Old Differential Equations (Tommy)

Males

$dS_m = -\lambda_m N_{sm} + (1 - \omega_m)N_m + \delta N_{im} - \mu N_{sm}$

$dI_m = \lambda_m N_{vm}(1 - \psi) + (\lambda_m N_{sm})N_m - \delta N_{im} - \mu N_{im}$

$dV_m = -\lambda_m N_{vm}(1 - \psi) + (\omega_m \mu N_m) - \mu N_{vm}$

Females

$dS_f = -\lambda_f N_{sf} + (1 - \omega_f)N_f + \delta N_{if} - \mu N_{sf}$

$dI_f = \lambda_f N_{vf}(1 - \psi) + (\lambda_f N_{sf})N_f - \delta N_{if} - \mu N_{if}$

$dV_f = -\lambda_f N_{vf}(1 - \psi) + (\omega_f \mu N_f) - \mu N_{vf}$

Updated Differential Equations

Contact Rate Differential Equations

Contact rates for females dependent on contact rate for males

Forcing dissortative heterosexual contact only

$$ce_{f.in} = \frac{ce_{m.in} \cdot m_{num}}{f_{num}}$$

External contact rate for females

$$ce_{f.ex} = ce_{m.ex}$$

Differential Equations for Infection Rates

Note: In these equations, subscript, x is used to denote variables that apply universally across both genders

External Infection Rate for Susceptibles

$$\lambda_{s.ex,x} = \tau \cdot ce_{ex} \cdot i_{prev}$$

Internal Infection Rate for Susceptibles

$$\lambda_{s.in,x} = \tau \cdot ce_{in} \cdot \frac{i_{num}}{n}$$

Total Susceptible Infection Rate

$$\lambda_{s,x} = \lambda_{s.ex,x} + \lambda_{s.in,x}$$

External Infection Rate for Vaccinated Individuals

$$\lambda_{v.ex,x} = \tau \cdot (1 - \psi) \cdot ce_{ex} \cdot i_{prev}$$

Internal Infection Rate for Vaccinated Individuals

$$\lambda_{v.in,x} = \tau \cdot (1 - \psi) \cdot ce_{in} \cdot \frac{i_{num}}{n}$$

Total Vaccinated Infection Rate

$$\lambda_{v,x} = \lambda_{v.ex,x} + \lambda_{v.in,x}$$

Internal Infection Rate

$$\lambda_{s.in} = \tau \cdot ce_{in} \cdot \frac{i_{num}}{n}$$

Total Suceptible Infection Rate

$$\lambda_s = \lambda_{s.ex} + \lambda_{s.in}$$

Vaccinated External Infection Rate

$$\lambda_{v.ex} = \tau \cdot (1 - \psi) \cdot ce_{ex} \cdot i_{prev}$$

Vaccinated Internal Infection Rate

$$\lambda_{v.in} = \tau \cdot (1 - \psi) \cdot ce_{in} \cdot \frac{i_{num}}{n}$$

Total Vaccinated Infection Rate

$$\lambda_v = \lambda_{v.ex} + \lambda_{v.in}$$

Differential Equations for Infection Rates (Same as above but with gender stratification) ** Susceptible Males**

$$\lambda_{m.s.ex} = \tau \cdot ce_{m.ex} \cdot i_{f.prev}$$

$$\lambda_{m.s.in} = \tau \cdot ce_{m.in} \cdot \frac{i_{f.num}}{f_{num}}$$

$$\lambda_{m.s} = \lambda_{m.s.ex} + \lambda_{m.s.in}$$

Susceptible Females

$$\lambda_{f.s.ex} = \tau \cdot ce_{f.ex} \cdot i_{m.prev}$$

$$\lambda_{f.s.in} = \tau \cdot ce_{f.in} \cdot \frac{i_{m.num}}{m_{num}}$$

$$\lambda_{f.s} = \lambda_{f.s.ex} + \lambda_{f.s.in}$$

Vaccinated Males

$$\lambda_{m.v.ex} = \tau \cdot (1 - \psi) \cdot ce_{m.ex} \cdot i_{f.prev}$$

$$\lambda_{m.v.in} = \tau \cdot (1 - \psi) \cdot ce_{m.in} \cdot \frac{i_{f.num}}{f_{num}}$$

