MATHEMATICAL ART

Brian Mintz
Grad Student Seminar,
Spring 2023

WHAT IS MATHEMATICAL ART?
OR, WHY SHOULD I CARE?
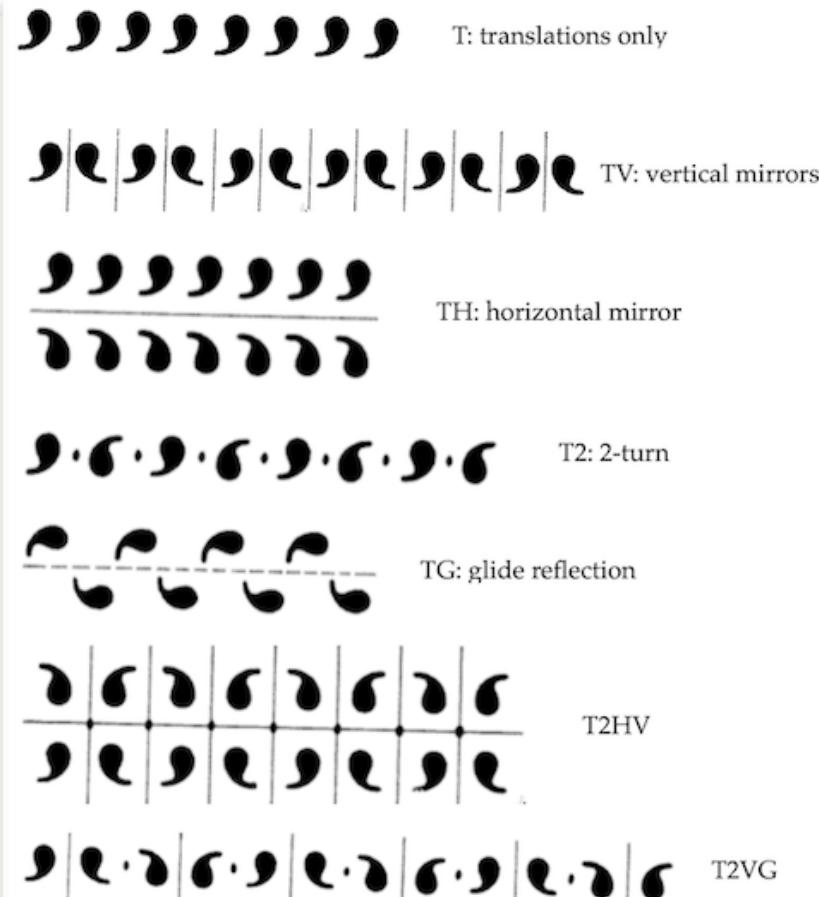
Few people get to see the creative side of mathematics. We can share the beauty of patterns and structure in math through traditional art media.

It's also great outreach, an easy answer to "I'm not a math person," or "why would anyone study math?"

This talk will explore some of the myriad ways math has been used in art, and hopefully encourage more people to make and talk about mathematical art!
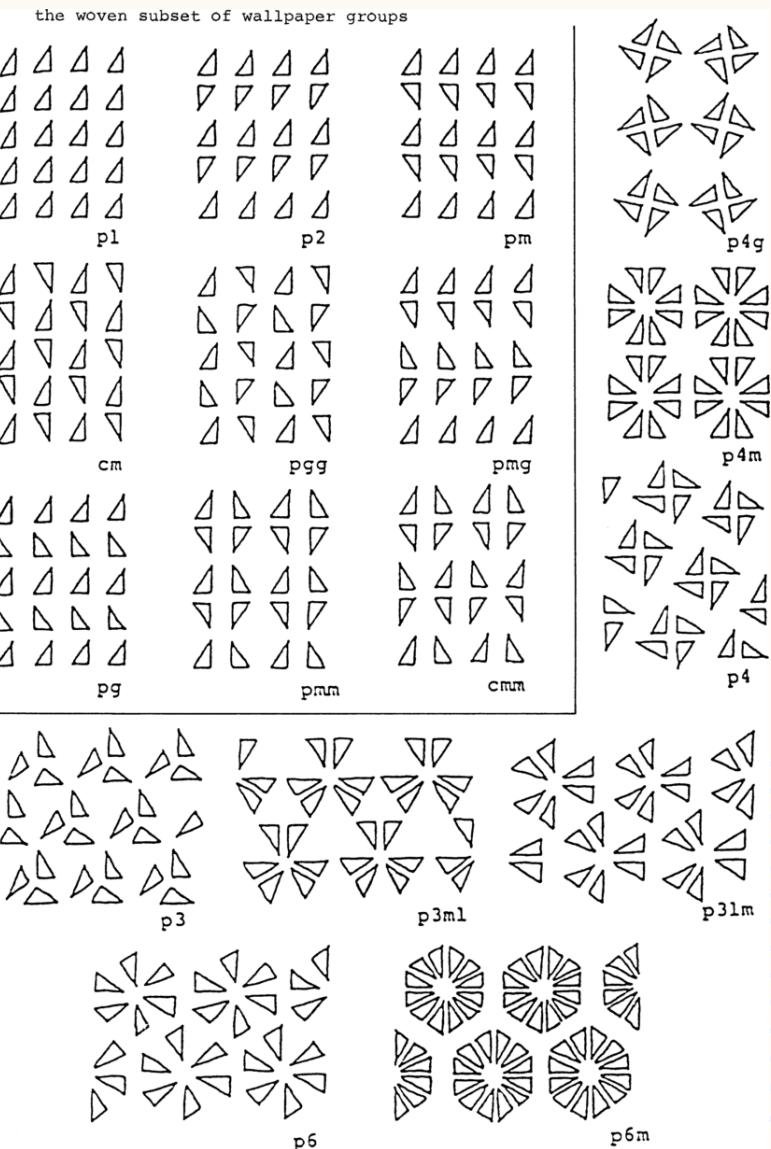
SYMMETRY GROUPS

The **symmetry group** of a set of points F in R^n is the group of isometries of R^n mapping F onto itself.

This is generated by translations, rotations, reflections, and glide reflections.

