Домашнее задание

8.1.29

$$\int \frac{dx}{x^2\sqrt{x}} = \int x^{-\frac{5}{2}}\,dx = \frac{x^{-\frac{5}{2}+1}}{-\frac{5}{2}+1} + C = \frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} + C = -\frac{2}{3\sqrt{x^3}} + C$$

8.1.30

$$\int \frac{dx}{x^2+3} = \frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{x}{\sqrt{3}} + C$$

8.1.31

$$\int \frac{1}{5^x}\,dx = \int 5^{-x}\,dx = \frac{5^{-x}}{\ln 5} + C = -\frac{1}{5^x \ln 5} + C$$

8.1.32

$$\int \frac{dx}{\sqrt{4-x^2}} = \arcsin\frac{x}{2} + C$$

8.1.33

$$\int \frac{dx}{\sqrt{x^2-1}} = \ln\left|x + \sqrt{x^2-1}\right| + C$$

8.1.34

$$\int \frac{dx}{x^2-25} = \frac{1}{10}\ln\left|\frac{x-5}{x+5}\right| + C$$

8.1.35

$$\int\left(x+\frac{2}{x}\right)^2dx = \int\left(x^2+2\cdot\frac{2}{x}\cdot x+\frac{4}{x^2}\right)dx =$$

$$= \int x^2dx + 4\int dx + 4\int\frac{dx}{x^2} = \frac{x^3}{3}+4x+\frac{x^{-2+1}}{-2+1}+C = \frac{x^3}{3}+4x-\frac{1}{x}$$

8.1.36

$$\int\frac{dx}{4x^2+1} = \frac{1}{4}\int\frac{dx}{x^2+\frac{1}{4}} = \frac{1}{4}\int\frac{dx}{x^2+\left(\frac{1}{2}\right)^2} = \cdot\frac{1}{4}\,\frac{1}{\frac{1}{2}}\,\mathrm{arctg}\,\frac{x}{\frac{1}{2}}+C = \frac{1}{2}\,\mathrm{arctg}\,2x+C$$

8.1.37

$$\int\left(7^x-\frac{8}{x}+4\cos x\right)dx = \int 7^xdx - 8\int\frac{dx}{x}+4\int\cos x\,dx = \frac{7^x}{\ln 7}-8\cdot\ln|x|-$$

$$4\sin x + C$$

8.1.38

$$\int\left(\frac{\sqrt{3}}{\cos^2 x}-\sqrt[3]{x}-\frac{2}{x^4}\right)dx = \sqrt{3}\int\frac{dx}{\cos^2 x}-\int x^{\frac{1}{3}}dx - 2\int x^{-4}dx =$$

$$= \sqrt{3}\cdot \mathrm{tg}\,x - \frac{x^{\frac{4}{3}}}{\frac{4}{3}} - 2\cdot\frac{x^{-3}}{-3}+C = \sqrt{3}\,\mathrm{tg}\,x - \frac{3\sqrt[3]{x^4}}{4}+\frac{2}{3x^3}+C$$

8.1.39

$$\int\frac{\sqrt{x}-3\sqrt[5]{x^2}+1}{\sqrt[4]{x}}dx = \int x^{\frac{1}{2}-\frac{1}{4}}dx - 3\int x^{\frac{2}{5}-\frac{1}{4}}+\int x^{-\frac{1}{4}}dx =$$

$$= \int x^{\frac{1}{4}}dx - 3\int x^{\frac{3}{20}}dx + \int x^{-\frac{1}{4}}dx = \frac{x^{\frac{5}{4}}}{\frac{5}{4}} - \frac{3\cdot x^{\frac{23}{20}}}{\frac{23}{20}} + \frac{x^{\frac{3}{4}}}{\frac{3}{4}}+C =$$

$$= \frac{4\sqrt[4]{x^5}}{5} - \frac{60\sqrt[20]{x^{23}}}{23} + \frac{4\sqrt[4]{x^3}}{3}+C$$

