Problem Set 4

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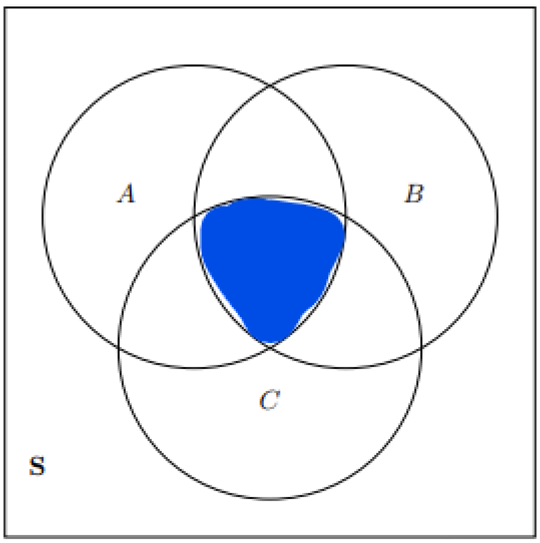
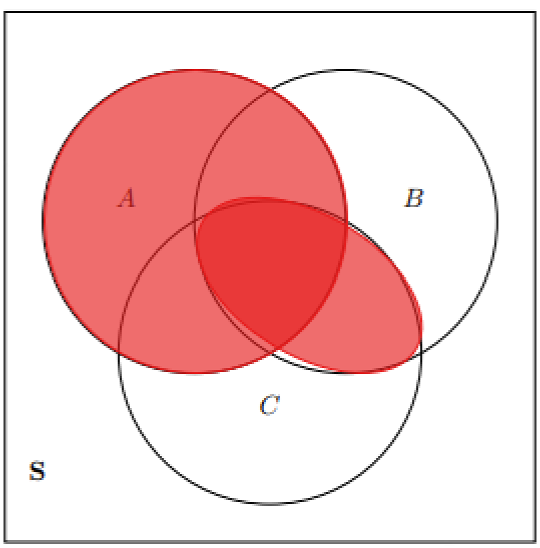
Task 1

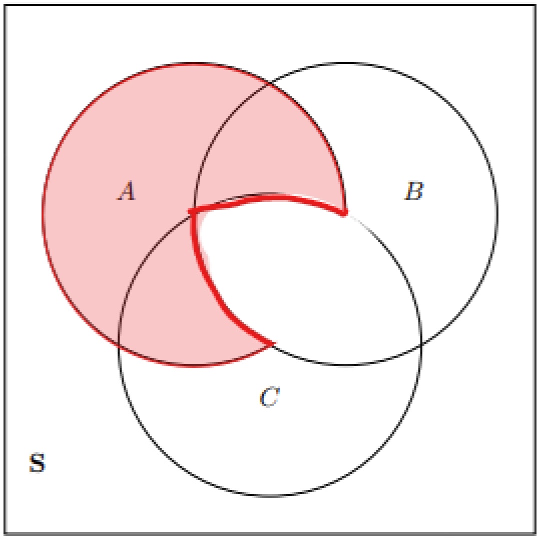
1. $(A \cap B \cap C)$



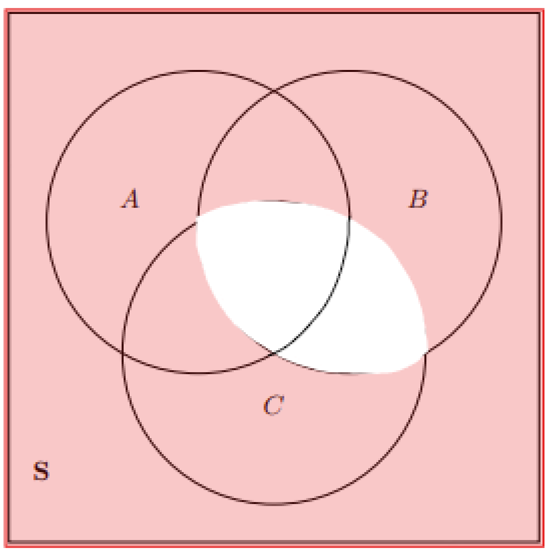
2. $A \cup (B \cap C)$



3. $A - (B \cap C)$



4. $A \cup (B \cap C)^c$



Task 2

1. $C = A \cup B$

C = {4, 7, 9, banana, cloud, 19, 21} - The union of A and B

2. $C = A \cap B$

C = {4, cloud, 19, 21} - The intersection of A and B

3. C= A - B

C = {9, banana} - All elements in A that are not in B

4. C= B - A

C = {7} - All elements in B that are not in A

5. C = { ∅}

There can be no elements in a null set.

