HW-4 bmullick.

Q 1.1) By fundamental matrix relation, we have :-

$\tilde{x}_2 F \tilde{x}_1 = 0$. From the figure we get, $\tilde{x}_1 = [0 \ 0 \ 1]^T$ and $\tilde{x}_2 = [0 \ 0 \ 1]^T$ where $\tilde{x}_1$, $\tilde{x}_2$ are the homogeneous coordinates of x_1 and x_2.

Rewriting above equation :-

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$\therefore F_{33} = 0$. Hence Proved.

Q 1.2) As mentioned, the translation is parallel to the X-axis

The translation matrix $t = [t_x, 0, 0]^T$

The cross product matrix :-

$$t_{cross} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

The pure rotation matrix :-

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The essential matrix :- $E = t_{cross} R = t_{cross}$

$$\text{Thus} \quad E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

The epipolar lines are given by the following equation:-

$$l_1^T = \tilde{x}_2^T E \quad \text{and} \quad l_2^T = \tilde{x}_1^T E^T$$

$$\tilde{x}_2 = \begin{bmatrix} x_2 & y_2 & 1 \end{bmatrix}^T \qquad \tilde{x}_1 = \begin{bmatrix} x_1 & y_1 & 1 \end{bmatrix}^T$$

$$\therefore \; l_1^T = \begin{bmatrix} x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & t_x & -t_x y_2 \end{bmatrix}$$

and

$$l_2^T = \tilde{x}_1^T E^T = \begin{bmatrix} 0 & -t_x & t_x y_1 \end{bmatrix}$$

Thus line 1 : $t_x y - t_x y_2 = 0$ and
line 2 : $t_x y - t_x y_1 = 0$ are both
<u>parallel to the x-axis</u>

Q1.3) Let the real world point $p = [x \; y \; z]^T$. At two different time, the corresponding points in the frame of reference camera are:-

$$p_1 = R_1 p + t_1 \qquad p_2 = R_2 p + t_2$$

$$p = R_1^{-1}(p_1 - t_1)$$

substituting this in the previous equation of P_2, we get :-

$$P_2 = R_2 R_1^{-1}(P_1 - t_1) + t_2$$

$$= R_2 R_1^{-1} P_1 - R_2 R_1^{-1} t_1 + t_2$$

Here $\boxed{R_2 R_1^{-1} = R_{rel}}$ ~~R_2~~ and $\boxed{-R_2 R_1^{-1} t_1 + t_2 = t_{rel}}$.

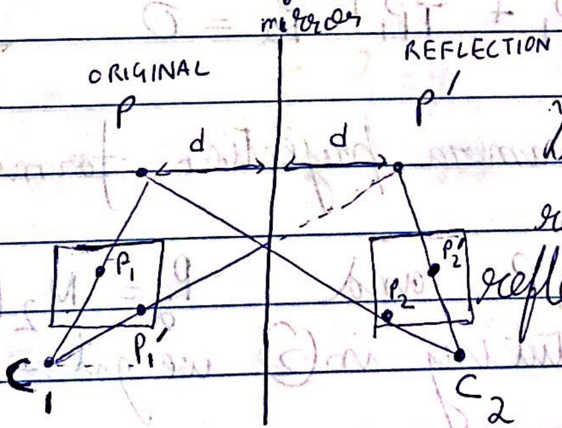
The Essential Matrix formulation is :-

$$E = t_{rel} \times R_{rel}$$

Fundamental Matrix $\Rightarrow F = (K^{-1})^T E K^{-1}$

$$\Rightarrow F = (K^{-1})^T (t_{rel} \times R_{rel}) K^{-1}$$

Q 1.4)



The point in the real world is P and its reflection on the mirror is P'. C_1 is the camera in the real 3D world and C_2 can be assumed to be a virtual camera

→ Here P_1 is the 2D image coordinate of P on camera C_1

→ P_1' is the 2D image coordinate of P' on C_1

→ P_2 is the 2D image coordinate of P on C_2

→ P_2' is the 2D image coordinate of P' on C_2

Since all points of P and P_2' are equidistant w.r.t to the mirror, the transformation between P and P' is pure translation i.e; $R = I$. Let the transformation matrix be T.

Now we can write this transformation as:-

$$P' = TP$$

Similarly, the camera matrix has the same transformation. Thus we get : $M_2 = TM_1$.

