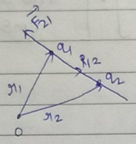
Coulomb's Law and Electric field Intensity

- static charges $\longrightarrow$ electricfield
- Uniform motion charge $\rightarrow$ magnetic field

application: electric power transmission
x-ray machines
CRT
medical instruments (Medical)
Electrochemical machines (industry)
separation of materials (agriculture)

Coulomb's law:

point charge q_1 & q_2



$$\vec{F} \propto \frac{q_1 q_2}{r^2}$$

$$\vec{F} = \frac{K Q_1 Q_2}{R^2}$$

$$K = \frac{1}{4\pi\varepsilon_0} = 9\times10^{9}\ Nm^2/C^2 \qquad \varepsilon_0 = 8.85\times10^{-12}\ F/m$$

$$\vec{F_{12}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\, \vec{a_{R12}}$$

$$\vec{R_{12}} = \vec{r_2} - \vec{r_1}$$

$$R = |\vec{R_{12}}|$$

$$\vec{a_{R12}} = \frac{\vec{R_{12}}}{|\vec{R_{12}}|} = \frac{(\vec{r_2}-\vec{r_1})}{R} = \frac{\vec{r_2}-\vec{r_1}}{|\vec{r_2}-\vec{r_1}|}$$

so, $$\vec{F_{12}} = \frac{Q_1 Q_2 (\vec{r_2}-\vec{r_1})}{4\pi\varepsilon_0 R^3} = \frac{Q_1 Q_2 (\vec{r_2}-\vec{r_1})}{4\pi\varepsilon_0 |\vec{r_2}-\vec{r_1}|^3}$$

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- charges are static.
- vaccume

practicaly exarple: CRO, speakers, atoms (nuclear)

electric field intensity: force per unit charge when placed in the electric field.

$$\vec{E} = \frac{\vec{F}}{Q} = \frac{Q}{4\pi\varepsilon_0 R^2}\, \vec{a_R}$$

when $\vec{Q} > 0$ then direction of field & force will same.

$$\vec{E} = \frac{Q\vec{R}}{4\pi\varepsilon_0 R^3} = \frac{Q(\vec{r}-\vec{r'})}{4\pi\varepsilon_0 |(\vec{r}-\vec{r_1})^3|}$$

$$= \frac{1}{4\pi\varepsilon_0}\sum_{K=1}^{N} \frac{Q_K(\vec{r}-\vec{r_K})}{|\vec{r}-\vec{r_K}|^3}$$

$\vec{E}$ due to different charge distributions:

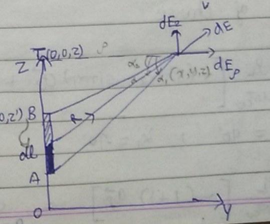
(i) $\overset{+}{\cdot}$ Q	(ii) ρ_l + + + +	(iii) ρ_s + + + +	(iv) ρ_v + + + +
Point charge	line charge	surface charge	volume charge
Unit: (C)	(C/m)	(C/m²)	(C/m³)

$$dQ = \rho_L\, dl \Rightarrow Q = \int_L \rho_L\, dl \rightarrow E = \int_L \frac{\rho_L\, dl}{4\pi\varepsilon_0 R^2}\vec{a_R}$$

$$dQ = \rho_s\, ds \Rightarrow Q = \int_s \rho_s\, ds \rightarrow E = \int_s \frac{\rho_s\, ds}{4\pi\varepsilon_0 R^2}\vec{a_R}$$

$$dQ = \rho_v\, dV \Rightarrow Q = \int_V \rho_v\, dV \rightarrow E = \int_V \frac{\rho_v\, dv}{4\pi\varepsilon_0 R^2}\vec{a_R}$$

