MAP COLORING PROBLEM

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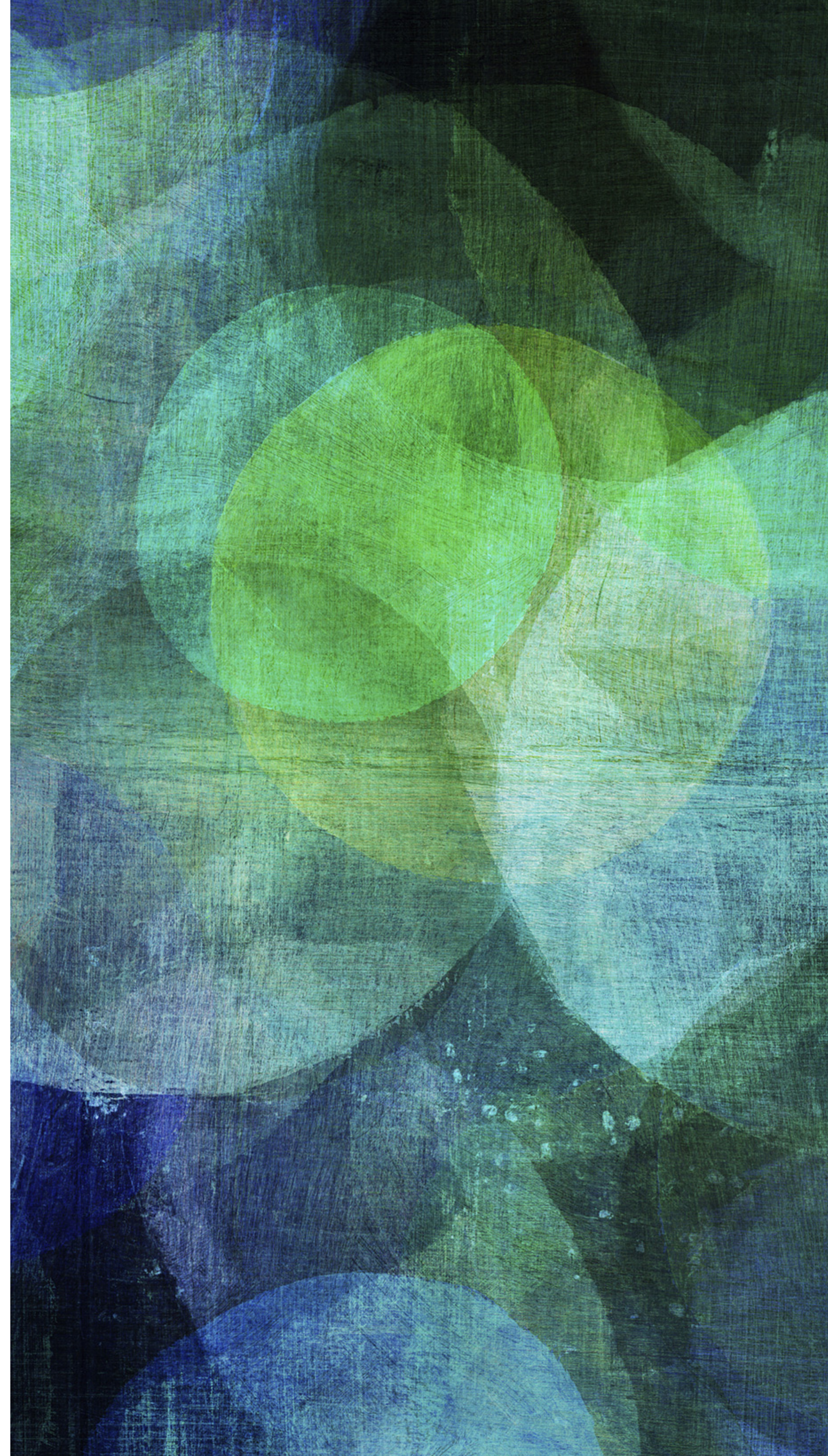
