Investigation of the Epstein Model for Civil Violence

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1 Introduction

History is full of events where local, small scale protests have escalated to immense proportions,in some cases even managing to overturn a country's regime. It is hence useful for nations to be able to get a basic understanding of the spreading of such events.

The purpose of this project is to model civil violence using an agent based computational approach. More precisely, we aim to study the dynamics of a decentralized insurgency versus a centralised authority. We will see that complex dynamics can arise even from a fairly basic rule set.

2 Epstein model: general rebellion against central authority [1]

This model involves two different actors: **agents** are members of a population and can be in two different states: active (they take part in the rebellion) or **quiet** (they are inactive); whereas **cops** represent the forces of the central authority which arrest rebellious agents. All these actors are moving on a two-dimensional square lattice, where the boundary conditions can be freely chosen (periodic or fixed). The motivation for any agent to become rebellious or not is based on a reward function: the decision to rebel is made on the basis of a personal *grievance* and a perceived *risk* of being arrested.

Agents Several parameters determine the state of an agent (active or quiet). In this simple model, the level of grievance can be expressed as a function of the following quantities:

- The agent's perceived *hardship* (H_i), for instance physical or economical. We take it as a constant value during the simulation, and different for each agent. It will be randomly drawn from the uniform distribution on the open interval $]0, 1[$.
- The perceived *legitimacy* of the regime (L), representing the faith of the agents in the regime. It is equal across all agents and will at first be assumed to be constant during a simulation and can take its value the same open interval $]0, 1[$.

Having defined these two important quantities, the *level of grievance* G_i of any agent i is assumed to be the product of the perceived hardship and perceived illegitimacy:

$$G_i = H_i(1 - L) \tag{1}$$

where the index i refers to a personal variable (contrary to L which is a global variable).

Obviously, the decision for an agent to rebel depends on more than their grievance. An agent will only rebel if it perceives its grievance to be so great that it is worth it to rebel, despite the risk of being arrested. The risk of being arrested is derived from the number of cops and active agents in an agents' vicinity. To define this more precisely:

- An agent has a *vision* (v), which will define which tiles the agent is able to see. The agents' *vision* corresponds to the number of lattice positions in one given direction (north, west, south, east) that the agent is able to inspect. So, if v is 4, then an agent can see the 9×9 square, with itself at the centre. It is exogenous and equal across agents. The fact that the vision is limited (can only take finite values) ensures the locality of information. This way, agents do not know the state of the whole lattice in its globality.

- The *cop-to-active ratio* $(C/A)_v$ denotes the ratio between the number of cops and the number of active agents within the vision v of an agent. Note that A is at least equal to 1 because the agent always counts himself as active when computing this ratio. The calculation of this ratio is a contentious issue: in the original paper by Epstein [1], integer division was used: $(\frac{C}{A})_{\text{integer division}} = 0$ for $A > C$. This is equivalent to rounding down the result of $(C/A)_v$, or *flooring* it. This is discussed further in chapter 4 (Consequences of the flooring and attempts to correct)

- The agent's estimated *arrest probability* (P) is then given by $P = 1 - \exp[-k(C/A)_v]$ where $k = \ln 10$ ensures a plausible estimate of $P = 0.9$ when $(C/A)_v = 1$. In considering whether or not to rebel, the agent asks himself how likely he is to be arrested if he goes active, and will act in function of this probability. The larger the ratio $(C/A)_v$, the higher the probability P. *Flooring* the ratio $(C/A)_v$ will lead to an estimated arrest probability $P = 0$ in the case $(A > C)_v$.

- The agent's level of *risk aversion* (R_i) determines the risk an agent is willing to accept when deciding to become rebellious. In a population some agents may accept a larger risk than others, and exactly rthese risk-neutal agents may become the catalyst for a large scale rebellion. It is heterogeneous across agents, and is assumed to be randomly drawn from the uniform distribution $U(0,1)$. It also stays fixed for the agent's life time. For any agent i, the larger its risk aversion, the less risk it accepts when deciding whether to rebel.

- The agent's *net risk* (N_i) is finally defined as the product of their risk aversion and estimated arrest probability: $N_i = R_i P$. This quantity is logical in the sense that, when considering whether or not to rebel, a risk-neutral agent (low R) won't care what the estimated arrest probability is, whereas a risk-averse agent (high R) will care. The smaller the net risk, the more likely the agent to rebel.

Now that all the necessary quantities are defined, the agent action rule reads as follows: If, for any agent, the difference $G - N$ exceeds some threshold $T \geq 0$, then that goes active. Otherwise, they will be quiescent. The agent action rule can be summarised by:

$$\text{\textit{Agent rule A: If } } G_i - N_i > T\text{\textit{, be active. Otherwise, be quiet.}} \tag{2}$$

T is typically set at some small positive value.

Cops and Jail The cops are much easier to simulate, since they only require one parameter and one action rule. Similarly to agents, the *cop vision* (v^*) corresponds to the number of lattice positions in one given direction (north, west, south, east) that the cop is able to inspect. It is exogenous and equal across cops. In order to keep the cop vision local, v^* must also be small beside the lattice size. The cops have one simple rule of behavior:

$$\textit{Cop rule C: Inspect all sites within } v^* \textit{ and arrest a random active agent in these sites.} \tag{3}$$

When a cop acts, he arrests an active agent if there is at least one within his vision. The arrested agent then "goes to prison" for J steps (or iterations), where J is randomly drawn from $U(0, J_{max})$. "Going to prison" in principle means that the agent becomes inactive for J steps. J_{max} is defined as the maximal term and will affect the dynamics in important ways by removing actives from circulation for various durations.