Dim	Name	Size
1	Frieze	7
2	Wallpaper	17
3	Crystal / space	230

the woven subset of wallpaper groups
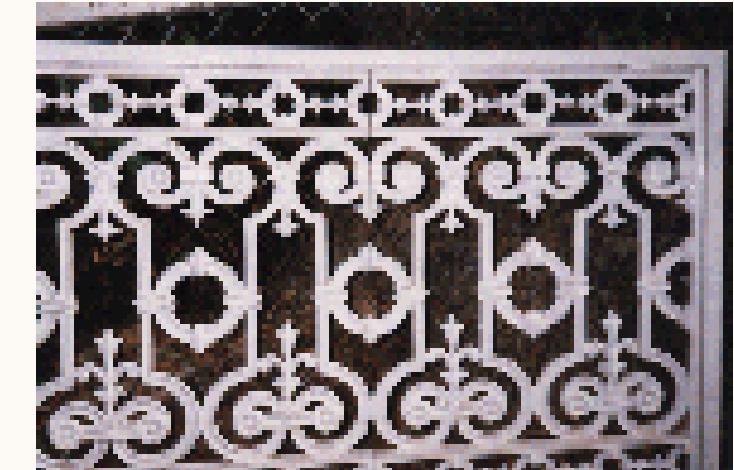
FRIEZE PATTERNS

Lorelei Koss

Table 1: *Percentages of artifacts found in each frieze symmetry class*
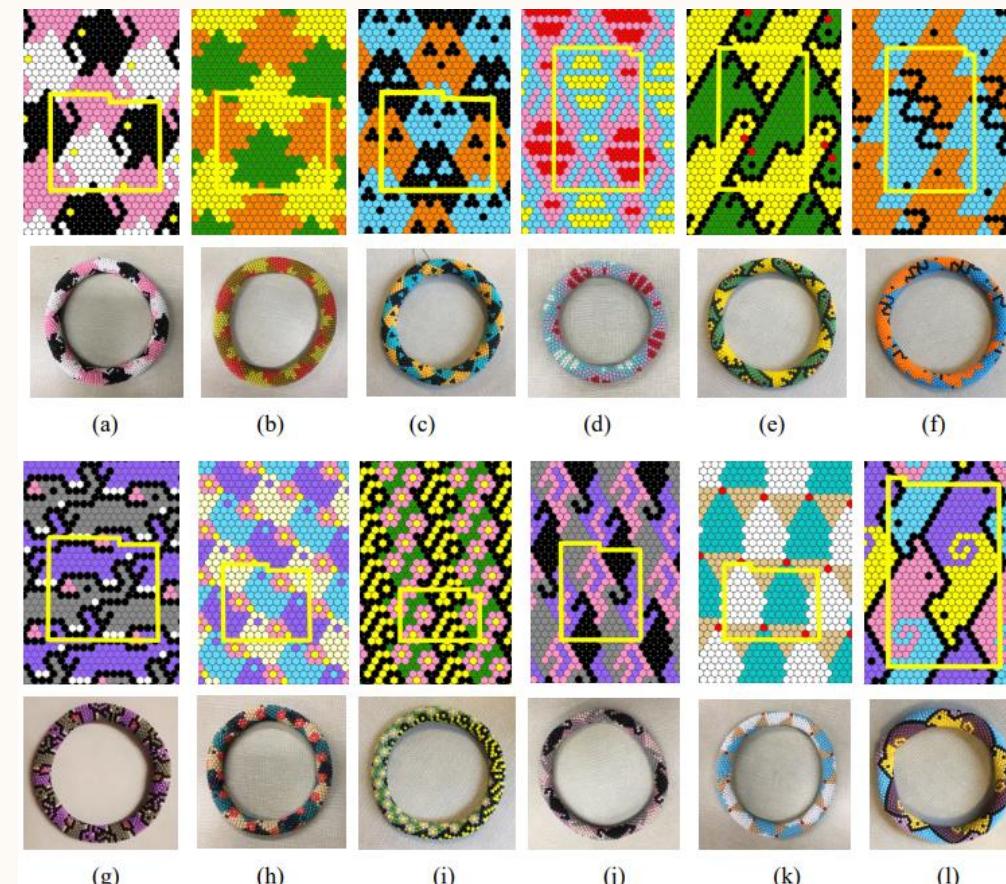
Artifacts	pmm2	pm11	pma2	p112	p1m1	p111	p1a1
European Folk Costumes	37	32	12	5	3	9	2
Lao Textiles	48	20	0	2	19	11	0
Han Textiles	21	23	19	7	3	19	7
Miao Textiles	47	13	26	6	1	4	2
Starkweather Pueblo Pottery	4	13	9	59	2	13	0
Begho Pipes	72	10	4	8	4	2	1
Peranakan Porcelain	1	68	1	10	0	18	1
Pirgí Friezes	41	19	8	11	9	4	9
Ming Porcelain	6	42	8	13	1	18	13
Saudi Arabia Mosques	23	46	1	2	7	17	3
Tonga Handbags	21	42	4	2	2	30	2
Averages from 11 previous studies	29	30	8	11	5	13	4

