

# Lecture 2

White-box Testing and  
Structural Coverage

(see Amman and Offut, Chapter 2)

# White-box Testing

## (aka. Glass-box or structural testing)

- An error may exist at one (or more) **location(s)**
  - Line numbers
  - Boolean tests
  - Expressions etc.
- *If tests don't exercise that (those) location(s) then error can never be observed*
- So **identify and exercise** locations
- No loops – **finitely many locations** – good!
- Loops – **infinitely many locations** – bad! cannot write infinite test suite
- Loops + branches – **exponential growth** in locations with loop depth – **very bad !!!!**

# Structural Testing

Structural testing is the process of exercising software with test scenarios written from the source code, not from the requirements.

Usually structural testing has the goal to exercise a minimum collection of (combinations of) locations.

How many locations and combinations are enough?

# Coverage

- *The size of a test suite is an unreliable indicator of the work achieved by testing.*
- **Coverage** refers to the extent to which a given testing activity has satisfied its objectives.
- *“Enough” testing is defined in terms of coverage rather than test suite size.*
- A major advantage of structural testing is that coverage can be easily and accurately defined.  
glassbox testing/whitebox testing
- Structural coverage measures

# Structural Testing – Problems!

good for unit testing but not good for scalability

- What about **sins of omission**? when you forgot to code stuffs
- Missing code = no path to go down!  
**Unimplemented requirements!**
- What about **dead code** – is a path possible? when you have a location, but cannot access it
- How to avoid **redundant testing**?
- What about testing the **user requirements**?  
functional statement cannot be tested by testing the line, since the testing is based on source code
- How to handle **combinatorial explosion** of locations and their combinations?

# Problems with requirements-based testing

- A test set that meets requirements coverage is not necessarily a **thorough** test set
- Requirements may not contain a **complete and accurate** specification of all code behaviour
- Requirements may be **too coarse** to assure that all implemented behaviours are tested
- Requirements-based testing alone cannot confirm that code doesn't include **unintended functionality**. **Need structural testing too!**
- **Three main types of glass box testing** based on 3 types of coverage criteria.

# Coverage Criteria 1: Control flow

- Measure the **flow of control** between statements and sequences of statements
- Examples: *node coverage, edge coverage*.
- Mainly measured in terms of statement invocations (line numbers).
- Exercise main flows of control.
- Oldest and most common method

# Coverage Criteria 2: Logic

- Analyse the **influence of all Boolean variables**
- Examples: *predicate coverage, clause coverage, MCDC* (FAA DO178B)
- Exercise Boolean variables at control points.
- Modern method, and increasingly common



