­­

SIMILARITY RELATIONS:

SOLVING THE PROBLEMS WITH EXPECTED UTILITY THEORY

Kevin Bielawski

EC 329: Decision: Theories and Experiments

December 2, 13

Sources

* *The Adaptive Decision Maker[[1]](#footnote-1)*
* “Similarity and Decision-making under Risk”*[[2]](#footnote-2)*
* *Search and Satisficing[[3]](#footnote-3)*
* *Heuristics and Biases: The Psychology of Intuitive Judgment[[4]](#footnote-4)*

DRAFT

The theory of expected utility surrounds is similar to its predecessor, expected value, which is calculated by multiply a lottery’s outcomes my their respective probabilities, and summing the results. The figure that is found is the given lottery’s expected value. In expected utility theory, however, a lottery’s expected utility involves an additional layer, the decision-maker’s utility function. An example of which could be:

Instead of simply multiplying outcomes and probabilities, one multiplies the *utility* of an outcome by the probability of the occurrence of that outcome. This can be seen in the following example:

Unfortunately, expected utility does no explain all of the instances of preference found in experimentation. For example, the Allais paradox, first noted by Maurice Allais in 1953, illustrates a decision-maker facing risk. The paradox is found in the fact that the “vast majority” of participants prefer lotteries in accordance with expected utility theory in one experiment, but ignore the theory in another experiment. How could this be? This will be the focus of much of this paper.

Fortunately, much advancement in the field of decision theory has occurred since the problems with expected utility were uncovered. One such mark of progress can be seen in the work of Ariel Rubinstein. In his paper, “Similarity and Decision-making under Risk,” he outlines an alternative to expected utility. It is focused on the idea of the similarity between the outcomes and the probabilities of two lotteries and how that information is used in determining which of the two lotteries is more favorable.

*Section 1*

Daniel Bernoulli spearheaded the concept of expected utility in 1738. A child of expected value, the theory approximates an individual’s preferences under uncertainty. Refer to the example expected value calculation, below:

Imagine two lotteries:

* An outcome of $1000 with a probability of 0.9
* An outcome of $900 with a probability of 1.0

The expected values of these lotteries are (respectively):

* $900
* $900

That’s correct. By weighting the outcomes by the probabilities of the occurrences of those outcomes, we have determined that each of the two lotteries yield the same expected value. In different language, the average outcome of both lotteries is the same.

Based on only the expected value calculation, we can see that one would, in theory, be indifferent between the two lotteries.

While expected value can be a useful tool, Bernoulli’s advancement, namely expected utility, proves to deliver better estimations of preference. The rationale was simple: the value of a lottery shouldn’t be determined solely based on its monetary value. Rather, it should be derived from the utility that the given monetary value provides to the individual. A decision-maker is assigned a utility function, such as, , with being any given outcome. The function yields the utility gained from outcome, :

In the (above) example, it is clear that one derives more value from the first 400 dollars of an outcome than he or she derives from the next 600 dollars. This idea can be seen as an example of the theory of diminishing marginal utility.[[5]](#footnote-5) Additionally, considering individuals don’t necessarily value outcomes and their respective utilities, proportionately, expected value may not be an adequate means for determining preference. Such an example can be seen, below:

While the expected values of the two lotteries are the same, expected utility yields a much different result.

Although more useful to economists than its predecessor, expected utility has some issues. One such flaw is known as the Allais paradox. Let’s take a look.

Given the following lotteries,

most subjects stated their preference as such:

This poses an interesting problem, given that the expected utilities can be computed as:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Lottery** | **X** | **P** | **EV** | **EU** |
| 1 | 4000 | 0.8 | *3200* | 50.60 |
| 2 | 3000 | 1.0 | 3000 | *54.77* |
| 3 | 4000 | 0.2 | *800* | 12.65 |
| 4 | 3000 | 0.25 | 750 | *13.69* |

This table gives us considerable information. First, we can see that with respect to expected value, lotteries 1 and 3 should be preferred over their counterparts. When considering our other heuristic, expected utility, the preferences are switched. Lotteries 2 and 4 are now preferable. Unfortunately, these calculations do not agree with the results of experimentation. When deciding between lotteries 1 and 2, the decision makers favored the second, validating expected utility’s result. The choice between lotteries 3 and 4, however, results in the vast majority of participants choosing lottery 3, voiding the very heuristic just observed.