WALLPAPER SYMMETRIES IN BEADING
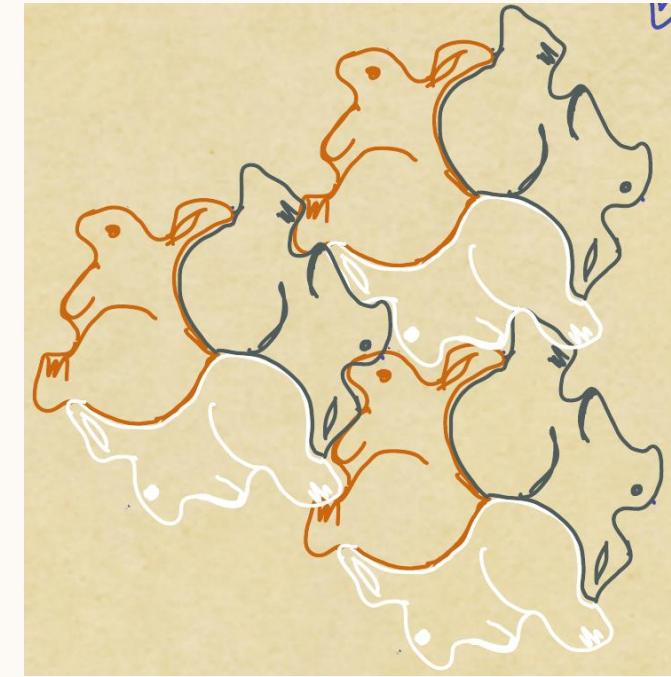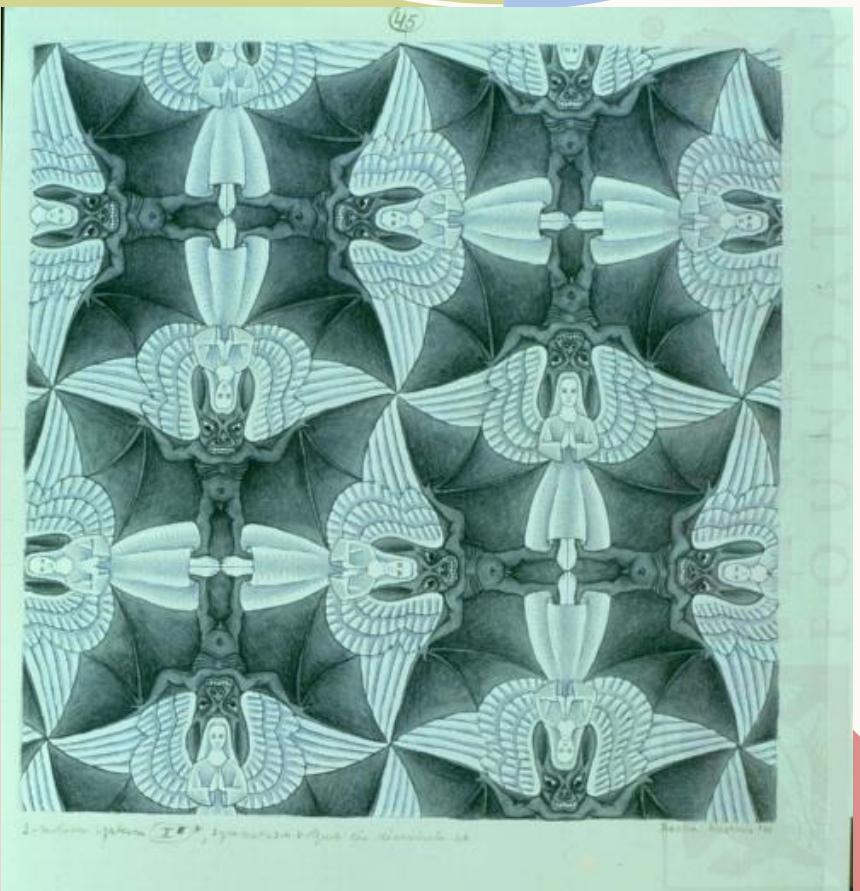
Ellie Baker and Susan Goldstine

(a) (b) (c) (d) (e) (f)

(g) (h) (i) (j) (k) (l)