Movement The rule concerning the movement is the same for both cops and agents:

$$\textit{Movement rule M: Move to a random site within your vision} \tag{4}$$

Runs All the explained previous quantities are sufficient to be able to run the model. The values of L, J_{max}, v, v^* must be chosen by the user, and as explained previously H_i and R_i are randomly set on an agent basis. The model runs as follows: at each step, an actor (cop or agent) is randomly selected, and, under rule (4), moves to a random site within his vision, where he acts in accords with rule (3) (if a cop) or with rule (2) (if an agent).

3 Implementation

In the Epstein paper [1], the way in which the model is explained leaves free interpretation on how to implement it.

The Epstein model has been implemented in a self-made python program, featuring a live Qt user interface (UI). The program consists of three files:

- *main.py*: This file contains the routine that runs the model and that allows it to interact with the UI.
- *toolbox.py*: This file contains the description of the agents involved, and some other relevant helper functions.

- *main.ui*: This Qt UI file contains the description of the UI, i.e. which UI objects there are and where they are placed on the screen.

The implementation follows an object oriented approach: individual agents are objects of either the *agent* or *cop* classes. All of these agents are stored in a large *agent list*. Alongside this list exists a *field*, which is a 2D array of single character strings. These strings represent all information needed by other agents about the lattice at a given position: it is either empty ('e'), or is occupied by an active ('a') or inactive ('i') agent, or a cop ('c'). Each agent keeps track of its own location, and when they act they update the *field* accordingly by manipulating the string at their location, and possibly the string at their destination when they move.

The motivation for this approach, where *field* only contains a single character string, is that searching such an array is much faster than searching an array of objects, where we are interested in a certain property of the objects. This is because *numpy* allows very fast, C-programmed, vectorized functions on arrays.

The Setup We start with an empty field. We randomly choose a required number of different locations, given by a requested density of agents and cops, for the agents. Then we instantiate agent objects, which place themselves on the field. During agent creation we randomly determine a personal hardship H_i and risk aversion R_i from a uniform distribution $U(0, 1)$.

The Epstein paper is not completely clear on what it considers to be an *iteration*. In the paper, one simulation step is defined as choosing a random agent and having them move and act. We define one *iteration* as the following:

One Iteration We randomly shuffle the order in which we let all the agents act one time step. Then we let the agents act and move in this order. The whole increment is an iteration. We then provide Qt the current field to be displayed on the screen and update the graph.

Agent Moving and Acting An agent moves in accordance to rule (4) and then acts in accordance to rule (2), however our particular implementation requires an extra step: When an agent is arrested, the string at its location on the field is changed from 'a' to 'i' by a cop. To detect this, an agent compares the string at its current location with the string it was last time it had acted; i.e. it detects if its string was changed by a cop. If it is arrested the agent randomly chooses a jail-time (according to J_{max}). Each subsequent time this agent will act, it subtracts one from it jail-time and then neither moves nor acts.

There are multiple times an agent or cop must inspect the other lattice-points within their vision: when calculating $(C/A)_v$, when moving, and when a cop looks for agents to arrest. These inspections can be reduced to two types, which allows some speed-up of this step which is computationally most expensive. When searching for an empty spot to move to, or searching an active agent to arrest, we randomly pick lattice sites to inspect within vision, and once we have found what we were looking for we stop searching. Searching in this random order and picking the first site serves two goals: we reduce the number of lattice sites we need to search[1] while still making sure the location we choose is random. When an agent needs to calculate $(C/A)_v$ there is no other choice but to inspect their whole vision and sum all 'c' and 'a' occurrences within vision.

The topology of the lattice is set by these two searching tasks, since they are the only way in which agents interact with the lattice: we can either cut off the field at the edges ('hard' boundaries) or we wrap around the field, meaning the field is a torus.

4 Verification of Epstein's results

The aim is to reproduce the results obtained in [1] to guaranty the reliability of the implementation presented previously.

Results by Epstein The main properties demonstrated by Epstein for the setup state-vs-rebels are the following:

- Agents demonstrate "individual deceptive behaviour" [1]: at a given individual level of grievance, agents alternate between the active and inactive state depending on the proximity of cops.
- At high agent density (or low cop density), outbursts are more frequent. There is a high spatial variability in the activity.
- Rebels' activity has a "peaked" behaviour: most of the time the amount of active agents is low, but it is interrupted by sudden and sharp peaks. The peaking frequency is variable but follows an exponential-decay like pattern.
- The activity level is highly correlated with the change of perceived legitimacy rate, more than with the amplitude of the variation itself.

[1]In the edge case where there are no empty lattice sites, this approach is actually slightly slower as simply searching every lattice site in vision, as we also shuffled the order first now.

