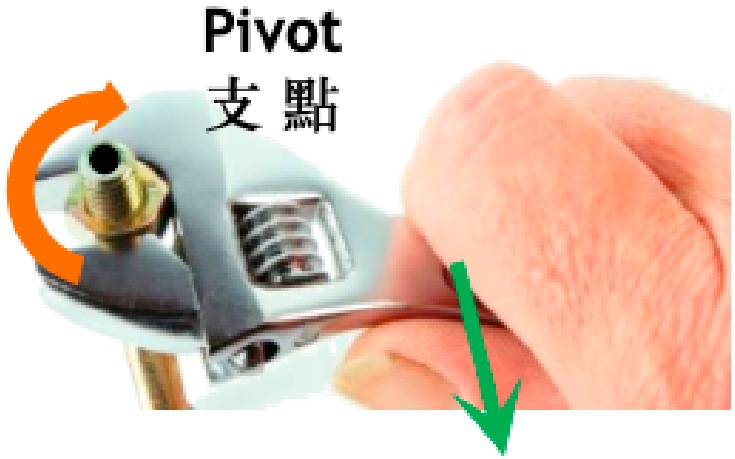
第三課

力的轉動效應 Turning effect of forces

周末班

力矩 Moment of force/ torque

- 在剛體（有固定形狀和大小的物體）中，力能令剛體圍繞一點轉動。
  In a rigid body (an object with a fixed shape and size), forces can let a rigid body rotate about a point.
- 作用力促使物體繞著支點轉動的趨向。
  Tendency of rotation on a pivot by an external force.



力矩 Moment of force

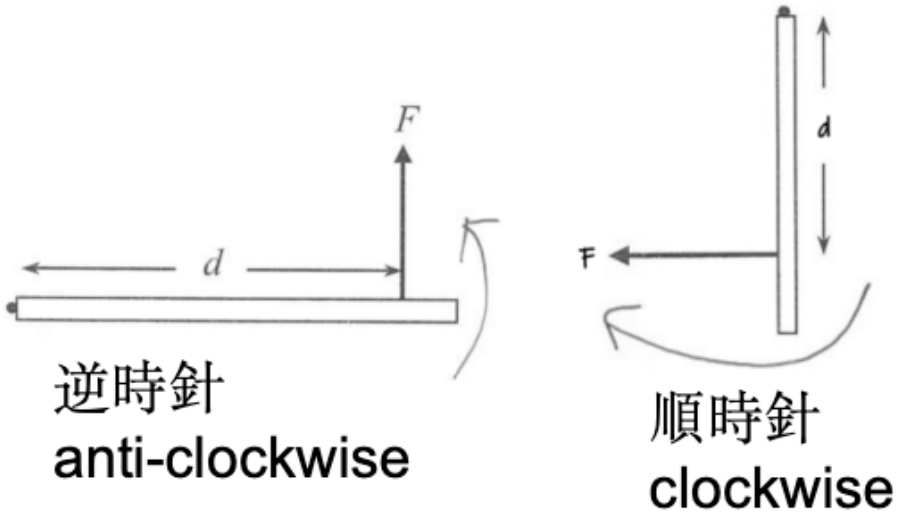
- 力矩是**矢量**，單位是牛頓米（N m）。
  Torques are **vectors**, its unit is Newton-meter (N m).
- 力矩的**方向**可以是順時針或逆時針，視乎它會令物體向哪個方向轉動。
  The **directions** of torques can be clockwise or anti-clockwise, it depends on how the force spins the object.
- 力矩（moment of force) 又稱轉矩 (turning moment)。
- 而支點 (pivot) 又稱支軸或轉動軸心。

力矩 moment of force

力矩量值 magnitude of moment of force

$$\tau = d \times F \tag{1}$$

- τ [N m] = 力矩。Torque.
- F [N] = **施力**。**Applied force**.
- d [m] = 施力至支點的**垂直距離**。
  Perpendicular distance of applied force from the pivot.

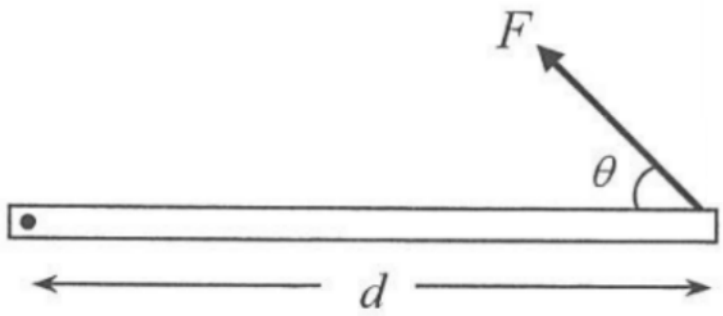


力矩 moment of force

當力矩的方向和剛體的幾何產生角度時，力矩的公式需要加上一個 $\sin\theta$ 來表達。
When the direction of the torque is at an angle to the geometry of the object, the formula for torque needs to include an additional $\sin\theta$ term.

