Quantum phase estimation
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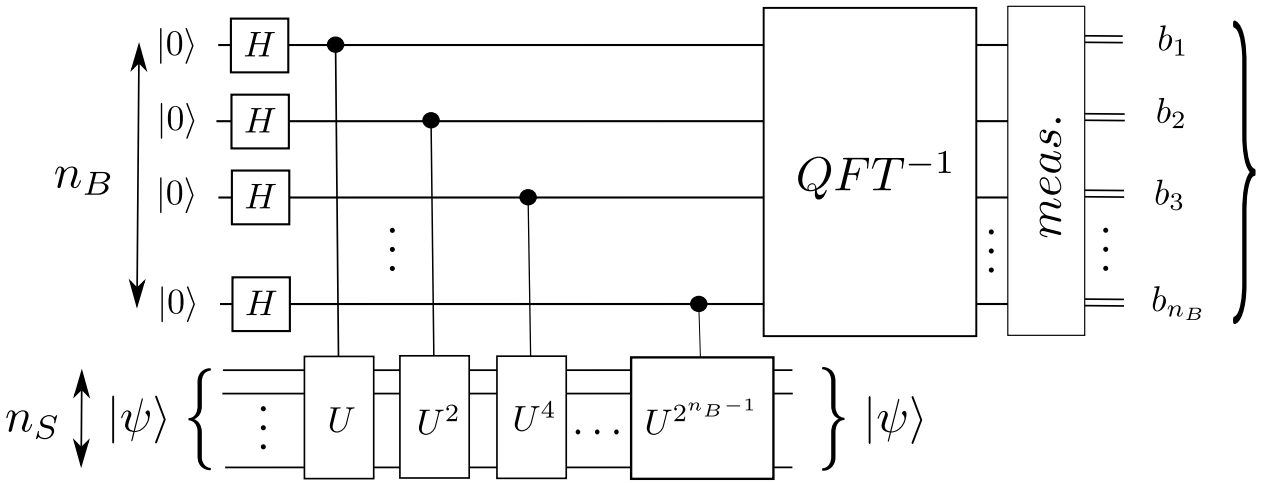
An example from chemistry

December 27, 2020

quantum phase estimation: original form



$$\phi = 0.b_1 b_2 b_3 \cdots b_{n_B} = \sum_k \frac{b_k}{2^k} = \frac{b_1}{2} + \frac{b_2}{4} + \cdots$$

With $|\psi\rangle$ **eigenvector** of U with **eigenvalue** $e^{-2i\pi\phi}$,

$(U|\psi\rangle = e^{-2i\pi\phi}|\psi\rangle,\ 0 \leq \phi < 1)$

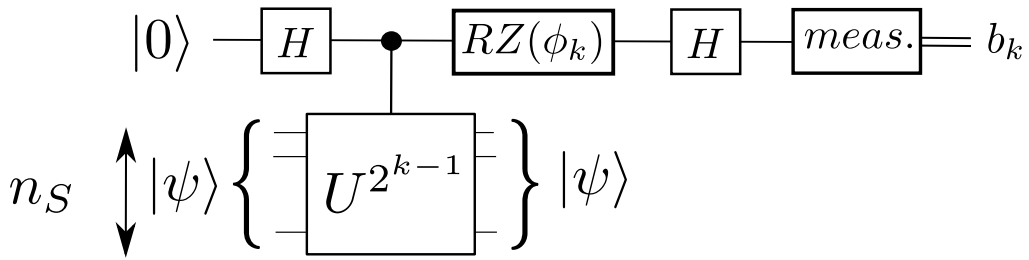
The circuit above allows to compute the n_B most significant bits of ϕ in a **binary decomposition**.

Template for *all* quantum algorithms with **exponential speedup**. (i.e with quantum circuits of **polynomial** size for classically-intractable problems)

iterative quantum phase estimation: what we will use

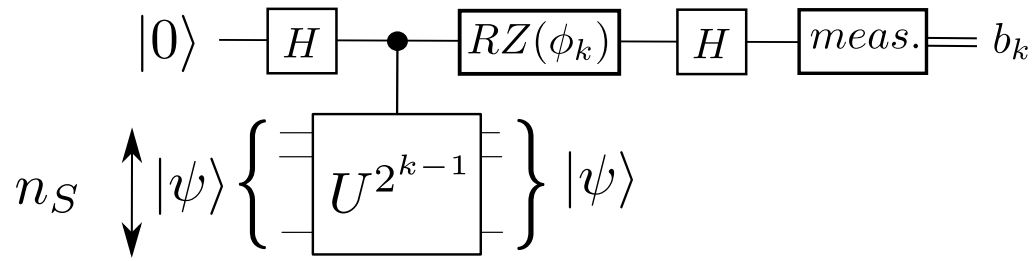
for k in **range**(n_B, 0, -1)[1]:

1. compute $\phi_k = \sum_{l=k+1}^{n_B} \frac{b_l}{2^{l-k+1}}$
2. execute the following circuit:



With $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ and $RZ(\phi_k) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi_k} \end{pmatrix}$

[1] Python style



Recall that $\phi = 0.b_1 \cdots b_{n_B}$ in binary decomposition and $\phi_k = \sum_{l=k+1}^{n_B} \frac{b_l}{2^{l-k+1}} = 0.0 b_{k+1} \cdots b_{n_B}$.

