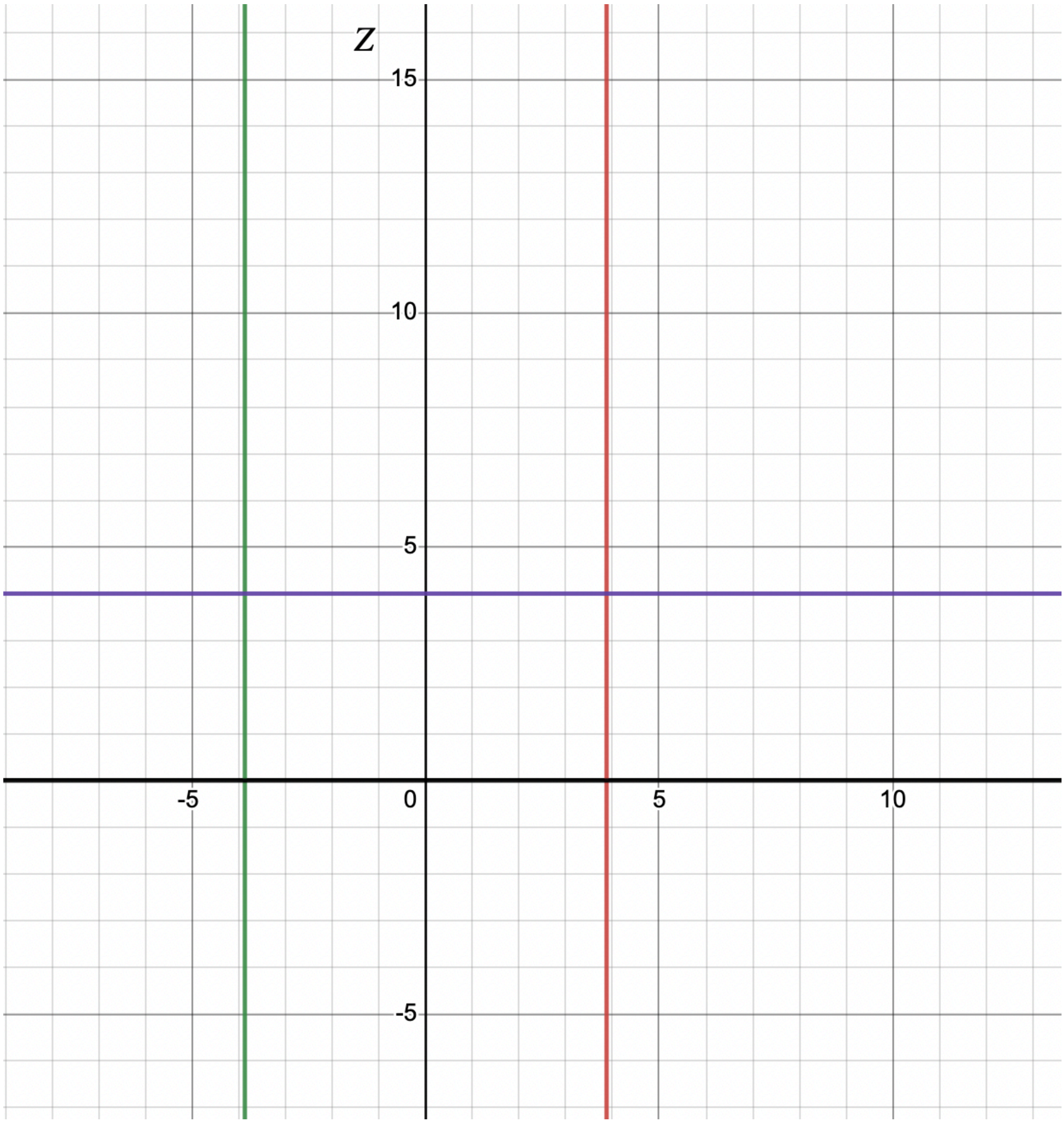
№9 М.А. Некрасов Артём 216

1. Привести тройной интеграл $\iiint_D f(x, y, z)dxdydz$ *к одному из повторных, где* $D = \{(x, y, z) | y^2 \le z \le 4, x^2 + y^2 \le 16\}$