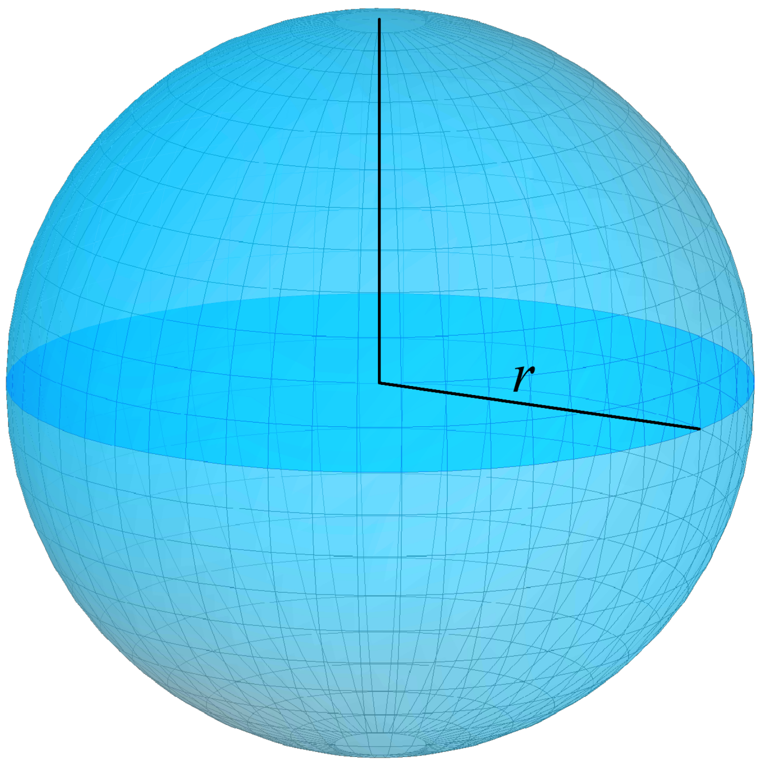
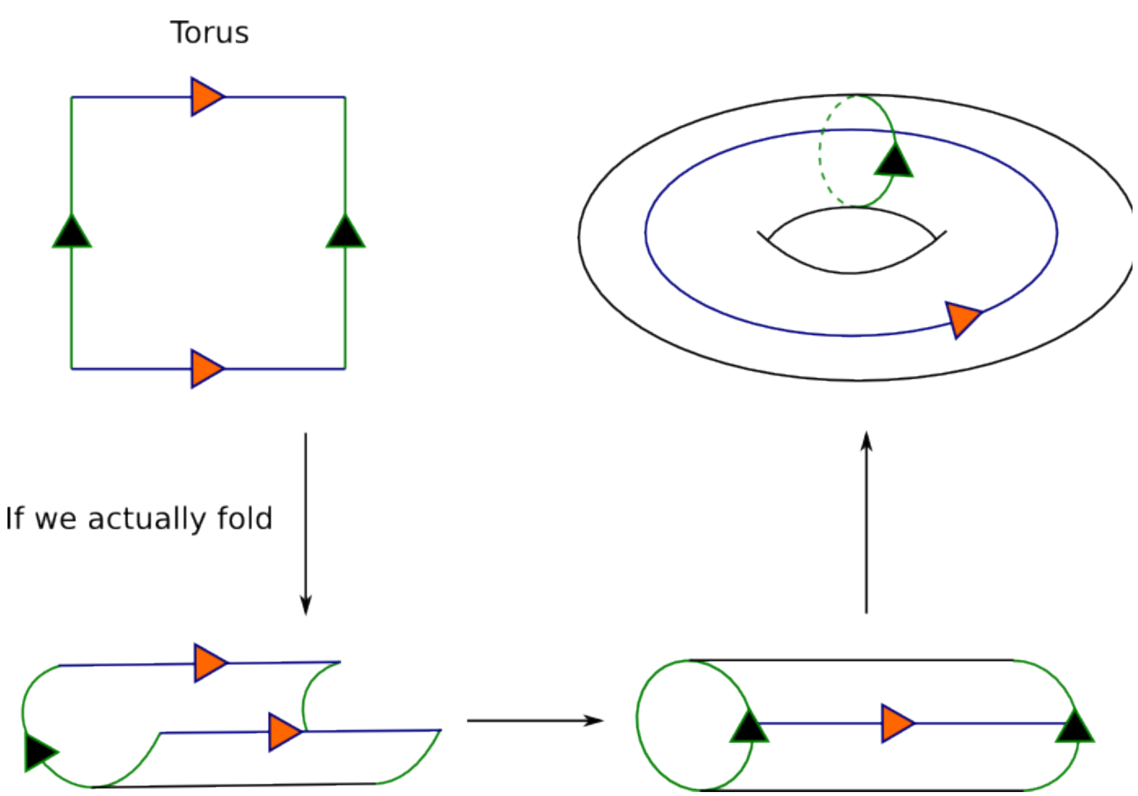