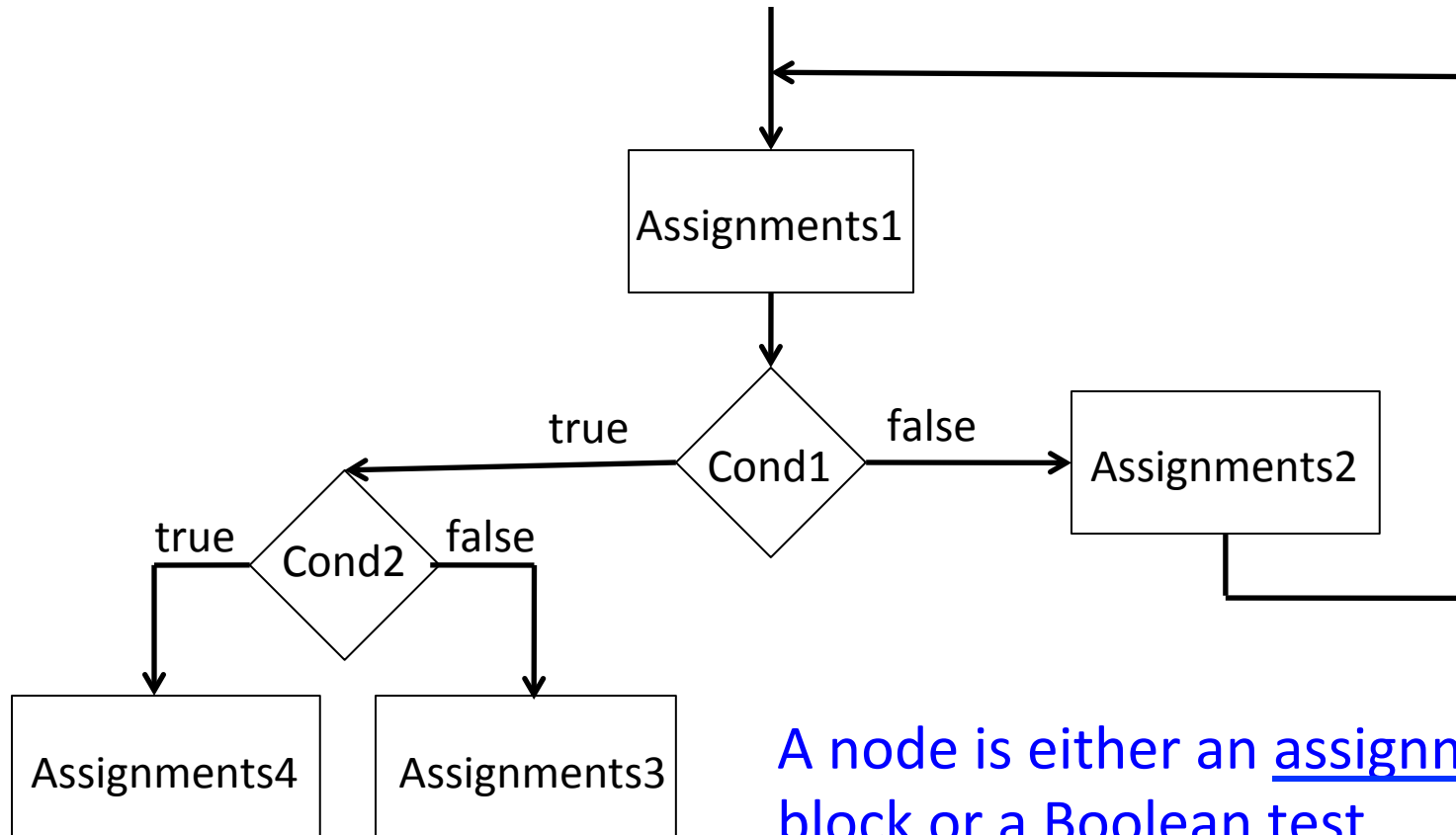
# Coverage Criteria 3: Data Flow

- Data flow criteria measure the **flow of data between variable assignments** (*writes*) and **variable references** (*reads*).
- Examples: *all-definitions, all-uses*
- Exercise **paths** between definition of a variable and its subsequent use.
- Still rather rare in industry

# Glass-box Test Requirements

- These are requirements on input values to satisfy a property: either
  - Graph-theoretic property (control & data flow)
  - data constraint (logic)
- Easy to define using graph theory
- Possible to automate generation by constraint solving
- Easy to measure coverage!
- Oracle is usually just crash (fail)/no crash(pass)
- Therefore they ignore functionality!

# Starting Point: a Condensation Graph aka. Flowchart



A node is either an assignment block or a Boolean test.

Every program statement is associated with exactly one node.

# Building condensation graphs

- **Boolean tests** can be from:
  - If-then-else statements `if (bexp) then .. else ..`  
boolean expression
  - If statements `if (bexp) ...`
  - Loops (of any kind) `while (bexp) ...`
- An **assignment block** contains consecutive
  - assignments `x = exp`
  - return statements `return exp`
  - procedure/method calls `myFunc(...)`
  - expressions e.g. `i++`

# Type 1 and Type 3 Coverage

- A **path** is a sequence of nodes  $n_0, \dots, n_k$  in a (condensation) graph  $G$ , such that each adjacent node pair,  $(n_i, n_{i+1})$  forms an edge in  $G$ .
- For type 1 and type 3 testing, a **test requirement**  $tr(.)$  is a **path**

# Covering a Graph Criterion C

**Definition:** Let  $TR$  be a set of test requirements demanded by a graph criterion  $C$ . A test suite  $TC$  satisfies  $C$  on graph  $G$  if, and only if for every test requirement  $p = n_0, \dots, n_k \in TR$ , there is at least one test case  $t$  in  $TC$  such  $t$  takes path  $p$ .