Eve Torrence
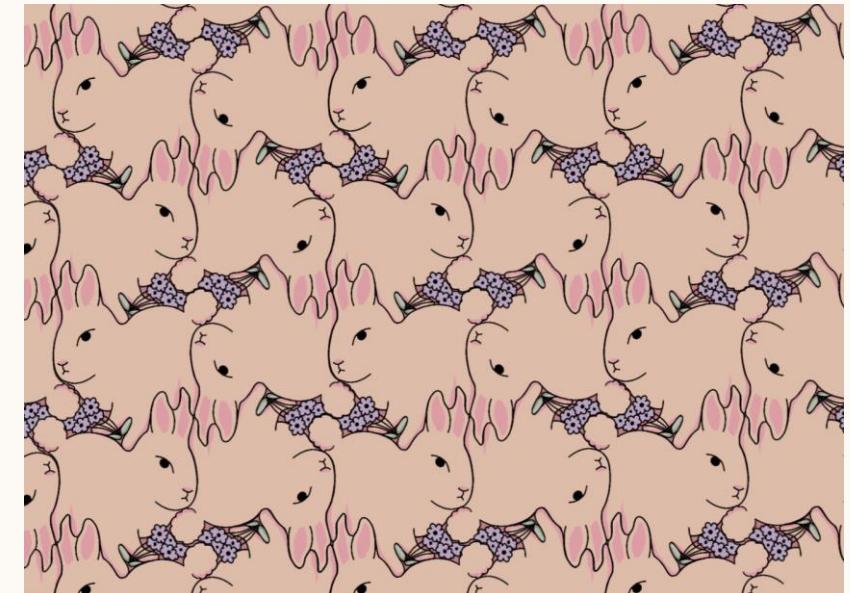
TILINGS

MC Escher
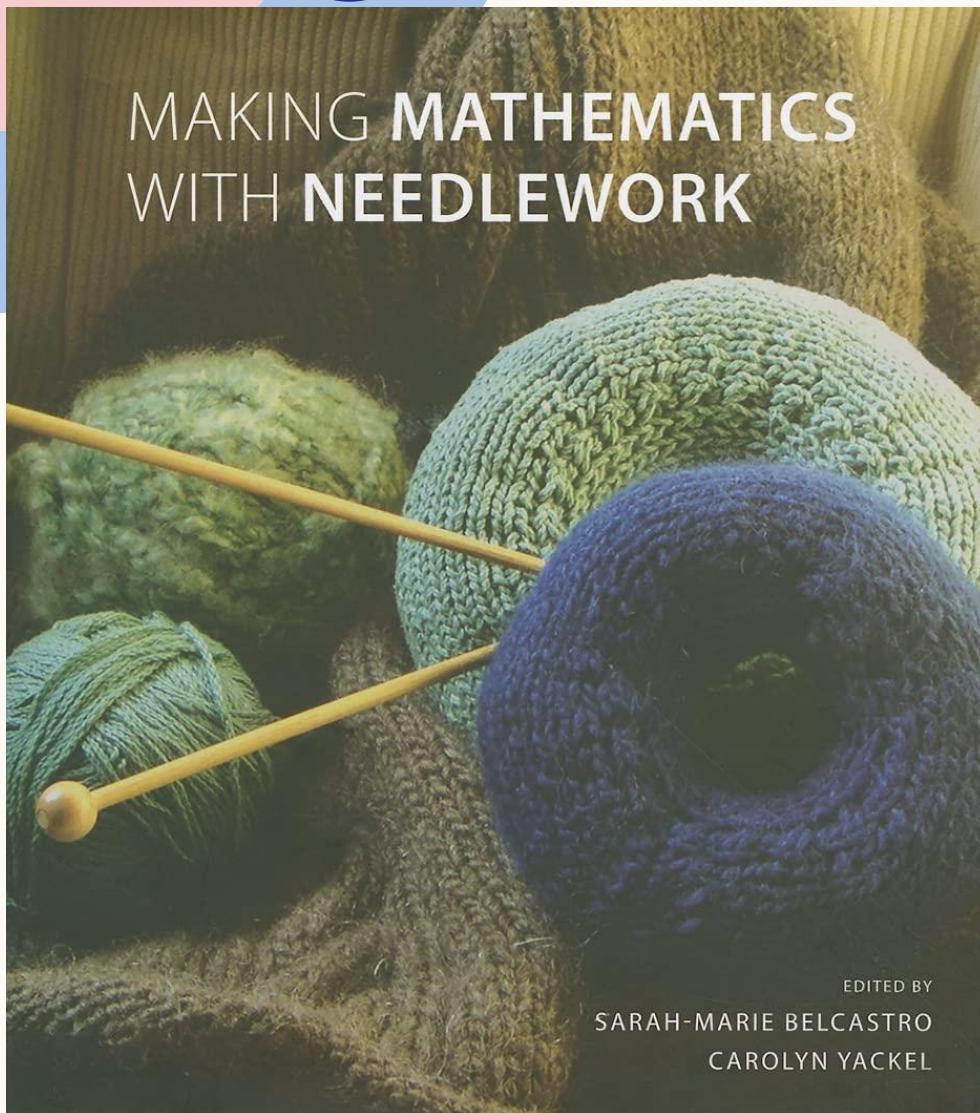
Jiayi Chen, Lucy Knight

KNITTING
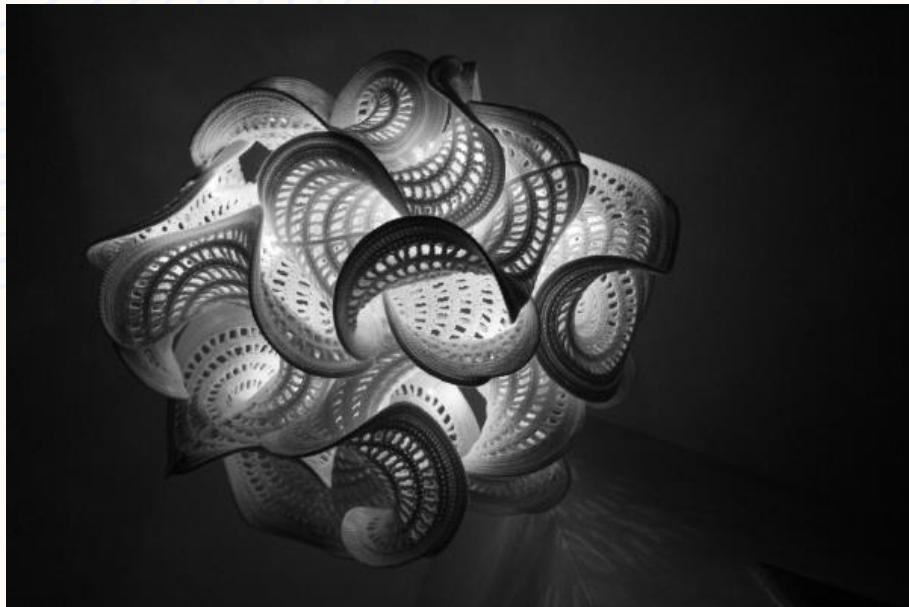
Austin Green

CROCHET

Shiying dong

Gabriele Meyer

Moira Chas
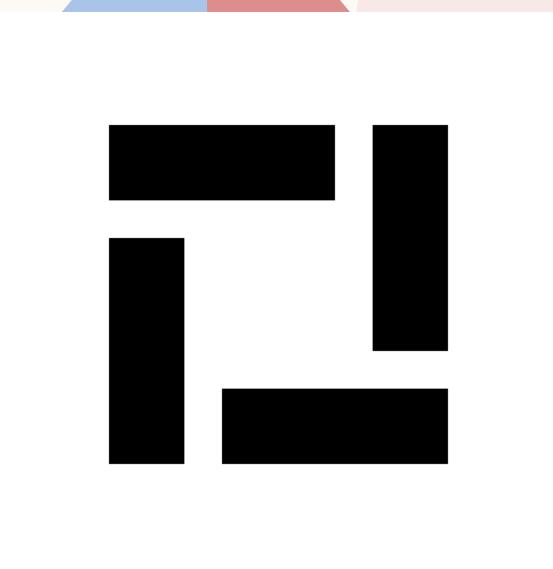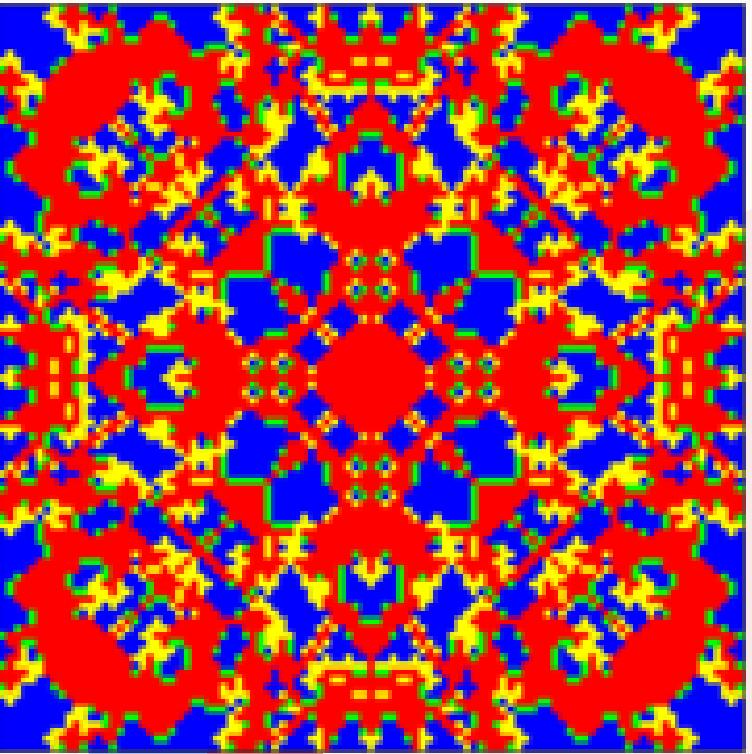
"Blue," by Daina Taimina (Cornell University, Ithaca, NY)