Now from the fundamental matrix equation:-

$P_2 F P_1' = 0$. Taking transpose we get: $P_1' F^T P_2 = 0$

Thus we get:-

$$P_2 F P_1' + P_1' F^T P_2 = 0 \quad \text{--- (1)}$$

since $P_1' = TP_1$ we rewrite (1) as:-

$$P_2 F T P_1 + T P_1 F^{T} P_2 = 0 \quad \text{--- (2)}$$

From the camera projection formulation, we get:-

$P_1 = M_1 P$ and $P_2 = M_2 P = TM_1 P$

substituting in (2) we get:-

$$TM_1 P F T M_1 P + T M_1 P F^T T M_1 P = 0.$$

$$\Rightarrow M_1 T P (F + F^T) M_1 T P = 0$$

Since $M_1 T P$ is $\neq 0$, we get:-

$(F + F^T) = 0$. Hence $F = -F^T$

ie Fundamental matrix is skew symmetric

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Part II

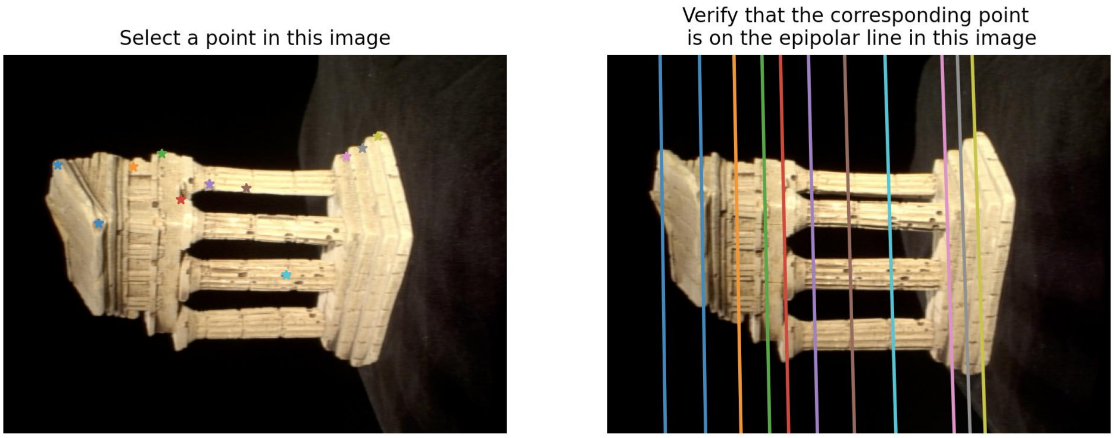
Q 2.1) The value of F matrix is as below:

```
[[ 9.80213860e-10 -1.32271663e-07  1.12586847e-03]
 [-5.72416248e-08  2.97011941e-09 -1.17899320e-05]
 [-1.08270296e-03  3.05098538e-05 -4.46974798e-03]]
```

By forcefully equating F[3,3] to 1 we get:

```
[[-2.19299581e-07  2.95926445e-05 -2.51886343e-01]
 [ 1.28064547e-05 -6.64493709e-07  2.63771739e-03]
 [ 2.42229086e-01 -6.82585550e-03  1.00000000e+00]]
```

Below shows the visualization:



Q 3.1)   E can be estimated from the F matrix. The value of E matrix is as below:

```
[[ 2.26587820e-03 -3.06867395e-01  1.66257398e+00]
 [-1.32799331e-01  6.91553934e-03 -4.32775554e-02]
 [-1.66717617e+00 -1.33444257e-02 -6.72047195e-04]]
```

Q 3.2) Let P_i be a 4×1 vector of a 3D coordinate in homogeneous form. Then, $C_1 P_i = \tilde{x}_{i_1}$ and $C_2 P_i = \tilde{x}_{i_2}$ where C_1 and C_2 are camera matrices.