- This definition implies 100% coverage.
- We can also have  $< 100\%$

# Why so Formal?

## Answers:

1. Sometimes coverage properties become very technical to define for reasons of accuracy.
2. Precise definitions can be automated to make **test case generation tools**.

# Type 1 : Graph Coverage

2.1. **Node Coverage (NC)** Each **reachable node** in  $G$  is contained in some path  $p \in TR$ .

"The modern line coverage"

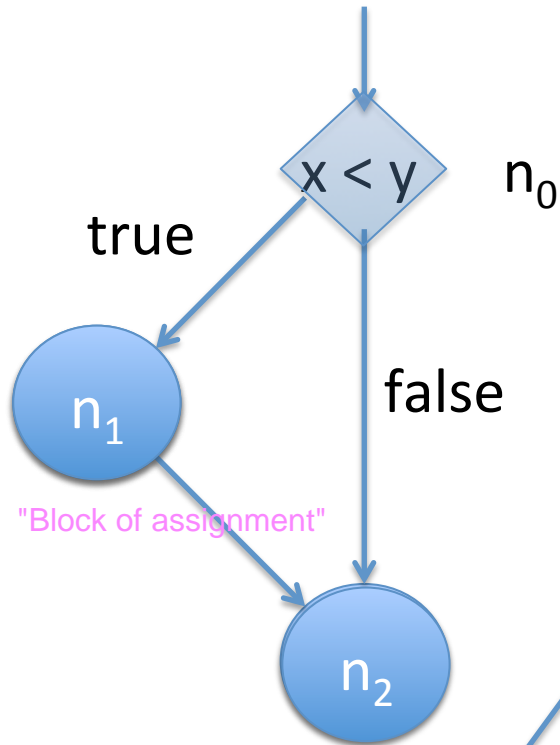
Myers : "NC is so weak that it is generally considered useless"

"Taking a particular branch. Hitting an edge with a certain path"

2.2. **Edge Coverage (EC)** Each **reachable path** of length  $\leq 1$  in  $G$  is contained in some path  $p \in TR$ .



# Node vs. Edge Coverage



Abstract test  
cases

$$p_1 = n_0, n_1, n_2$$

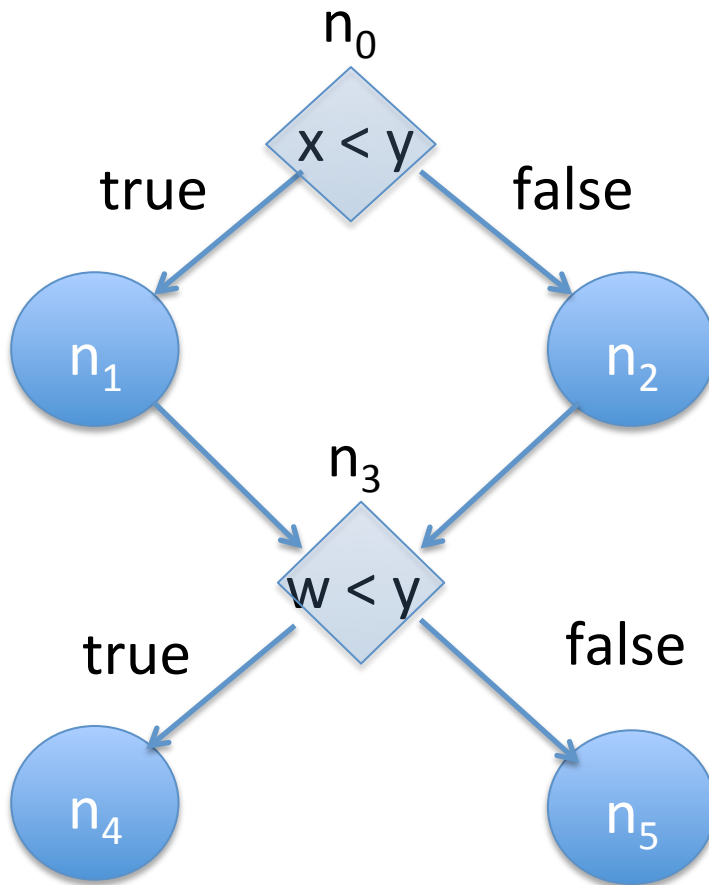
$$p_2 = n_0, n_2$$

$TS_1 = \{p_1\}$  satisfies  
node coverage

$TS_2 = \{p_1, p_2\}$  satisfies  
edge coverage

2.3 **Edge-Pair Coverage** ( $\text{EPC} = \text{EC}^2$ ) Each reachable path of length  $\leq 2$  in  $G$  is contained in some path  $p \in \text{TR}$ .

