

## 第三課

力的轉動效應 Turning effect of forces

周末班

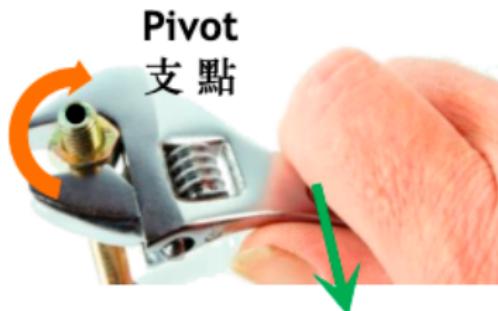
# 力矩 Moment of force/ torque

- 在剛體（有固定形狀和大小的物體）中，力能令剛體圍繞一點轉動。

In a rigid body (an object with a fixed shape and size), forces can let a rigid body rotate about a point.

- 作用力促使物體繞著支點轉動的趨向。

Tendency of rotation on a pivot by an external force.



# 力矩 Moment of force

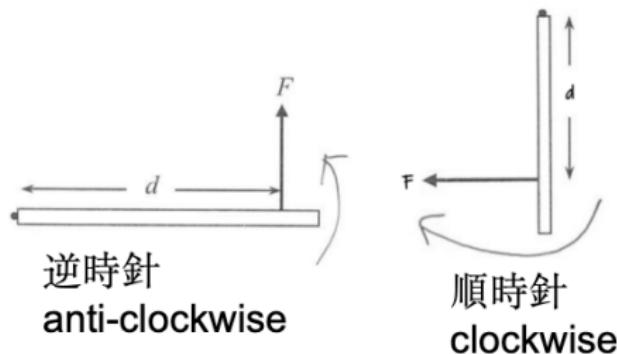
- 力矩是**矢量**，單位是牛頓米（N m）。  
Torques are **vectors**, its unit is Newton-meter (N m).
- 力矩的**方向**可以是順時針或逆時針，視乎它會令物體向哪個方向轉動。  
The **directions** of torques can be clockwise or anti-clockwise, it depends on how the force spins the object.
- 力矩 (moment of force) 又稱轉矩 (turning moment)。
- 而支點 (pivot) 又稱支軸或轉動軸心。

# 力矩 moment of force

## 力矩量值 magnitude of moment of force

$$\tau = d \times F \quad (1)$$

- $\tau$  [N m] = 力矩。Torque.
- $F$  [N] = 施力。Applied force.
- $d$  [m] = 施力至支點的垂直距離。  
Perpendicular distance of applied force from the pivot.



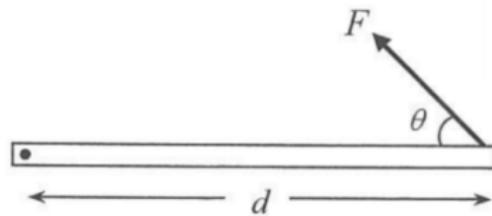
# 力矩 moment of force

當力矩的方向和剛體的幾何產生角度時，力矩的公式需要加上一個  $\sin \theta$  來表達。

When the direction of the torque is at an angle to the geometry of the object, the formula for torque needs to include an additional  $\sin \theta$  term.