Thus $C_1 P_i = \tilde{x}_{i_1}$ can be given as:-

$$\begin{bmatrix} C_{11} \\ C_{12} \\ C_{13} \end{bmatrix}_{3\times4} \begin{bmatrix} u_i \\ v_i \\ w_i \\ 1 \end{bmatrix} = \begin{bmatrix} x_{i_1} \\ y_{i_1} \\ 1 \end{bmatrix}$$

and $C_2 P_i = \tilde{x}_{i_2}$

$$\begin{bmatrix} C_{21} \\ C_{22} \\ C_{23} \end{bmatrix}_{3\times4} \begin{bmatrix} u_i \\ v_i \\ w_i \\ 1 \end{bmatrix} = \begin{bmatrix} x_{i_2} \\ y_{i_2} \\ 1 \end{bmatrix}$$

Thus we get:

$$C_{11} P_i = x_{i_1} \quad C_{12} P_i = y_{i_1} \quad C_{13} P_i = 1$$
$$C_{21} P_i = x_{i_2} \quad C_{22} P_i = y_{i_2} \quad C_{23} P_i = 1$$

Rearranging we have:-

$$(x_{i_1} C_{13} - C_{11}) P_i = 0$$

$$\cancel{x}(y_{i_1} C_{13} - C_{12}) P_i = 0$$

$$(x_{i_2} C_{23} - C_{21}) P_i = 0$$

$$(y_{i_2} \cdot C_{23} - C_{22}) P_i = 0$$

Thus $A_i = \begin{bmatrix} x_{i_1} C_{13} - C_{11} \\ y_{i_1} C_{13} - C_{12} \\ x_{i_2} C_{23} - C_{21} \\ y_{i_2} C_{23} - C_{22} \end{bmatrix}$