Line charge:



$$dQ = \rho_L \, dl = \rho_L \, dz \quad \text{—①}$$ [because dl is on z-axis]

$$Q = \int_{z_A}^{z_B} \rho_L \, dz \quad \text{—②}$$

from the figure

$$dl = dz' \quad \text{—③}$$

$$\vec{R} = -(0,0,z') + (x,y,z)$$

$$= x\vec{a_x} + y\vec{a_y} + (z-z')\vec{a_z} \quad \text{—④}$$

$$\vec{R} = \rho \vec{a_\rho} + (z-\bar{z})\vec{a_z} \quad \text{—⑤}$$

$$R^2 = x^2 + y^2 + (z-z')^2 = \rho^2 + (z-z')^2 \quad \text{—⑥}$$

$$\frac{\vec{a_R}}{R^2} = \frac{\rho\vec{a_\rho} + (z-z')\vec{a_z}}{[\rho^2 + (z-z')^2]^{3/2}} \quad \text{—⑦}$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0} \int \frac{\rho\vec{a_\rho} + (z-z')\vec{a_z}}{[\rho^2 + (z-z')^2]^{3/2}} \, dz' \quad \text{—⑧}$$

from the figure $\frac{\rho}{R} = \cos\alpha$

$$R = \rho\sec\alpha$$

$$[\rho^2 + (z-z')^2]^{1/2} = \rho\sec\alpha \quad \text{—⑨}$$

also; $z' = OT - \rho\tan\alpha$

$$dz' = -\rho\sec^2\alpha \, d\alpha \quad \text{—⑩}$$

sub. in equation ⑧

$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0} \int_{\alpha_1}^{\alpha_2} \frac{\rho^2\sec^3\alpha \, [\vec{a_\rho}\cos\alpha + \vec{a_z}\sin\alpha]}{\rho^3\sec^3\alpha} \, d\alpha$$

– infinite line of charge

$$\boxed{\vec{E} = \frac{-\rho_L}{\rho 4\pi\varepsilon_0}\left[-(\sin\alpha_2 - \sin\alpha_1)\vec{a_\rho} + (\cos\alpha_2 - \cos\alpha_1)\vec{a_z}\right]}$$

when $\alpha_2 = 90°$; $\alpha_1 = -90°$

$$\vec{E} = \frac{-\rho_L}{\rho 4\pi\varepsilon_0}\left[-(1+1)\vec{a_\rho}\right] = \frac{\rho_L}{\rho 2\pi\varepsilon_0}\vec{a_\rho}$$

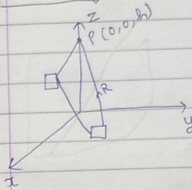
Date 25-july

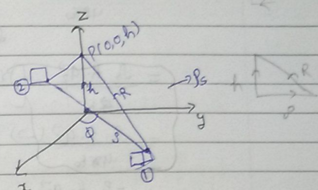
where $\rho_L \rightarrow$ line charge

$\rho \rightarrow$ radius of cylinder

$\varepsilon_0 \rightarrow$ permo of space

Surface charge:





$$dQ = \rho_s \, ds \quad \text{—①}$$

$$\vec{E} = \int_S \frac{\rho_s \, ds}{4\pi\varepsilon_0 R^2}\vec{a_R}$$

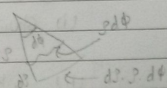
$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\vec{a_R} \quad \text{—②}$$

$$\vec{R} = -\rho\vec{a_\rho} + h\vec{a_z}$$

$$|\vec{R}| = \sqrt{\rho^2 + h^2}$$

$$\vec{a_R} = \frac{\vec{R}}{|\vec{R}|} = \frac{-\rho\vec{a_\rho} + h\vec{a_z}}{\sqrt{\rho^2 + h^2}}$$

$$dQ = \rho_s \, dS = \rho_s \, \rho \, d\rho \, d\phi$$



sub. in eqn ②

[$\vec{a_\rho}$ cancelled because for every $\vec{a_\rho}$ vector, there will always be $-\vec{a_\rho}$]

$$d\vec{E} = \frac{\rho_s \cdot \rho \, d\phi \, d\rho}{4\pi\varepsilon_0} \cdot \frac{-\rho\vec{a_\rho} + h\vec{a_z}}{(\rho^2 + h^2)^{3/2}} \quad \text{—⑦}$$