Task 3

1. P(A ∪ B ∪ C) if A, B, and C are mutually exclusive.

Because the sets are mutually exclusive, we can just add the probabilities, which is equal to 0.91

2. P(A ∪ B ∪ C) if A is mutually exclusive from B and C, but B and C overlap.

Because B and C are not mutually exclusive, we will have to subtract the intersection of B and C, or 0.05. This gives us:

P(A ∪ B ∪ C) = P(A) + P(B) + P(C) - P(B and C) = 0.86

Task 4

There are 24 possible combinations. Re-positioning the categories gives us essentially the permutation table of 4!. 4 policy types times 3 administration levels times 2 position types gives us $4 \cdot 3 \cdot 2 \cdot 1$, or 4!.

```
factorial(4)
```

```
## [1] 24
```

Task 5

1. When looking for the number of combinations of k from n, we can use:

$$C_n^k = \frac{P_n^k}{k!} = \frac{n!}{k!(n-k)!}$$

In this case we can plug in n=12 and k=5, which gives us 792 possible ways to make a 5 person committee from a 12 member assembly.

```
(factorial(12)/ (factorial(5)*factorial(7)))
```

```
## [1] 792
```

```r
# or

choose(12,5) #combinations
```

```
## [1] 792
```

```r
#Choose(n,k)
```

2. When finding the number of ways to list 6 names, we only need to find 6!. This gives us 720 possible ways to list six names on a ballot.

```r
factorial(6)
```

```
## [1] 720
```

Task 6

1.

$$P(Red) = (P(Red|Bag1) \cdot P(Bag1)) + (P(Red|Bag2) \cdot P(Bag2)) + (P(Red|Bag3) \cdot P(Bag3))$$

To find this we need to utilize the formula for conditional probabilities of a red marble given each bag is the one pulled from. Finding the probability of a red marble, regardless of bag, requires us to add the conditional probabilities together. This gives us a 60% probability of pulling a red when pulling from the bags at random.

```r
((1/3)*(3/4))+((1/3)*(3/5))+((1/3)*(9/20))
```

```
## [1] 0.6
```

2.

$$P(Bag1|Red) = \frac{P(Bag1 \cap Red)}{P(Red)}$$

Given that the probability of pulling from bag 1 is $\frac{1}{3}$, pulling a red from bag 1 is $\frac{3}{4}$, and the probability of pulling a red, as we discovered, was $\frac{3}{5}$, we can rewrite this formula as:

$$P(Bag1|Red) = \frac{\frac{1}{3} \cdot \frac{3}{4}}{\frac{3}{5}}$$

Which is equal to $\frac{5}{12}$, or roughly 0.417

```r
((1/4)/(3/5))
```

```
## [1] 0.4166667
```

Task 7

1. P(HHH) = $\frac{1}{8}$, or 0.125, as there are 8 possible combinations.

```r
0.5*0.5*0.5
```

```
## [1] 0.125
```

2. P(TTH $\cup$ THT $\cup$ HTT) $= \frac{3}{8}$ or 0.375, as each of the three combinations has a probability of $\frac{1}{8}$.

3. This question doesn't explicitly state that the first flip is heads, so I'm operating under the assumption that any of the flips in an observation can be heads. There is a $\frac{7}{8}$ chance an observation has at least one heads, and 4 out of the 8 possible observations have at least 2 heads, so the formula would be:

$$P(HH|H = \frac{\frac{4}{8}}{\frac{7}{8}})$$

   Which is $\frac{4}{7}$, or roughly 0.57

```r
((4/8)/(7/8))
```

```
## [1] 0.5714286
```