8.1.40

$$\int\left(0{,}7\cdot x^{-0{,}1}+0{,}2\cdot(0{,}5)^x\right)dx = 0{,}7\int x^{-\frac{1}{10}}dx + 0{,}2\int\left(\frac{5}{10}\right)^x dx =$$

$$= \frac{7}{10}\cdot\frac{x^{\frac{9}{10}}}{\frac{9}{10}}+\frac{2}{10}\cdot\frac{\left(\frac{5}{10}\right)^x}{\ln\left(\frac{5}{10}\right)}+C = \frac{7\cdot\sqrt[10]{x^9}}{9}+\frac{2\cdot(0{,}5)^x}{10\cdot\ln(0{,}5)}+C$$

8.1.41

$$\int(5\,\mathrm{sh}\,x-7\,\mathrm{ch}\,x+1)dx = 5\int\mathrm{sh}\,x\,dx - 7\int\mathrm{ch}\,x\,dx+\int dx =$$

$$= 5\,\mathrm{ch}\,x - 7\,\mathrm{sh}\,x + x + C$$

8.1.42

$$\int (x^2-1)(\sqrt{x}+4)\,dx = \int (x^2\sqrt{x}+4x^2-\sqrt{x}-4)\,dx =$$

$$= \int x^{\frac{5}{2}}dx + 4\int x^2 dx - \int \sqrt{x}\,dx - 4\int dx = \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{4x^3}{3} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 4x = +C$$

$$= \frac{2\cdot x^{\frac{7}{2}}}{7} + \frac{4x^3}{3} - \frac{2x^{\frac{3}{2}}}{3} - 4x + C$$

8.1.43

$$\int \frac{7-\sqrt{x^2+\pi}}{\sqrt{x^2+\pi}}\,dx = 7\int \frac{dx}{\sqrt{x^2+\pi}} - \int dx = 7\ln\left|x+\sqrt{x^2+\pi}\right| - x + C$$

8.1.44.

$$\int \left(\frac{\sqrt{x}-5}{x}\right)^3 dx = \int \left(\frac{x\sqrt{x}-15x+75\sqrt{x}-125}{x^3}\right) dx = \int x^{\frac{3}{2}-3}dx - 15\int x^{1-3}dx + 75\int x^{\frac{1}{2}-3}$$

$x^{-\frac{3}{2}}$, x^{-2}, $x^{-\frac{5}{2}}$

$$-125\int x^{-3}dx = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} - 15\cdot\frac{x^{-1}}{-1} + 75\cdot\frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} - 125\,\frac{x^{-2}}{-2} + C =$$

$$= -\frac{2}{\sqrt{x}} + \frac{15}{x} - \frac{\cancel{150}\ 50}{\cancel{3}\sqrt{x^3}} + \frac{125}{2x^2} + C$$

8.1.45

$$\int \sin 7x\,dx = \left[\int f(ax+b)\,dx = \frac{1}{a}\cdot F(ax+b)\right] = \frac{1}{7}\cdot(-\cos 7x) + C$$

8.1.46

$$\int \sqrt[5]{2x-8}\,dx = \int (2x-8)^{\frac{1}{5}}dx = \frac{1}{2}\cdot\frac{(2x-8)^{\frac{6}{5}}}{\frac{6}{5}} + C = \frac{5(2x-8)^{\frac{6}{5}}}{12}$$

8.1.47 $$\int (1-4x)^{2001}dx = -\frac{1}{4}\cdot\frac{(1-4x)^{2002}}{2002} + C$$

8.1.48 $$\int \frac{dx}{9x+7} = \frac{1}{9}\cdot\ln|9x+7| + C$$

8.1.49

$$\int \frac{dx}{(6x+11)^4} = \int (6x+11)^{-4}dx = \frac{1}{6}\cdot\frac{(6x+11)^{-3}}{-3} + C = -\frac{1}{18(6x+11)^3} + C$$

