

Final Exam

Stat 567

Winter 2017

due March 21, 5pm

1. Let $f(x, y) = \begin{cases} c(x+y), & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$ be the joint p.d.f. of the random variables X and Y .
(same as midterm problem)
P.175 plus (Section 2.5.3) p.50

Find the correlation between X and Y .

2. A and B play a series of games with A winning each game with probability p . The overall winner is the first player to have won two more games than the other.

tree diagram for prob. 5

(a) Find the probability that A is the overall winner.

(b) Find the expected number of games played.

3. The number of customers entering a store on a given day is Poisson distributed with mean $\lambda = 10$. The amount of money spent by a customer is uniformly distributed over $(0, 100)$. Find the mean and variance of the amount of money that the store takes in on a given day.
compounding, sum of i.i.d.s

4. The random variable X has the density function $f(x) = 2kxe^{-2x^2}$, $x > 0$. Let $Y = X^2$. Find the density of Y .
transformation to Y

5. If X is a random variable such that $E[X] = 3$ and $E[X^2] = 13$, determine a lower bound for the probability $P(-2 < X < 8)$.
4 limit theorems use one of 'em

6. Assume that X_1, X_2, \dots, X_{10} are i.i.d. normal random variables with mean 70 and standard deviation 15. Find the joint probability that the sample mean lies between 68 and 72 and the sample standard deviation is less than 16.
(02/16 lecture) look at \bar{x} and s rules (P.71 Ross)

7. For each of the following transition matrices, (i) determine the communication classes, (ii) determine which states are transient or recurrent, and (iii) find the limit of P^n as $n \rightarrow \infty$.

(a) $P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

use spectral
decomp and
look at eigenvalues
 $\rightarrow \infty$

same as
class exercise on
TVs

(b) $P = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

(c) $P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$