力矩量值 magnitude of moment of force

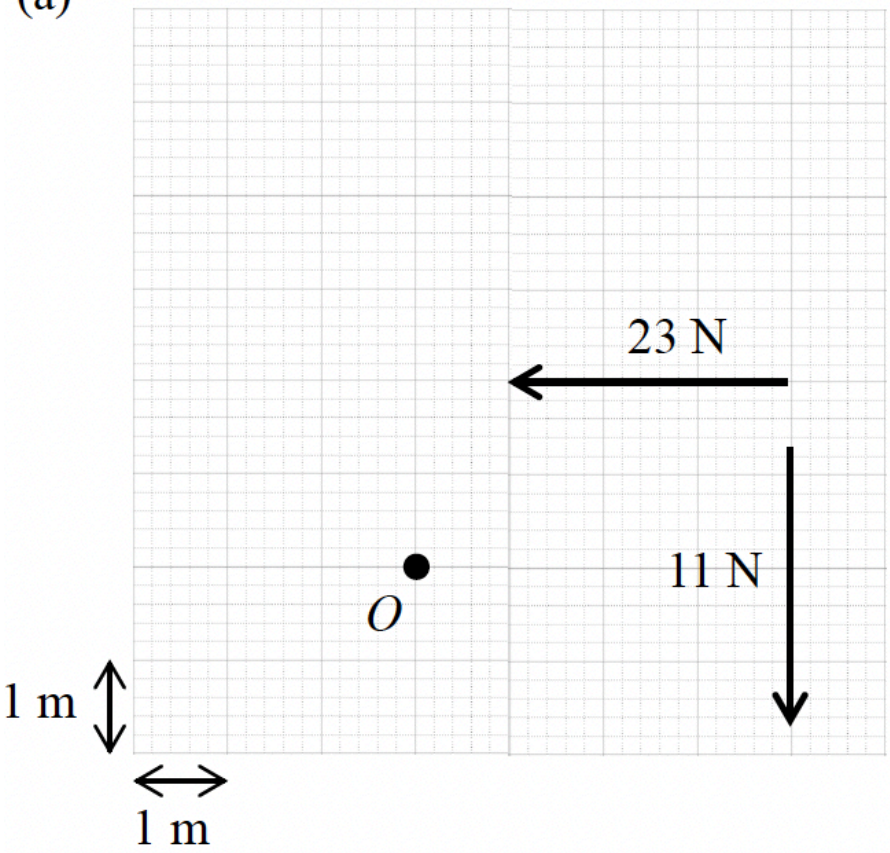
$$\tau = d \times F \times \sin\theta \tag{2}$$



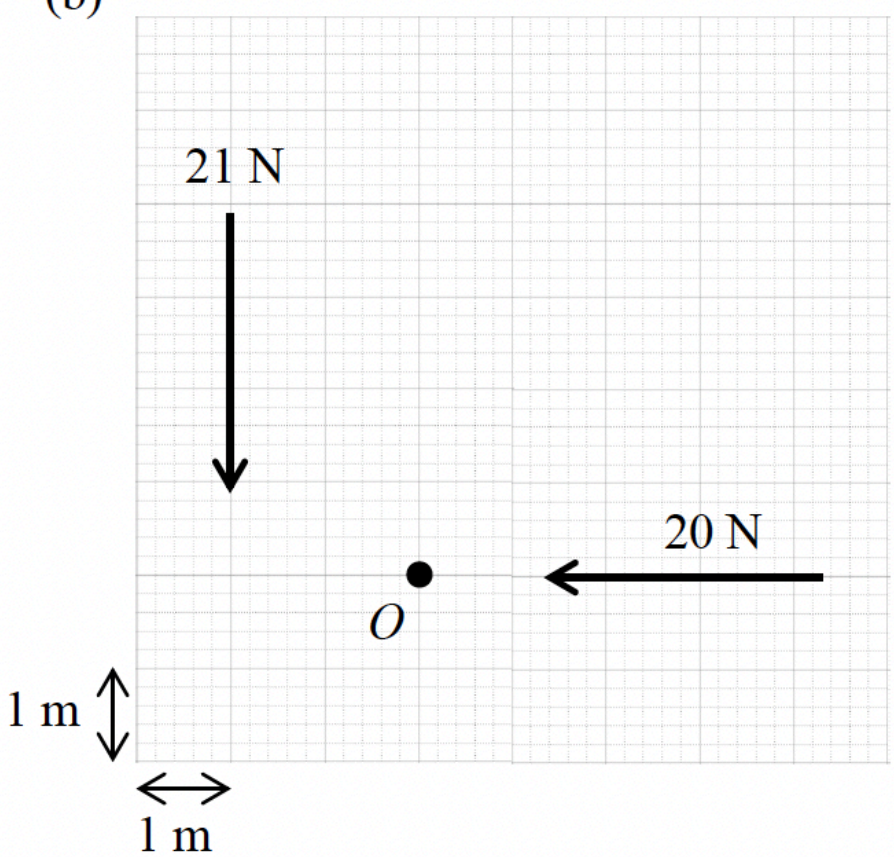
例題 Example

對於以下各力的系統中，求對 O 的**合力矩量值**及**方向**。
For each of the following systems of force, find the **magnitude** and the **direction** of **resultant moment** about O.

(a)



(b)