Reproducibility The results from the reproduced model are expected to be in agreement with the ones obtained by Epstein. However, the peaking behaviour is not obtained, only fluctuations are observed (e.g. in figure 1), left. It appears that the only way to recreate the results from the paper is to apply a flooring function when computing the cop-to-active ratio. This means that in the published paper, the division (C/A) was an integers' division, not a floating-point division. A simulation ran with exact same parameters as previously, but with an integers' division, is given figure 1, right. In the latter corrected simulation, the peaking behaviour stressed by Epstein is

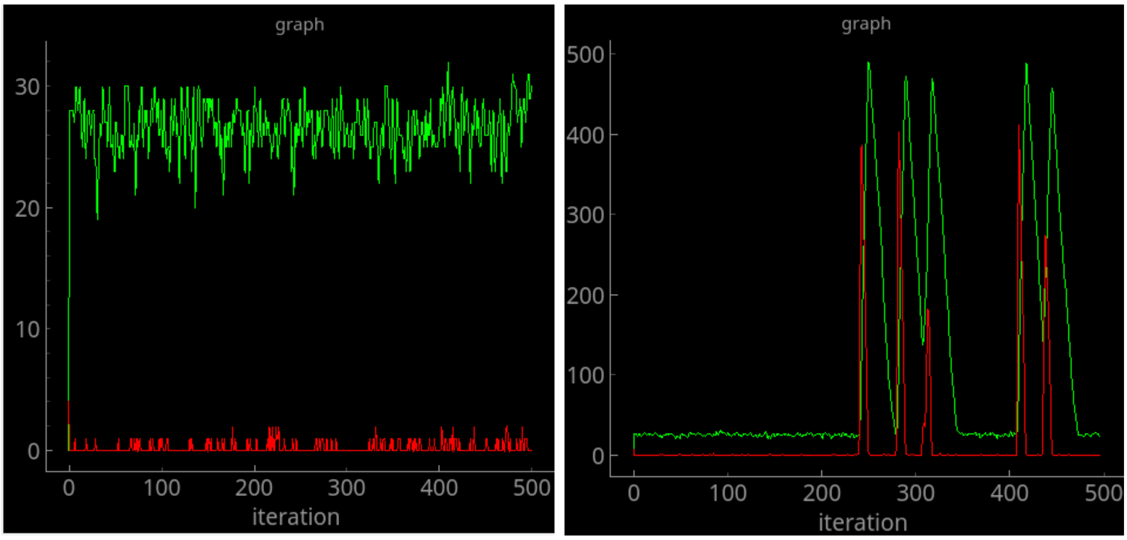

Figure 1: Simulation with parameters $L = 0.79$, $v_{\text{cop}} = v_{\text{agent}} = 7$, grid size $n = 40$, densities $\rho_{\text{cops}} = 0.04$, $\rho_{\text{agents}} = 0.7$, $J_{\max} = 30$, $T = 0.1$. Red curve: active agents, green curve: jailed agents.

clearly visible: the active agents' number jumps from less than 10 to nearly 400. This result is easy to explain with the entire division. Indeed, the decision to go active (rule 2) depends strongly on the agent's net perceived risk N. However, this risk is zero when the estimated arrest probability P is null, i.e. when the cop-to-active ratio C/A is equal to zero. Whilst this condition should imply that there is no cop in the vision of the agent, it is actually enough to have more active agents than cops ($\lfloor C/A \rfloor = 0$). Not only is this requirement easy to meet by the fact that the cops' density is much lower than the agents' density, it also has a cascade effect since the more the agents become active, the more the others are likely to become so too, considering they calculate that becoming active is risk-free.

To ensure the coherence between the model developed here (with the flooring) and Epstein's publication, another analysis is made. One interesting property of the peaked behaviour is the time between two outbreaks. Following Epstein's terminology we will call this the *waiting time*. Letting the model run for a long time (200000 iterations) we made a histogram of the waiting time (fig. 2). We see a sharp increase to some maximum, and then what appears like an exponential drop-off. This in agreement with

Epstein's findings. Letting the model run for longer should smooth out the distribution, however the decision was made that computing time was best used for variations of Epstein model, instead of very precisely recreating the Waiting time distribution.

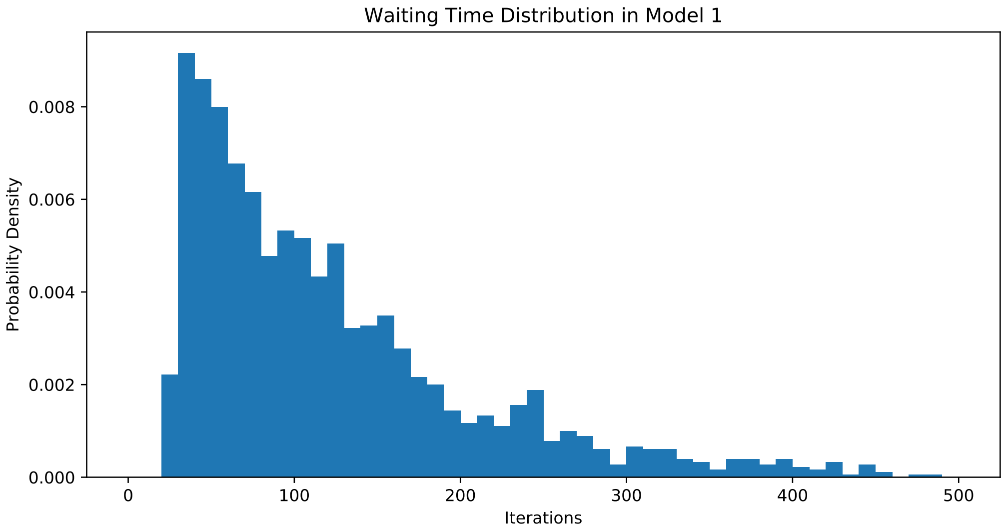

Figure 2: Waiting time distribution. The x-axis shows represents the time between two successive outbreaks.