8.1.50

$$\int \frac{dx}{25x^2+1} = \frac{\operatorname{arctg} 5x}{5} + C$$

8.1.51

$$\int 3^{2-11x}\,dx = -\frac{1}{11}\cdot\frac{3^{2-11x}}{\ln(3)} + C$$

8.2.52

$$\int \frac{dx}{\sqrt{4x^2-1}} = \frac{1}{2}\cdot\ln\left|2x+\sqrt{4x^2-1}\right| + C$$

8.1.53

$$\int \sin^2 3x\,dx = \int \frac{1-\cos 6x}{2}\,dx = \frac{1}{2}\int dx - \frac{1}{2}\int \cos 6x\,dx =$$

$$= \frac{x}{2} + \frac{\sin 6x}{12} + C$$

8.1.54

$$\int \cos^2 8x\,dx = \int \frac{1+\cos 16x}{2}\,dx = \frac{1}{2}\left(x + \frac{1}{16}(-\sin 16x)\right) + C =$$

$$= \frac{x}{2} - \frac{\sin 16x}{32}$$

8.1.55

$$\int tg^2 x\,dx = \int \frac{1-\cos 2x}{1+\cos 2x}\,dx = \left[\begin{array}{l} 1+\cos 2\alpha = 1+\cos^2\alpha - \sin^2\alpha = \\ = 2\cos^2\alpha \end{array}\right]$$

$$= \int \frac{1-\cos 2x}{2\cos^2 x}\,dx = \frac{1}{2}\int \frac{dx}{\cos^2 x} - \int \frac{\cos 2x}{\cos^2 x}\,dx =$$

$$= \frac{tg x}{2} + \int \frac{\sin^2 x}{\cos^2 x}\,dx - \int dx = \frac{tg x}{2} - x + \int tg^2 x\,dx$$

$$= \int \sec^2 x - 1\,dx = tg x - x + C$$

(8.1.56.)

$$\int \frac{4x+1}{x-5}dx = 4\int \frac{x}{x-5}dx + \int \frac{dx}{x-5} = \ln|x-5| + 4\int \frac{(x-5)+5}{(x-5)} =$$

$$= 4\int dx + 20\int \frac{dx}{x-5} + \ln|x-5| + C = 21\ln|x-5| + 4x + C$$

(8.1.57)

$$\int (3\,tgx - 2\,ctgx)^2 dx = \int (9tg^2x - 2\cdot 2\cdot 3\cdot \underbrace{tg\cdot ctg}_{1} + 4ctg^2x)dx =$$

$$= 9\int tg^2x\,dx - 12\int dx + 4\int ctg^2x\,dx = 9(tgx + x) - 12x + 4(-ctg - x) + C$$

$$= 9tgx + 9x - 12x - 4ctgx - 4x + C = 9tgx - 4ctgx - 7x + C$$

(8.1.58)

$$\int \frac{4\sqrt{1-x^2} + 3x^2}{x^2-1}dx = 4\int \frac{\sqrt{1-x^2}}{x^2-1} + 3\int x^2 dx =$$

$$\left[4\int \frac{\sqrt{1-x^2}\cdot\sqrt{1-x^2}}{(x^2-1)(\sqrt{1-x^2})}dx = -4\int \frac{\cancel{(x^2-1)}}{\cancel{(x^2-1)}(\sqrt{1-x^2})}dx = -4\int \frac{dx}{\sqrt{1-x^2}} \right] =$$

$$+C$$

$$= -4\int \frac{dx}{\sqrt{1-x^2}} + \cancel{3}\frac{x^3}{\cancel{3}} = -4\arcsin x + x^3 + C$$

(8.1.59)

$$\int \frac{\cos 2x\,dx}{\sin^2x\cos^2x} = \int \frac{\cos^2x}{\sin^2x\cancel{\cos^2x}}dx - \int \frac{\cancel{\sin^2x}}{\cancel{\sin^2x}\cos^2x}dx = -ctgx - tgx + C$$

(8.1.60)

$$\int \frac{\sin 2x}{\cos x}dx = \int \frac{2\sin x\cancel{\cos x}}{\cancel{\cos x}}dx = 2\int \sin x\,dx = -2\cos x + C$$