例題 Example

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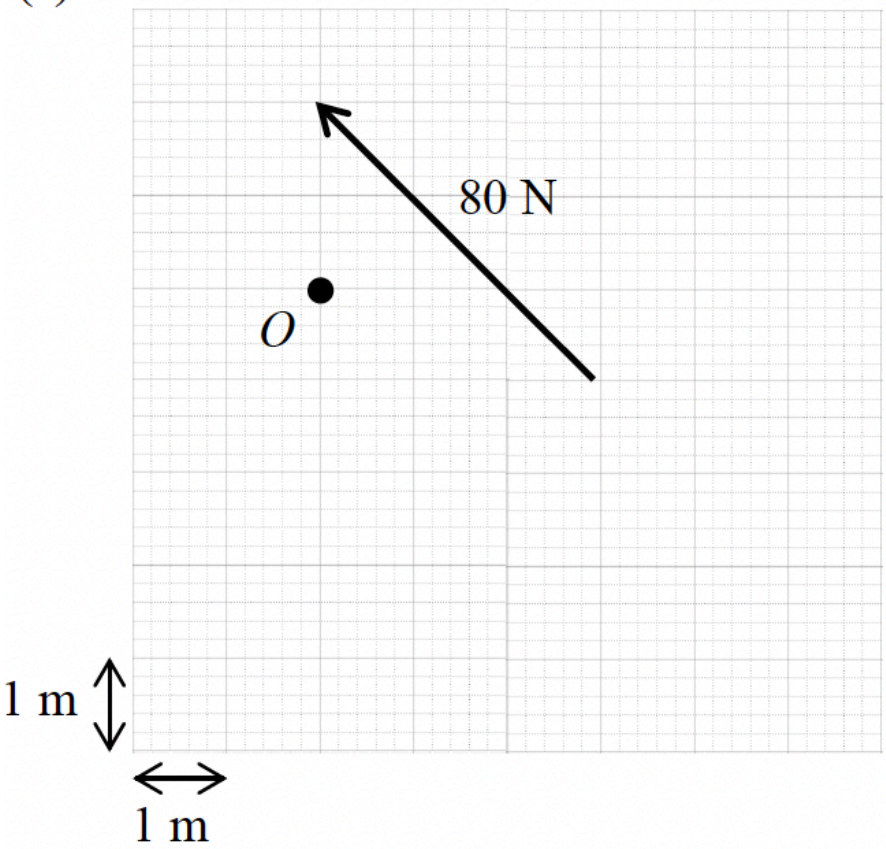
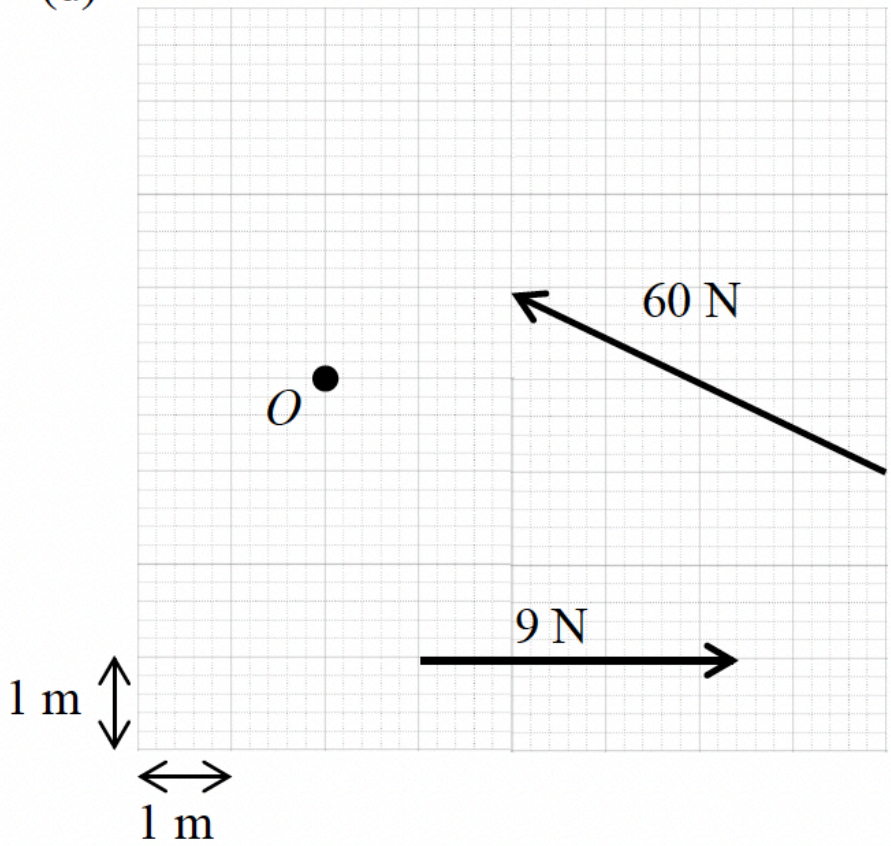
(c)

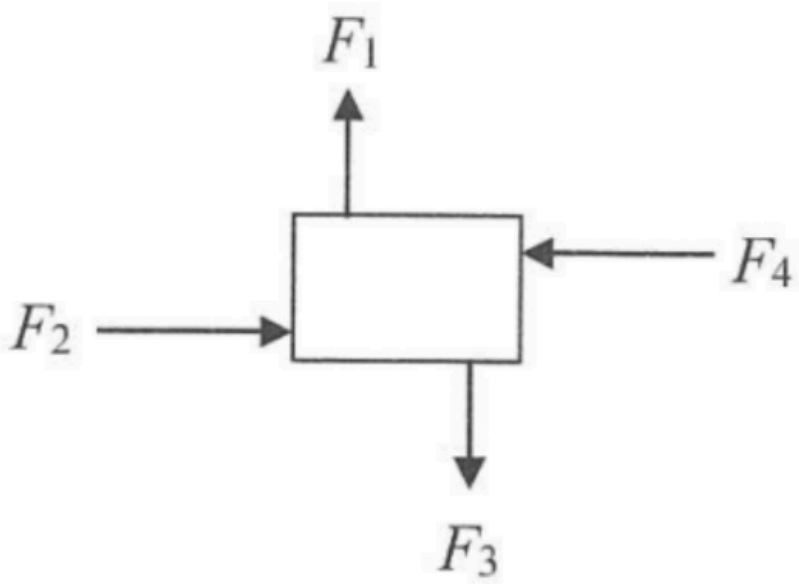


(d)