- Clearly we can continue this beyond 1,2 to  $\text{EC}^n$
- Combinatorial explosion in TR size!
- $\text{EC}^n$  doesn't deal with loops, which have unbounded length.



$$p_1 = n_0, n_1, n_3, n_4$$

$$p_2 = n_0, n_2, n_3, n_5$$

$$p_3 = n_0, n_2, n_3, n_4$$

$$p_4 = n_0, n_1, n_3, n_5$$

$TS_1 = \{p_1, p_2\}$  satisfies **edge coverage**

$TS_2 = \{p_1, p_2, p_3, p_4\}$  satisfies **edge-pair coverage**

# Simple Paths

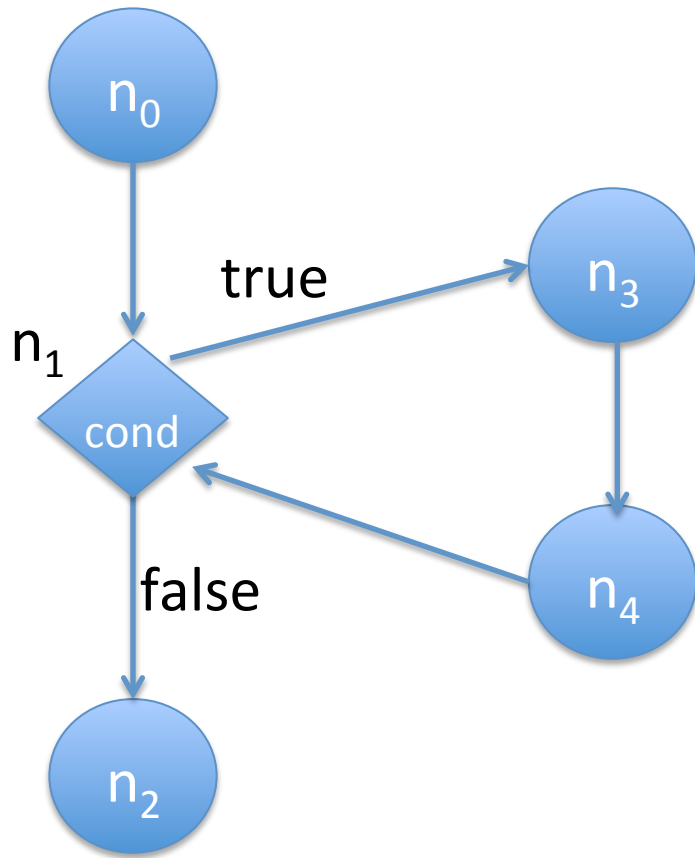
- How to deal with code loops?
- A path  $p$  is **simple** if it has no repetitions of nodes other than (possibly) the first and last node.
- *So a simple path  $p$  has no internal loops, but may itself be a loop*
- **Problem**: there are **too many** simple paths, since many are just sub-paths of longer simple paths.

# Prime Paths

if add anything more, will not be simple anymore

- A path  $p$  is **prime** iff  $p$  is a maximal simple path i.e.  $p$  cannot be extended without losing simplicity.
- This cuts down the number of cases to consider

2.4. **Prime Path Coverage (PPC)** Each reachable prime path in  $G$  is contained in some path  $p \in TR$ .



