Homework 3

Mathematical Optimization for AI

Due Date : 16:00 AM, 16 Jun 2020

1. Consider the SVM with soft margin

$$\text{minimize} \quad \frac{1}{2}\|w\|_2^2 + C\sum_{i=1}^{N}\xi_i$$

$$\text{subject to} \quad y_i(w\cdot x_i + b) \geq 1-\xi_i, \quad \xi_i \geq 0, \quad i=1,2,\cdots,N$$

Show that the dual problem of SVM with soft margin is the following. **(10pts)**

$$\text{maximize}_\lambda \quad \sum_{i}^{N}\lambda_i - \frac{1}{2}\sum_{i=1}^{N}\sum_{i=1}^{N} y_i y_j \lambda_i \lambda_j \vec{x}_i^T \vec{x}_j$$

$$\text{subject to} \quad 0 \leq \lambda_i \leq C, \quad i=1,2,\cdots,N$$

$$\sum_{i=1}^{N}\lambda_i y_i = 0$$

2. Let $A \in \mathbb{R}^{n\times n}$ and A has a singular value decomposition $A = U\Sigma V^T$. Answer the following questions on SVD of A. **(10pts)**

(a) If $U = [u_1|u_2|\cdots|u_n]$, $V = [v_1|v_2|\cdots|v_n]$, $\Sigma = \mathrm{diag}(\sigma_1,\sigma_2,\cdots,\sigma_2)$, show that A can be expressed as the sum of rank-one matrix, that is,

$$A = \sum_{i=1}^{n}\sigma_i u_i v_i^T$$

(b) Find an eigenvalue decomposition of the $2n\times 2n$ matrix

$$\begin{bmatrix} O & A^T \\ A & O \end{bmatrix}$$

(Here, O is the $n\times n$ zero matrix.)

3. Find the singular value decomposition $A = U\Sigma V^T$ of the matrix (row vector) $A = [2\ 1\ -2]$. **(10pts)**

4. **(Python Example)** Let us consider the following table.

x	2.5	0.5	2.2	1.9	3.1	2.3	2	1	1.5	1.1
y	2.4	0.7	2.9	2.2	3.0	2.7	1.6	1.1	1.6	0.9

Write a code for finding principal axes of the given data. **(10pts)**