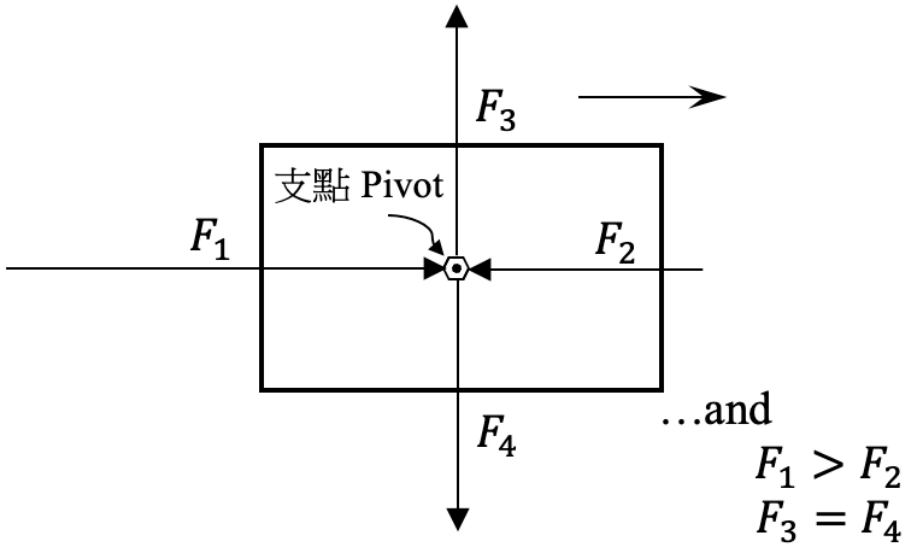
剛體平衡 Rigid body equilibrium

- 平移平衡 Translational equilibrium
  - 淨力 Net Force $= 0$



剛體平衡 Rigid body equilibrium

- 旋轉平衡 Rotational equilibrium
  - 淨力矩 Net torque = 0



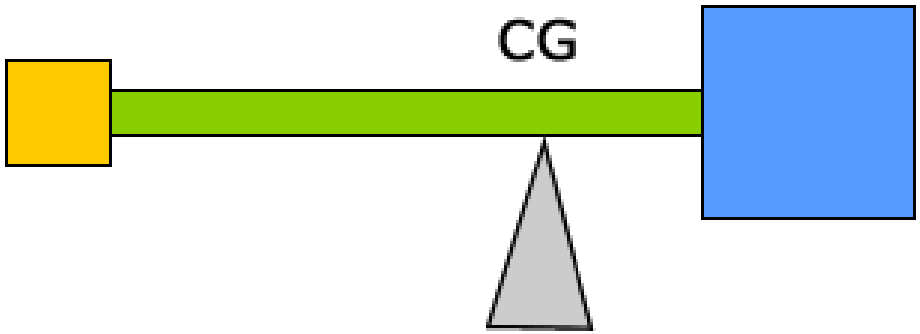
剛體平衡 Rigid body equilibrium

- **平衡**狀態或**靜止**狀態：
  Equilibrium or **at rest**:

$$= \begin{array}{c} \text{平移平衡} \\ \text{Translational} \\ \text{equilibrium} \\ + \\ \text{旋轉平衡 Rotational} \\ \text{equilibrium} \end{array}$$

重心 Center of gravity

- 每個剛體都有一個稱為重心（簡稱 CG）的固定點，剛體的全部重量好像都作用於這一點。
  Every rigid body has a fixed point called the centre of gravity (abbr. C.G.), where the object's weight seems to act at.



重心 Center of gravity

- 均勻且對稱的物體（例如密度和直徑都恆定不變的棒），重心位於它的中心，至於不規則物體的重心，則可用實驗方法找出。
  A uniform and symmetric object (e.g. a rod with uniform density and cross-section) has its C.G. at its centre, while the C.G. of an irregular object can be determined experimentally.

力矩題目計算步驟 Steps for problems

1. 畫出自由體圖。
   Draw the free body diagram.
2. 選擇合適的支點。
   Choose a suitable pivot for calculation.
3. 平衡順時針和逆時針的力矩。
   Balance clockwise and counter-clockwise moments.
4. 平衡不同方向的力。
   Balance forces of different directions.

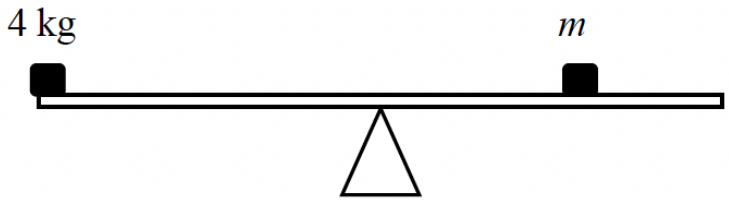
如何選擇合適的支點 How to choose a suitable pivot

- 在平衡狀態下，可以選擇任何一點作為支點。
  In equilibrium state, any point can be chosen as pivot point.
  - 計算過程中，所有通過支點作用的力會被忽略掉。
    All forces acting through pivot will be ignored in the calculation process.
- 在旋轉狀態下，只能選擇旋轉的支點進行計算。
  In rotating state, only the pivot of rotation can be chosen for calculation.

例題 Example

一塊質量可略的板水平放置在支撐點上。板上如圖放上質量分別為 4 kg 和 m 的小方塊後，仍保持水平。4 kg 與支撐點相距 80 cm，m 與支撐點相距 32 cm。

A wooden plank with negligible mass is put on a support. After putting a mass of 4 kg and m are put on the plank, the plank still keeps horizontal. The distance between the 4 kg and the support is 80 cm, that between m and the support is 32 cm.



例題 Example

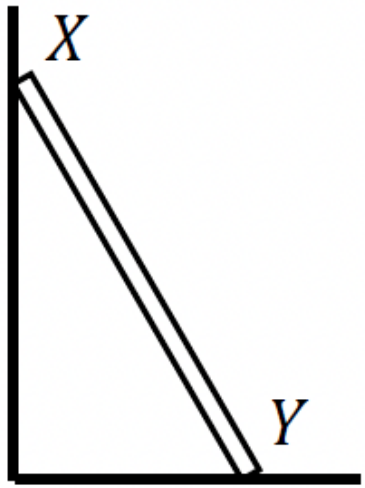
(a) 求 m 的值。
Find the value of m.

例題 Example

(b) 求支撐點對板施加的力的量值。
Find the magnitude of the force acting on the plank by the support.

例題 Example

圖中的梯子 XY 長 3 m，質量為 5 kg，斜倚牆上靜止不動，X 點離地 2 m。假設梯子是均質，且牆壁與梯子之間的摩擦力可略去不計。
The ladder XY in the figure has a length and mass of 3 m and 5 kg respectively, it leans on the wall and stays at rest, point X is 2 m from the ground. Assume the ladder is uniform and assume negligible friction between the wall and the ladder.



(a) 繪畫梯子的隔離體圖。
Sketch the free-body diagram of the ladder.