$$d\vec{E} = \frac{\rho_s \, \rho \, d\phi \, d\rho}{4\pi\varepsilon_0} \frac{h\vec{a_z}}{(h^2+\rho^2)^{3/2}} \quad \text{—⑧}$$

by Integration:

$$\vec{E} = \frac{\rho_s h \vec{a_z}}{4\pi\varepsilon_0} \int_0^{2\pi}\int_0^{\infty} \frac{\rho \, d\rho}{(h^2+\rho^2)^{3/2}} \, d\phi$$

$$= \frac{\rho_s \cdot h \cdot \vec{a_z}}{4\pi\epsilon_0} \times 2\pi \int_0^\infty \frac{\rho \, d\rho}{(\rho^2+h^2)^{3/2}}$$

$\rho^2 + h^2 = x$

$2\rho \, d\rho = dx$

$$= \frac{\rho_s \cdot h \cdot \vec{a_z}}{4\pi\epsilon_0} \cdot 2\pi \int_{h^2}^{\infty} \frac{dx}{2x^{3/2}}$$

$$= \frac{\rho_s \cdot h \cdot \vec{a_z}}{4\epsilon_0} \left[\frac{x^{-3/2+1}}{1-3/2}\right]_{h^2}^{\infty}$$

$$= \frac{\rho_s \cdot h \cdot \vec{a_z}}{4\epsilon_0}(-2)\left[x^{-1/2}\right]_{h^2}^{\infty}$$

$$= \frac{-\rho_s \cdot h \cdot \vec{a_z}}{2\epsilon_0} \times \frac{-1}{h}$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\epsilon_0}\vec{a_z}}$$ its valid when $h>0$

* $\vec{E} = \frac{\rho_s}{2\epsilon_0}\vec{a_N}$ Normal vector / Unit vector to the surface

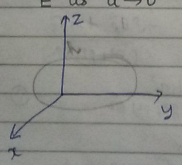
Example A circular ring of radius 'a' carries a uniform charge ρ_L C/m & its placed on the x-y plane with axis the same as z-axis.

(i) Show that $\vec{E}(0,0,h) = \frac{\rho_L \cdot a \cdot h}{2\epsilon_0 [h^2+a^2]^{3/2}} \cdot \vec{a_z}$

$dE/dh=0$ (ii) what values of h gives the max. value of $\vec{E}$

(iii) if the total charge on the ring is Q. find $\vec{E}$ as $a \to 0$

Sol:



$$d\vec{E} = \frac{dQ \, \vec{a_R}}{4\pi\epsilon_0 r^2}$$

$$dQ = d\phi \cdot a \cdot \rho_L$$

$$dL = a \, d\phi$$

$$dE = \frac{\rho_L \, d\phi \, a \, [h\vec{a_z} - a\vec{a_\rho}]}{4\pi\epsilon_0 (a^2+h^2)^{3/2}}$$

$$E = \frac{\rho_L \, a \, h \, \vec{a_z} \, (2\pi)}{4\pi\epsilon_0 (a^2+h^2)^{3/2}}$$

$$dQ = \rho_L \, dL = \rho_L (a \, d\phi)$$

$\because \vec{a_R} = \vec{R}/|\vec{R}|$ $\because \vec{R} = h\vec{a_z} - a\vec{a_\rho}$

$$\int dE = \int_0^{2\pi} \frac{\rho_L \, a \, d\phi}{4\pi\epsilon_0} \cdot \frac{h\vec{a_z} - a \cdot \vec{a_\rho}}{(h^2+a^2)^{3/2}}$$

$$E = \frac{\rho_L \cdot a}{4\pi\epsilon_0} \frac{h a_z}{(h^2+a^2)^{3/2}} \int d\phi = \frac{\rho_L \, a \, h}{2\epsilon_0 (a^2+h^2)^{3/2}} \vec{a_z}$$