Prime Paths =  
*Maximal simple paths =*

$(n_0, n_1, n_2),$   
 $(n_0, n_1, n_3, n_4),$   
 $(n_1, n_3, n_4, n_1),$   
 $(n_3, n_4, n_1, n_3),$   
 $(n_4, n_1, n_3, n_4),$   
 $(n_3, n_4, n_1, n_2)$

$P_1 = (n_0, n_1, n_2)$

$P_2 = (n_0, n_1, n_3, n_4, n_1, n_2)$

$P_3 = (n_0, n_1, n_3, n_4, n_1, n_3, n_4, n_1, n_2)$

every prime paths can be found in  
 either P1 or P3

$TS_1 = \{p_1, p_3\}$  satisfies  
 prime path coverage

$TS_2 = \{p_1, p_2\}$  doesn't satisfy  
 prime path coverage! (why?)

# Computing Prime Paths

- One advantage is that the set of all prime paths can be computed by a simple dynamic programming algorithm
- See Amman and Offut Chpt. 2 for details
- Then test cases can be derived manually (heuristic: start from longest paths?) or automatically.

**2.7. Complete Path Coverage (CPC)** Every reachable path in  $G$  is contained in some path  $p \in TR$ .

Infeasible if  $G$  has infinitely many paths

**2.8. Specified Path Coverage (SPC)** Every reachable path in a set  $S$  of test paths is contained in some path  $p \in TR$ . Here  $S$  is supplied as a **parameter**.

**Example heuristic.**  $S$  contains paths that traverse every **loop free path**  $p$  in  $G$  and every **loop** in  $G$  both **0** and **1** times.



## Type 2 : Logic Coverage

- Graph and data flow coverage force execution of certain paths (branches) through code.
- They don't necessarily exercise **different ways of taking the same branch**.
- We can **partition** the number of ways to be finite and coverable.
- Since a Boolean condition may contain many **Boolean** and **data** (**int, float, object**) variables, we consider **exercising the condition in different ways**.

# Clauses and Predicates

- A **clause** is a Boolean valued expression with no Boolean valued sub-expression (i.e. **indivisible atomic**)
- Examples:  $p$ ,  $\text{myGuard}$ ,  $x==y$ ,  $x<=y$ ,  $x>y$   
boolean variable      binary predicate such as ==, <=, >
- A **predicate** is a Boolean combination of clauses using Boolean operators e.g.  $\&$ ,  $\mid$ ,  $!$ , eager operator
- Let  $P$  be a set of predicates
- For  $p \in P$ , let  $C_p$  be the set of all clauses in  $p$ .

For each predicate in  $P$

eager operator vs lazy operator  
e.g.  $\&$ ,  $\mid$  vs e.g.  $\&\&$ ,  $\mid\mid$

e.g.  $P: \{A \mid B \mid C, A \& B \& C, A \mid B \& C\}$

$p: A \mid B \mid C$

$C_p: \{A, B, C\}$

# Type 2 Coverage

- For logic coverage, a test requirement **tr** is a logical constraint on input data values

# Covering a Logic Criterion $C$

**Definition**: Let  $TR$  be a set of test requirements demanded by a logic criterion  $C$ . A test suite  $TC$  satisfies  $C$  if, and only if for every test requirement  $\Phi \in TR$ , there is at least one test case  $t$  in  $TC$  such  $t$  satisfies  $\Phi$ .

- Again this definition implies 100% coverage.
- We can also have  $< 100\%$

# Logic Coverage Measures

- Look at some well known coverage models
- Increasingly sophisticated and subtle,  
so that can really dig in to the structure of the code
- Powerful for exactly these reasons!
- Should produce better testing results?  
Does better theory led to better practices?
- Since a test requirement is a constraint, it may not be solvable i.e. **dead code**

# Predicate Coverage

- **3.12 Predicate Coverage (PC)** For each predicate  $p \in P$ , the set  $TR$  contains: (1) a requirement that implies  $p$  is reached and evaluates to **true**, and (2) a requirement that implies  $p$  is reached and evaluates to **false**.
- Example:  $p = a \mid b$
- $TR1 \ P = \text{true}, \quad TC1 = (a = T, b = F)$
- $TR2 \ P = \text{false}, \quad TC2 = (a = F, b = F)$
- Notice here we never test for  $b = T$

It is already 100% predicate coverage.