The total force of infection for vaccinated males:

$$\lambda_{m.v} = \lambda_{m.v.ex} + \lambda_{m.v.in}$$

Vaccinated Females

$$\lambda_{f.v.ex} = \tau \cdot (1 - \psi) \cdot ce_{f.ex} \cdot i_{m.prev}$$

$$\lambda_{f.v.in} = \tau \cdot (1 - \psi) \cdot ce_{f.in} \cdot \frac{i_{m.num}}{m_{num}}$$

The total force of infection for vaccinated females:

$$\lambda_{f.v} = \lambda_{f.v.ex} + \lambda_{f.v.in}$$

Additional Differential Equations (Gender Neutral)

Note: In these equations, subscript, x is used to denote variables that apply universally across both genders.

Alpha is used for age.in and age.out.

Susceptible Unexposed (For individuals aged 11-18 years):

$$\frac{dS_{x,ue}}{dt} = 0.5 \cdot (1 - \omega_x \cdot \chi) \cdot \nu \cdot n - \alpha_{\text{in}} \cdot S_{x,ue} - \mu_{ue} \cdot S_{x,ue}$$

Susceptible Exposed

For sexually active individuals:

$$\frac{dS_{x,e}}{dt} = \alpha_{\text{in}} \cdot S_{x,ue} - \mu \cdot S_{x,e} - \alpha_{\text{out}} \cdot S_{x,e} - \lambda_{x,s} \cdot \zeta_s \cdot S_{x,e} - \lambda_{x,s} \cdot (1 - \zeta_s) \cdot S_{x,e} + \delta_{s,a} \cdot I_{x,a} + \delta_{s,s} \cdot I_{x,s} - \kappa \cdot S_{x,e}$$

Infected Asymptomatic

$$\frac{dI_{x,a}}{dt} = -\alpha_{\text{out}} \cdot I_{x,a} - \mu \cdot I_{x,a} + \lambda_{x,s} \cdot (1 - \zeta_s) \cdot S_{x,e} - \delta_{s,a} \cdot I_{x,a}$$

Infected Symptomatic

$$\frac{dI_{x,s}}{dt} = -\alpha_{\text{out}} \cdot I_{x,s} - \mu \cdot I_{x,s} + \lambda_{x,s} \cdot \zeta_s \cdot S_{x,e} - \delta_{s,s} \cdot I_{x,s}$$

Vaccinated Unexposed

$$\frac{dV_{x,ue}}{dt} = 0.5 \cdot \omega_x \cdot \chi \cdot \nu \cdot n - \alpha_{\text{in}} \cdot V_{x,ue} - \mu_{ue} \cdot V_{x,ue}$$

Vaccinated Exposed/Sexually Active

$$\frac{dV_{x,e}}{dt} = \alpha_{\text{in}} \cdot V_{x,ue} - \alpha_{\text{out}} \cdot V_{x,e} - \mu \cdot V_{x,e} - \lambda_{x,v} \cdot \zeta_v \cdot V_{x,e} - \lambda_{x,v} \cdot (1 - \zeta_v) \cdot V_{x,e} + \delta_{v,a} \cdot I_{x,a} + \delta_{v,s} \cdot I_{x,s} + \kappa \cdot S_{x,e}$$

Infected Asymptomatic (Vaccinated)

$$\frac{dI_{x,v,a}}{dt} = -\alpha_{\text{out}} \cdot I_{x,v,a} - \mu \cdot I_{x,v,a} + \lambda_{x,v} \cdot (1 - \zeta_v) \cdot V_{x,e} - \delta_{v,a} \cdot I_{x,v,a}$$

Infected Symptomatic (Vaccinated)

$$\frac{dI_{x,v,s}}{dt} = -\alpha_{\text{out}} \cdot I_{x,v,s} - \mu \cdot I_{x,v,s} + \lambda_{x,v} \cdot \zeta_v \cdot V_{x,e} - \delta_{v,s} \cdot I_{x,v,s}$$