

Homework 1

(#1)

Due : 2020.04.06. 10:00

1. Show that a half space

$$\{x \mid a^T x \leq b\}$$

is convex but not affine. ($a, x, b \in \mathbb{R}^n$)

2. Cauchy-Schwarz inequality ^{외워둬기!!!} For any $x, y \in \mathbb{R}^n$ and norm

$\|\cdot\|: \mathbb{R}^n \rightarrow \mathbb{R}$, Show that the following inequality holds.

$$\|x\| \|y\| \geq |x \cdot y|$$

3. Assume that $f: \mathbb{R}^n \rightarrow \mathbb{R}$, $A \in \mathbb{R}^{n \times m}$, and $b \in \mathbb{R}^n$. If

f is a convex function, then show that $g: \mathbb{R}^m \rightarrow \mathbb{R}$ by

$$g(x) = f(Ax + b)$$

is also a convex function.

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4. Find conjugate function f^* for the following f . #2

(a) $f(\lambda) = \log\left(\sum_{i=1}^n e^{\lambda_i}\right)$, $\lambda = (\lambda_1, \lambda_2)$

(b) $f(\lambda) = \|\lambda\|$, $\lambda = (\lambda_1, \lambda_2)$

(c) $f(\lambda) = \lambda^p$ where $p \geq 1$, $\lambda > 0$

5. Define $g(\lambda) = f(\lambda) + c^T \lambda + d$, where f is convex. Express

g^* in terms of f^* (and c, d).