Def^n $\{f_n\}$ defined on E and $\{f_n(x)\}$ converges $\forall x \in E$. Then

$$f(x) = \lim_{n\to\infty} f_n(x) \quad \text{for } x \in E$$

$\{f_n\}$ converge to f $\boxed{\text{point-wise}}$

Def^n $\lim_{t\to x} \lim_{n\to\infty} f_n(t) = \lim_{n\to\infty} \lim_{t\to x} f_n(t)$ if limit of a sequence of continuous functions is continuous.

Which is $\boxed{\text{not necessarily true}}$

UNIFORM CONVERGENCE

Def^n $\{f_n\}$ defined on E $\boxed{\text{converges}}$ $\boxed{\text{uniformly}}$ to function f $\Leftrightarrow \forall \epsilon > 0, \exists N$ st $\forall n \geq N$, $|f_n(x) - f(x)| \leq \epsilon$ $\forall x \in E$

clearly, $\forall \{f_n\}$ uniformly convergent, $\{f_n\}$ pointwise convergent

Def^n let $f(x) = \sum_{n=1}^{\infty} f_n(x)$.

$\sum_{n=1}^{\infty} f_n(x)$ $\boxed{\text{converges uniformly}}$ on E if $\{s_n\}$ where $s_n(x) = \sum_{i=1}^{n} f_i(x)$ converges uniformly on E.

Thm $\{f_n\}$ converges uniformly on E $\Leftrightarrow \forall \epsilon > 0, \exists N \in \mathbb{N}$ st $\forall m, n \geq N$, $\forall x \in E$, $|f_n(x) - f_m(x)| \leq \epsilon$

(cauchy condition)

* remember that cauchy sequence in a metric space is convergent to point in ~~space~~ compact MS

Thm Let $\lim_{n\to\infty} f_n(x) = f(x)$ $\forall x \in E$. and $M_n = \sup_{x\in E} |f_n(x) - f(x)|$

Then $f_n \to f$ uniformly on E $\Leftrightarrow M_n \to 0$ as $n \to \infty$

Thm suppose $\{f_n\}$ and $|f_n(x)| \leq M_n$.

then $\sum f_n$ converges uniformly if $\sum M_n$ converges

U.C AND CONTINUITY

Thm $\{f_n\} \to f$ uniformly on $E \subset X$ metric space. x a limit point of E.

Let $\lim_{t \to x} f_n(t) = A_n$. Then $\{A_n\}$ converges and $\lim_{t \to x} f(t) = \lim_{n \to \infty} A_n$

$\Rightarrow \lim_{t \to x} \lim_{n \to \infty} f_n(t) = \lim_{n \to \infty} \lim_{t \to x} f_n(t)$

Thm $\{f_n\}$ continuous functions on E and $f_n \to f$ uniformly on $E \Rightarrow f$ continuous on E

Thm K compact and

(a) $\{f_n\}$ continuous on K

(b) $\{f_n\}$ converges pointwise to a continuous f on K

(c) $f_n(x) \geq f_{n+1}(x) \ \forall x \in K, n = 1, 2, \ldots$

Then $f_n \to f$ uniformly on K

Def^n pointwise bounded : $|f_n(x)| < \phi(x) \ \forall n$

uniformly bounded : $|f_n(x)| < M \quad \forall n$