# Clause Coverage

- **3.13 Clause Coverage (CC)** For each predicate  $p \in P$ , and each clause  $c \in C_p$  the set  $TR$  contains: (1) a requirement that implies  $c$  is reached and evaluates to **true**, and (2) a requirement that implies  $c$  is reached and evaluates to **false**.
- Example:  $p = a \vee b$   $p$  is a predicate,  $a$  and  $b$  makes 2 clauses
- TR1  $a = \text{true}$ , TC1 =  $(a = T, b = F)$
- TR2  $a = \text{false}$ , TC2 =  $(a = F, b = T)$
- TR3  $b = \text{true}$ , TC3 = TC2
- TR4  $b = \text{false}$ , TC4 = TC1 100% clause coverage
- Notice  $p$  is always **true**, so we satisfy **CC** but not **PC**
- So **PC** and **CC** are **independent coverage criteria**.

# Distributive vs. non-distributive

- Note: these definitions of PC and CC are **non-distributive**, i.e. We don't take all combinations of all predicate or clause values. (Can be unsolvable combinations even without dead code!)
- (Non-distributive) **PC** – linear growth.
- (Non-distributive) **PC** implies **EC**, but not **EC<sup>n</sup>** for **n ≥ 2**.
- **Distributive PC** - exponential growth.



# Brute Force Approach to PC & CC

- **3.14 Combinatorial Coverage (CoC)** For each predicate  $p \in P$ , and every possible truth assignment  $\alpha$  to the clauses  $C_p$  of  $p$  the set  $TR$  contains a requirement which implies  $p$  is reached and the clauses evaluate to  $\alpha$ .

A | B | C | p1 | p2 | p3 ... | pn

-----  
F | F | F  
F | F | T  
F | T | F  
F | T | T  
T | F | F  
T | F | T  
T | T | F  
T | T | T

- CoC implies both PC and CC.
- CoC is also called multiple condition coverage (MCC).
- Too strong? Too many test cases? Use less?

# Active Clause Coverage

Example:  $p = a \mid b$

$TC1 = (a = T, b = T), TC2 = (a = F, b = F)$

$(TC1, TC2)$  satisfies both PC and CC

- Effect of  $a$  on its own and  $b$  on its own are never considered.
- Notice  $b = T$  masks the effect of  $a$  (and vice versa)
- But  $b = F$  completely exposes the effect of  $a$  (vice versa)
- We say that  $a$  **determines**  $p$  in this latter case
- *Can we find something **more expressive than PC or CC** but **less expensive than CoC** which handles this?*  
not enough  
too much
- Use notion of **active clause** which **determines** the overall predicate value

# Determination

**Definition:** Given a clause  $c$  in a predicate  $p$  we say that  $c$  determines  $p$  under assignment  $\alpha$  iff changing the value of  $c$  under  $\alpha$  (and only this value) changes the truth value of  $p$ .

values chosen for other clauses must be the same

The idea is that in some contexts (assignments)  $c$  “has complete control” of  $p$ , and we should test this context.

Notice determination is a local property depending only on a truth table.

Below, when  $G1(C) = G1(D) = G1(E) = T$  then  $D$  determines  $P$   
When  $R1(C) = R1(D) = R1(E) = F$  then  $D$  again determines  $P$ .

C	D	E	P	
T	T	T	T	G1
F	T	T	T	
T	F	T	F	G2
F	F	T	T	
T	T	F	T	
F	T	F	F	R1
T	F	F	T	
F	F	F	T	R2

# Refined Clause Coverage Models

**3.43 Active Clause Coverage (ACC)** For each predicate  $p \in P$  and each clause  $c \in C_p$  which determines  $p$  (under some  $\alpha$ ), the set  $TR$  contains two requirements for  $c$ :  $c$  is reached and evaluates to **true**, and  $c$  is reached and evaluates to **false**.

**Example:** For ACC of clause  $D$  above there are 4 possible pairs of test cases

$(G1, G2), (R1, R2), (G1, R2), (G2, R1)$

# Ambiguity

- ACC can be seen as **ambiguous**.
- Do the other clauses get the same assignment when **c** is true and **c** is false, or can they have different assignments?
- We may not be able to **isolate** individual clauses
- Problems of **masking**, **logical overlap** and **side-effects** (e.g. **variable synonyms**) between clauses
- Consider e.g. **p = (x>10) -> (x>0)**
- If the first clause is set to true the second can never be false.

