Problem Set 10

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$$H_0 : \sigma_A^2 = \sigma_B^2$$

$$H_1 : \sigma_A^2 \neq \sigma_B^2$$

1

```
a<- c(170, 172, 185, 173, 169, 175, 171, 170 )
b<- c(165, 167, 164, 166, 168, 165, 167, 166 )
an<-length(a)
adf<-(an-1)
bn<-length(b)
bdf<-(bn-1)
abar<- mean(a)
bbar<- mean(b)


print(c(abar,bbar))
```

```
## [1] 173.125 166.000
```

2

```
avar<- var(a)
bvar<- var(b)
print(c(avar,bvar))
```

```
## [1] 26.696429  1.714286
```

3

```r
F<-avar/bvar
print(F)
```

```
## [1] 15.57292
```

4

```r
alpha <- 0.01
f.critical <- qf(1-alpha/2, df1 = (an-1), df2 = (bn-1))
print(f.critical)
```

```
## [1] 8.885389
```

5

Given that our F-statistic (15.57292) is greater than our F-critical (8.885389), we should reject the null hypothesis. We have evidence that our sample variances are significantly different from one another.

6

We can say that our variances are statistically different, so we must use the differences in means test that accounts for different variances.

7

Our null hypothesis states that there is no statistical difference between the mean donations from Regions A and B. Our alternative hypothesis would be that there is some difference between the mean donations. We are not claiming a direction with the alternative.

$$H_0 : \bar{A} = \bar{B}$$

$$H_1 : \bar{A} \neq \bar{B}$$

8

$$t = \frac{\bar{A} - \bar{B}}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

$$t = \frac{173.125 - 166}{\sqrt{\frac{26.69643}{8} + \frac{1.714286}{8}}}$$

$$t = \frac{7.125}{\sqrt{3.337054 + 0.2142857}} = \frac{7.125}{\sqrt{3.55134}} = \frac{7.125}{1.8845} = 3.780844$$

9

To find the critical value, we need to find the degrees of freedom first, since the variances between the samples differs. After we find the degrees of freedom (7.895305), we can find the critical value. In this case, our critical value is 2.905637.

$$df = \frac{(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2})^2}{\frac{(\frac{s_1^2}{n1})^2}{n_1 - 1} + \frac{(\frac{s_2^2}{n2})^2}{n_2 - 1}}$$

$$df = \frac{(\frac{26.69643}{8} + \frac{1.714286}{8})^2}{\frac{(\frac{26.69643}{8})^2}{7} + \frac{(\frac{1.714286}{8})^2}{7}}$$

$$df = \frac{12.61201}{1.597407} = 7.895305$$

```
num<- (((26.69643/8)+(1.714286/8))^2)
den<- ((((26.69643/8)^2)/7)+(((1.714286/8)^2)/7))
df<-num/den
print(df)
```

```
## [1] 7.895305
```

```
tcrit<-qt(1-alpha, df=df)
print(tcrit)
```

```
## [1] 2.905637
```

10

Our T-statistic (3.780844) is greater than our critical value (2.905637), so we can reject the null hypothesis. Our data suggests that there is a significant difference between the mean donations in Region A and Region B.

11

Our P-value is 0.005513292, which means that the probability of obtaining a T-statistic at least as extreme as the one we found, assuming that the null hypothesis is correct, is roughly 0.55%, or about $\frac{1}{181}$.

```
t<-3.780844

pvalue<-2*(pt(-abs(t), df))
print(pvalue)
```

```
## [1] 0.005513292
```

12

Looks like we match up!

```r
t.test(a,b)
```

```
##
##  Welch Two Sample t-test
##
## data:  a and b
## t = 3.7808, df = 7.8953, p-value = 0.005513
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##   2.769285 11.480715
## sample estimates:
## mean of x mean of y
##   173.125   166.000
```