Daina Taimina
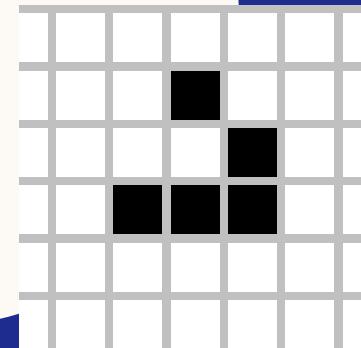
ALGORITHMIC ART
Spatial Games
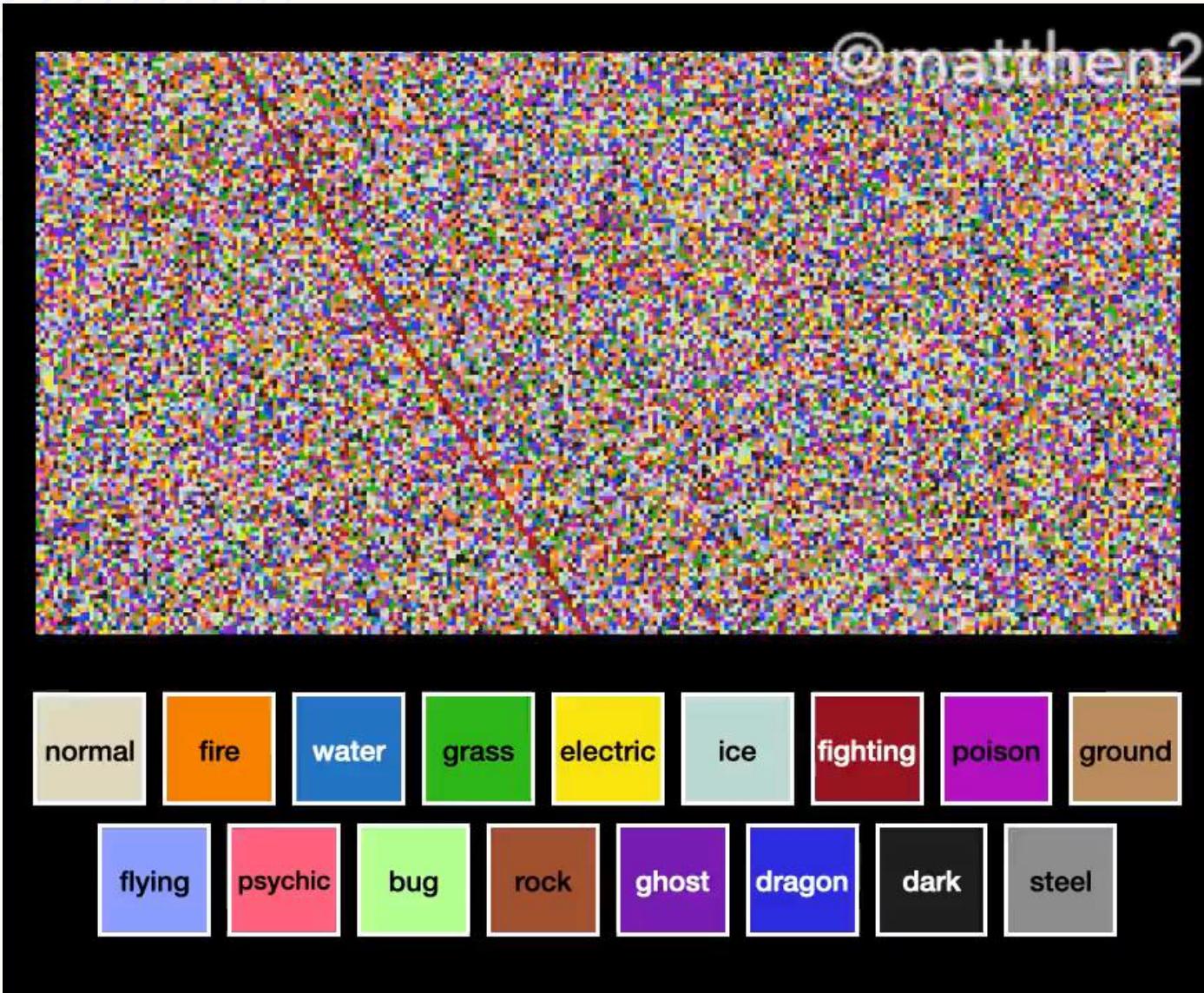
Christoph Hauert

van Dommelen, van Kreveld, and Urhausen

Table 1: *Different systems for figures with a simple procedural definition. The dimension refers to the typical dimension for the image.*

| System	Example	Result	Dim.	Properties
Parametrized curve	Lissajoux	curve	2	continuous
	Pendulum	curve	2	continuous
	Spirograph	curve	2	continuous
	Guilloché	curve	2	continuous
	Spirolateral	curve	2	continuous, piecewise-linear
Cellular automata	Game of Life	grid	2	discrete-time
	Sierpiński triangle	grid	2	1D automaton with history
Grammar-based	L-system	shape (object)	2, 3	parallel replacement
	Shape grammar	shape (object)	2, 3	serial or parallel replacement
	Koch snowflake	shape	2	parallel replacement, fractal
Coupled equation system	Hénon map	strange attractor	2	chaotic, discrete-time, quadratic
	Lorenz system	strange attractor	3	chaotic, continuous, quadratic
	Mandelbrot, Julia set	colored plane	2	fractal, iterated function, complex plane
	Spiroplot	*point plot, trace*	*2*	*discrete-time, linear, multi-point state*

LivingRPS, Peter Ganunis

Pokemon spatial game, Matt Henderson

KUMIHIMO

- A Japanese method of braiding.
- Kongō Gumi: 16 strands, 2 colors.
- Joshua Holden counted all symmetric patterns with de Bruijn's generalization of the Pólya enumeration theorem.