(ii) $\frac{dE}{dh} = 0$

$$\frac{\rho_L \, a \, a_z}{2\epsilon_0} \frac{d}{dh}\left[\frac{h}{(a^2+h^2)^{3/2}}\right] = 0$$

$$\frac{(a^2+h^2)^{3/2} \times 1 - h \times \frac{3}{2}(a^2+h^2)^{1/2} \cdot 2h}{(a^2+h^2)^3} = 0$$

$$(a^2+h^2) - 3h^2 = 0$$

$$a^2 = 2h^2$$

$$h = \pm a/\sqrt{2}$$

(iii) $Q = \int_L \rho_L \, dl$ (Point charge)

$$= \rho_L \cdot (2\pi a)$$

$$\rho_L = \frac{Q}{2\pi a}$$

so; $\vec{E} = \frac{Q}{4\pi\epsilon_0 h^2}\vec{a_z}$

Example: the finite sheet $0 \le y \le 1$ on the $z=0$ plane has a charge density $\rho_s = xy(x^2+y^2+25)^{3/2}$ nC/m². ; $0 \le x \le 1$

find (a) total charge on sheet

(b) Electric field at (0, 0, 5)

(c) the force experience by a -1mC charge located at (0, 0, 5).

Sol:

Q:- 6.81 nC

$$Q = \int_S \rho_s \, ds$$

$$= \int_0^1 \int_0^1 x(x^2+y^2+25)^{3/2} dx \cdot y \, dy$$

$$= \frac{1}{2}\cdot\frac{2}{5}\int_0^1 y\,dy \left[(y^2+25+1)^{5/2} - (y^2+25+0)^{5/2}\right]$$

$$= \frac{1}{5}\cdot\frac{2}{7}\times\frac{1}{2} \left[(27)^{7/2} - (26)^{7/2} - (26)^{7/2} + (25)^{7/2}\right]$$

$$= 33.15\ nC$$

(b) $$E = \int \frac{\rho_s\, ds\, \vec{r}}{4\pi\varepsilon_0 r^3}$$

$$\vec{r} = (0,0,5) - (x,y,0) = (-x,-y,5)$$

$$F = \int_0^1\int_0^1 \frac{xy\,\cancel{(x^2+y^2+25)^{3/2}}\,(-x a_x - y a_y + 5 a_z)}{4\pi\varepsilon_0\,\cancel{(x^2+y^2+25)^{3/2}}}\,dx\,dy$$

$$= 9\left(\frac{-1}{6}, \frac{-1}{6}, \frac{5}{4}\right) = (-1.5, -1.5, 11.25)\ \frac{V}{m}$$

(c) $$F = qE$$

$$= (-1\ mC)(-1.5, -1.5, 11.25)\ V/m$$

$$= (1.5, 1.5, 11.25)\ mN$$

Date 27-july

ELECTRIC FLUX DENSITY, GAUSS LAW & DIVERGENCE

flux: lines coming out of a point / surface.

electric lines → electric ~~field intensity~~ flux

EFD (electric field density) = $\frac{\text{electric field}}{\text{Area}}$

$\vec{D} = \varepsilon_0 \vec{E}$ → electric field intensity

$\vec{D}$ ↓ flux density; ε_0 ↓ Permitivity

$\psi = \int_s \vec{D}\cdot d\vec{s}$ — webes = C/m^2

ψ ↓ total electric flux — lines from a charge which passes through a surface in which the charge is enclosed.

- $\vec{D}$ is function of charge distribution but is independent of medium.

  as $D = \varepsilon_0 E$ ⇒ all the formulas of D is independent ε_0 and denpendent on Q.

Ex: Determine $\vec{D}$ at (4,0,3) if there is a point charge -5π mC at (4,0,0) and a line charge 8π mC/m along y-axis.