<u>Решение.</u>

$y = y_0 -$ фиксируем:

$$\begin{cases} y_0^2 \le z \le 4 \\ x^2 + y_0^2 \le 16 \end{cases}$$



$$\iiint_D f(x,y,z)dxdydz = \int_2^2 \left(\int_{-\sqrt{16-y_0^2}}^{\sqrt{16-y_0^2}} \left(\int_{y_0^2}^{4} f(x,y,z)dz\right)dx\right)dy$$

2. Изменить порядок интегрирования в повторном интеграле всеми возможными способами:

$$\int_0^1 dx \int_0^{\sqrt{1-x^2}} dy \int_0^{x^2+y} f(x,y,z)dz$$

<u>Решение.</u>

xyz: $\int_0^1 dx \int_0^{\sqrt{1-x^2}} dy \int_0^{x^2+y} f(x,y,z)dz = \int_0^1 dx \int_0^{\sqrt{1-x^2}} dy \int_0^{x^2+y} f(x,y,z)dz$

xzy: $\int_0^1 dx \int_0^{\sqrt{1-x^2}} dy \int_0^{x^2+y} f(x,y,z)dz = \int_0^1 dx \left(\int_0^{x^2} dz \int_0^{\sqrt{1-x^2}} f(x,y,z)dy + \int_{x^2}^1 dz \int_{z-x^2}^{\sqrt{1-x^2}} f(x,y,z)dy\right)$

yxz: $\int_0^1 dx \int_0^{\sqrt{1-x^2}} dy \int_0^{x^2+y} f(x,y,z)dz = \int_0^1 dy \int_0^{\sqrt{1-y^2}} dx \int_0^{x^2+y} f(x,y,z)\,dz$

yzx: $\int_0^1 dx \int_0^{\sqrt{1-x^2}} dy \int_0^{x^2+y} f(x,y,z)dz = \int_0^1 dy \left(\int_0^{y} dz \int_0^{\sqrt{1-y^2}} f(x,y,z)dx + \int_y^{1-y^2+y} dz \int_{\sqrt{z-y}}^{\sqrt{1-y^2}} f(x,y,z)dx\right)$

zxy: $\int_0^1 dx \int_0^{\sqrt{1-x^2}} dy \int_0^{x^2+y} f(x,y,z)dz = \int_0^1 dz \left(\int_{\sqrt{z}}^1 dx \int_0^{\sqrt{1-x^2}} f(x,y,z)dy + \int_0^{\sqrt{z}} dx \int_0^{\sqrt{1-x^2}} f(x,y,z)dy\right)$

zyx: $\int_0^1 dx \int_0^{\sqrt{1-x^2}} dy \int_0^{x^2+y} f(x,y,z)dz = \int_0^1 dz \left(\int_0^{\sqrt{1-z}} dy \int_{\sqrt{z}}^1 f(x,y,z) + \int_0^1 dy \int_0^{\sqrt{z}} f(x,y,z)dx\right)$

3. Вычислить интеграл:

$$\int_0^4 dz \int_{-z}^{z} dx \int_0^{\sqrt{z^2-x^2}} z^2xy^2dy$$

<u>Решение.</u>

$I)$ $\int_0^{\sqrt{z^2-x^2}} z^2xy^2dy = z^2x \int_0^{\sqrt{z^2-x^2}} y^2dy = z^2x \cdot \frac{y^3}{3}\Bigg|_0^{\sqrt{(z^2-x^2)}} = z^2x \cdot \frac{\sqrt{(z^2-x^2)^3}}{3}$