## 力矩量值 magnitude of moment of force

$$\tau = d \times F \times \sin \theta \quad (2)$$

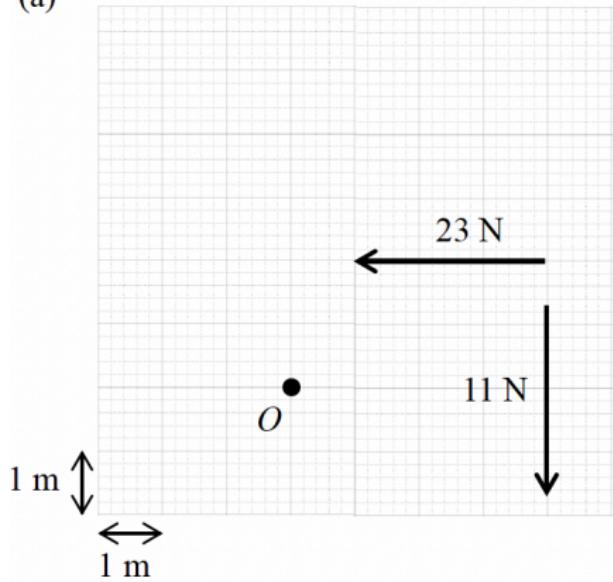


# 例題 Example

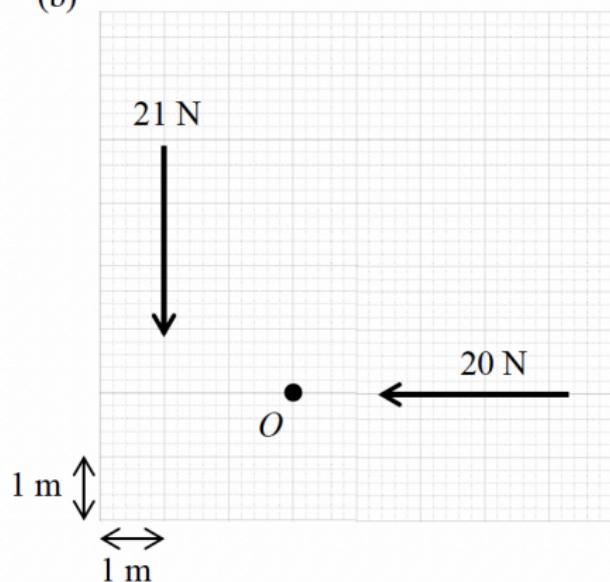
對於以下各力的系統中，求對 O 的**合力矩量值及方向**。

For each of the following systems of force, find the **magnitude** and the **direction of resultant moment** about O.

(a)



(b)

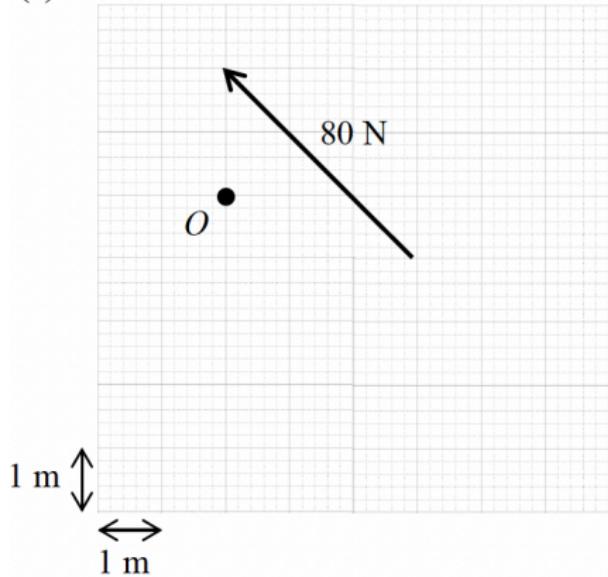


# 例題 Example

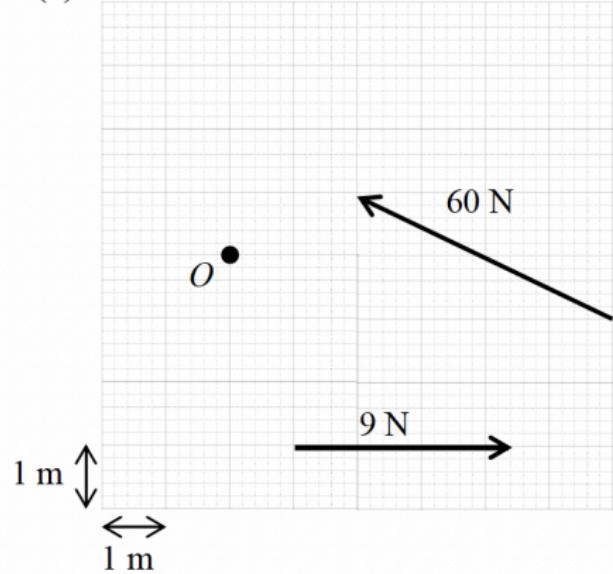
對於以下各力的系統中，求對 O 的**合力矩量值及方向**。

For each of the following systems of force, find the **magnitude** and the **direction of resultant moment** about O.

