Department of Computer Science | FCUP

Algorithms (CC4010) | 2021/2022

Test (26.11.2021) | *duration: 2h*

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1. [1.5] Is $\Theta(\log_2 n) = \Theta(\log_{100000} n)$? Explain.

2. [2.5] Which is the difference between *quicksort* and *randomized quicksort*? Which is the worst case time complexity of *quicksort* and the expected time complexity of *randomized quicksort*? Why for *randomized quicksort* no specific input elicits the worst-case behavior?

3. [2.5] Explain why the asymptotic time complexity of Insertion Sort is $O(n^2)$ and of Selection Sort is $\Theta(n^2)$. Start by presenting the main ideas of the two methods.

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4. [6.0] Given an array x of n integers, SELECT(x, a, b, k) returns the k^{th} **largest** element of $x[a], \ldots, x[b]$. Suppose that all elements of x are distinct, $x[1]$ is the first element, $1 \leq a \leq b \leq n$ and $1 \leq k \leq b - a + 1$. Assume that SELECT(x, a, b, k) is correct and MYPARTITION **uses** $x[a]$ **as pivot and runs deterministically**, in-place, and in time $\Theta(b - a + 1)$.

SELECT(x, a, b, k)

1. **if** $a = b$ **then return** $x[a]$
2. $t =$ MYPARTITION(x, a, b)
3. $p = t - a + 1$
4. **if** $p = k$ **then return** $x[t]$
5. **if** $p < k$ **then**
6.   **return** SELECT$(x, t + 1, b, k - p)$
7. **return** SELECT$(x, a, t - 1, k)$

a) Let $x[i] = s_i$, for $1 \leq i \leq n$, be the state of x in line 1. Which is the state of x in **line 4** and the value of t? Recall that SELECT(x, a, b, k) returns the k^{th} **largest** element of $x[a], \ldots, x[b]$.

b) Could the time complexity of SELECT$(x, 1, n, n)$ be $\Omega(n^2)$ in the worst case? Which is the time complexity of SELECT(x, a, b, k) in the worst case and in the best case? Explain.

c) Under the condition stated on **a)**, justify the correctness of SELECT.

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d) Which are the differences to *quickselect* and *median of five medians*?

5. [2.0] Given the sequence $v_1, \ldots, v_n$ of the vertices of a n-vertex convex polygon P, in **counterclockwise order** (CCW), we want to check whether a point q belongs to the interior of P. Assume v_1 is the vertex with the smallest y-value. Explain the main steps of the $O(\log_2 n)$ algorithm for solving the problem.

6. [2.5] Explain why radix sort requires the auxiliary sorting algorithm to be a stable sorting algorithm. Is radix sort itself a stable sorting algorithm?

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7. [3.0] Let $P = \{p_1, p_2, p_3, \ldots, p_n\}$ be a set of n points in the plane. Assume that p_1 is the point with the smallest y-value, P is sorted in **strictly <u>decreasing</u> order** of polar angle w.r.t. p_1 (there are no ties), and we want to report the vertices of the convex hull in **<u>clockwise</u> order (CW)**, starting from p_1.

Using pseudocode, write the algorithm (by adapting Graham-scan). Explain why it is correct and which is the time complexity in this case.

Master theorem:

Let $a \geq 1$ and $b > 1$ be constants, let $f(n)$ be a function, and let $T(n)$ be defined on the nonnegative integers by the recurrence $T(n) = aT(n/b) + f(n)$, where we interpret n/b to mean either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$. Then $T(n)$ has the following asymptotic bounds:

1. If $f(n) = O(n^{\log_b a - \varepsilon})$ for some constant $\varepsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.
2. If $f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \log_2 n)$.
3. If $f(n) = \Omega(n^{\log_b a + \varepsilon})$, for some constant $\varepsilon > 0$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) = \Theta(f(n))$.