$II)$ $\int_{-z}^{z} z^2x \cdot \frac{(z^2-x^2)^{\frac{3}{2}}}{3}dx = \frac{z^2}{3}\int_{-z}^{z} x\sqrt{z^2-x^2}\,dx \Rightarrow \left|\begin{matrix} t = z^2-x^2 \\ dt = -2xdx \\ dx = -\frac{1}{\quad}dt \end{matrix}\right| \Rightarrow -\frac{z^2}{6}\int_0^0 \sqrt{t}dt = 0$

$$\left. dx = \frac{dt}{-2x} \right]$$

$$III) \int_0^4 0dz = 0$$

4. Вычислить интеграл:

$$\int_0^1 dx \int_x^1 dy \int_y^1 e^{z^3} dz$$

<u>Решение.</u>

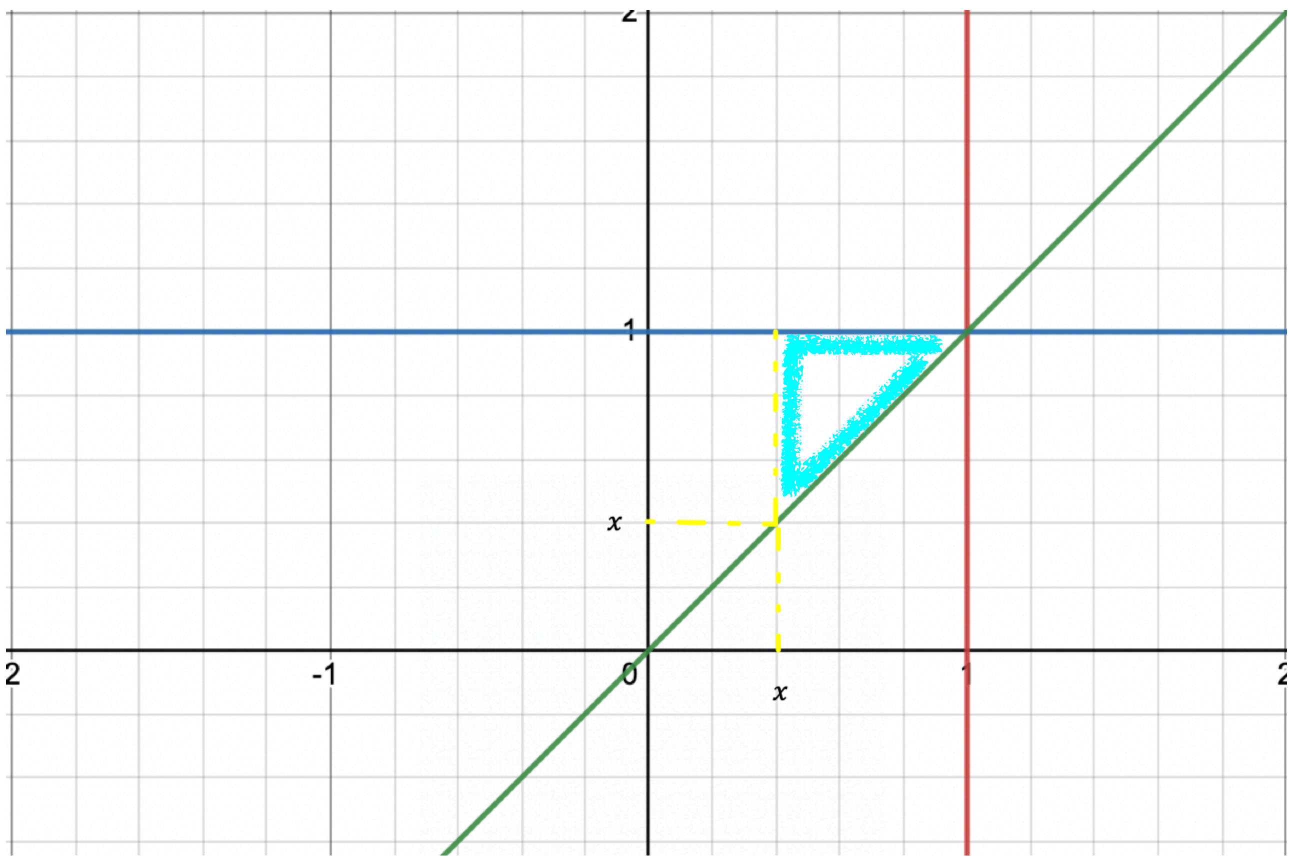
$$\int_0^1 dx \int_x^1 dy \int_y^1 e^{z^3} dz = \int_0^1 dx \int_x^1 dz \int_x^z e^{z^3} dy = \int_0^1 dx \int_x^1 e^{z^3}(z-x) dz = \int_0^1 dz \int_0^z e^{z^3}(z-x) dx =$$

