Stat 567
HW#1

13. $$P(\text{win}) = \sum_{i=2}^{12} P(\text{win} \mid \text{roll } i)\, P(\text{roll } i)$$

If $i = 2, 3, 12$, $P(\text{win} \mid \text{roll } i) = 0$

If $i = 7, 11$, $P(\text{win} \mid \text{roll } i) = 1$

If $i = 4$, $$P(\text{win} \mid \text{roll } 4) = \frac{3}{36} + \frac{27}{36}\cdot\frac{3}{36} + \left(\frac{27}{36}\right)^2 \frac{3}{36} + \cdots$$

P(roll another 4)   P(roll something other than 4 or 7)

$$= \frac{\frac{3}{36}}{1 - \frac{27}{36}} = \frac{3}{9}$$

Similarly, $$P(\text{win} \mid \text{roll } 5) = \frac{4/36}{1 - \frac{26}{36}} = \frac{4}{10}$$

$$P(\text{win} \mid \text{roll } 6) = \frac{5/36}{1 - 25/36} = 5/11$$

$P(\text{win} \mid \text{roll } 8) = P(\text{win} \mid \text{roll } 6)$,
$P(\text{win} \mid \text{roll } 9) = P(\text{win} \mid \text{roll } 5)$,
$P(\text{win} \mid \text{roll } 10) = P(\text{win} \mid \text{roll } 4)$

So $$P(\text{win}) = \frac{1}{36}(0) + \frac{2}{36}(0) + \frac{3}{36}\left(\frac{3}{9}\right) + \frac{4}{36}\left(\frac{4}{10}\right) + \frac{5}{36}\left(\frac{5}{11}\right) + \frac{6}{36}(1)$$
$$+ \frac{5}{36}\left(\frac{5}{11}\right) + \frac{4}{36}\left(\frac{4}{10}\right) + \frac{3}{36}\left(\frac{3}{9}\right) + \frac{2}{36}(1) + \frac{1}{36}(0) = .4929$$

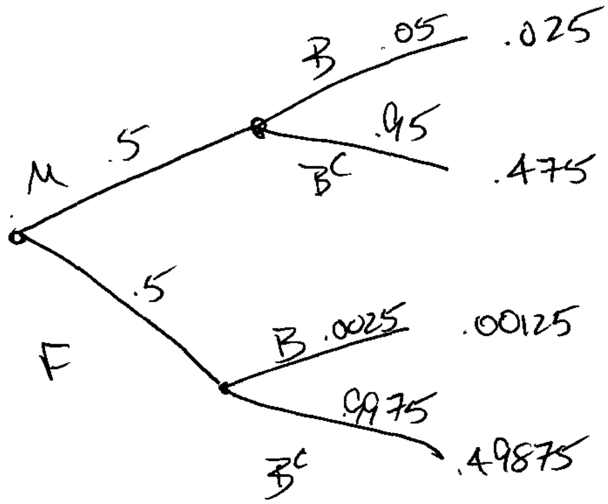
16 a) $P(\text{at least one } 6) = P(16, 26, 36, 46, 56, 66, 65, 64, 63, 62, 61)$

$= \frac{11}{36}$

b) $P(\text{at least one } 6 \mid \text{different faces}) = \dfrac{P(\text{at least one } 6 \cap \text{diff. faces})}{P(\text{diff. faces})}$

$= \dfrac{\frac{10}{36}}{30/36} = \frac{1}{3}$

21.



	B	B^C	
M	.025	.475	.5
F	.00125	.49875	.5
	.02625	.97375	1

$P(M \mid B) = \dfrac{P(MB)}{P(B)} = \dfrac{.025}{.02625} = \dfrac{20}{21} = .9524$

30. $P(B) = .7 \quad P(G) = .4$, B and G independent.

a) $P(G \mid \text{exactly } 1) = \dfrac{P(G \cap \text{exactly } 1)}{P(\text{exactly } 1)} = \dfrac{P(G \cap B^c)}{P(G \cap B^c) + P(G^c \cap B)}$

$= \dfrac{(.4)(.3)}{(.4)(.3) + (.6)(.7)} = \dfrac{2}{9} = .\overline{2}$

30b) $P(G \mid \text{at least } 1) = \dfrac{P(G \cap \text{at least } 1)}{P(\text{at least } 1)} = \dfrac{P(G)}{P(G \cup B)}$

$= \dfrac{.7}{.7 + .4 - .28} = \dfrac{7}{82} = .8537$

36. $P(\text{black}) = P(\text{black} \mid \text{Box}1)\,P(\text{Box}1) + P(\text{black} \mid \text{Box}2)\,P(\text{Box}2)$

$= (.5)(.5) + \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{4} + \frac{1}{3} = \frac{7}{12} = .58\overline{3}$