Consequences of the flooring and attempts to correct

The effect of the flooring is significant on the results, and determine whether there is or not any major outbreaks of violence. In terms of modeling, this flooring behaviour corresponds to non linear (jumps) risk perception and a complete fear inhibition as soon as enough agents are involved. This behaviour could describe a demonstration-like activity, where the personal risks are very low. The jail term would then need to be reinterpreted as an obligation to return to daily activities (work for instance). The "uprising" should not be deeply binding, i.e. easily joined and easily left to match the spiking pattern. One example that could fit this description is the Climate strike: a regular pattern of strikes during weekends that settle after a few hours, where a part of the population is willing to join as soon as a movement is started, and there is no risk, only a time cost that takes over once the week days are reached again.

Since a strike is not the object studied here, but rather civil violence with higher personal risk, the model might need a correction. Given the spiking-like behaviour linked with legitimacy jumps, one correction attempt is to modify the behaviour of L. In Epstein's model, L is kept constant. However, perceived legitimacy fluctuates in real world conflicts (release of information on corruption, martyrs, first deaths during uprisings, or on the contrary upcoming elections, efficient state propaganda etc). To account for these fluctuations in the model, a random variation of L is added at each iteration: $L = L_0(1 + 1/3 \cdot \beta)$, with $\beta \in U([-1, 1])$ and $L \in]0, 1[$. However, even if the

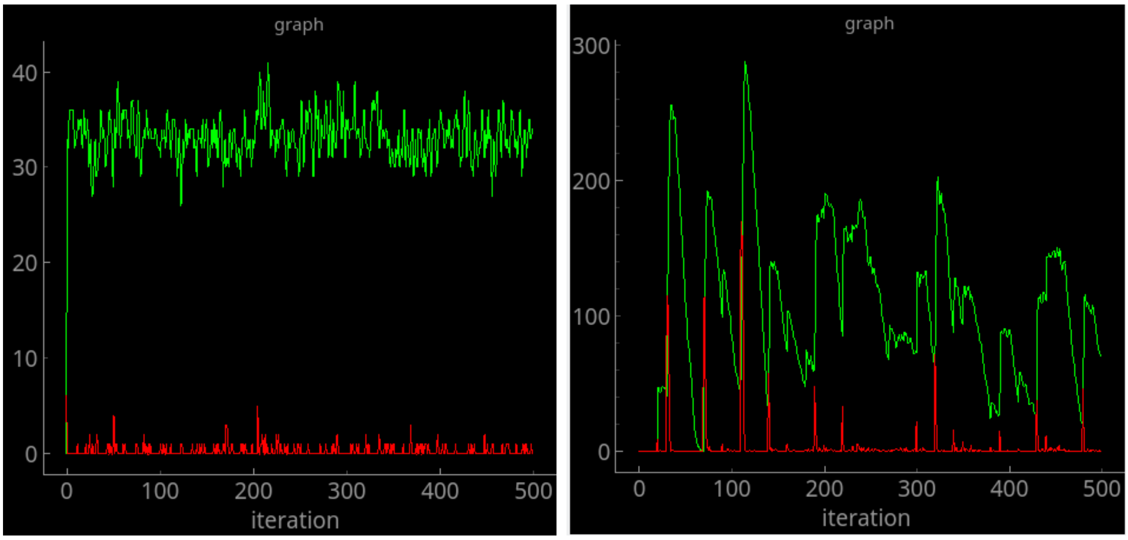

Figure 3: Parameters as for fig. 1. $L_0 = 0.79$. For the right graph, $\tau = 10$. Red curve: active agents, green curve: jailed agents.

amplitude variation of the activity of the agents is increased, no spiking behaviour is observed (fig. 3, left). In a second time, the same variation of L is implemented but the value is updated only every τ steps. This could represent the fact that the access to information is only periodic. The result is shown figure (fig. 3, right). Therefore, alongside with the strike-like hypothesis, a spiking behaviour could also be the result of a situation of civil violence in a state where access to information is limited. Some information coming from both the rebellious part and the state might create some contradictory fluctuations at discrete time steps and thereby rebellious outbreaks.

5 Extensions

5.1 Global access to information

The model designed by Epstein et al. considers that information propagates only within the vision radius of the agents. However, this discards the influence of media or any global communication platform as social media for instance. In order to simulate the contribution of these new communication channels, an additional term has been added to the arrest probability (P). This additional term depends on the ratio of cops on active agents on the whole map. This is motivated by the fact that inactive agents will be more likely to become active if they know that in most of the agents are active, even if based on their direct surroundings their willingness to act is below threshold. The modified perceived arrest probability is given by:

$$P' = P - \alpha \cdot \exp[-\kappa(C_{\text{total}}/A_{\text{total}})]$$

with $\kappa = k - \ln(\alpha)$ and α is small (typically $\alpha \in [0, 0.1]$). With this choice the behaviour with respect to local and global unrest is coherent. The choice of κ allows to have a probability shift of the order of α^2, which is negligible.