(c)

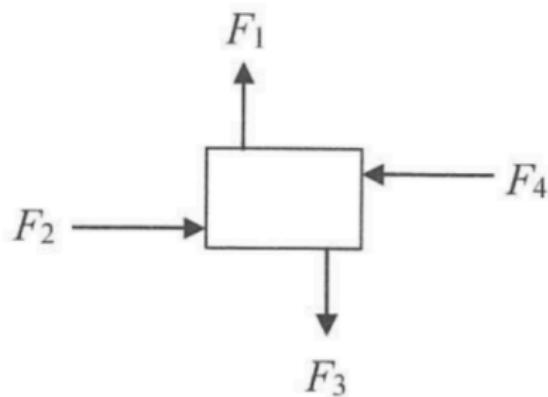


(d)



# 剛體平衡 Rigid body equilibrium

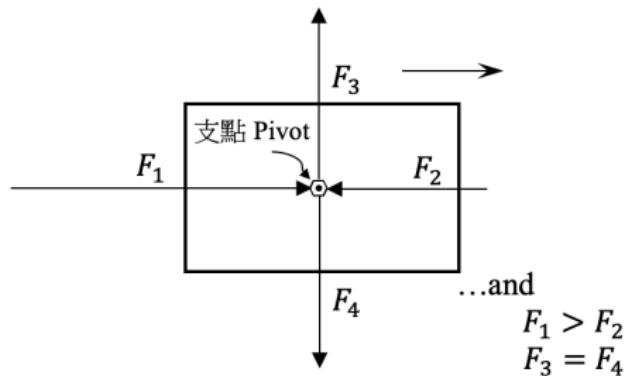
- 平移平衡 Translational equilibrium
  - 淨力 Net Force = 0



# 剛體平衡 Rigid body equilibrium

- 旋轉平衡 Rotational equilibrium

- ▶ 淨力矩 Net torque = 0



# 剛體平衡 Rigid body equilibrium

- 平衡狀態或靜止狀態：  
Equilibrium or at rest:

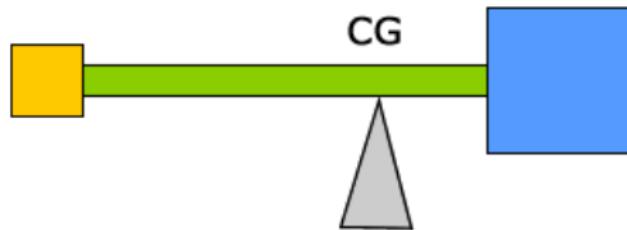
平移平衡  
Translational  
equilibrium  
= +  
旋轉平衡 Rotational  
equilibrium



# 重心 Center of gravity

- 每個剛體都有一個稱為重心（簡稱 CG）的固定點，剛體的全部重量好像都作用於這一點。

Every rigid body has a fixed point called the centre of gravity (abbr. C.G.), where the object's weight seems to act at.



# 重心 Center of gravity

- 均勻且對稱的物體（例如密度和直徑都恆定不變的棒），重心位於它的中心，至於不規則物體的重心，則可用實驗方法找出。

A uniform and symmetric object (e.g. a rod with uniform density and cross-section) has its C.G. at its centre, while the C.G. of an irregular object can be determined experimentally.



# 力矩題目計算步驟 Steps for problems

① 畫出自由體圖。

Draw the free body diagram.

② 選擇合適的支點。

Choose a suitable pivot for calculation.

③ 平衡順時針和逆時針的力矩。

Balance clockwise and counter-clockwise moments.

④ 平衡不同方向的力。

Balance forces of different directions.

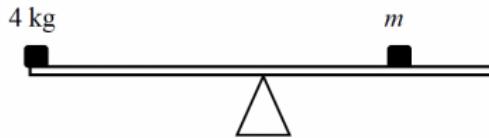
# 如何選擇合適的支點 How to choose a suitable pivot

- 在平衡狀態下，可以選擇任何一點作為支點。  
In equilibrium state, any point can be chosen as pivot point.
  - ▶ 計算過程中，所有通過支點作用的力會被忽略掉。  
All forces acting through pivot will be ignored in the calculation process.
- 在旋轉狀態下，只能選擇旋轉的支點進行計算。  
In rotating state, only the pivot of rotation can be chosen for calculation.