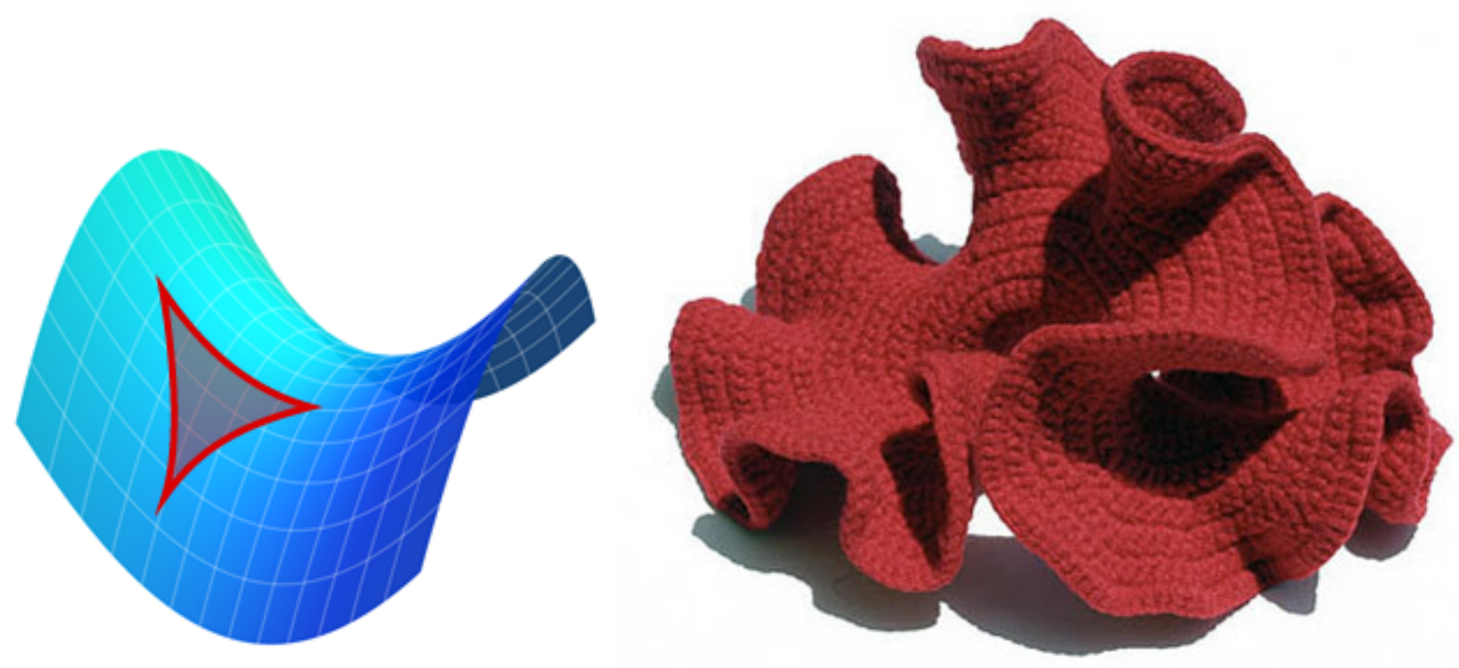
WHAT IS A SURFACE?



A sphere with radius r

- Generalization of a plane (possibly curved)
- Dimension = 2
- Genus = Number of holes on a surface