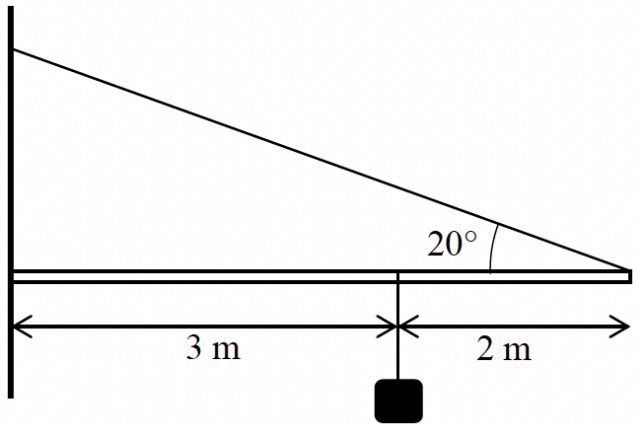
例題 Example

(b) 設地面作用於梯子的合力為 R，求 R 的量值和方向。
Let the resultant force acting on the ladder be R, find the magnitude and direction of R.

例題 Example

一根質量為 8 kg 的勻質木棒其中一端光滑鉸接在牆上，另一端以輕繩如圖懸掛。繩與水平之間的夾角是 $20°$ 。在木棒上掛上一個質量為 5 kg 的物件。

A uniform wooden rod with mass 8 kg is hinged smoothly at one end on the wall and is hung by a light thread on the wall at the other end as shown. The thread is $20°$ above horizontal. A 5 kg object is hung on the rod.



求繩子及牆壁對木棒施加的力的量值。

Find the magnitudes of the forces acting on the rod.

例題 Example

sol.

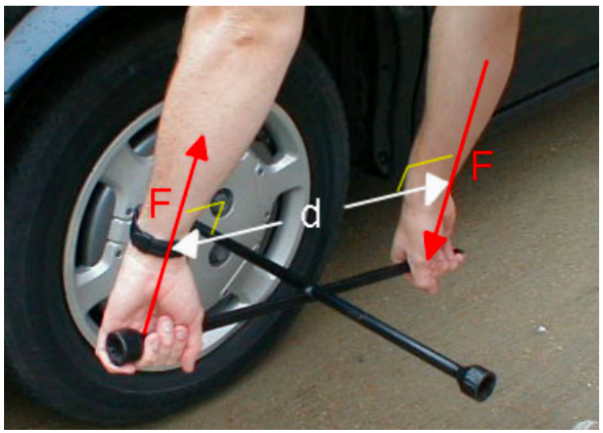
力偶 Couples

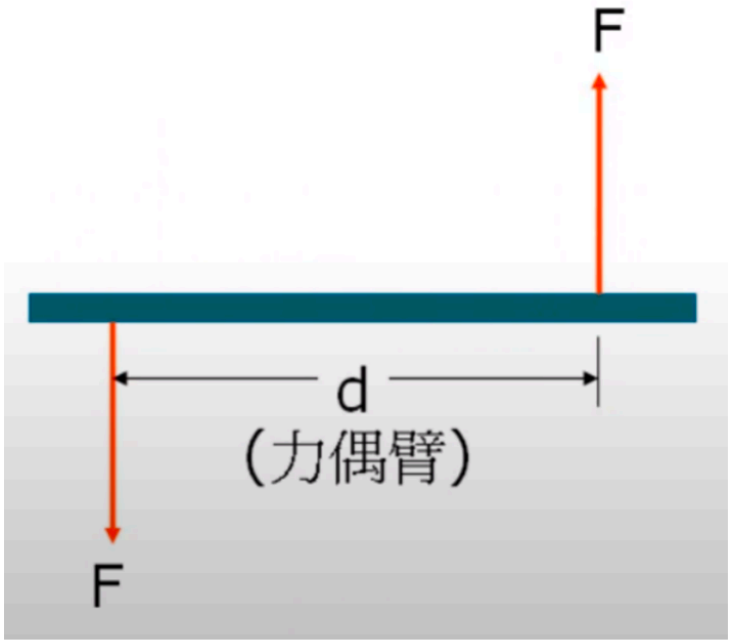
- 只有淨力矩，沒有淨力。
  There is only net moment, no net force.
- 物體只會在原地打轉，不會平移移動。
  The object will only rotate in place, it will not undergo translational motion.