The reprojection error after triangulation comes to $\boxed{727.897}$

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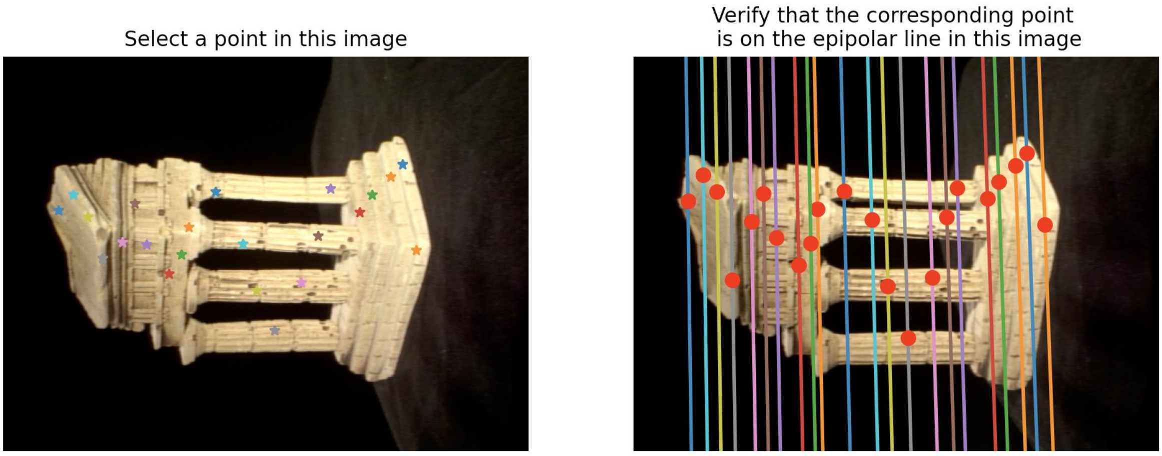
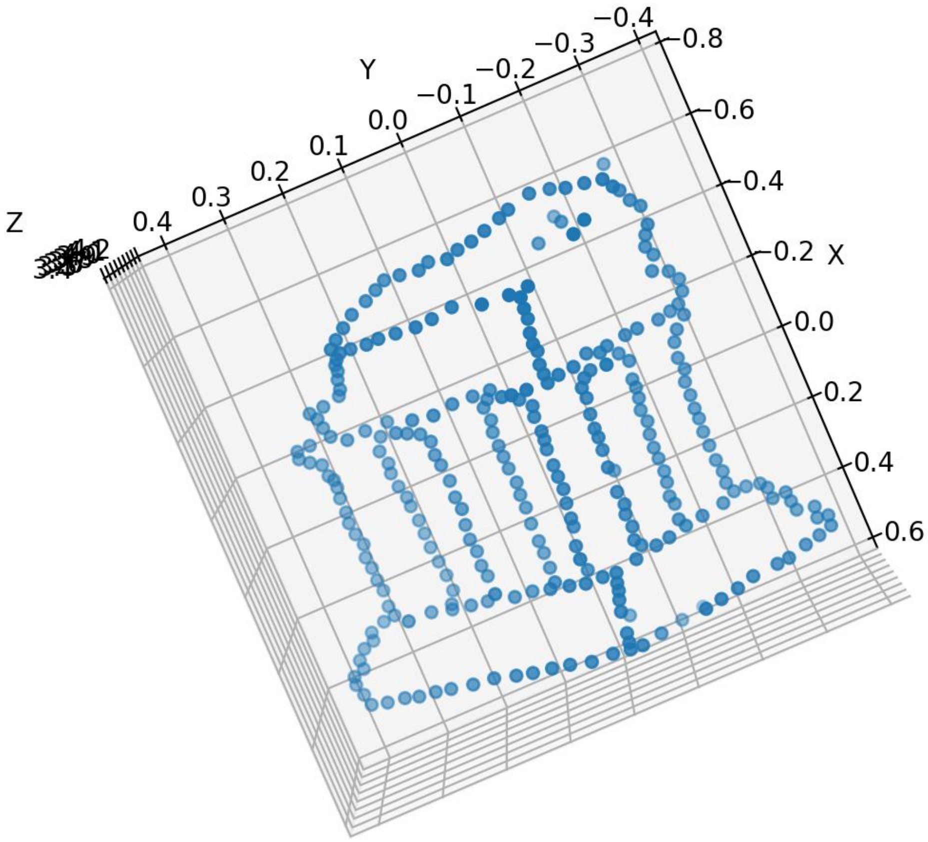
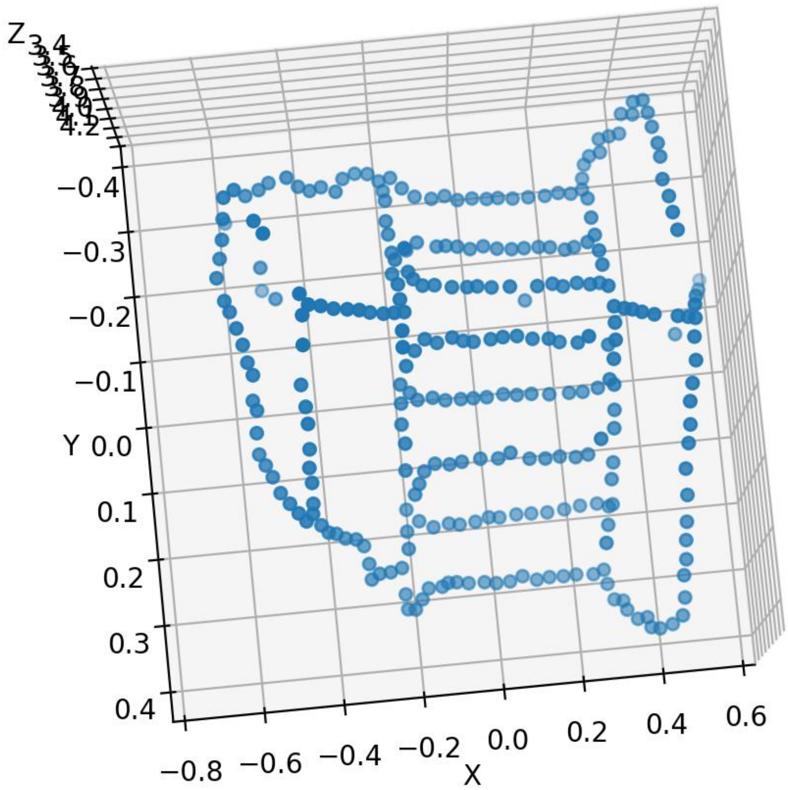
Q 4.1)