# Different Values

- **3.15 General Active Clause Coverage (GACC)** For each predicate  $p \in P$  and each clause  $c \in C_p$  which determines  $p$ , the set  $TR$  contains two requirements for  $c$ :  $c$  is reached and evaluates to **true**, and  $c$  is reached and evaluates to **false**. The values chosen for the other clauses  $d \in C_p, d \neq c$ , need not be the same in both cases.

referring to the truth value in C, D, E in slide 36

**Example:** For GACC of clause D above there are 4 possible pairs of test cases  
(G1,G2), (R1,R2), (G1,R2), (G2,R1)

# GACC Problem: Clause Correlation

- One problem is that GACC does not imply PC.

Consider the predicate  $p = a \leftrightarrow b$

For some  $\alpha$ ,  $a$  determines  $p$  (so does  $b$ ) so let:

TC1:  $a = T$ ,  $b = T$  so  $p = \text{true}$

TC2:  $a = F$ ,  $b = F$  so  $p = \text{true}$

For this test suite  $p$  never becomes false so PC is not satisfied although GACC is!

- Here the correlation between  $a$  and  $b$  is explicit in the condition, but it may be implicit in the code.



# Combining GACC and PC

## 3.16 Correlated Active Clause Coverage (CACC)

For each predicate  $p \in P$  and each clause  $c \in C_p$  which determines  $p$ , the set  $TR$  contains two requirements for  $c$ :  $c$  is reached and evaluates to **true**, and  $c$  is reached and evaluates to **false**. The values chosen for the other clauses  $d \in C_p, d \neq c$ , must cause  $p$  to be **true** in one case and **false** in the other.

**Example:** For CACC of clause  $D$  above there are 2 possible test suites  $(G1, G2), (R1, R2)$ .

# Why not solve using Determination?

is about keeping the colour constant

## 3.15 Restricted Active Clause Coverage (RACC)

For each predicate  $p \in P$  and each clause  $c \in C_p$  which determines  $p$ , the set  $TR$  contains two requirements for  $c$ :  $c$  is reached and evaluates to **true**, and  $c$  is reached and evaluates to **false**. The values chosen for the other clauses  $d \in C_p$ ,  $d \neq c$ , must be the same in both cases.

refer to slide 35

**Example:** For RACC of clause  $D$  above there are 2 possible test suites  $(G1, G2)$ ,  $(R1, R2)$ .

# Determination is really a global property!

Consider the code snippet

```
x := y;  
...  
if (x>0 or y>0 ) then ...
```

For predicate  $p = (x > 0 \mid y > 0)$  it looks like PC using RACC is possible from a determinacy analysis of  $(a \mid b)$  where  $a = x > 0$  and  $b = y > 0$ .

But this is misleading, since here  $x$  and  $y$  are effectively **synonyms** and RACC is not satisfiable at all.

Amman and Offut give a different kind of example, based (essentially) on **logical overlap of clauses**.

# Safety Critical Testing: MCDC

- In [DO-178B](#), the [Federal Aviation Authority](#) (FAA) has mandated a minimum level of logic coverage for level A (highest safety) avionic software.
- “*Modified Condition Decision Coverage*” (MCDC)
- Has been some confusion about this definition
- “*Unique Cause MCDC*” (original definition) is RACC
- “*Masking MCDC*” (new definition) is CACC

# Type 3: Data Flow Coverage

- A **definition** of a variable  $v$  is any statement that writes to  $v$  in memory
- A **use** of  $v$  is any statement that reads  $v$  from memory.
- A path  $p = (n_1, \dots, n_k)$  from a node  $n_1$  to a node  $n_k$  is **def-clear** for  $v$  if for each  $1 < j < k$  node  $n_j$  has no statements which write to  $v$

If at least one node  $n_j$  has statement which write to  $v$ , it is not def-clear