The implementation is not modified, only two values are added to the simulation to keep track of the global amount of cops and agents. Indeed, computing this value at each iteration would increase the complexity quadratically with the lattice size. All the agents have a reference to these values. Hence, when an agent changes its state, the count of active agents is updated for all agents.

Observations There are two main observations in the scope of this modified model: firstly the reactions are exacerbated by an increased global communication (quantified by α), and secondly the active agents tend to be more spread over the map.

The modification in the reactions is shown figure 4. This curve is the result of the following process: set the legitimacy to 1 until stabilisation, modify the value of alpha (without effect on the state since $L = 1$), then suddenly drop the legitimacy to 0.9. The first drop was done at time step $t = 10$ with $\alpha = 0$, the second at $t = 200$ with $\alpha = 0.05$, the third at $t = 825$ with $\alpha = 0.1$. When the global information factor is

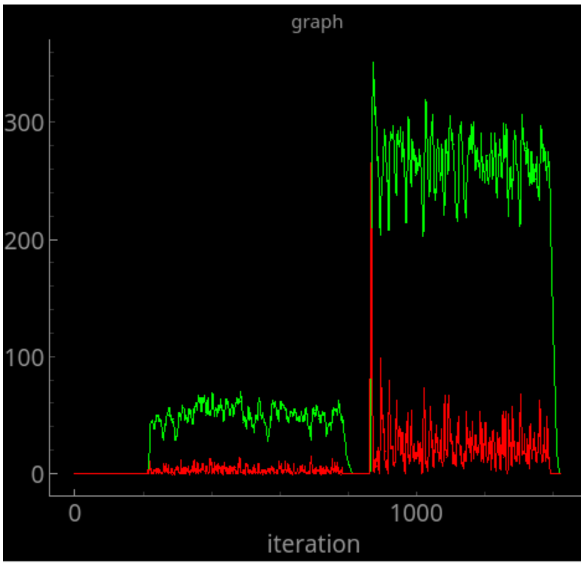

Figure 4: Effect of a perceived legitimacy drop from 1 to 0.9 with successively $\alpha = \{0, 0.05, 0.1\}$

not being accounted for, no agent becomes active. When α increases, not only does the response of the agents increase, but also the amplitude of the variations. It appears that having access to global communication channels makes the behaviour of the agents less predictable, and that behaviours go faster to the extremes. Cutting those channels would then be an efficient way to diminish the spiking probability.

To visualise the spatial distribution of the agents over the map in a quantified way would require an extensive graph correlation analysis. Here only some qualitative results are given. 6 simulations are ran with $L = 0.8$. 3 of them are done with $\alpha = 0$, three are done with $\alpha = 0.1$. A snapshot of the map is taken at $t = 300$, which is an arbitrary iteration values but identical for all the simulations in order to not induce a bias other that the one due to fluctuations. These maps are shown figure 5, with on the first row no global communication, and on the second $\alpha = 0.1$.

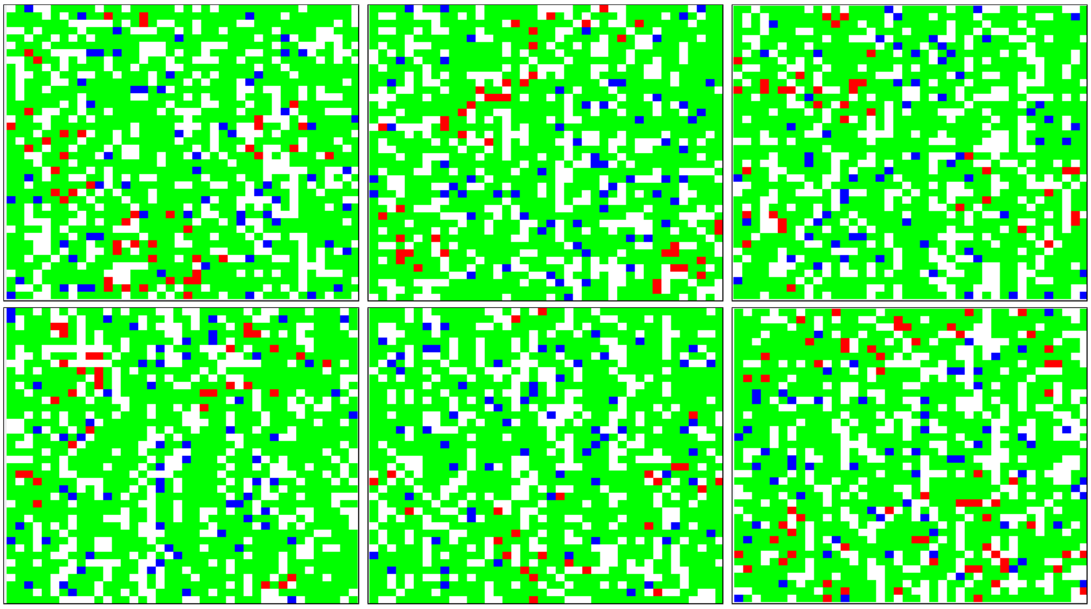

Figure 5: Simulations with no global communication (top) or with $\alpha = 0.1$ (bottom). Green: inactive agent, red: active agent, red: cop.