## 例題 Example

一塊質量可略的板水平放置在支撐點上。板上如圖放上質量分別為  $4\text{ kg}$  和  $m$  的小方塊後，仍保持水平。 $4\text{ kg}$  與支撐點相距  $80\text{ cm}$ ， $m$  與支撐點相距  $32\text{ cm}$ 。

A wooden plank with negligible mass is put on a support. After putting a mass of  $4\text{ kg}$  and  $m$  are put on the plank, the plank still keeps horizontal. The distance between the  $4\text{ kg}$  and the support is  $80\text{ cm}$ , that between  $m$  and the support is  $32\text{ cm}$ .



# 例題 Example

(a) 求  $m$  的值。

Find the value of  $m$ .

# 例題 Example

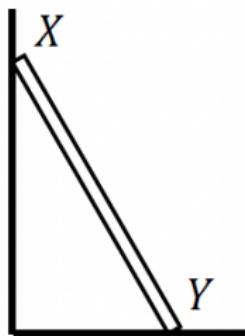
(b) 求支撐點對板施加的力的量值。

Find the magnitude of the force acting on the plank by the support.

## 例題 Example

圖中的梯子 XY 長 3 m，質量為 5 kg，斜倚牆上靜止不動，X 點離地 2 m。假設梯子是均質，且牆壁與梯子之間的摩擦力可略去不計。

The ladder XY in the figure has a length and mass of 3 m and 5 kg respectively, it leans on the wall and stays at rest, point X is 2 m from the ground. Assume the ladder is uniform and assume negligible friction between the wall and the ladder.



- (a) 繪畫梯子的隔離體圖。

Sketch the free-body diagram of the ladder.

## 例題 Example

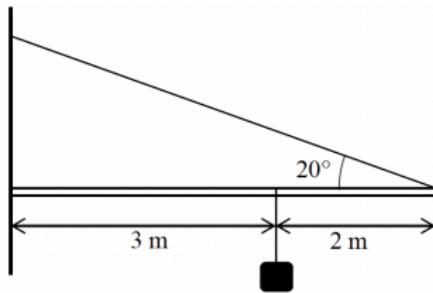
(b) 設地面作用於梯子的合力為  $R$ ，求  $R$  的量值和方向。

Let the resultant force acting on the ladder be  $R$ , find the magnitude and direction of  $R$ .

# 例題 Example

一根質量為 8 kg 的勻質木棒其中一端光滑鉸接在牆上，另一端以輕繩如圖懸掛。繩與水平之間的夾角是  $20^\circ$ 。在木棒上掛上一個質量為 5 kg 的物件。

A uniform wooden rod with mass 8 kg is hinged smoothly at one end on the wall and is hung by a light thread on the wall at the other end as shown. The thread is  $20^\circ$  above horizontal. A 5 kg object is hung on the rod.



求繩子及牆壁對木棒施加的力的量值。

Find the magnitudes of the forces acting on the rod.

# 例題 Example

sol.

# 力偶 Couples

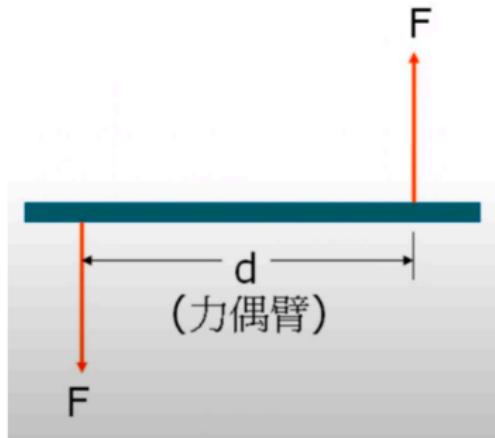
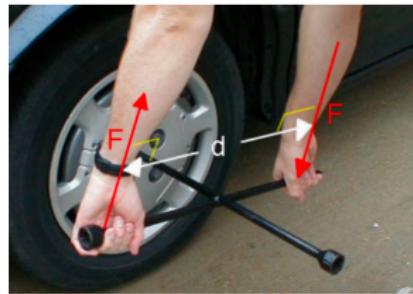
- 只有淨力矩，沒有淨力。  
There is only net moment, no net force.
- 物體只會在原地打轉，不會平移移動。  
The object will only rotate in place, it will not undergo translational motion.