*Section 2*

In his paper, “Similarity and Decision-making under Risk”, Ariel Rubinstein explains a solution to some of the problems with expected utility theory. He understands that the Allais paradox reveals properties of the decision-making scheme used in certain situations: if outcomes, x1 and x2 are “similar”, the decision making process is then reverted to the relationship between p1 and p2. If the given probabilities are also similar, a different heuristic is used. If not, the lottery with the more favorable probability is preferred. Let’s take a look at our lotteries:

As we can see, the probabilities p1 and p2 are not “similar”. While 80% certainty represents fairly good odds, there is still a 20% chance of winning nothing. It is known that individuals are very much risk averse and the opportunity of a certain bet makes a probability of only 80% appear paltry in comparison. Additionally, the outcomes, x1 and x2, are seen to be distinct such that probabilities wouldn’t become the deciding factor in regards to preference. Because of these parameters, expected utility proves sufficient in determining favorability among lotteries.

Venturing on to the comparison between lottery L3 and L4, we can see that only the probabilities have changed. In fact, they have been divided by four, resulting in p3 and p4 equaling 0.2 and 0.25, respectively. Now, the difference between probabilities is smaller and considering neither a 20% nor a 25% chance is anywhere near “definite”, p3 and p4 can be seen as similar. Because the probabilities can be treated as alike, preference can be focused upon the lotteries’ respective outcomes. Just as before, 4000 is considerably larger than 3000. The significant difference in outcomes causes one to prefer lottery L3.

Based on the thought experiment outlined in the previous paragraphs, Rubinstein presented a procedure[[6]](#footnote-6), denoted as “\*”, which can be seen, below.

1. Let L1=(x1, p1) and L2=(x2, p2) represent two lotteries
2. If x1>x2 and p1>p2, L1>L2
   1. Similarly, if x2>x1 and p2>p1, L2>L1
3. If p1~p2 and x1!~x2 and x1>x2, L1>L2
4. If x1~x2 and p1!~p2 and p1>p2, L1>L2
5. If neither the outcomes nor the probabilities are similar, choose a different heuristic[[7]](#footnote-7)

Put simply, if either the two outcomes or the two probabilities of the two lotteries are similar enough that the decision-maker can treat them as approximately the same, he or she can base his or her preference solely on the other, dissimilar factors. According to Rubinstein, “The main finding is that *similarities* on both the prize and the probability dimensions result in a “unique” preference which is consistent with similarities and the above procedure.”[[8]](#footnote-8)

If one is to think about how he or she actually makes decisions, expected utility probably does not enter one’s mind. (At least not at first.) Additionally, it is probably unlikely that only a single heuristic is used and is applied uniformly in all circumstances. In the work, *The Adaptive Decision Maker[[9]](#footnote-9)*, by Payne, Bettman, and Johnson, it is said that when an “agent” is tasked with making a decision, he or she has at his or her disposal a wide array of choice heuristics that could prove useful in formulating that decision. Let us illustrate this concept with an example:

Imagine yourself walking into a coffee shop. Upon entrance, you walk to the counter and begin to decide as to what type of drink you wish to purchase. What type of drink do you choose? You have many options, and similarly, there are many routes you can take in choosing the drink you will eventually enjoy (or dislike, for that matter). Here are a few of the seemingly limitless possible decision-making techniques:

* + Choose the same drink that you bought, previously. You enjoyed a 12oz Americano, yesterday, and you think that drink will grant you the most enjoyment, today.
  + Choose a completely new drink that you have never tried, before. While you enjoyed your coffee, yesterday, you are excited to try something new.
  + Select the drink that the barista prefers. He or she spends a lot of time creating coffee drink and would probably have some unique insight into what would be enjoyable e.g. what type of beans are “good”, today.
  + Ask your friend, “What are you getting?” Choose the drink that he or she decides to order. Perhaps you are indecisive.
  + Select the coffee drink based on any of the many metrics available to you. These can include volume, price, perceived caffeine content, temperature, and many other specifications.

Although, in this paper, we are primarily discussing choice in terms of lotteries, this example is helpful in convincing one of the myriad decision-making tools available when forced to into a situation familiar to many of us.