力偶 Couples

力偶的淨力矩 Net moment of couples

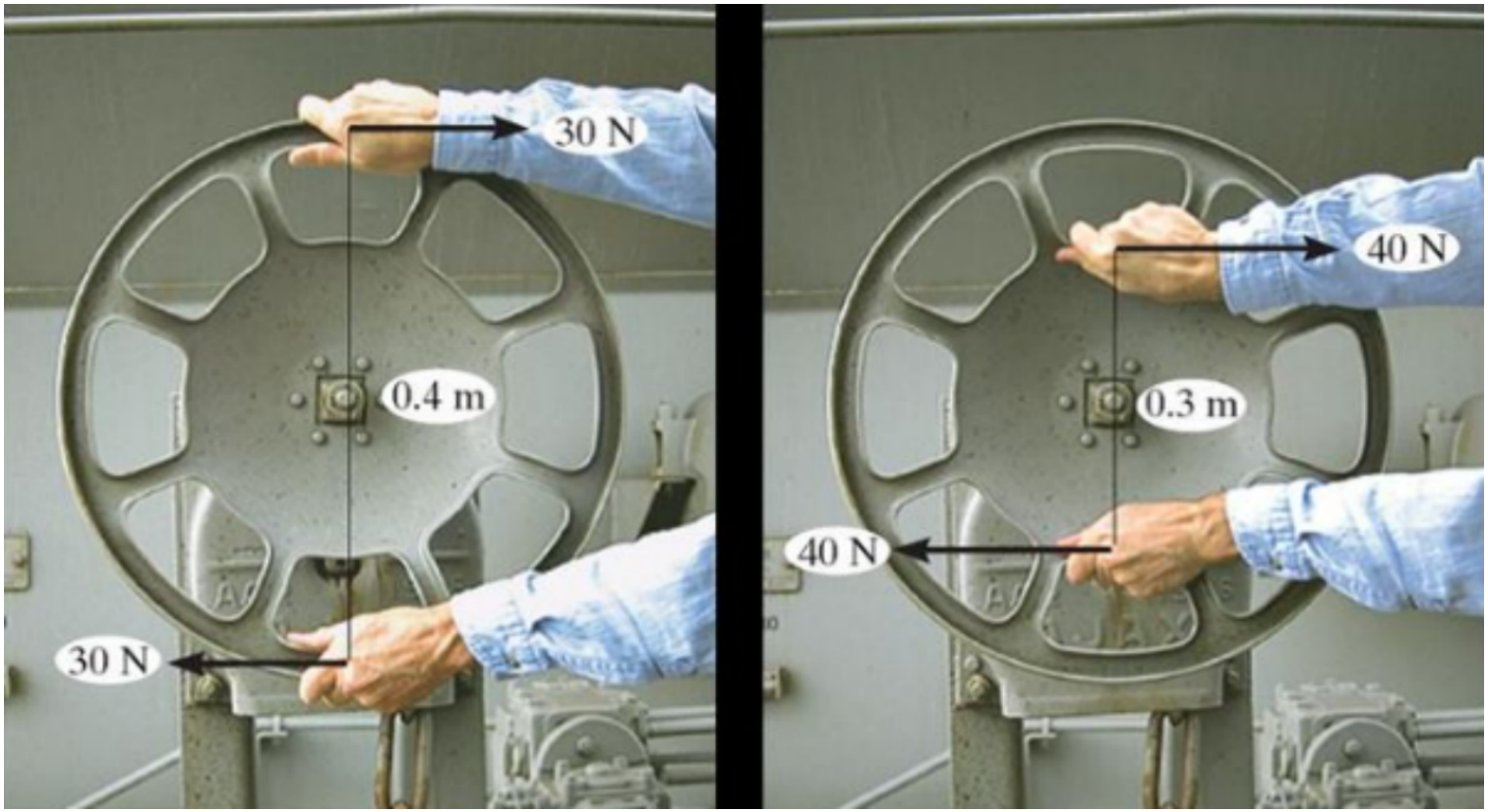
$$\tau = d \times F \tag{3}$$





力偶 Couples

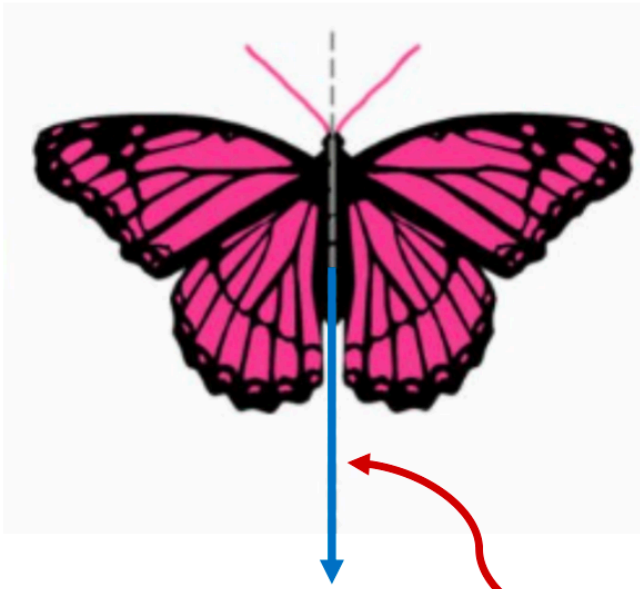
- d 越大，所需要的 F 越小。
  As d increases, the required F decreases.



找重心的一些方法 Methods to find C.G.

- 找對稱特徵 (幾何中心) By symmetry (Geometric center)
- 鉛垂線 Using plumb lines
- 計算出來 By calculation (不考 Out-syl)

找對稱特徵 By symmetry



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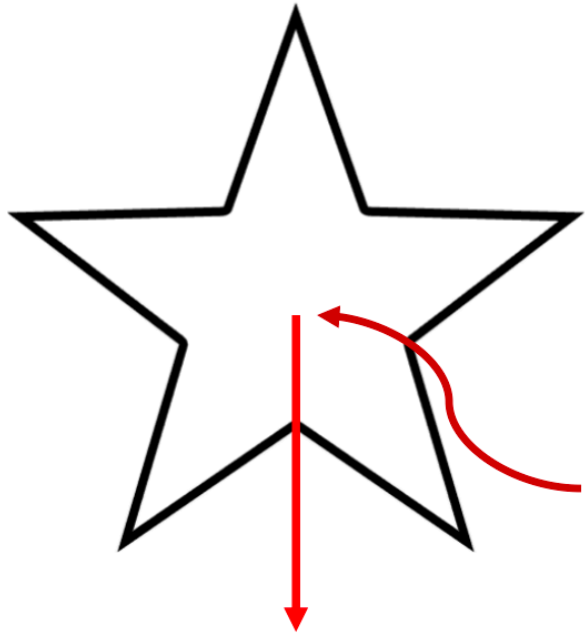
重心在這條直線上
C.G. is along
the line