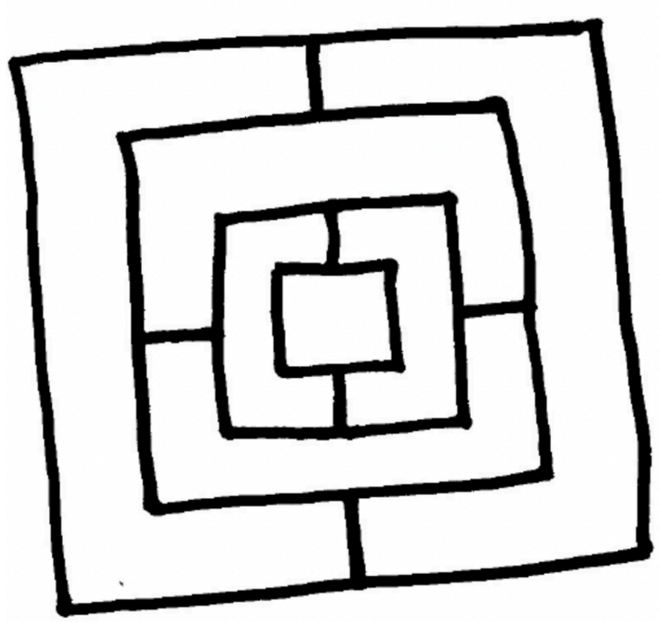


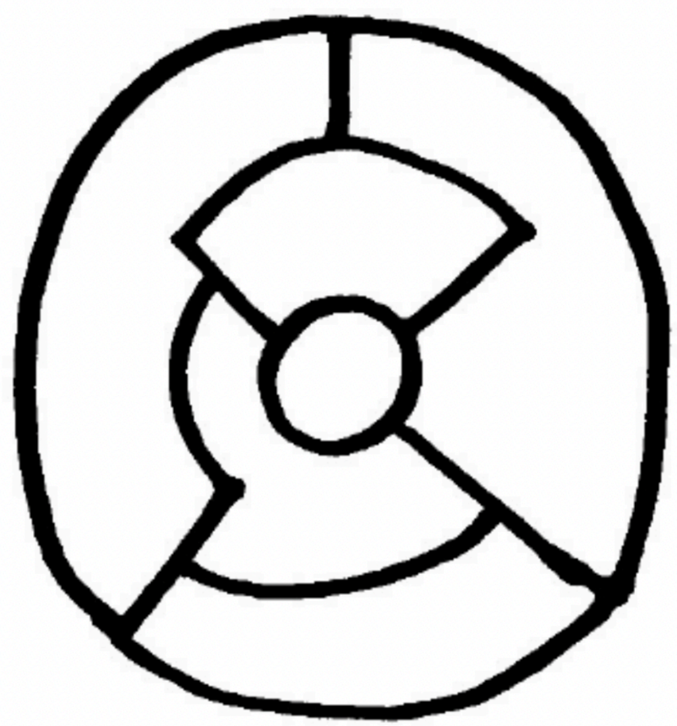


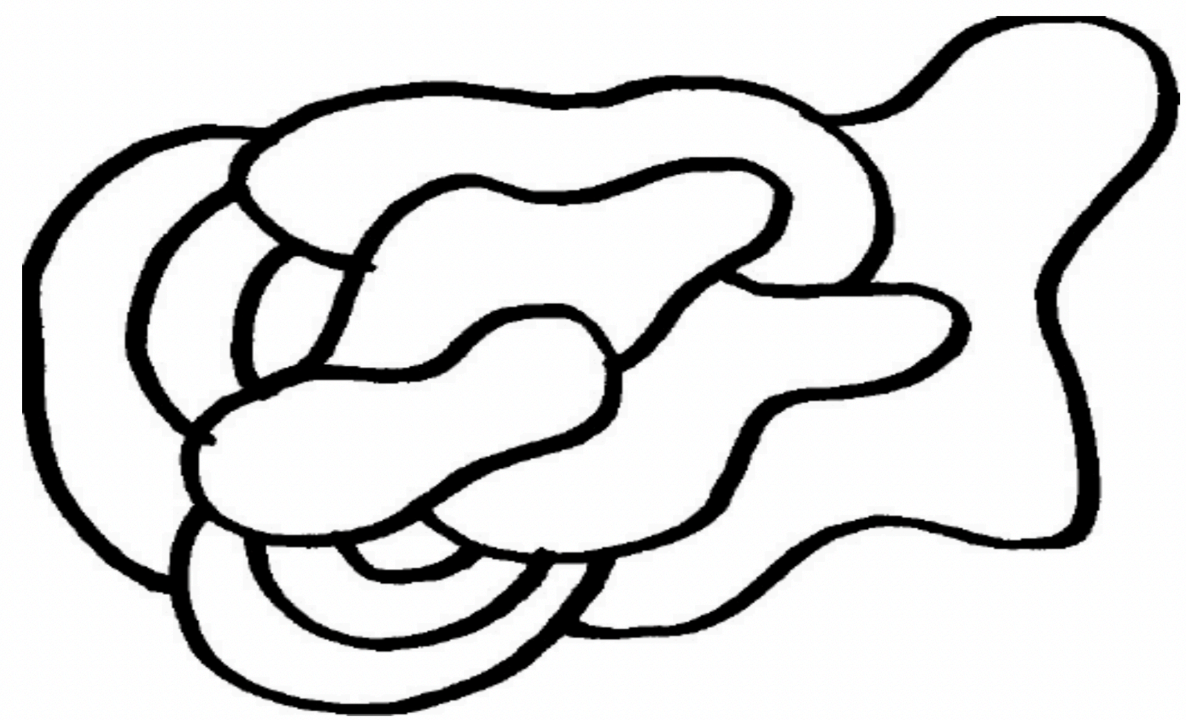
(Hyperbolic triangle and crochet

MAP COLORING PROBLEM

- A map decomposes a surface into non-overlapping regions
- Color the following maps so that no regions with the same color are next to each other