Sol: $$E(\text{Point charge}) = \vec{E_1} = \frac{Q}{4\pi\varepsilon_0 R^2}\,a_R$$

$$\vec{R} = (4,0,3) - (4,0,0) = (0,0,3)$$

$$= \frac{-5\pi\times10^{-3}}{4\pi\varepsilon_0\times 9}\,\frac{(0,0,3)}{3}$$

$$D = \varepsilon_0 E$$

Date 29-july
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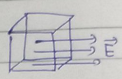
Gauss law :

~~-when symmetricity is presence~~

Gauss's law is basically a way to find the electric field intensity, provided symmetry exists.

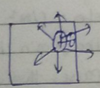
: **advantage** of gauss's law over coulomb's law ⇒ reduced complexity while ~~showing~~ solving and finding electric field intensity.

$$\boxed{\Phi = \frac{Q_{enc}}{\varepsilon_0}}$$

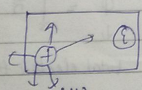


flux would be zero when:
↳ charge is zero
↳ equal no. of pos. and neg. charges are there

-



(1)



(11)

fig(1) & fig(2) ; flux is same.

-

$\Psi = Q_{enclosed}$

$\Phi = \oint_s d\Psi = \oint_s \vec{D}\cdot d\vec{s}$ — (2)

$Q_{enclosed} = \oint_S \vec{D}\cdot d\vec{s}$

$\oint \rho_v$

Gauss's law in ~~integral~~ form :

$$\boxed{\int_V \rho_v \, dv = \oint_S \vec{D}\cdot d\vec{s} \quad — (3)}$$

↳ prove

$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{a_R}$

$\vec{D} = \varepsilon_0 \vec{E}$

$\vec{D} = \frac{Q \vec{a_R}}{4\pi r^2}$

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$\vec{D_s} = \frac{Q \vec{a_R}}{4\pi a^2}$ [spherical cordinates ; radius = a

$ds = r^2 \sin\theta \, d\theta \, d\phi$ differential surface

$= a^2 \sin\theta \, d\theta \, d\phi$

so; $D_s \cdot ds = \frac{Q}{4\pi a^2} \cdot a^2 \sin\theta \, d\theta \, d\phi$

by Integrating

$= \int_0^{\pi}\int_0^{2\pi} \frac{Q}{4\pi} \sin\theta \, d\theta \, d\phi$

$= \frac{Q}{4\pi} \times 2\pi \int_0^{\pi} \sin\theta \, d\theta$

$\oint_s \vec{D}\cdot d\vec{s} = \frac{Q}{2}[\cos\theta]_{\pi}^{0} = \frac{Q}{2}[1+1] = Q$

- $Q_{enclosed}$ of electric flux crossing the surface as Q is enclosed in the surface.

Application of Gauss's law : $Q = \oint \vec{D}\cdot d\vec{s}$

for find the total charge
↳ $d\vec{s}$ either normal and tangential to the surface
normal → 0°
tangential → 90°
↳ whether symmetry exist or not.

① **point charge :**

radius of sphere = a

total charge $Q = \oint \vec{D_s}\cdot d\vec{s}$ $\vec{a}\cdot\vec{b} = ab\cos\theta = ab\cos(0)$

ds is the normal to the sphere surface always

$= \oint_{sph} D_s \, ds$

$= D_s \oint_{sph} ds$

$= D_s \oint_{sph} r^2 \sin\theta \, d\theta \, d\phi$

$= D_s \cdot r^2 \int_0^{\pi} \sin\theta \, d\theta \cdot 2\pi$

$= D_s \cdot r^2 \cdot 2\pi \left[-\cos\theta\right]_0^{\pi}$

$Q = 4\pi r^2 D_s$

$D_s = \dfrac{Q}{4\pi r^2}$

flux density $\quad \vec{D} = \dfrac{Q}{4\pi r^2} \vec{a_R}$

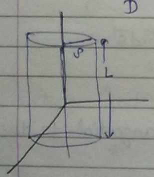
field intensity $\quad \vec{E} = \dfrac{Q}{4\pi\epsilon_0 r^2} \vec{a_r}$

Conditions – field is symmertical & radially directed outwards

(2) line charge:

- cylindrical surface
- $d\rho$ components is available

$\vec{D} = \vec{D_\rho} \vec{a_\rho}$



radius $= \rho$

$z = 0$ to $z = l$

$Q = \oint_{cyl} \vec{D_s} \cdot \vec{ds}$

$= \int_{sides} \vec{D_s} \cdot \vec{ds} + \int_{top} \vec{D_s} \cdot \vec{ds} + \int_{bottom} \vec{D_s} \cdot \vec{ds}$

$\cos 90^\circ \quad \cos 90^\circ$

$Q = \int_{sides} \vec{D_s} \cdot \vec{ds} = D_s \int_{sides} \vec{ds}$

$= D_s \cdot \int \rho \, d\phi \, dz$

$= D_s \cdot \rho \, 2\pi \int_0^L dz$

$Q = D_s \cdot \rho (2\pi L)$

$D_s = \dfrac{Q}{\rho 2\pi L}$

$Q = \rho_L \cdot L$

$\boxed{D_s = \dfrac{\rho_L}{2\pi\rho}}$ → ~~electric field~~ flux denesity

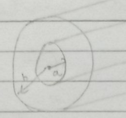
$\boxed{E_\rho = \dfrac{\rho_L}{2\pi\epsilon_0 \rho}}$ electric field intensity

$\Rightarrow \quad \psi = Q_{enclosed} = \vec{E} \cdot \vec{S} = ES\cos\theta$

when $\theta = 0^\circ$; $\quad \psi_{max}$

$\theta = 90^\circ$; $\quad \psi_{min}$

(3) co-axial wire:



inner radius $= a$

outer radius $= b$

ρ_s → charge distribution on outer surface of inner cord.

I. when $a < \rho < b$

$Q = D_s \cdot 2\pi\rho L$ —— (1)

inner conductor $\quad Q = \int_{z=0}^{L} \int_{\phi=0}^{2\pi} \rho_s \, a \, d\phi \, dz = 2\pi a L \rho_s$ —— (2)

(1) = (2)

$D_s 2\pi\rho L = 2\pi a L \rho_s$

$D_s = \dfrac{a\rho_s}{\rho}$

$\boxed{\vec{D_s} = \dfrac{a\rho_s}{\rho} \vec{a_\rho}} \quad a < \rho < b$

charge per unit length $\quad \rho_s = \dfrac{\rho_L}{2\pi a}$

$\boxed{\vec{D_s} = \dfrac{\rho_L}{2\pi\rho} \vec{a_\rho}}$ same as line charge

- flux density is independent of medium (ϵ_0)

Date: 1-Aug

charge on outer cylinder

$$\phi_{outer\ cy} = -2\pi a L \rho_{s\ inner\ cy}$$

$$2\pi b L \rho_{s\ outer\ cy.} = -2\pi a L \rho_{s\ inner\ cy.}$$

$$\rho_{s\ outer} = \frac{-a}{b} \rho_{s\ inner}$$

II. when $\rho > a$ & $\rho > b$

then flux density $= 0$

because changes on surface (+ve) & (-ve) $= 0$

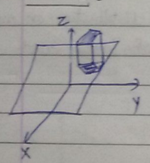
($D_s = 0$) (effect is nullified)

III. when $\rho < a$

then flux density $= 0$

($D_s = 0$)

④ Sheet:



$$\psi = Q_{enclosed}$$

$$Q = \rho_s \cdot ds$$

$$Q = \int \vec{D} \cdot d\vec{s}$$

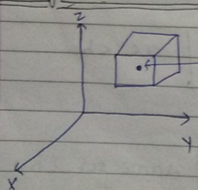
$$= D_z \int ds$$

$$\rho_s\, ds = D_z (2ds)$$

$$D_z = \frac{\rho_s}{2}$$

$$\boxed{\vec{D} = \frac{\rho_s}{2} \vec{a_z}}$$

Application of Gauss' law differential volume element:



- when Charge distribution is not-symmetrical

$$\vec{D} = D_{x0} \vec{a_x} + D_{y0} \vec{a_y} + D_{z0} \vec{a_z}$$

$$Q = \oint_S \vec{D} \cdot d\vec{s}$$

$$= + \int_{front} + \int_{back} + \int_{top} + \int_{bottom} + \int_{left} + \int_{right}$$

front side: $\int_{front} = \vec{D}_{front}\, \Delta\vec{s}_{front}$ (x-is perpendicular)

$$= \vec{D}_{front}\, \Delta y \Delta z\, \vec{a_x}$$

$$= D_{front}\, \Delta y\, \Delta z$$

$$D_{x,front} = D_{x0} + \frac{\Delta x}{2} \frac{\partial D_x}{\partial x}$$

$$\int_{front} = \left(D_{x0} + \frac{\Delta x}{2} \frac{\partial D_x}{\partial x} \right) \Delta y\, \Delta z$$

back side: $\int_{back} = \left(-D_{x0} + \frac{\Delta x}{2} \frac{\partial D_x}{\partial x} \right) \Delta y\, \Delta z$

$$\int_{front} + \int_{back} = \frac{\partial D_x}{\partial x} \cdot \Delta x\, \Delta y\, \Delta z$$

so; $\oint_S \vec{D} \cdot d\vec{s} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \right) \Delta x\, \Delta y\, \Delta z$

$\because \Delta V = \Delta x\, \Delta y\, \Delta z$

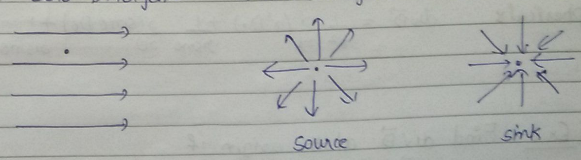
$$\boxed{\oint \vec{D} \cdot d\vec{s} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \right) \Delta V}$$

Partial diff of D (flux density)

Divergence:

how much this vector is flowing out from surface.

↳ Zero Divergence ↳ +ve Divergence ↳ -ve Divergence



- magnetic mono-poles does not exist.

Date 3-Aug

$$\nabla = \frac{\partial}{\partial x}\vec{a_x} + \frac{\partial}{\partial y}\vec{a_y} + \frac{\partial}{\partial z}\vec{a_z}$$

- The divergence of A at a given point P is the constant flux per unit volume as the volume about P.

$$\text{div } A = \nabla \cdot A = \lim_{\Delta v \to 0} \frac{\oint_s A \cdot ds}{\Delta v}$$

- divergence of the vector field A at a given point as a measure of how much the field diverges or emanates from the point.

$$* \quad \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \frac{\oint_s \vec{D}\cdot d\vec{s}}{\Delta v} = \frac{\Phi}{\Delta v}$$

(volume is shrinking)

$$= \lim_{\Delta v \to 0} \frac{\oint_s \vec{D}\cdot d\vec{s}}{\Delta v} = \lim_{\Delta v \to 0} \frac{\Phi}{\Delta v}$$

$$= \lim_{\Delta v \to 0} \rho_v$$

- "divergence" of any vector $\vec{A}$, outflow of flux from a closed surface as volume shrinks. results obtained as scalar.

$$\text{div}\vec{A} = \lim_{\Delta v \to 0} \frac{\oint_s \vec{A}\cdot d\vec{s}}{\Delta v}$$

rectangular: $\text{div }\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$

cylindrical: $\text{div }\vec{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$

spherical: $\text{div}\vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta D_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}$

Ex:- Find div $\vec{D}$ at the origin if

$$\vec{D} = e^{-x}\sin y\,\vec{a_x} - e^{-x}\cos y\,\vec{a_y} + 2z\,\vec{a_z}$$

Sol: $\text{div}\vec{D} = -e^{-x}\sin y + e^{-x}\sin y + 2$

$= 2 \quad C/m^3$

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Maxwell's first equation:

- for only electrostatics

$$\text{div }\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= \lim_{\Delta v \to 0} \frac{\oint_s \vec{D}\cdot d\vec{s}}{\Delta v}$$

$$\boxed{\text{div }\vec{D} = \rho_v} \text{ — (A)}$$

↳ its also called as point form of Gauss' law / Differential eqn form of Gauss' law

* Gauss law ← integral form of maxwell 1st eqn

charge enclosed by the surface = flux

Vector operator ∇:

$$\nabla = \frac{\partial \vec{a_x}}{\partial x} + \frac{\partial \vec{a_y}}{\partial y} + \frac{\partial \vec{a_z}}{\partial z}$$

$$\nabla \cdot \vec{D} = (\quad\quad)(D_x\vec{a_x} + D_y\vec{a_y} + D_z\vec{a_z})$$

$$\text{div }\vec{D} = \frac{\partial}{\partial x}D_x + \frac{\partial}{\partial y}D_y + \frac{\partial}{\partial z}D_z$$

Divergence theorem:

Gauss's law $\quad \Phi = \oint_s \vec{D}\cdot d\vec{s}$

$$\int \rho_v\, dv = \oint_s \vec{D}\cdot d\vec{s}$$

from eqn (A)

$$\boxed{\int \nabla\cdot\vec{D}\, dv = \oint_s \vec{D}\cdot d\vec{s}}$$

" Divergence theorem : the integral of normal component of any vector field over a closed surface is equal to the integral of the divergence of this vector field through out the volume ~~and~~ enclosed by the closed surface.

Ques- { given a vector, proved divergence surface

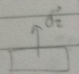
Example: A non uniform surface charge density of $\frac{5\rho}{\rho^2+1}$ nC/m² lies in the plane $z = 2$; where

$\rho < 5$

$$\rho_s = \begin{cases} 5\rho/\rho^2+1 & ; \rho \leq 5 \\ 0 & ; \rho > 5 \end{cases}$$

How much electric flux leaves the circular region $\rho < 5$; $z = 2$:

Sol:



$\rho d\rho d\phi$
ds

$$\Phi = \oint Q_{enclosed}$$

$$\Rightarrow \Phi = \int d\Phi$$

$$= \int \rho_s \cdot ds$$

$$= \int \rho_s . \rho \, d\rho \, d\phi$$

$$= \int_0^{2\pi} \int_0^5 \frac{5\rho^2}{\rho^2+1} d\rho \, d\phi$$

$$= 2\pi \cdot 5 \int_0^5 \frac{\rho^2+1-1}{1+\rho^2} d\rho$$

$$= 10\pi \left[\rho - \tan^{-1}\rho\right]_0^5$$

$$= 10\pi \left[5 - \tan^{-1}(5)\right]$$

$$= 113.39 \text{ nC/m}^2$$

Example: Given that $\vec{D} = z\rho\cos^2\phi \, \vec{a_z}$ C/m²

calculate the charge density at $(\overset{\rho}{1}, \overset{\phi}{\pi/4}, \overset{z}{3})$

the total charge enclosed by the cylinder of radius $\rho = 1$m with $-2 \leq z \leq 2$m

Sol:

~~$x = \rho\cos\phi = 1$~~

~~$y = \rho\sin\phi = \pi/4$~~

$$\rho_v = \nabla \cdot \vec{D}$$

$$= \rho\cos^2\phi$$

$$= \rho \cdot \cos^2(\pi/4) = 1/2 \text{ C/m}^3$$

$\frac{4\pi}{3}$

$$\Phi = \oint_V \rho_v \, dV$$

$$= \frac{1}{2} \int \rho^2 d\phi \, dz \, d\rho$$

$$= \frac{1}{2} \times \frac{\pi}{4} \int_{-2}^{2} \int_0^1 \rho^2 d\rho \, dz$$

$$= \frac{\pi}{8} \left(\frac{\rho^3}{2}\right)_0^1 \int_{-2}^{2} dz$$

$$= \frac{\pi}{8} \times \frac{1}{3} (2+2) = \frac{\pi}{}$$

$$=$$