Evolution of the system:

$$|0\rangle|\psi\rangle \underset{H}{\rightarrow} \left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|\psi\rangle = \left(\frac{|0\rangle|\psi\rangle+|1\rangle|\psi\rangle}{\sqrt{2}}\right)$$

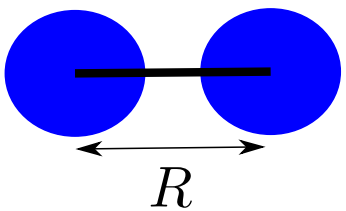
$$\underset{\bullet-U}{\hookrightarrow} \left(\frac{|0\rangle|\psi\rangle+e^{-2i\pi\cdot 2^{k-1}\phi}|1\rangle|\psi\rangle}{\sqrt{2}}\right) = \left(\frac{|0\rangle+e^{-2i\pi\cdot 0.b_k\cdots b_{n_B}}|1\rangle}{\sqrt{2}}\right)|\psi\rangle$$

$$\underset{RZ(\phi_k)}{\hookrightarrow} \left(\frac{|0\rangle+e^{-2i\pi\cdot 0.b_k}|1\rangle}{\sqrt{2}}\right)|\psi\rangle \underset{H}{\rightarrow} |b_k\rangle|\psi\rangle$$

Our use-case: ground state energy calculation

Given $\mathcal{H}$, Hamiltonian of target system (e.g a molecule), we want to compute its ground state energy.

Actually, usually $\mathcal{H}$ depends on some structural/geometric parameters (e.g H_2 dissociation curve)



$\rightarrow$

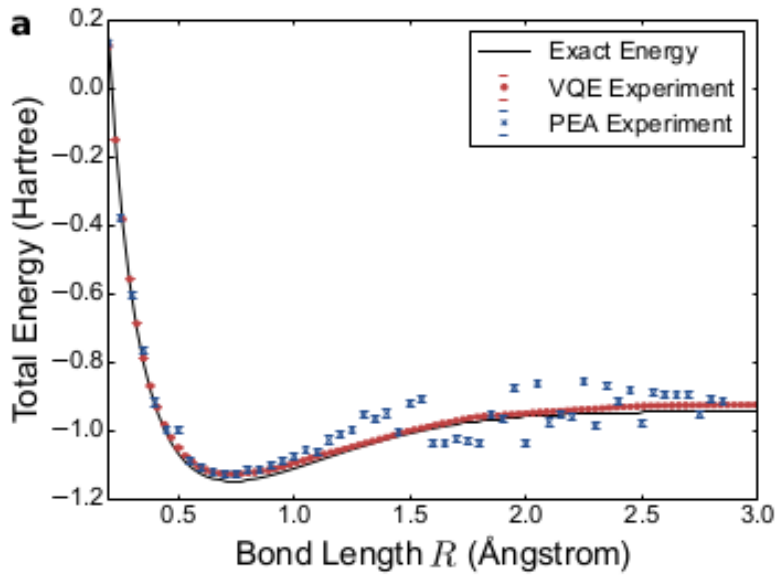
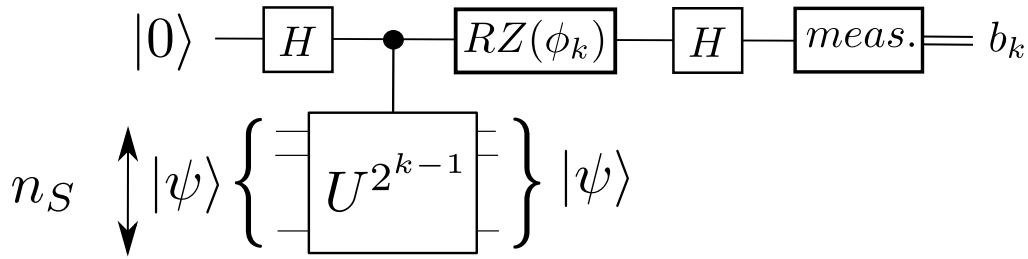


FIG. 3. **Computed H_2 energy curve and errors.** (a) Ene

From [1]. This is what we will reproduce today.



We will therefore work with:

- $U = e^{-i\mathcal{H}dt}$, for some value of dt to be chosen.
- $|\psi\rangle$ the ground state of H, whose energy E we want to compute.
- we then have $U|\psi\rangle = e^{-iEdt}|\psi\rangle = e^{-2i\pi\frac{Edt}{2\pi}}$

Problem 1: the algorithm only allows us to compute $\phi = \frac{Edt}{2\pi} \bmod [1]$ $(\in [0, 1])$

Following [2], we will use an *interval guess* $[E_{min}, E_{max}]$ for E. Then $\mathcal{H} \leftarrow \mathcal{H} - E_{min} \cdot I$ ensures $E > 0$ and choosing $dt = \frac{2\pi}{E_{max}}$ ensures $\frac{Edt}{2\pi} < 1$

Other problems

Problem 2 In general we do not have access to the ground state.
$\rightarrow$use an **ansatz**[2] with theoretical good overlap with ground state.

Problem 3 How do we implement $U^{2^k} = e^{-i\mathcal{H}2^k dt}$?
$\rightarrow$ usually $\mathcal{H}$ comes as a sum of simpler terms $\sum_l \mathcal{H}_l$ (e.g Pauli products) for which $e^{-i\mathcal{H}_l dt}$ is simple to implement.
$\rightarrow$ and use **Trotter's formula** [2]

$$e^{-i\left(\sum_l \mathcal{H}_l\right)dt} \simeq \left(\prod_l e^{-\frac{i\mathcal{H}_l dt}{p}}\right)^p$$

with p integer, as large as possible.

[2] easy-to-prepare approximation


Peter JJ O'Malley, Ryan Babbush, Ian D Kivlichan, Jonathan Romero, Jarrod R McClean, Rami Barends, Julian Kelly, Pedram Roushan, Andrew Tranter, Nan Ding, et al.
Scalable quantum simulation of molecular energies.
Physical Review X, 6(3):031007, 2016.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Simulation of electronic structure hamiltonians using quantum computers.
Molecular Physics, 109(5):735–750, 2011.