# 力偶 Couples

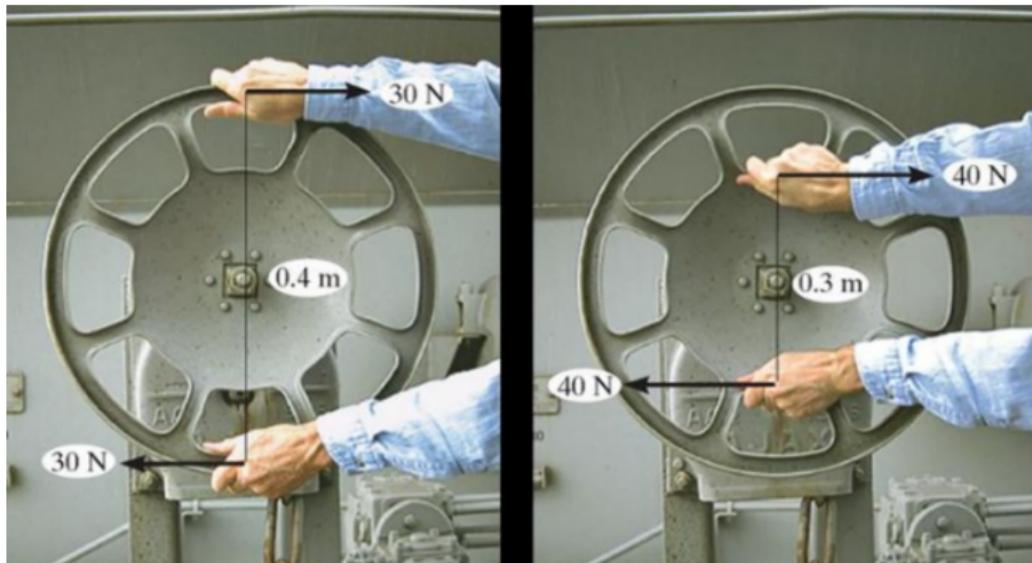
力偶的淨力矩 Net moment of couples

$$\tau = d \times F \quad (3)$$



# 力偶 Couples

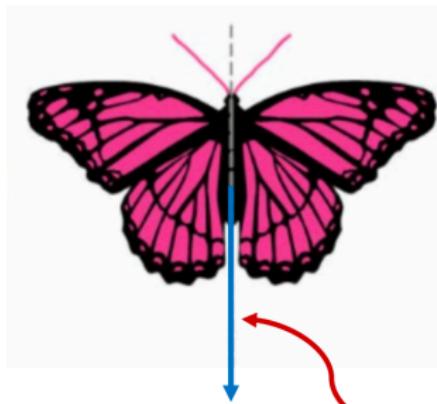
- $d$  越大，所需要的  $F$  越小。  
As  $d$  increases, the required  $F$  decreases.



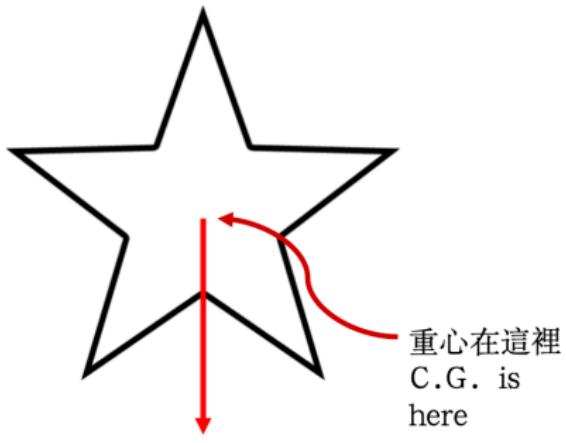
# 找重心的一些方法 Methods to find C.G.

