PS5

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Question 1

1. We are looking for,

$$P(\neg Late \cap Traffic \cap \neg Rain)$$

which we could also understand as,

$$P(\neg Late|Traffic \cap \neg Rain) * P(Traffic|\neg Rain) * P(\neg Rain)$$

We know that $P(Late|Traffic \cap \neg Rain) = \frac{1}{4}$, so it's complement, or $P(\neg Late|Traffic \cap \neg Rain) = \frac{3}{4}$. It was given that $P(Traffic|\neg Rain) = \frac{1}{4}$ and we can assume that $P(\neg Rain) = \frac{2}{3}$. Multiplying these probabilities gives us a probability of $\frac{1}{8}$.

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(3/4)*(1/4)*(2/3)
```

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## [1] 0.125
```

2. The total probability of being late is the sum of the instances that we are late, whether that be with rain and traffic, with rain and no traffic, etc. You can see how I tracked that in the chunk below, but the sum of the probabilities of being late, and therefore the probability of being late, is ~ 0.229, or $\frac{11}{48}$.

```
#P(R,T,L) = (1/3)(1/2)(1/2) = 1/12
#P(R,T^c, L) = (1/3)(1/2)(1/4) = 1/24
#(R^C, T, L) = (2/3)(1/4)(1/4) = 1/24
#(R^c, T^c, L) = (2/3)(3/4)(1/8) = 1/16
(1/12)+(1/24)+(1/24)+(1/16)
```

```
## [1] 0.2291667
```

```
#(32/192) + (12/192)= (44/192) = (11/48)
```

3. Since we are again working with a conditional probability, we know:

$$P(Rain|Late) = \frac{P(Rain \cap Late)}{P(Late)}$$

We have found P(Late), so we just need to find the joint probability, $P(Rain \cap Late)$. In the last exercise I found that the 2 instances of it raining and we being late had a probability of $\frac{1}{12}$ and $\frac{1}{24}$. This gives us a joint probability of

$$P(Rain \cap Late) = (\frac{1}{12} + \frac{1}{24}) = \frac{1}{8}$$

So, we end up with

$$P(Rain|Late) = \frac{P(Rain \cap Late)}{P(Late)} = \frac{\frac{1}{8}}{\frac{11}{48}} = \frac{6}{11}$$

Question 2

We are looking for the probability of disease given a positive test. We know the probability of having the disease, but not the probability of a random person testing positive. We can use Bayes' rule to solve this.

P(Disease|Positive)= $\frac{P(Positive|Disease)P(Disease)}{(P(Positive|Disease)P(Disease))+(P(Positive|Disease^c)P(Disease^c))}$

Substituting our given probabilities gives us:

$$\frac{P(\frac{99}{100})P(\frac{1}{10{,}000})}{P(\frac{99}{100})P(\frac{1}{10{,}000}) + P(\frac{1}{50})P(\frac{9{,}999}{10{,}000})}$$

Which is equal to ~ 0.0049.

```r
((99/100)*(1/10000))/(((99/100)*(1/10000))+((1/50)*(9999/10000)))
```

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## [1] 0.004926108
```

Question 3

1. Because it is asking for the number of successes from a given n, this is a binomial distribution.

2. Because this question is asking for the probability of more than 2 supporters, we could also understand this as the $P(X \geq 3)$. To find this, we can simply subtract $P(X \leq 2)$ from 1. The probability of selecting 3 or more Green party supporters from a randomly selected group of 10 people from state A is ~ 0.617.

$$F(x) = P(X \geq 3) = 1 - (\sum_{k=0}^{x} \binom{10}{2} p^2 (1-p)^8)$$

```r
pbinom(2, size=10, prob = 0.30) -> p2

1-p2
```

```
## [1] 0.6172172
```

Question 4

1. Y = {0,1,2}, as the probability of Y taking a value outside of this set is 0.
2. Y $\neq$ 1.5, but can take the values of 2, so it's just P(y=2), or $\frac{5}{12}$.
3. This can be simplified to P(Y=1 $\cup$ Y=2), so $\frac{3}{4}$.
4. You could rewrite this as P(Y=0 $\cup$ Y=1), which is just $\frac{7}{12}$, or ~ 0.58.
5. I read this as just simplifying to P(Y=2), which = $\frac{5}{12}$

6.