Now, let us apply this coffee-choosing example to the thoughts of Rubinstein. As discussed, previously, his methodology is determinant on similarities between the outcomes or probabilities of two, or presumably, more than two, lotteries. This concept proves helpful when outcomes are alike, but probabilities are much different, or vice versa. Such can be seen as logical, considering if one is tasked with the decision between $99 with certainty or $100 dollars with only a 50% likelihood, the best outcome would probably be clear to most people. The idea of losing out on a single dollar isn’t a particularly painful one, especially when contrasted by a 50/50 chance of going home empty-handed. Unfortunately, Rubinstein’s procedure proves fruitless when the lotteries, themselves, are very similar to one another, or when they are particularly dissimilar to one another.[[10]](#footnote-10) This poses a problem.

(In this next section, I hope to explore the edge cases in which Rubinstein’s hypotheses fail. I could use some guidance with this section. Additionally, I hope to elaborate on the lottery calculator I’ve developed, which can be found at <http://blwsk.com/lottery-solver.html>. When we discuss the paper, I’d like to provide some background into what my thought process was when designing the app, and how my algorithm works. It’s fairly rudimentary, of course, but I think there is room for improvement.)

*Section 3*

Why should economists care about the problems set forth by these experiments? This question can be answered by identifying the significance of the shortcomings of expected utility.

As noted previously, expected utility fails to account properly for a decision-maker’s risk aversion, which can be seen identified in the Allais paradox. Given the following sets of lotteries,

* L1 = (4000, 0.8) ; L2 = (3000, 1)
* L3 = (4000, 0.2) ; L4 = (3000, 0.25)

expected utility theory suggests that the preference should be displayed as

In experimentation, however, these preferences do not prove to be accurate. Instead, L4 is preferred to L3 by the “vast majority of subjects.” This problem proves to be a serious one, especially considering the significance of the topic of lotteries to areas such as insurance and gambling.[[11]](#footnote-11)

Although speaking of expected utility’s negative qualities is of interest in its own right, it is also necessary to highlight why solving these problems is relevant. As outlined, previously, in this paper, the conclusions of experimentations such as that of the Allais paradox can be remedied given certain alternatives to pure expected utility. Namely, the invention of similarity relations can be used to more aptly model one’s preferences under risk.[[12]](#footnote-12)

*Section 4*

A question of the instructor’s choosing

1. John W. Payne, James R. Bettman, and Eric J. Johnson. *The Adaptive Decision Maker* (Cambridge: Cambridge University Press, 1993) [↑](#footnote-ref-1)
2. Ariel Rubinstein. “Similarity and Decision-making under Risk,” *Journal of Economic Theory* (1968) [↑](#footnote-ref-2)
3. Andrew Caplin, Mark Dean, and Daniel Martin. “Search and Satisficing” [↑](#footnote-ref-3)
4. Thomas Gilovich, Dale Griffin, Daniel Kahneman. *Heuristics and Biases: The Psychology of Intuitive Judgment* (Cambridge: Cambridge University Press, 2002) [↑](#footnote-ref-4)
5. Diminishing marginal utility [↑](#footnote-ref-5)
6. I have created a web application, found at <http://blwsk.com/lottery-solver.html> , that models this procedure. There are some bugs that will be fixed, soon. [↑](#footnote-ref-6)
7. Rubinstein does not specify the nature of this last “step.” [↑](#footnote-ref-7)
8. Ariel Rubinstein. “Similarity and Decision-making under Risk,” *Journal of Economic Theory* (1968) [↑](#footnote-ref-8)
9. John W. Payne, James R. Bettman, and Eric J. Johnson. *The Adaptive Decision Maker* (Cambridge: Cambridge University Press, 1993) [↑](#footnote-ref-9)
10. In other words, if x1 ~ x2 and p1 ~ p2, Rubinstein’s procedure will not work. Similarly, if x1!~x2 and p1!~p2, the procedure will not work. [↑](#footnote-ref-10)
11. In my final paper, I want to emphasize the significance of lotteries in relation to areas of interest such as insurance. Would this be an appropriate path for the final section of the paper? [↑](#footnote-ref-11)
12. Is it appropriate to refer to preferences, (broadly defined) as those of an individual? Or, rather, do preferences, such as those found in this paper, refer to those displayed by the “vast majority” of participants? [↑](#footnote-ref-12)