- 找對稱特徵 (幾何中心) By symmetry (Geometric center)
- 鉛垂線 Using plumb lines
- 計算出來 By calculation (不考 Out-syl)

# 找對稱特徵 By symmetry



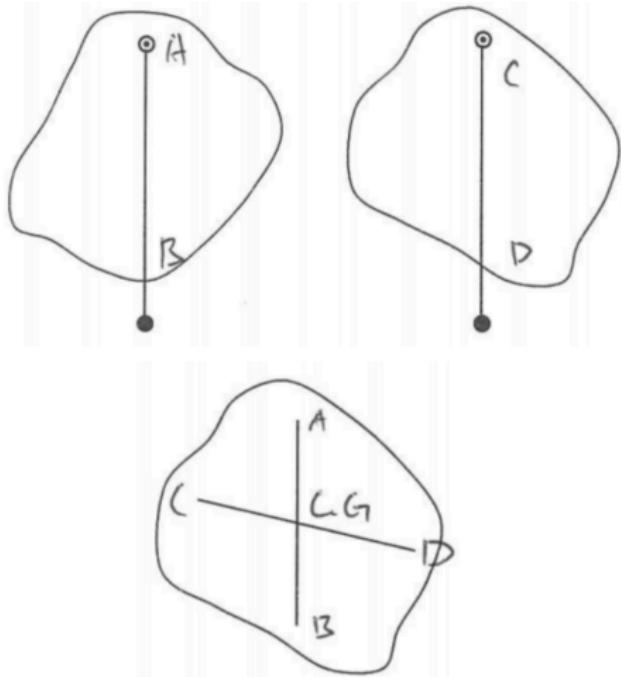
重心在這條直線上  
C.G. is along  
the line



重心在這裡  
C.G. is  
here

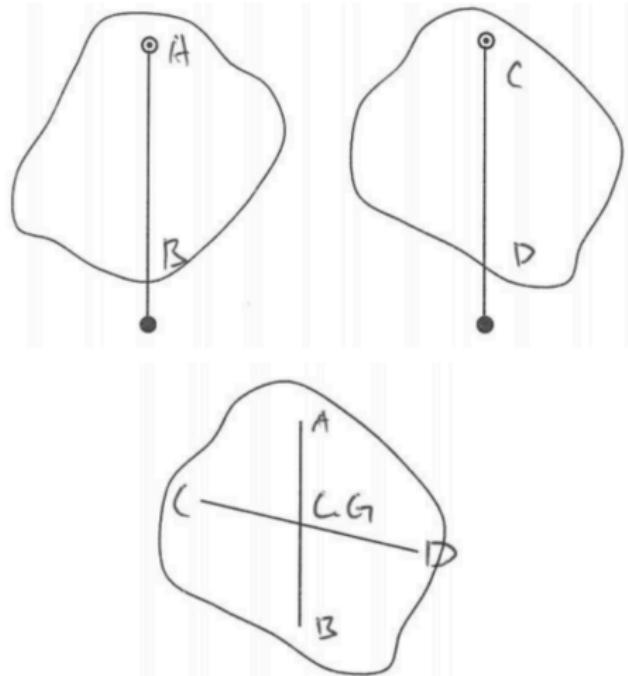
# 鉛垂線 Using plumb lines

- 把鉛垂線懸掛在點 A。  
Hang the plumb line at point A.
- 重心必須在線上的某一點。  
The center of gravity must be at a certain point on the line.
- 旋轉物件，把鉛垂線懸掛在另一點 C。  
Rotate the object, hang the plumb line at another point C.
- 重心 = 兩條線的交點。  
The center of gravity = the intersection point of the two lines.



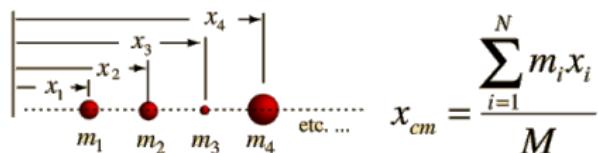
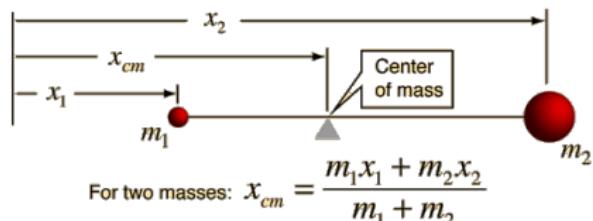
# 鉛垂線 Using plumb lines

- 把鉛垂線懸掛在點 A。  
Hang the plumb line at point A.
- 重心必須在線上的某一點。  
The center of gravity must be at a certain point on the line.
- 旋轉物件，把鉛垂線懸掛在另一點 C。  
Rotate the object, hang the plumb line at another point C.
- 重心 = 兩條線的交點。  
The center of gravity = the intersection point of the two lines.