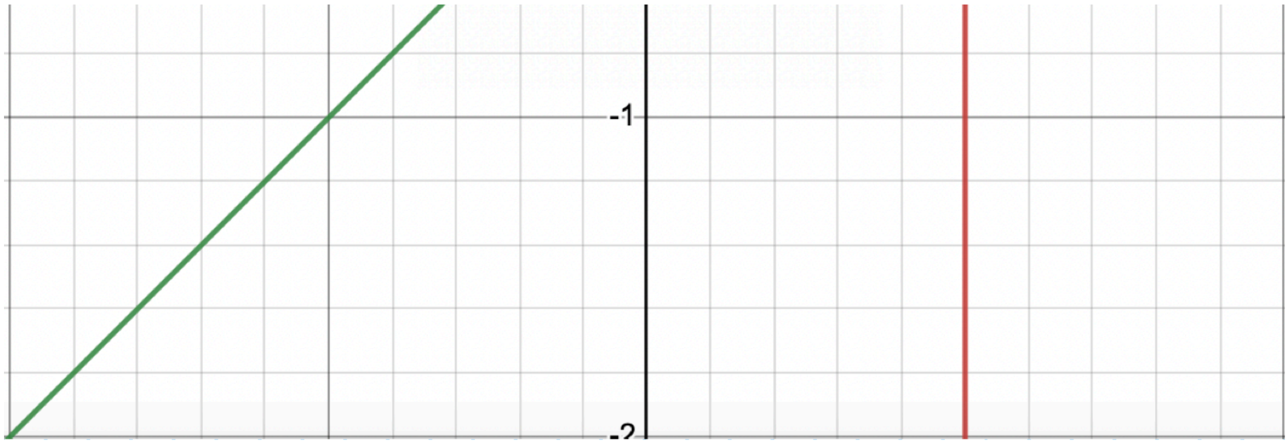
$$= \int_0^1 dz \left(\int_0^z z e^{z^3} - \int_0^z x e^{z^3} \right) dx = \int_0^1 z^2 e^{z^3} dz - \int_0^1 e^{z^3} \cdot \frac{z^2}{2} dz = \int_0^1 z^2 e^{z^3} dz - \frac{1}{2}\int_0^1 e^{z^3} z^2 dz = \frac{1}{2}\int_0^1 e^{z^3} z^2 dz \Rightarrow$$

$$\begin{bmatrix} du = 3z^2 dz \\ dz = \dfrac{du}{3z^2} \end{bmatrix} \Rightarrow \frac{1}{2}\int_0^1 \frac{e^u}{3} du = \frac{1}{2} \cdot \frac{1}{3} \cdot (e^1 - e^0) = \frac{e}{6} - \frac{1}{6} \int_0^1 dx \int_x^1 dy \int_y^1 e^{z^3} dz = \int_0^1 dx \int_x^1 dz \int_x^z e^{z^3} dy =$$