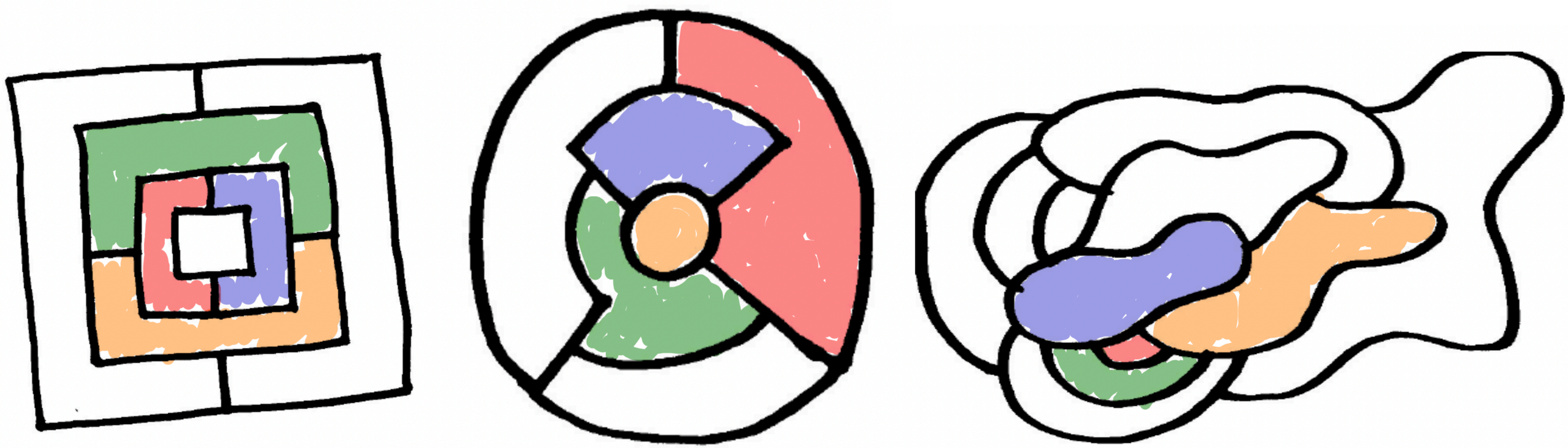




MAP COLORING PROBLEM

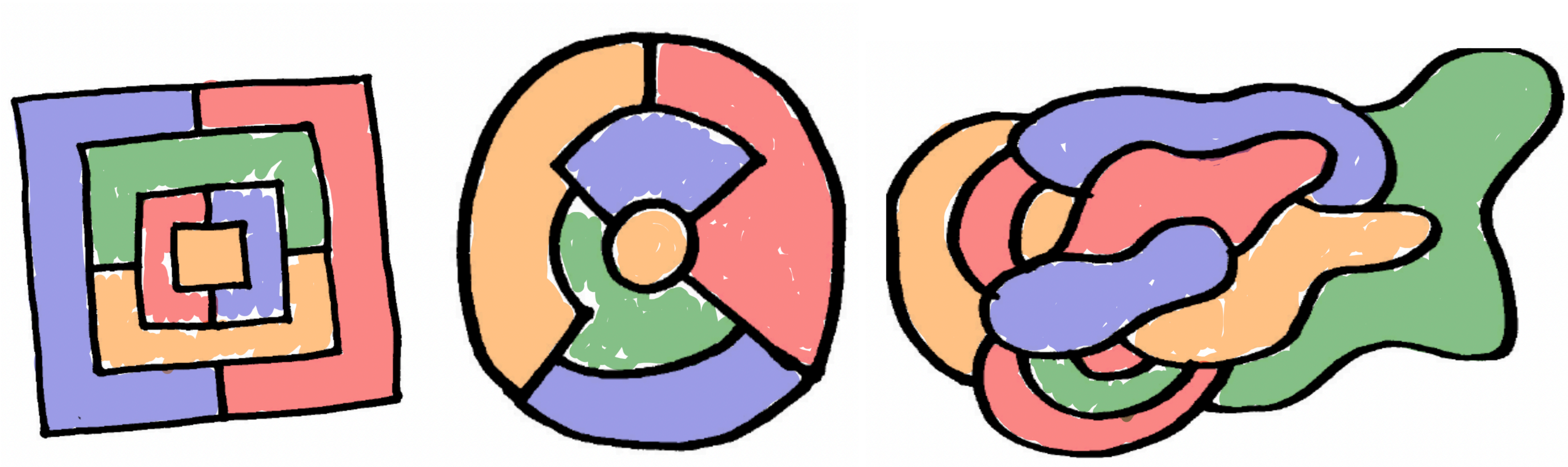
- In each of these maps, we can find 4 regions, all are neighbors of each other

