Problem Set 6

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Question 1

1.

$$E(X) = \sum xp(x) = (P(x=1)1) + (P(x=2)2) + (P(x=3)3)$$

Substitute our probabilities and x's.

$$((0.5*1) + (0.4*2) + (0.1*3)) = 1.6$$

E(x)=1.6, or $\frac{8}{5}$

```
(0.5*1)+(0.4*2)+(0.1*3)
```

```
## [1] 1.6
```

2.

Var(X) =

$$Var(X) = E[(X-x)^2] = E(x^2) - [E(x)]^2$$

To find $E(x^2)$

$$(P(x=1)1^2) + (P(x=2)2^2) + (P(x=3)3^2) = ((0.5*1^2) + (0.4*2^2) + (0.1*3^2)) = 3$$

To find $E(x)^2$

$$[E(x)]^2 = 1.6^2 = 2.56$$

Finally, we substitute back into the formula for variance and find Var(x)=0.44.

$$3 - (1.6^2) = \frac{75}{25} - \frac{64}{25} = \frac{11}{25} = 0.44$$

```
(0.5*(1^2))+(0.4*(2^2))+(0.1*(3^2))-> a
1.6^2 -> b
a-b
```

```
## [1] 0.44
```

Standard deviation is simply the square root of the variance, or

$$SD(X) = \sigma = \sqrt{\sigma^2} = \sqrt{Var(X)} = \sqrt{0.44} \approx 0.66$$

```r
sqrt(0.44)
```

```
## [1] 0.663325
```

3.

I simply substituted $f(x_i)$ for every x_i.

$$Ef(x) = \sum(\frac{2}{x_i})p(x_i) = \sum(0.5 * \frac{2}{1}) + (0.4 * \frac{2}{2}) + (0.1 * \frac{2}{3}) \approx 1.4\overline{66}$$

```r
(0.5*(2/1))+(0.4*(2/2))+(0.1*(2/3))
```

```
## [1] 1.466667
```

Question 2

1. We can use substitution to find E(x) from E(f(x)). We can then distribute the E. The expectation of some constant is always the constant, so we can drop E(7).

$$E(f(x)) = E(-3x + 7) = -3E(x) + E(7) = -3E(x) + 7$$

Substitute the given E(f(x))= 1,

$$1 = -3E(x) + 7$$

Move constants to one side,

$$1 - 7 = -3E(x)$$

Divide by -3,

$$-6 = -3E(x)$$

And we get E(x) = 2.

$$2 = E(x)$$

2.

We know that $Var(x) = E(f(x)^2) - (E(x))^2$, and we have already found E(x), so we know that $(E(x))^2 = 2^2$, or 4. We need to find $E(f(x)^2)$.

$$E(f(x)^2) = E(-3x+7)^2 = E(9x^2 - 42x + 49) = 9E(x^2) - 42E(x) + 49$$

Substitute E(x)=2,

$$9E(x^2) - 42(2) + 49 = 9E(x^2) - 84 + 49$$

Set equation equal to 9,

$$9 = 9E(x^2) - 84 + 49$$

Move over the constants,

$$44 = 9E(x^2)$$

Divide by 9

$$\frac{44}{9} = E(x^2)$$

Now that we have found both $E(f(x)^2)$ and $E(x)^2$, we can find Var(x)

$$Var(x) = \frac{44}{9} - \frac{36}{9} = \frac{8}{9}$$

Question 3

1. We know the formula for the PDF of a uniform variable, so we can substitute our bounds into that formula and find the PDF.

$$PDF = \frac{1}{b-a} = \frac{1}{7-1} = \frac{1}{6}$$

so,

$$F(X) = \begin{cases} \frac{1}{6} & : 1 \leq x \leq 7 \\ 0 & : otherwise \end{cases}$$

R check:

```r
dunif(2, min=1, max=7)
```

```
## [1] 0.1666667
```

2. We also know the function for the CDF, so we can again input our bounds.

$$F(X) = \begin{cases} 0 & : x < 1 \\ \frac{x-1}{6} & : 1 \le x < 7 \\ 1 & : x \ge 7 \end{cases}$$

R check:

```
punif(2, min=1, max=7)
```

```
## [1] 0.1666667
```

```
(2-1)/6
```

```
## [1] 0.1666667
```

Question 4

First let's solve the integral of $3x^2$.

$$\int_a^b 3x^2 dx = 3\frac{x^{2+1}}{2+1} = 3\frac{x^3}{3} = x^3|_a^b$$

I understood the conditional probability to be:

$$\frac{P(\frac{1}{3} < X \le \frac{2}{3})}{P(X > \frac{1}{3})} = \frac{x^3|_{\frac{1}{3}}^{\frac{2}{3}}}{x^3|_{\frac{1}{3}}^{1}} = \frac{(\frac{2}{3})^3 - (\frac{1}{3})^3}{1^3 - (\frac{1}{3})^3} = \frac{\frac{8}{27} - \frac{1}{27}}{1 - \frac{1}{27}} = \frac{\frac{7}{27}}{\frac{26}{27}} = \frac{7}{26}$$

We find the probability of intersection via the integral, solving for our bounds and subtracting the lower bound from the upper bound, and divide that by the probability of the given observation.So $P(X \le \frac{2}{3}|X > \frac{1}{3}) = \frac{7}{26}$