It appears that when the global communication is non existent, the active agents gather preferentially in clusters, and that few to any cops are present in those clusters. This is a direct consequence of the way the model has been defined: the agents become active if other around are active or if there is a low risk of being arrested. On the other hand in the simulations with global communication, it appears that the agents are more spread over the map. They still group preferentially in clusters, but some isolated agents are also visible, as well as clusters with cops in the neighbourhood. This underlines the fact that when localized communication is possible, for instance through social media, isolated agents are more likely to become active even if their neighbours are not.

5.2 Cops motion improvement

Previously, under rule (4), a cop moved randomly to a free site within its vision. This action is realistic in the limit where no active agents lie within the cop vision. When a cop is aware of an active agent in their vision, it would make more sense if the cop moved in the direction of this active agent, as it would be likely to find more active agents in that direction. Movement rule (4) becomes then (for cops):

Movement rule M: Move in the direction of the greatest number of active agents. (5)

The exact implementation of this rule can be done in multiple ways; one could bias the movement of the cop in the direction of the active agent somehow. We will take a simpler approach however:

Move to a random location in the quarter of vision that has the most active agents (6)

We used implementation C.1 to compare the number of active agents over time with and without this movement rule. Implementing this extension to the code and comparing it to the basic model leads to figure 6. The parameters of the simulation are given in appendix A. The observations show a clear difference. On one hand, the cops' clever

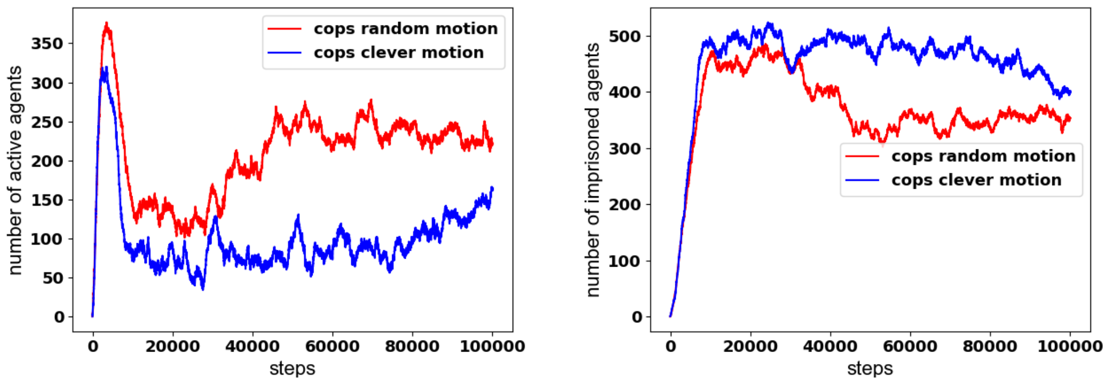

Figure 6: Influence of clever cop motion on the number of active and jailed agents.

motion reduces the number of active agents in the lattice, and on the other hand, it increases the number of jailed agents. These results are consistent because they are related to each other: the higher the number of jailed agents, the lower the amount of active agents on the lattice. Therefore, these results confirm the trend that was expected.

We also tested this rule in implementation 1 (ch. 3) (which displays Epstein's peaked behaviour), where we measured the movement rules' impact by looking at the how the waiting time distribution is changed. The results (fig. 7) show that the mean waiting time is slightly longer, however the tail of the distribution is also much larger than without the smart movement.

Thus, in the aim of calming down any rebellion, the way that cops move has an non-negligible influence: moving in the direction of the greatest number of active agents would lead to a larger number of arrested rebellious individuals.

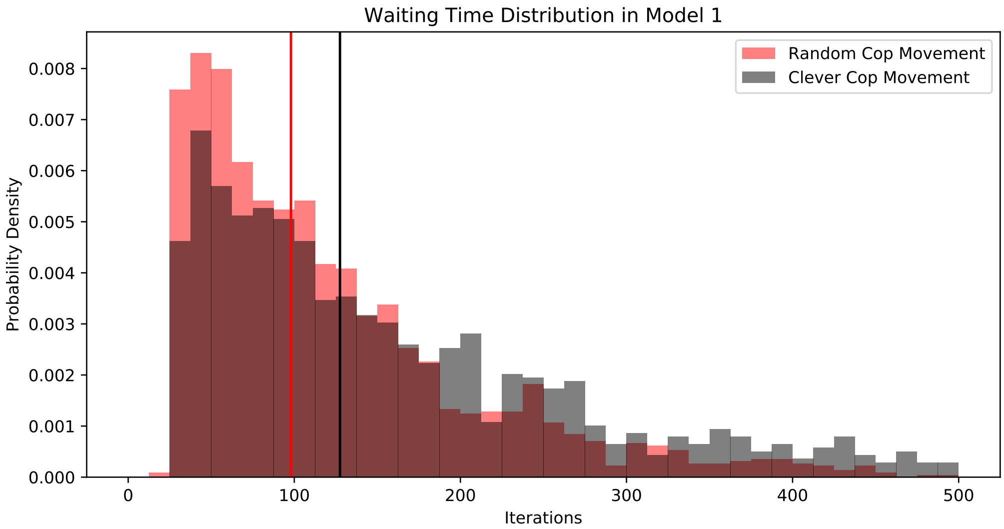

Figure 7: Waiting time distribution of smart cop movement compared to random movement. Medians are indicated with vertical lines. With the smart movement enabled the mean waiting is shifted slightly, however the tail of the distribution is also bigger.