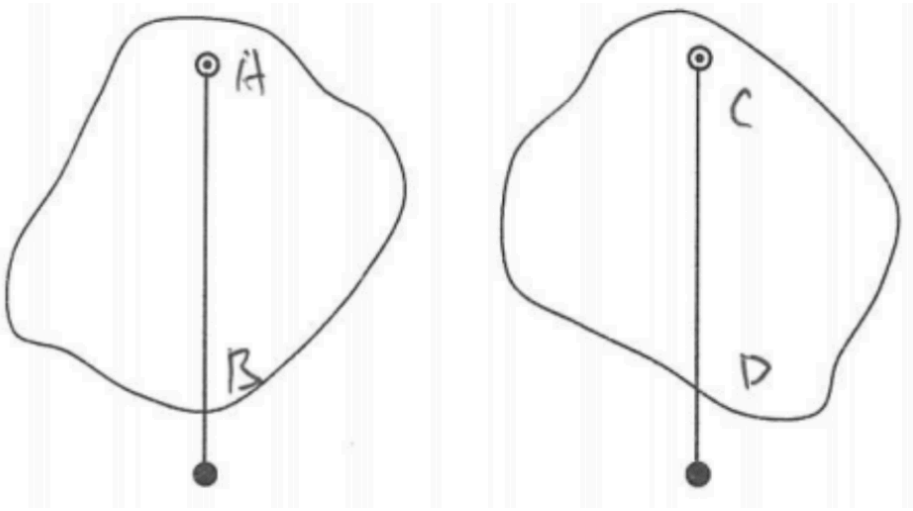


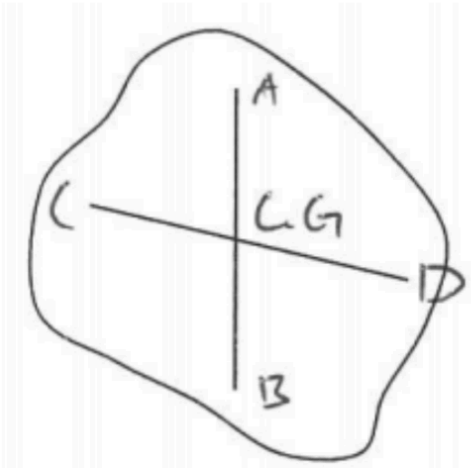
mg

重心在這裡
C.G. is
here

鉛垂線 Using plumb lines

- 把鉛垂線懸掛在點 A。
  Hang the plumb line at point A.
- 重心必須在線上的某一點。
  The center of gravity must be at a certain point on the line.
- 旋轉物件，把鉛垂線懸掛在另一點 C。
  Rotate the object, hang the plumb line at another point C.
- 重心 = 兩條線的交點。
  The center of gravity = the intersection point of the two lines.





鉛垂線 Using plumb lines

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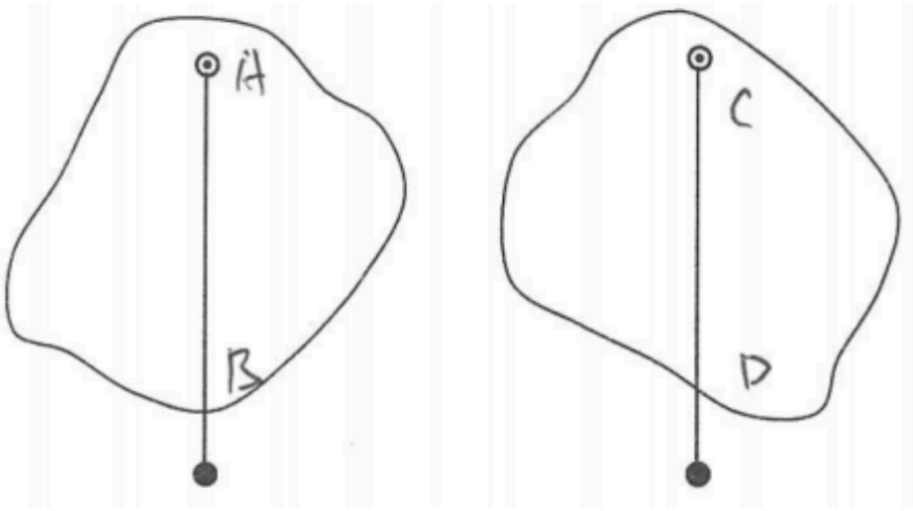

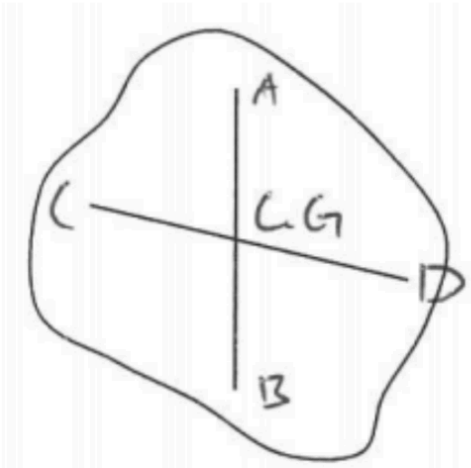

直接計算出來 By calculation (Out-syl)

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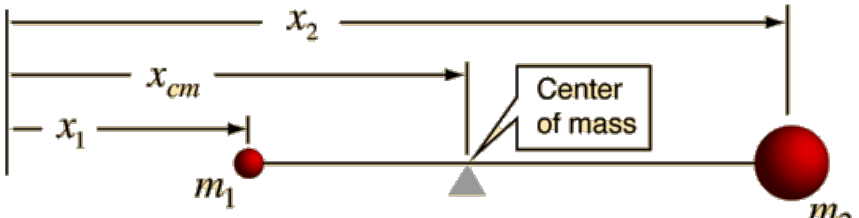
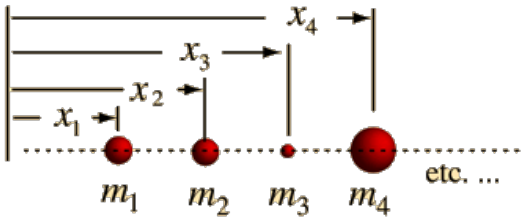