Spots	1	2	3	4
#	1	8	21	72
Spots	5	6	7	8
#	147	280	375	257
Total	1161			
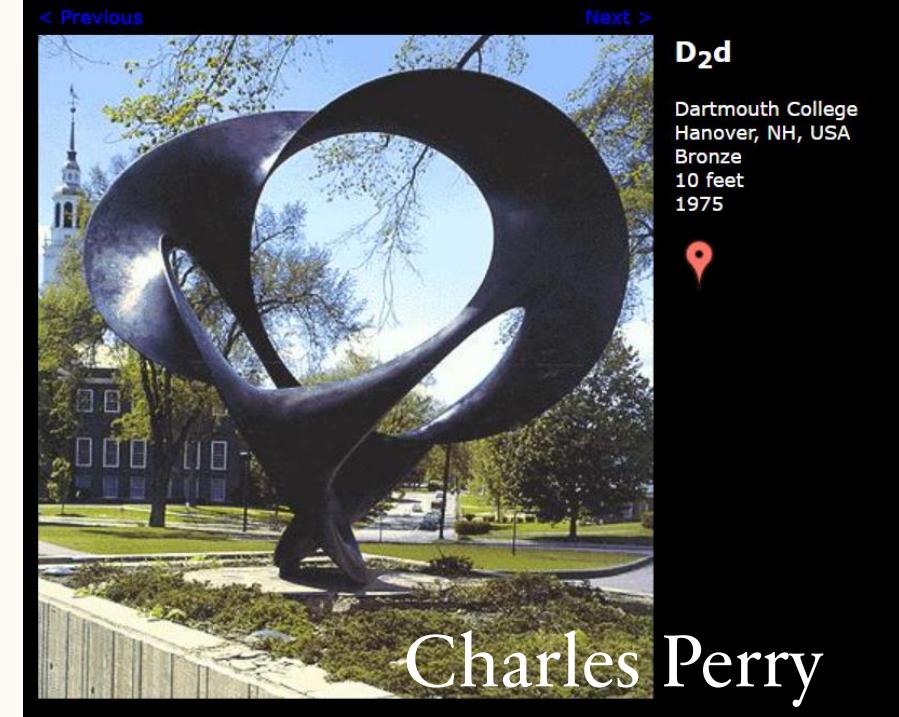
Joshua Holden
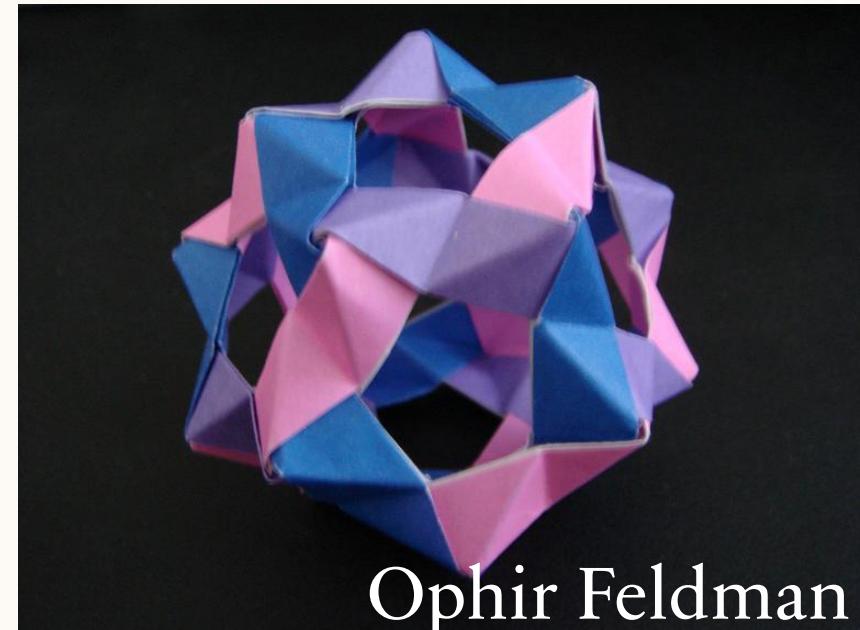
SCULPTURE