5.3 Agent Reactions

5.3.1 Motivation and Explanation

We aim to implement an additional interaction: We let agents respond to another nearby agent being arrested. In particular we aim to add a negative 'reaction': a given agent can be expected to be more aggrieved and perhaps encouraged to be active when one of his neighbors has been (in their opinion unjustly) arrested. It can be argued that this extension adds more realism to the model: it makes sense for an active agent's entourage to either seek revenge for their loss or to continue his 'fighting legacy.' The French 'Resistance' or Soviet 'Partisan' movements during WW2 could be the corresponding real life examples.

The implementation is as follows: when a cop jails an active agent, this induces one of the passive neighboring agents (within the arrested agent's visibility v_a) to become aggrieved ($G \to G' = 0.9$) and active with a certain non-zero probability p.

5.3.2 Observations

(All simulation parameters are given in the appendix)

We deem the active agent count N_a to be an appropriate observable and plot it vs simulation cycles for different reaction probabilities p. This is depicted in figure 5.3.2. A simulation cycle corresponds to an entire randomized batch of agents and cops being activated sequentially, one at a time.

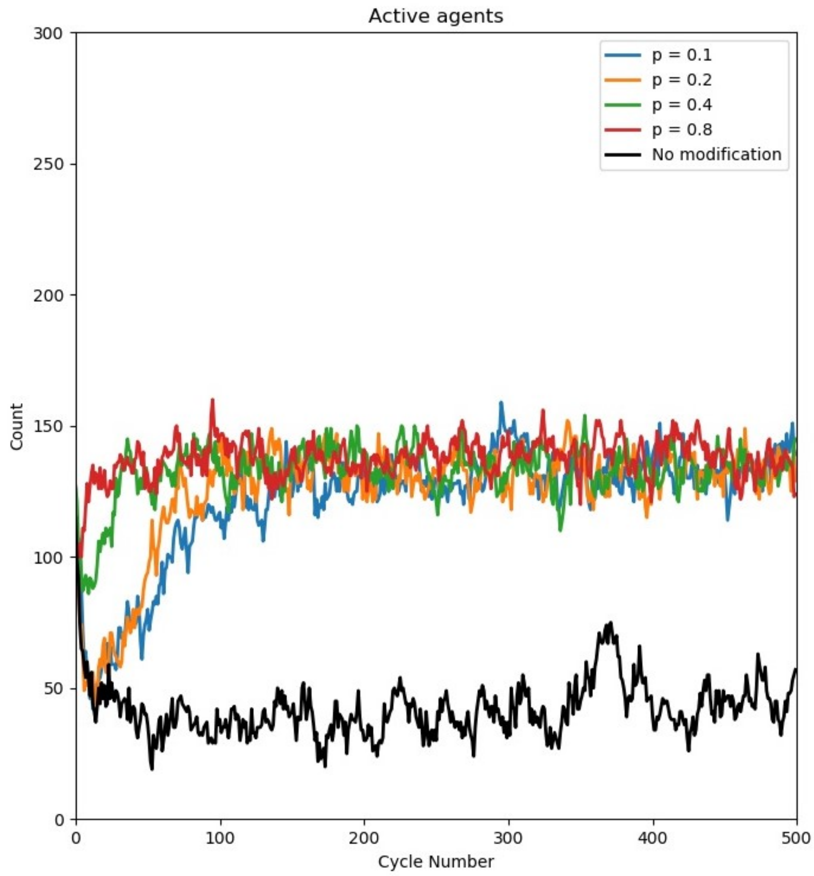

Figure 8: Active Agent Count N_a vs Cycle Number for a range of p's

We observe a number of things. Firstly, adding the modification significantly alters the average 'steady-state' active agent count: the latter increases from approximately 40 to 130. Secondly, we note that the modified dynamics are very similar: they strikingly saturate at the same 'steady-state' N_a value of 130 irrespective of the value of p! The latter seems to only correlate with the initial rate of growth towards equilibrium, which is an interesting result.

Although, we observe a saturation of the active agent population towards a common value, we might still ask how the active agents are distributed in space, and whether

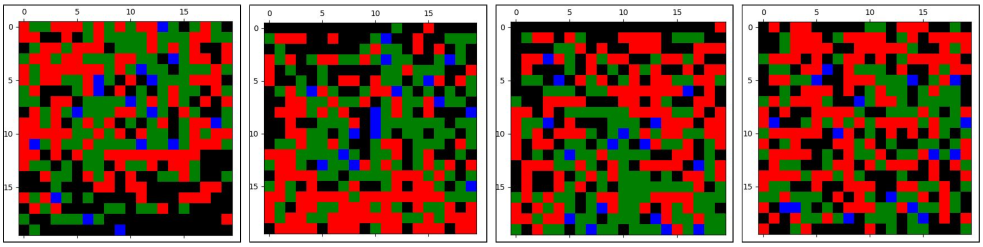

Figure 9: State profiles for $p = 0.2, 0.4, 0.6, 0.8$ (left to right)

their spatial distribution might vary with p. The initial hypothesis is that it might, as it seems reasonable to expect that for larger p's, we should see larger clusters on average, as more neighbors are more likely to get activated. We could thus perform a clustering study, i.e. analyze cluster sizes of active agents, for different p's (Figure 9) Here, the red squares represent the active agents, green the quiescent agents, blue the cops, and black the unoccupied spaces.