➡ We need at least 4 colors!



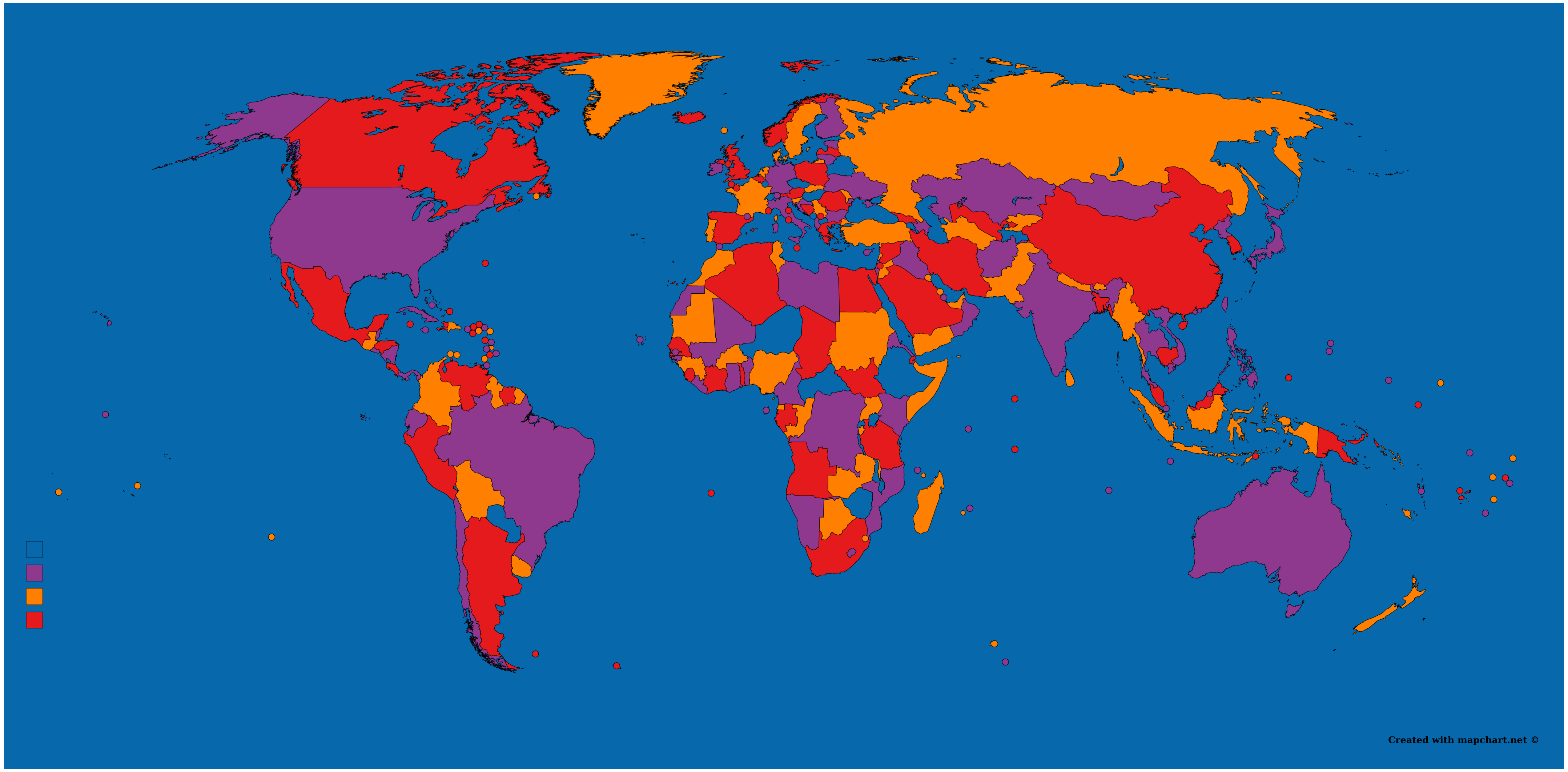
MAP COLORING PROBLEM

- In fact, 4 colors **always** suffice with these maps.