Henry Segerman

George Hart

< Previous   Next >

D₂d

Dartmouth College
Hanover, NH, USA
Bronze
10 feet
1975

Charles Perry

Ophir Feldman

PUZZLES

Hanayama

Henry Segerman

Disc

Mobius

TEMARI

COOKING
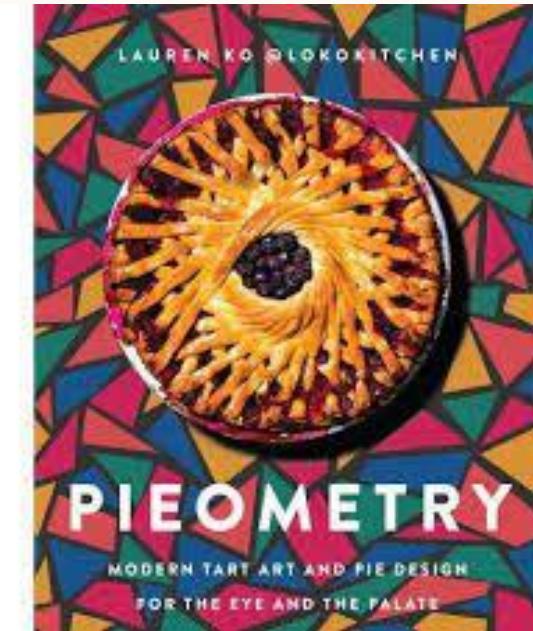
Joy Hsiao

Wafer paper (top left), gum paste (bottom left), dark and white chocolates
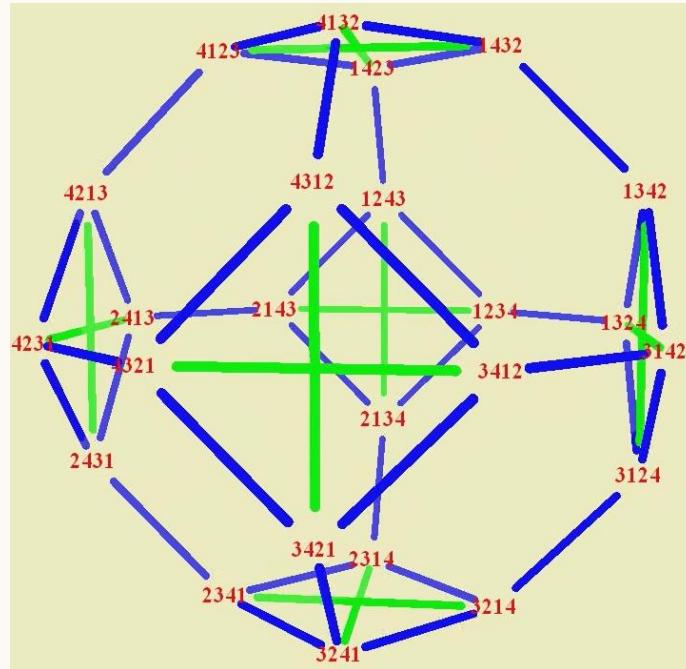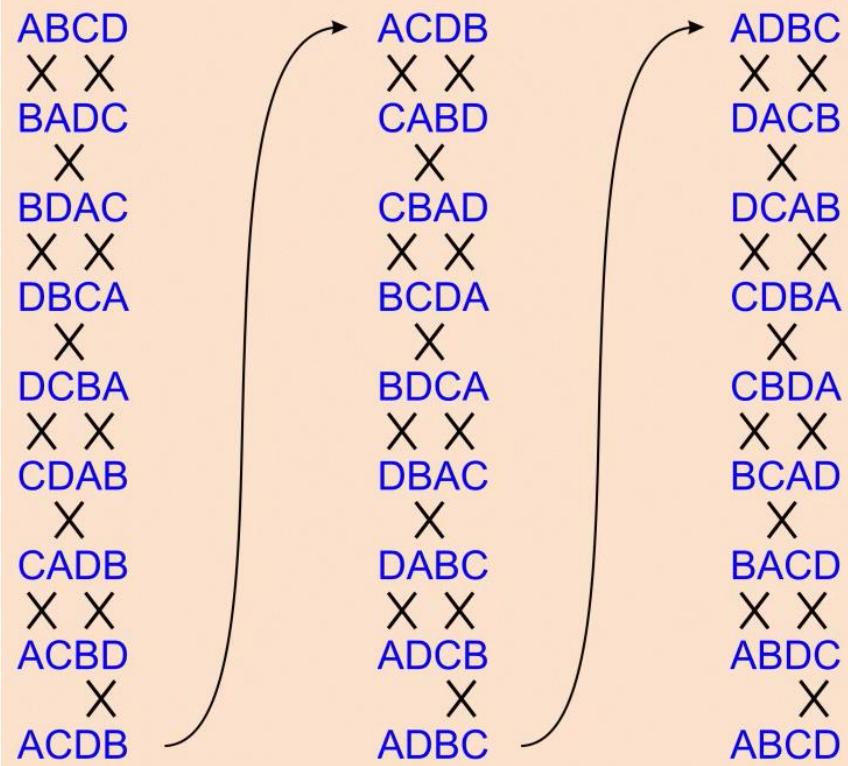
Lizzie Buchanan, Beth-Anne Castellano, Brian Mintz, and Alex Wilson.

Artic Circle theorem Pie
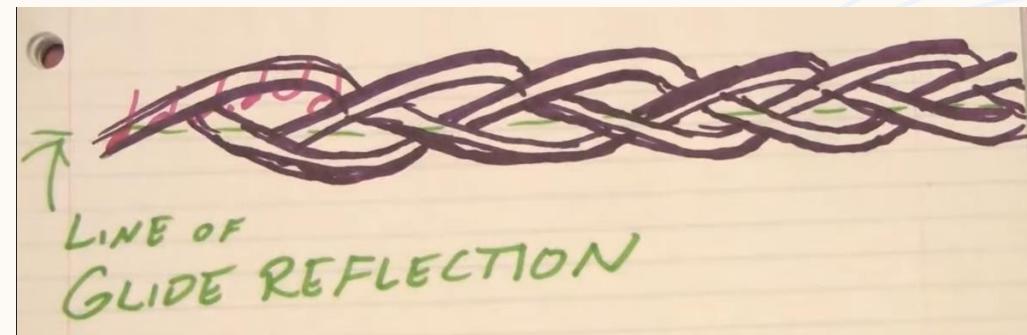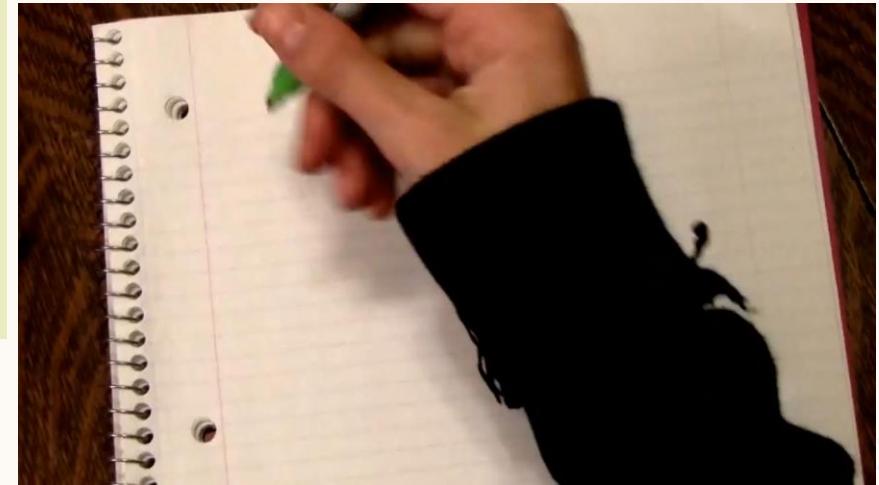
Cookie Shapes! Vihart