The initial hypothesis, that for larger p's, we should see larger clusters on average is not verified, as we observe a certain invariance of cluster size with p. The exact reason for this is unknown and to be determined.

6 Conclusion

In summary, we have seen that complex dynamics can arise even from a fairly basic rule-set, and have managed to replicate some key results (such as periodic conflict outbursts) of the Epstein model, while rectifying one of it's flaws (poorly defined cop/agent ratio). We have also attempted to extend the scope of the model by implementing additional interactions (such as global information, intelligent cop movement and agents' reactions) and have observed some additional interesting dynamics.

References

[1] Epstein, J. M. (2002). Modeling civil violence: An agent-based computational approach. Proceedings of the National Academy of Sciences, 99(suppl 3), 7243-7250.

Appendix

A Cops motion improvement : Simulation Parameters

Both simulations have been run with fixed boundary conditions (like walls at the edges of the quadratic lattice, whose size is 40×40). The rebellion threshold is set to $T = 0.1$, legitimacy of the regime to $L = 0.75$. Given a number of step $N = 10^5$, we chose a maximal jail term equal to a tenth of its value: $J_{max} = N/10 = 10^4$ (warning: here the jail term is forced to be set to a large value because, for any jailed agent, it decreases by one *at each step*). We set both agent and cop vision $v = v^* = 3$. Finally, the population density was set to 0.7 and cop density to 0.075 (among the population).

B Agent Reactions: Simulation Parameters

Topology: fixed bcs, on a 20x20 grid. Total population density = 0.7, cop population density = 0.04, perceived legitimacy $L = 0.7$, agent/cop visibility = 3 (equal), rebellion threshold $T = 0.1$, cycle number = 500, maximum jail term $J_{\max} = 15$ and coefficient $\alpha = 0$.

C Additional Implementations

C.1 Implementation 2

The second implementation, as well written in python, consists of three files:

- *Epstein-model.py* : this file is composed of one unique function which takes all the needed parameters of Epstein model, runs the model and returns the number of active and jailed agents at each step/iteration.
- *functions.py* : this files contains all the required functions to run *Epstein-model.py.*
- *simulations.py* : this file runs the simulation. This is in this file that one can set the parameters and get results. One provides the parameters of the model and this file plots the following results: number of active and jailed agents as a function of time.

In this implementation, each agent is a class (which contains all his features). Agents and cops move on the two dimensional square lattice with fixed boundary conditions (as if a wall encircled the lattice). When a rebellious agent is arrested, he goes directly to prison, implemented as a dynamic list (and the agent is thus taken of the field and replaced by an empty space). Once he has served his sentence, the agent is removed from prison and released in an random free site on the lattice. Furthermore, the vision of both agents and cops is represented by square boxes, whose side is equal to twice the vision.

At the beginning, all agents are set quiet. In this implementation, the notion of iteration (or step) is much different than the previous one. Indeed, one iteration corresponds here to : pick randomly an agent or a cop, make him move in accordance to his specific movement rule, and make him act (arrest an active agent if a cop, update his state if an agent).

D Period of the legitimacy fluctuations

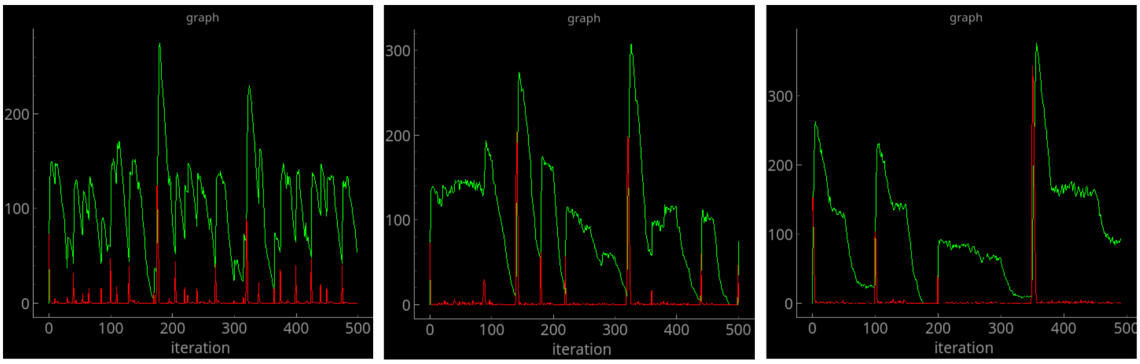

Figure 10: Parameters as for fig. 1. $L_0 = 0.79$. The periods are from left to right $\tau = 5, 20, 50$. Red curve: active agents, green curve: jailed agents.

As mentioned in section 4, one possible correction of the model is to add fluctuations to the legitimacy at discrete time steps (period τ). The dependency of the behaviour with the periods is visible figure 10. The peaking frequency seems to increase with the fluctuation frequency. For $\tau = 50$ time steps, a steady state is reached before any other disturbance occurs, which is visible by the plateaus of the number of jailed agents.

It would be interesting to do a peaks-frequency analysis and determine the quantitative behaviour over a large number of simulations as a function of the L fluctuations. There would probably be a saturation effect or other non-linearities. The influence of the other parameters should also be studied as has been done partially in the paper by Epstein.