MAP COLORING PROBLEM

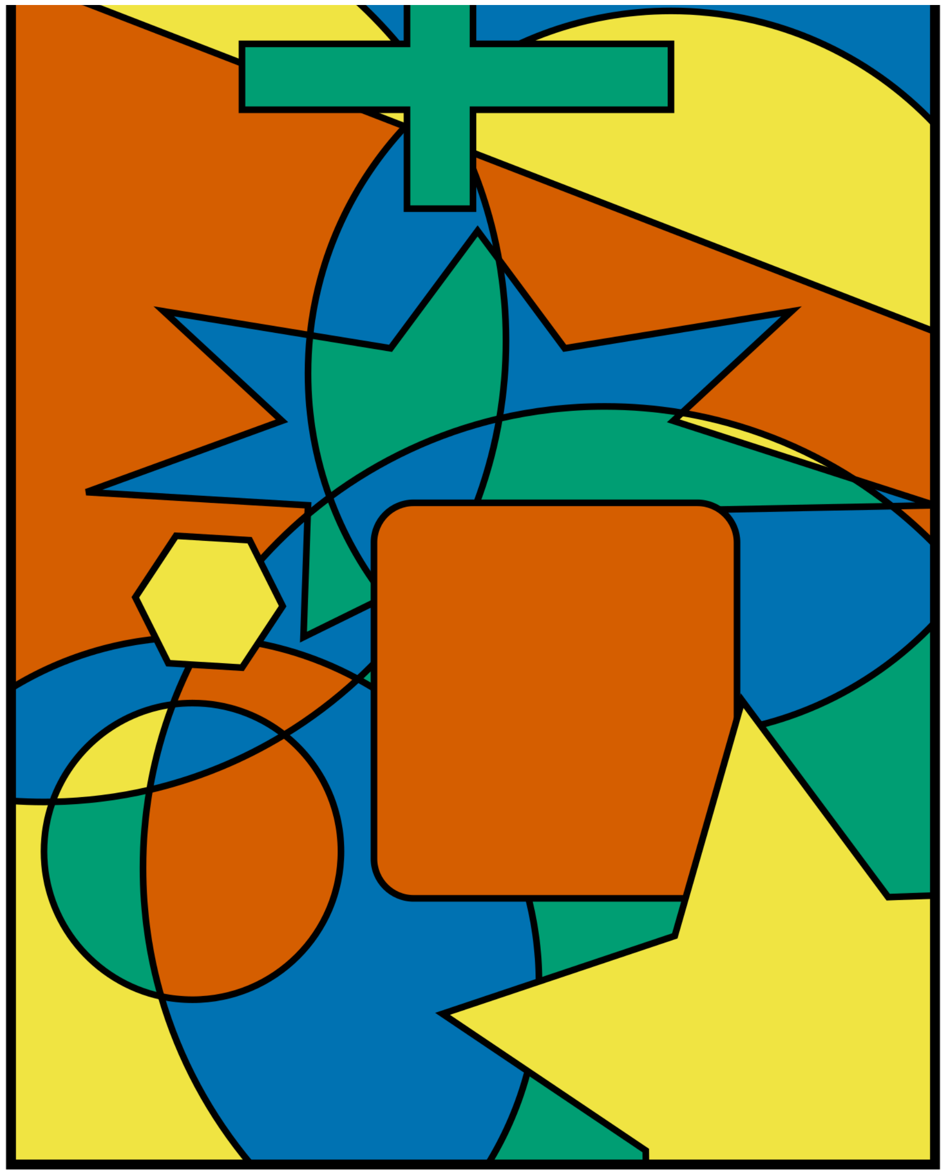
- How many colors do we need to color the regions of any map on earth so that no two adjacent regions have the same color?