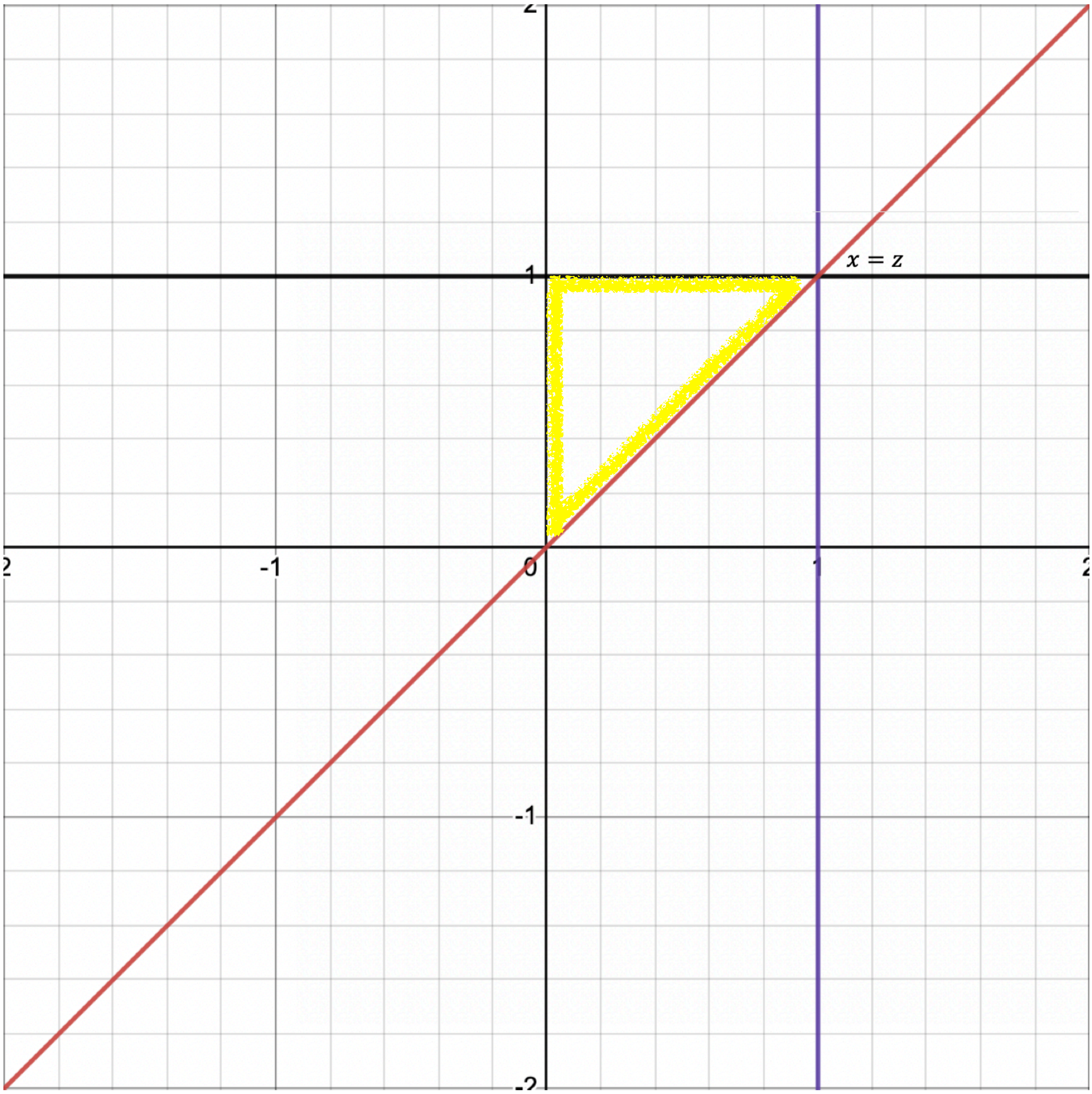
$$= \int_0^1 dx \int_x^1 e^{z^3}(z-x) dz = \int_0^1 dz \int_0^z e^{z^3}(z-x) dx = \int_0^1 dz \left(\int_0^z z e^{z^3} - \int_0^z x e^{z^3} \right) dx = \int_0^1 z^2 e^{z^3} dz - \int_0^1 e^{z^3} \cdot \frac{z^2}{2} dz =$$

$$= \int_0^1 z^2 e^{z^3} dz - \frac{1}{2}\int_0^1 e^{z^3} z^2 dz = \frac{1}{2}\int_0^1 e^{z^3} z^2 dz \Rightarrow \begin{bmatrix} du = 3z^2 dz \\ dz = \dfrac{du}{3z^2} \end{bmatrix} \Rightarrow \frac{1}{2}\int_0^1 \frac{e^u}{3} du = \frac{1}{2} \cdot \frac{1}{3} \cdot (e^1 - e^0) = \frac{e}{6} - \frac{1}{6}$$





-1
-2



2
1
$x = z$
-1
0
1
2
-1
-2