For two masses: $x_{cm} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

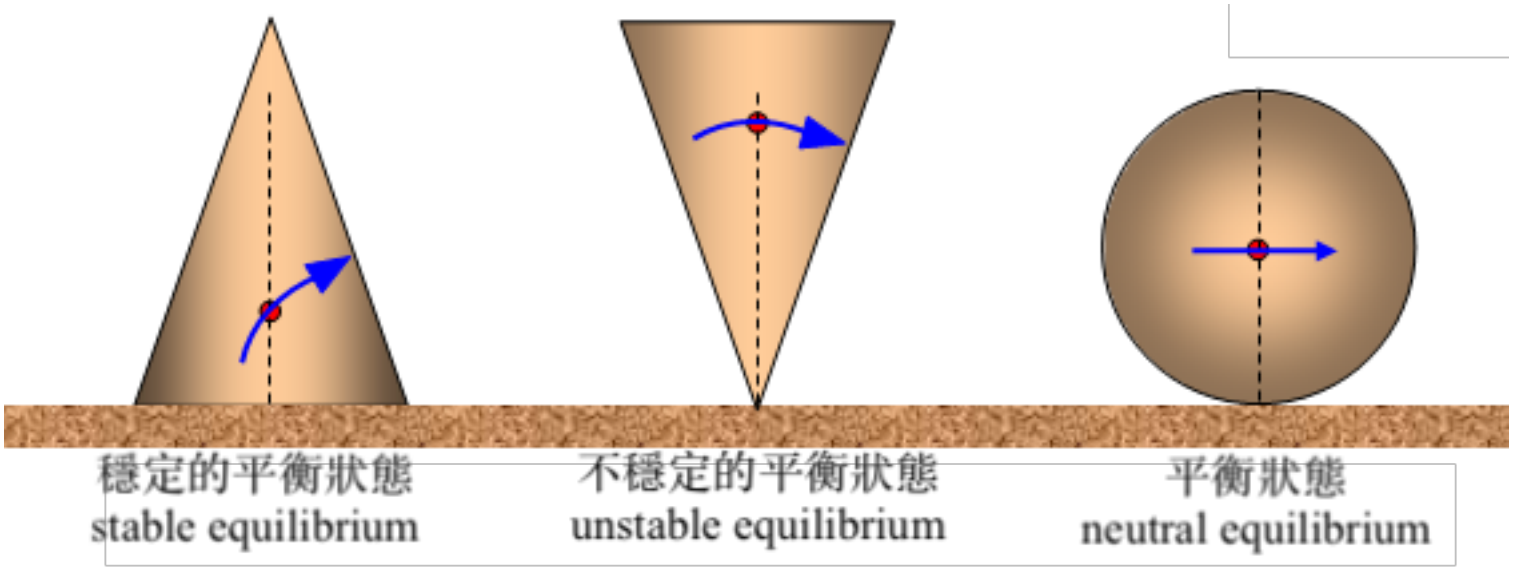


$$x_{cm} = \frac{\sum_{i=1}^{N} m_i x_i}{M}$$

$$x_{cm} = \lim_{\Delta m \to 0} \frac{\sum_{i=1}^{N} \Delta m_i x_i}{M} = \frac{\int_0^M x\, dm}{M}$$

平衡狀態的穩定性 Equilibrium stability

- 給予一個小的推力，穩定平衡會趨向回到原始位置，而不穩定平衡則會傾倒。
  Given a small push, a stable equilibrium would tend to go back to its original position, while unstable equilibrium would topple.



平衡狀態的穩定性 Equilibrium stability

- 一般而言，可以通過增加底座尺寸和降低重心位置來提高穩定性。In general, better stability can be achieved by increasing the base size and decreasing the C.G. position.

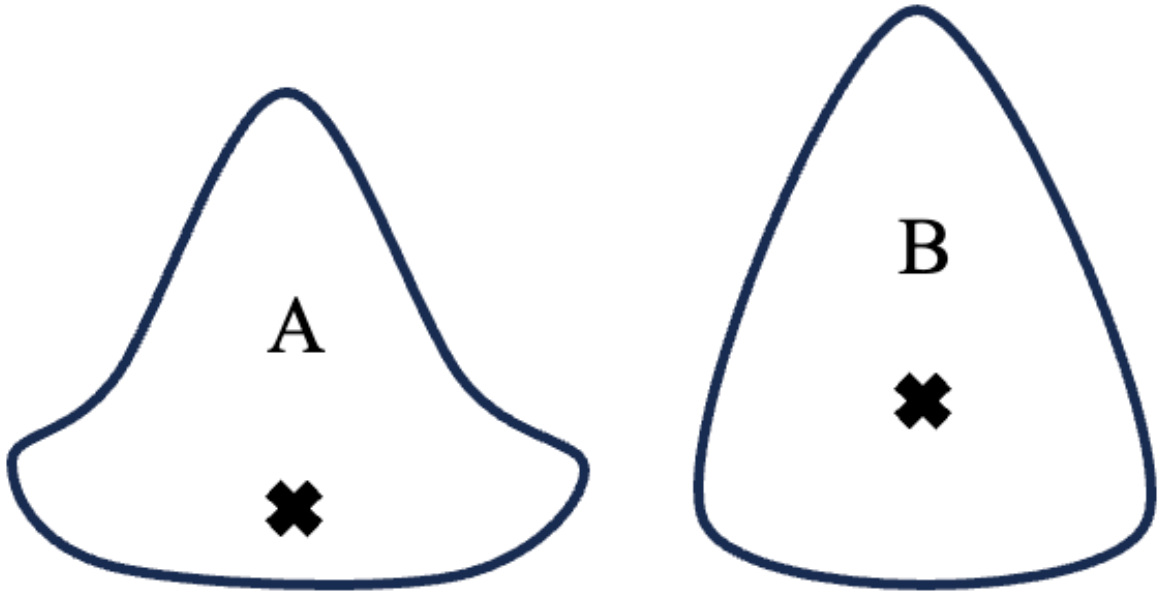


Figure: A 比 B 更不容易被推倒，因為 A 具有較大的底座和較低的重心。A is more difficult to topple than B because of its larger base and lower C.G. position.

滑雪 skiing

- 姿勢使重心靠下和靠後，防止向前傾倒。
  Positioning the center of gravity lower and further back helps prevent forward tipping.

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