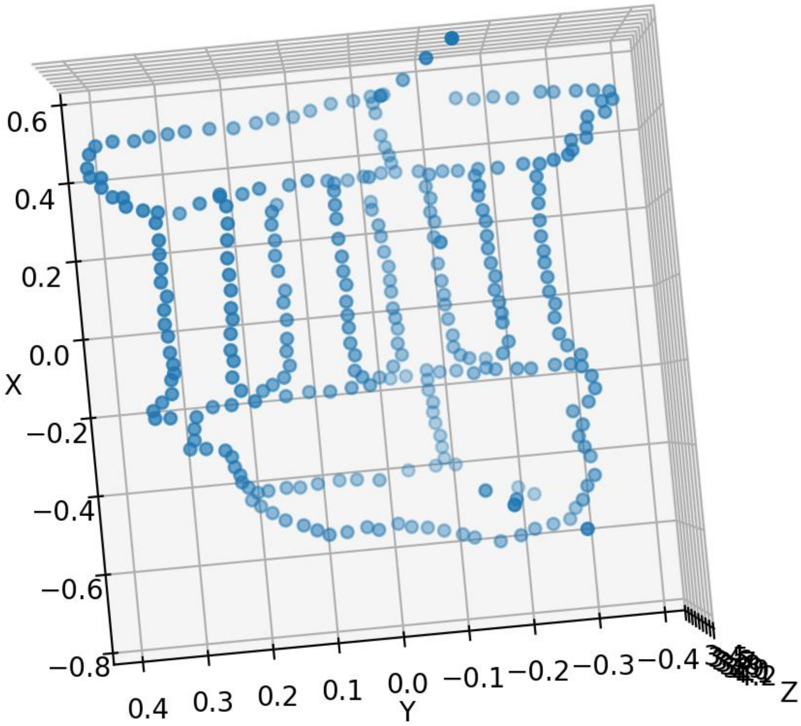
The window size was chosen to be 6 x 6. Below is the GUI visualization with chosen x1, y1 points from the left image and corresponding points x2,y2 with epipolar lines on the right image.

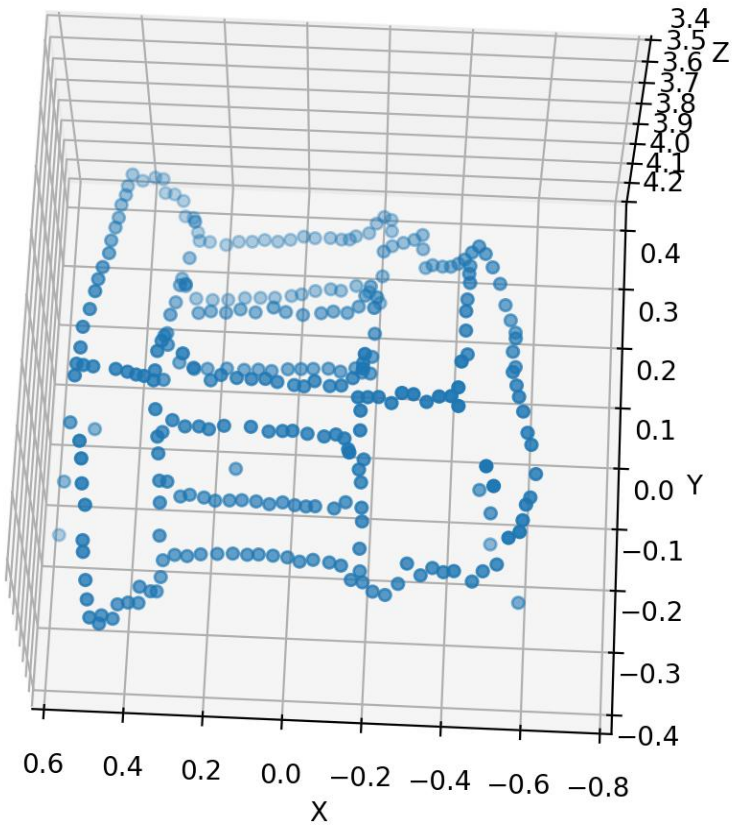


Q 4.2) 3D Visualization:



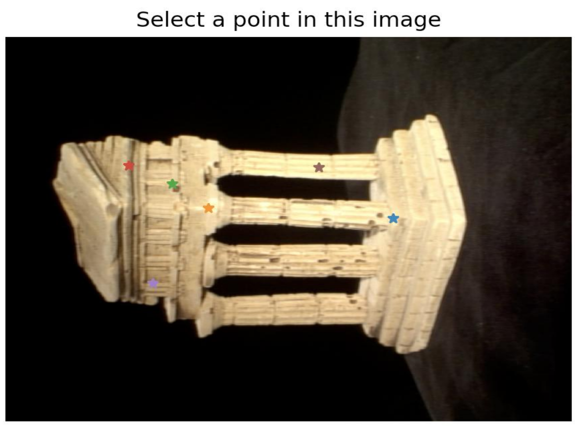

Y
−0.4
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
0.4
X
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
Z