直接計算出來 By calculation (Out-syl)

不考不考不考：

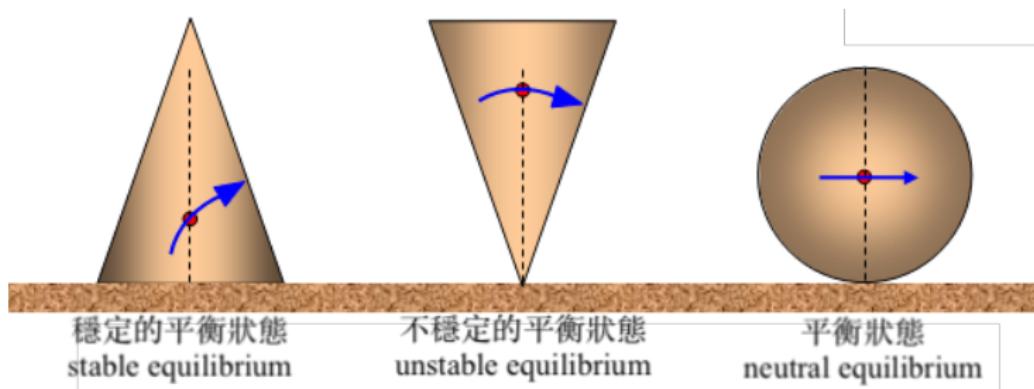


$$x_{cm} = \lim_{\Delta n \rightarrow 0} \frac{\sum_{i=1}^N \Delta m_i x_i}{M} = \frac{\int_0^M x dm}{M}$$

# 平衡狀態的穩定性 Equilibrium stability

- 給予一個小的推力，穩定平衡會趨向回到原始位置，而不穩定平衡則會傾倒。

Given a small push, a stable equilibrium would tend to go back to its original position, while unstable equilibrium would topple.



穩定的平衡狀態  
stable equilibrium

不穩定的平衡狀態  
unstable equilibrium

平衡狀態  
neutral equilibrium

# 平衡狀態的穩定性 Equilibrium stability

- 一般而言，可以通過增加底座尺寸和降低重心位置來提高穩定性。  
In general, better stability can be achieved by increasing the base size and decreasing the C.G. position.

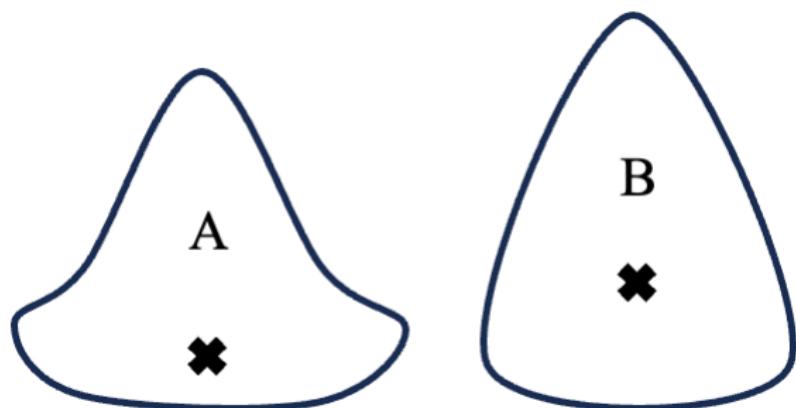


Figure: A 比 B 更不容易被推倒，因為 A 具有較大的底座和較低的重心。A is more difficult to topple than B because of its larger base and lower C.G. position.

# 滑雪 skiing

- 姿勢使重心靠下和靠後，防止向前傾倒。  
Positioning the center of gravity lower and further back helps prevent forward tipping.



# 不倒翁 roly poly toy