4-colored world map

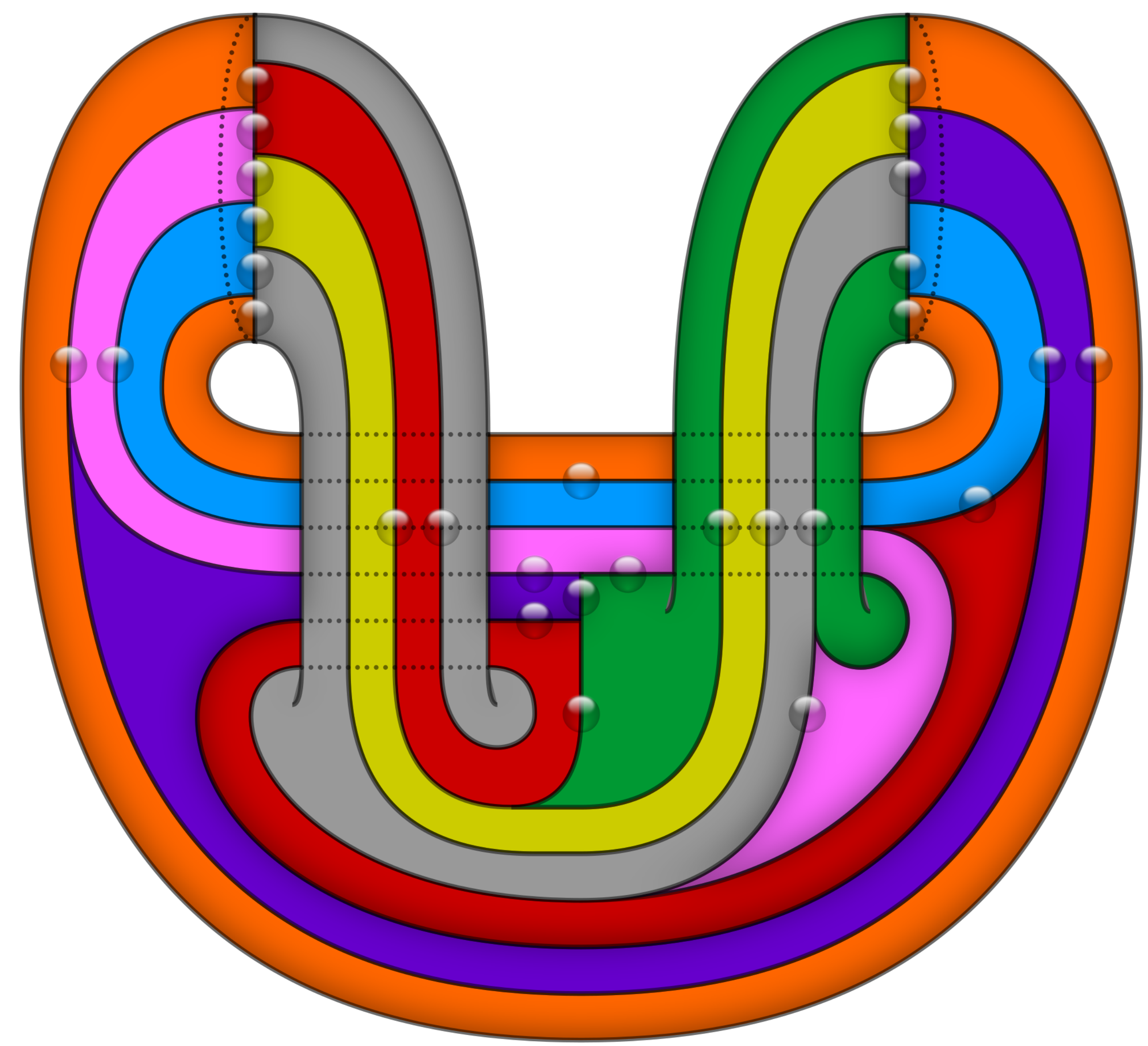
FOUR COLOR THEOREM

- Conjectured by South African mathematician and botanist Francis Guthrie (1852)
- Many incorrect proofs appeared afterwards
  - Alfred Kempe (1879)
  - Percy John Heawood (1890)
    - Five Color Theorem
- Proven by Kenneth Appel & Wolfgang Haken (1976) using computer



HEAWOOD CONJECTURE

- Generalization of the Four Color Theorem:
    - The number of colors needed to color a map on a surface with genus $g \geq 0$ so that no two adjacent have the same color is
    - $\left\lfloor \dfrac{7 + \sqrt{1 + 48g}}{2} \right\rfloor$
- Proven by Gerhard Ringel & Ted Youngs (1976) using current graphs



8-colored 2-tori (genus $g = 2$)

MAP COLORING WITH CROCHETS

7-colored torus

Another 7-colored torus

8-colored 2-tori (front and back)

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Let’s make a 7-colored torus ourselves!