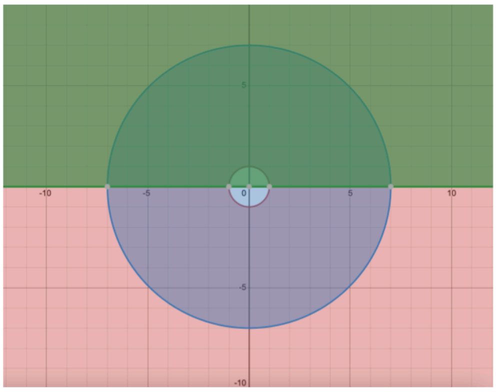
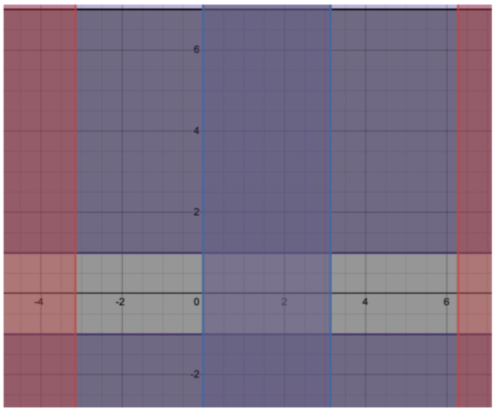
№10 М.А. Некрасов Артём 216

Вычислить интегралы:

1. $\iint_{\substack{1 \le x^2+y^2 \le 49 \\ y \ge 0}} \frac{ln(x^2+y^2)}{x^2+y^2} dxdy$

<u>Решение.</u>

$$\begin{cases} x^2+y^2=1 \\ x^2+y^2 \le 49 \\ y \ge 0 \end{cases} \Rightarrow \begin{cases} 1 \le r^2 \le 49 \\ r\sin\varphi \ge 0 \\ r > 0 \\ \varphi \in [0,\pi] \end{cases} \Rightarrow \begin{cases} r^2 \ge 1 \\ r^2 \le 49 \\ \sin\varphi \ge 0 \\ r > 0 \\ \varphi \in [0,\pi] \end{cases}$$

$$\iint_{\substack{1 \le x^2+y^2 \le 49 \\ y \ge 0}} \frac{ln(x^2+y^2)}{x^2+y^2} dxdy = \int_0^\pi d\varphi \int_1^7 \frac{2\ln r}{r^2} r\,dr = 2\int_0^\pi d\varphi \int_1^7 \frac{\ln r}{r^2} r\,dr ⊜$$

$$\int_1^7 \frac{\ln r}{r^2} r\,dr = \int_1^7 \frac{\ln r}{r} dr = \begin{bmatrix} \ln r = t \\ \frac{1}{r} dr = dt \\ dr = rdt \end{bmatrix} = \int_0^{\ln 7} t\,dt = \frac{t^2}{2}\Big|_0^{\ln 7} = \frac{\ln^2 7}{2}$$

$$⊜\ 2\int_0^\pi \frac{\ln^2 7}{2} d\varphi = \ln^2 7\,\pi$$

2. $\iint_{x^2+y^2 \le 2x} \frac{x\,dxdy}{\sqrt{4-x^2-y^2}}$

<u>Решение.</u>

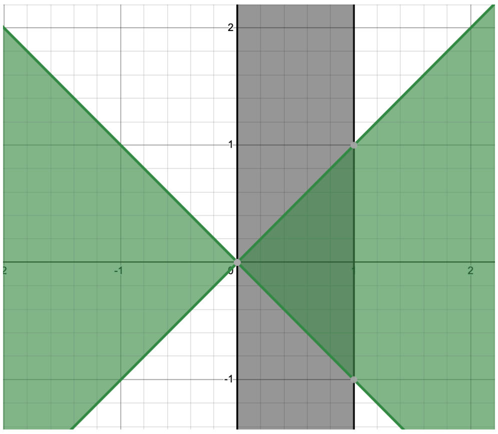
Место для уравнения.

3. $\iiint_{\substack{x^2+y^2 \le z^2 \\ 0 \le z \le 1}} (z-xy)dxdydz$

<u>Решение.</u>

Цилиндрическая замена координат:

$$\begin{cases} x = r\cos\varphi \\ y = r\,sin\varphi \\ z = h \\ r > 0 \\ \varphi \in [0,2\pi) \end{cases} \Rightarrow \begin{cases} r^2 \le h^2 \\ 0 \le h \le 1 \end{cases} \Rightarrow -h \le r \le h$$



$$\iiint\limits_{\substack{x^2+y^2\le z^2 \\ 0\le z\le 1}} (z - xy)dxdydz = \int_0^{2\pi} d\varphi \int_0^1 dh \int_0^h (h - r^2\cos\varphi\cdot\sin\varphi) r\,dr$$

$$I)\int_0^h hr\,dr - \int_0^h (-r^3\cos\varphi\sin\varphi)dr = \frac{1}{2}hr^2 + cos\,\varphi\,sin\,\varphi\frac{1}{4}\cdot r^4\ \Big|_0^h = \frac{1}{2}h(h^2-0) + \frac{1}{4}cos\,\varphi\,sin\,\varphi\,(h^4-0) =$$

$$= \frac{1}{2}h^3 + \frac{1}{4}cos\,\varphi\,sin\,\varphi\,h^4$$

$$II)\ \int_0^1 \frac{1}{2}h^3dh + \int_0^1 \frac{1}{4}cos\,\varphi\,sin\,\varphi\,h^4dh = \frac{1}{2}\cdot\frac{1}{4}h^4 + \frac{1}{4}cos\,\varphi\,sin\,\varphi\cdot\frac{1}{5}h^5\Big|_0^1 = \frac{1}{8} + \frac{1}{20}cos\,\varphi\,sin\,\varphi$$

$$III)\ \int_0^{2\pi}\frac{1}{8}d\varphi + \int_0^{2\pi} cos\,\varphi\,sin\,\varphi\,d\varphi = \begin{bmatrix} t = \sin\varphi \\ dt = \cos\varphi\,d\varphi \end{bmatrix} = \frac{1}{8}\cdot 2\pi + \frac{1}{20}\int_0^0 tdt = \frac{\pi}{4}$$

4. $\iiint\limits_{\substack{x^2+y^2+z^2\ge 1 \\ x^2+y^2+z^2\le 2z}} z^2dxdydz$

<u>Решение.</u>