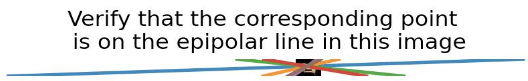

X
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
Y
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
Z



Q)5.1

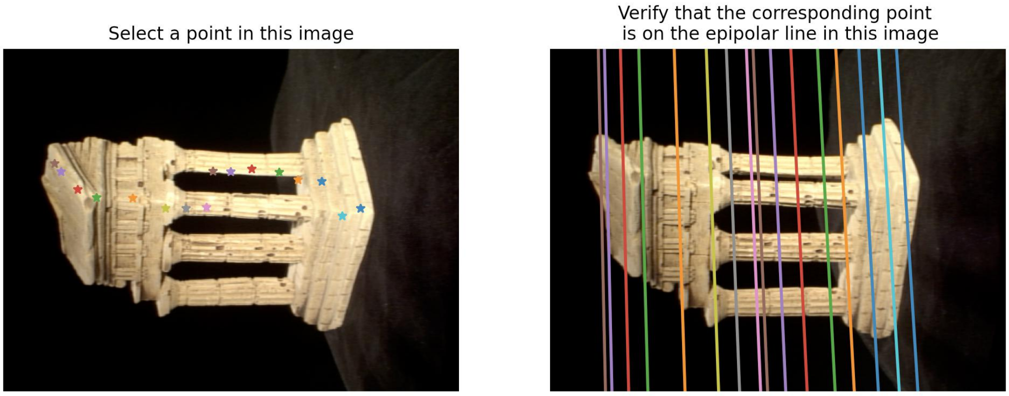
Using the noisy dataset without RANSAC we visualise:





For using RANSAC, Geometric distance between point and epipolar line is taken as the error metric. The points for which the distance came to less than the tolerance value were selected as inliers.

The visualisation with RANSAC at maxiteration = 1000 and tolerance = 0.5:

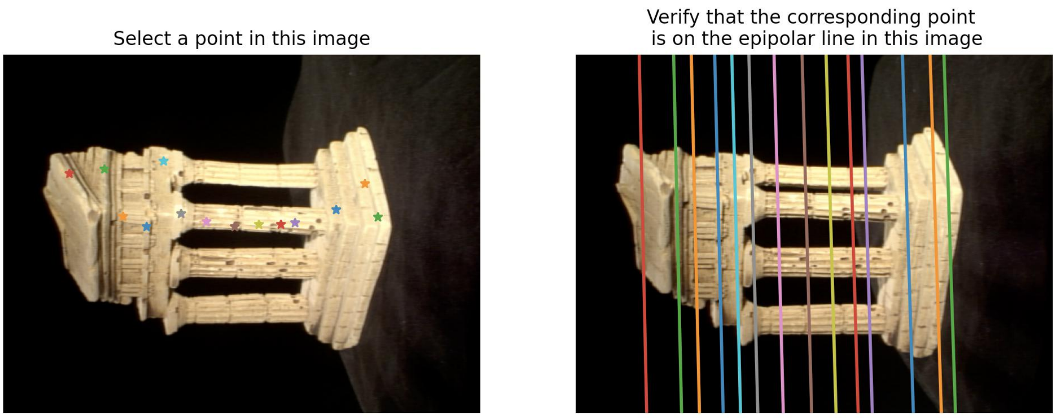


The total number of inliers was found to be 71.

The F matrix corresponding to this is:

```
F RANSAC
[[-8.89506777e-09 -8.57788671e-08  1.12571142e-03]
 [-1.01377037e-07  2.90655595e-09 -7.87190847e-06]
 [-1.07700337e-03  2.48612955e-05 -4.84867168e-03]]
```

The visualisation with RANSAC at maxiteration = 1000 and tolerance = 1:



The total number of inliers was found to be 103.

The F matrix corresponding to this is:

```
F RANSAC
[[ 3.09184450e-09  1.59587861e-07 -1.12434509e-03]
 [ 2.78583871e-08 -3.58431464e-09  1.87233008e-05]
 [ 1.07847784e-03 -3.64911093e-05  4.78679892e-03]]
```

Q 5.3)

The number of inliers was found to be 106 and the reprojection error was 109.84789338763112. Below is the   point scatter obtained after bundle adjustment.