MUSIC
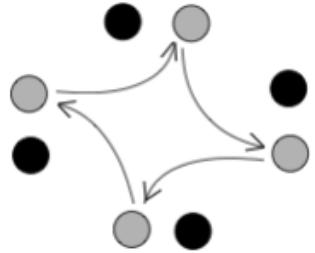
Sound Braid, Victoria Hart

Change Ringing
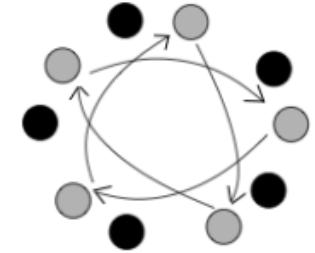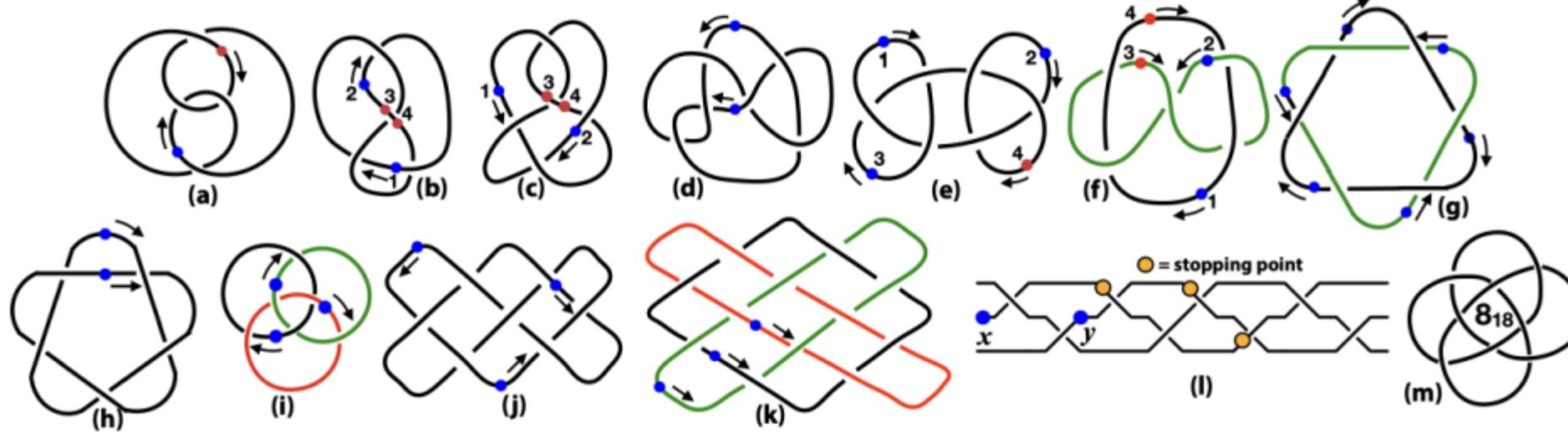
DANCE

Christine von Renesse

(a) (Simplified) Follower's Movements in Dame (b) (Simplified) Follower's Movements in Dame Dos
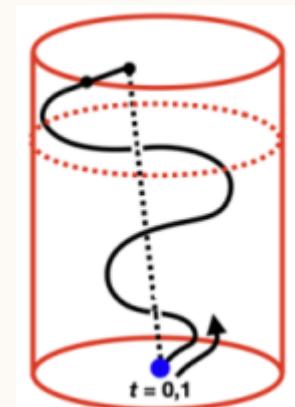
Figure 2: Dame and Dame Dos

Maypole

Karl Schaffer

Figure 9: Variety of links and knots and their danceability.

POETRY

from: **Quando Che'l Cubo** by Tartaglia

When the cube and the added things
Result in a certain number, find
Two numbers whose difference is this one.

$$[x^3 + ax = b]$$
$$[u - v = b]$$

You should always use these two numbers
Provided their product is equal
Precisely the cube of a third of the thing.

$$[uv = (a/3)^3]$$

Then make it a general rule to subtract the
Cube-roots from each other, in order
To obtain the main thing as your answer.

$$[x = \sqrt[3]{u} - \sqrt[3]{v}]$$

Legato Gelato

Susan Gerofsky

Adepts pasted sateen
The palest pastel petals —
To please those senators asleep to treason

to podiums of the senate,
stapled plates to pleats
whose elapsed duties suited them too well

They tended dented sacred cedars
Dropping a peremptory crusty curtsy
Sirens applied rinses
Ochres thicken
Bruise earth's rubies
Silver livers sliver
She poises her burden
Serves and severs verses,
As the lifter of fares

that scared cadres had chopped with chesty scythe
to entrap a curt parent
to resins
kitchen chores
and buries busier hearts
the risen siren
of burned posies
stayed steady
fears and filters safer trifles.
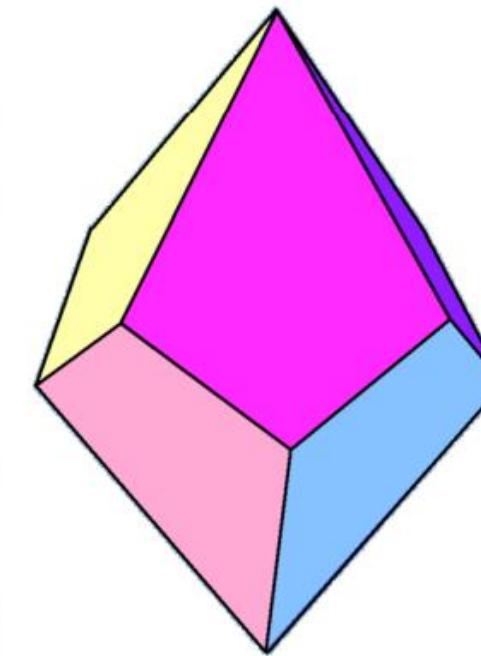
WHY MATH ART? IT'S FUN!

Andrea Hawksley

Makoto Nakamura

Figure 3: *The otterhedron (left) is made from 8 stuffed otters arranged like a tetragonal trapezohedron (right).*

Tetragonal Trapezohedron ©Tomruen, CC BYSA 4.0

MATH CAN BE ADDED TO ANYTHING, WHAT COULD YOU MAKE?

THANK YOU!

This presentation is available on my website if you'd like to follow up with any of these:
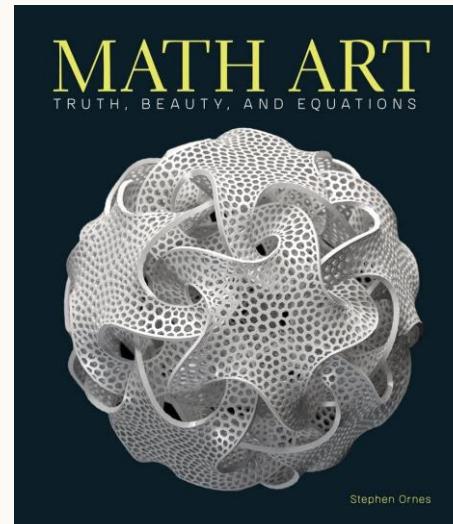
I'm also happy to chat more!

A FEW REFERENCES

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AND MUCH MORE...

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