

!Program is to calculate the position of a wave in space and time using 1-D wave equation
 $u_{tt} = A^2 u_{xx}$, where $u(x,t)$ is a function of x and t that tells the displacement of wave at position x and time t

!The program calculating the wave generated by a tightly stretched string at both ends of length L

!Also it is assumed that at time $t = 0$ the wave is at equilibrium

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program wave
  real,parameter :: PI = 3.14159
  real :: string_length, time_end, x_step, t_step, phase_vel ,lambda
  real :: wave_vector, ang_vel

  !Here string_length is the ending point of string i.e. the length of string
  !time_end is the final time
  !x_step is the step size of x
  !t_step is the step size of t
  !phase_vel is the constant in the 1D wave equation

  integer :: nt, nx
  !nt and nx are the number of steps in t and x respectively

  real, dimension(:,,:), allocatable :: y
  !y will be containing the the displacement of the x at time t

  real, dimension(:,,:), allocatable :: ya
  !ya is the analytic solution:  $y_a(x,t) = \sin(kx)\cos(\omega t)$ 

  integer :: i, j
  !just two variable to move through the matrix

  write(*, '("The following program is to calculate the position of a wave generated
by a stretched string is space-time)")')
  write(*, '("Using 1-D Wave Equation", /, 15X, "u_tt = phase_vel^2 u_xx", /, /)')
  write(*, '("Enter the length of the string : ")', advance = "no")
  read *, string_length
  write(*, '("Enter the final time t : ")', advance = "no")
  read *, time_end
  write(*, '("Enter the step size in x : ")', advance = "no")
  read *, x_step
  write(*, '("Enter the step size in t : ")', advance = "no")
  read *, t_step
  write(*, '("Enter the constant A : ")', advance = "no")
  read *, phase_vel

  ! Calculate the number of time and distance points
  nt = time_end/t_step + 0.5
  nx = string_length/x_step + 0.5
  print *, "nx = ", nx, ", nt = ", nt

  ! Wave vector and angular velocity
  ! We are assuming the wave is the fundamental so the wavelength is  $2L$  ,  $k =$ 
 $2\pi/\lambda$  ,  $2L/n = \lambda$  ,  $k = n\pi/L$  :
  wave_vector = 7*PI/string_length
  ang_vel = phase_vel*wave_vector
  print *, "wave vector = ", wave_vector, ", angular velocity = ",ang_vel

  !Make the sure the value is 1 or less than 1 otherwise it will shoot to infinity
  lambda = phase_vel*phase_vel*t_step*t_step/(x_step*x_step)
  print *, "Lambda = ", lambda

  ! This does the analytic calculation
  allocate(ya(0:nx, 0:nt))
  do i = 0, nx
    do j = 0, nt
      ya(i, j) = sin(wave_vector*i*x_step)*cos(ang_vel*j*t_step)
    end do
  end do

  ! Write the analytic data

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open(10, file = "wave_a.txt")
do j = 0, nt
  do i = 0, nx
    write(10, '(F0.4, 1X, F0.4, 1X, F0.4)') x_step*i, ya(i,j)
  end do
  write(10, '(/)')
end do
close(10)

! Now the numerical calculation
! the 1 is added to both nt and nx because the initial points are also needed to be
mapped
! as the string is stretched from the both ends and initially the string is at
equilibrium thus  $u(0,t) = u(L,t) = 0$ 
! this is boundary value
allocate(y(0:nx, 0:nt))

do j = 0, nt
  y(0,j) = 0
  y(nx, j) = 0
end do

! set the 1st initial values:  $y(x,0) = \sin(kx)$ 
do i = 1, nx - 1
  y(i,0) = sin(wave_vector*i*x_step)
end do

! set the 2nd initial values:  $y(x,t) = \sin(\omega t + kx)$ 
do i = 1, nx-1
  y(i,1) = sin(wave_vector*i*x_step)*cos(ang_vel*t_step)
end do

! calculating the value of  $u(x,t)$ 
do j = 1, nt-1
  do i = 1, nx-1
    y(i,j+1) = 2*y(i,j) + lambda*(y(i+1,j)-2*y(i,j) + y(i-1,j)) - y(i,j-1)
  end do
end do

open(19, file = "wave.txt")
do j = 0, nt
  do i = 0, nx
    write(19, '(F0.4, 1X, F0.4, 1X, F0.4)') x_step*i, y(i,j)
  end do
  write(19, '(/)')
end do

! All finished so free the arrays we allocated
deallocate(ya)
deallocate(y)

end program wave

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