# Definition/Use Paths (du-paths)

no repetition apart from start and end

- A **du-path** w.r.t.  $v$  is a simple path

$$p = (n_1, \dots, n_k)$$

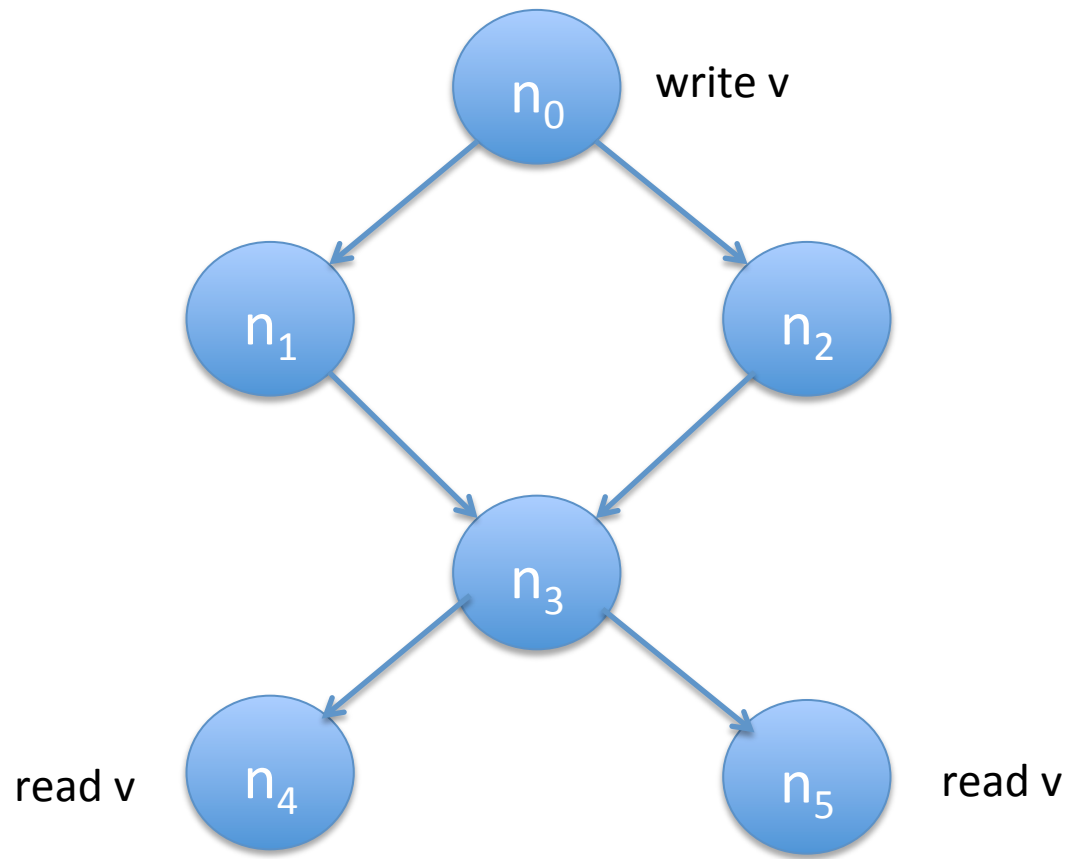
- Such that:

1. A statement in  $n_1$  writes to  $v$
2. Path  $p$  is def-clear for  $v$
3. A statement in  $n_k$  reads  $v$

- $du(n, v)$  = set of all du-paths wrt  $v$  starting at  $n$  write to v
- $du(m, n, v)$  = set of all du-paths wrt  $v$  starting at  $m$  and ending at  $n$

# Data flow coverage models

- **2.9. All-defs Coverage (ADC)** For each def-path set  $S = \text{du}(n, v)$  the set  $TR$  contains at least one path  $d$  in  $S$ .
- **2.10 All-uses Coverage (AUC)** For each def-pair set  $S = \text{du}(m, n, v)$  the set  $TR$  contains at least one path  $d$  in  $S$ . AUC contains ADC
- **2.11 All-du-paths Coverage (ADUPC)** For each def-pair set  $S = \text{du}(m, n, v)$  the set  $TR$  contains every path  $d$  in  $S$ .



All-defs =  $\{(n_0, n_1, n_3, n_4)\}$

All-uses =  $\{(n_0, n_1, n_3, n_4)$   
 $(n_0, n_1, n_3, n_5)\}$

All-du-paths =  $\{(n_0, n_1, n_3, n_4)$   
 $(n_0, n_1, n_3, n_5),$   
 $(n_0, n_2, n_3, n_4),$   
 $(n_0, n_2, n_3, n_5)\}$