Name **SOLUTIONS**

Quiz 1

Please write all answers on these pages.

1. Consider the two-variable logical OR function:

x_1	x_2	class
0	0	−1
0	1	1
1	0	1
1	1	1

(a) Draw a perceptron with two input nodes, one bias node, and one output node, and label it with weights so that it correctly classifies all four possible inputs for this function.

(Note: You don't have to apply the perceptron learning rule here; just figure out the weights yourself by looking at the function.)
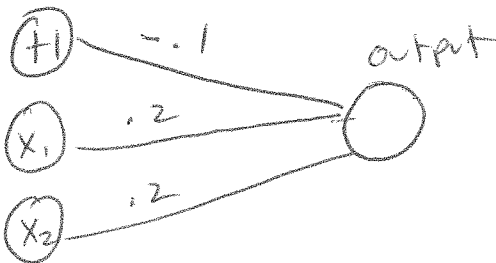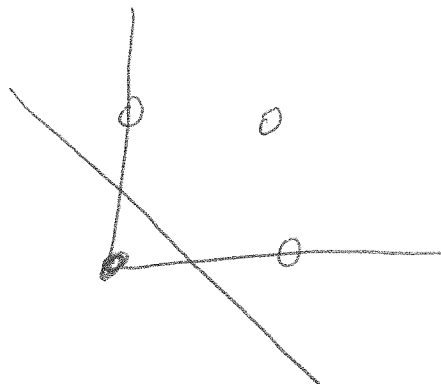
(other solutions are possible)

(b) Write the equation of the separation line for your perceptron in slope-intercept form, and sketch the line on a graph. Include the four points of the OR function to show that your line correctly separates the positive and negative examples.

$$-.1 + .2 x_1 + .2 x_2 = 0$$

$$x_2 = -x_1 + \tfrac{1}{2}$$

2. Consider the perceptron below. What class would it assign to the three-dimensional vector $(-1, -1, -3)$? Show your work.

+1 $\longrightarrow$ -0.7

x_1 $\longrightarrow$ -0.8

x_2 $\longrightarrow$ -0.1

x_3 $\longrightarrow$ 0.1

output
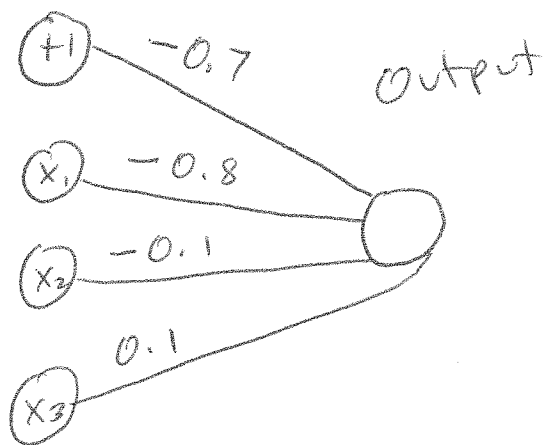
$$class = sgn\left(-.7 + .8 + .1 - .3\right)$$

$$= -1$$

3. Suppose you have the perceptron of Question 2, and the following training set of two examples:

Example:	Target:
(1, 1, 3)	+1
(1, –2, 2)	–1

$$class = sgn(-.7-.8-.1+.3) = -1$$
$$class = sgn(-.7-.8+.2+.2) = -1$$

(a) What is the accuracy of the perceptron on this training set?

$$accuracy = \frac{1}{2}$$

(b) Apply the perceptron learning rule with stochastic gradient descent for one epoch of training. (I.e., change each of the weights after processing each training example.) Use a learning rate $\eta = 0.1$. What are the weights after one epoch?

Example 1: Output $= -1$ Target $= 1$ $(t - o) = 2$

$$\Delta w_0 = 0.1 (2)(1) = .2$$
$$\Delta w_1 = 0.1 (2)(1) = .2$$
$$\Delta w_2 = 0.1 (2)(1) = .2$$
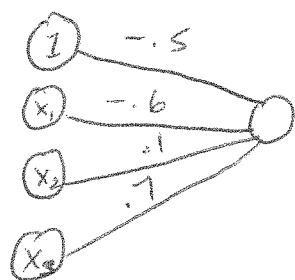$$\Delta w_3 = 0.1 (2)(3) = .6$$

New weights after Example 1:
$$w_0 = -.5$$
$$w_1 = -.6$$
$$w_2 = .1$$
$$w_3 = .7$$

Perceptron after example 1:

Example 2: Output $= sgn(-.5-.6-.2+1.4)$
$= 1$

$(t-o) = -2$

$$\Delta w_0 = 0.1 (-2)(1) = -.2$$
$$\Delta w_1 = 0.1 (-2)(1) = -.2$$
$$\Delta w_2 = 0.1 (-2)(-2) = .4$$
$$\Delta w_3 = 0.1 (-2)(2) = -.4$$

New weights after Example 2:
$$w_0 = -.7$$
$$w_1 = -.8$$
$$w_2 = .5$$
$$w_3 = .3$$

(c) What is the accuracy on the training set after one epoch of training?

$$O_1 = sgn(-.7-.8+.5+.9) = -1$$
$$O_2 = sgn(-.7-.8-1+.6) = -1$$

$$Accuracy = \frac{1}{2}$$

4. Consider the table below, which reports, for four instances, the actual class of each instance and the score given by a classifier to that instance.

Instance	Actual Class	Score
1	Positive	0.1
2	Positive	0.05
3	Negative	−0.05
4	Positive	−0.1

For each value of the threshold, fill in the table below with the accuracy, precision, recall, TPR (true positive rate), and FPR (false positive rate) of the classifier.

(If the score is equal to or greater than the threshold, the classifier assigns "positive", else it assigns "negative".)

Threshold	Accuracy	Precision	Recall	TPR	FPR
$+\infty$	1/4	0	0	0	0
.1	1/2	1	1/3	1/3	0
0	3/4	1	2/3	2/3	0
−.05	1/2	2/3	2/3	2/3	1
$-\infty$	3/4	3/4	1	1	1

(a) Sketch the **Precision /Recall** curve for these points:
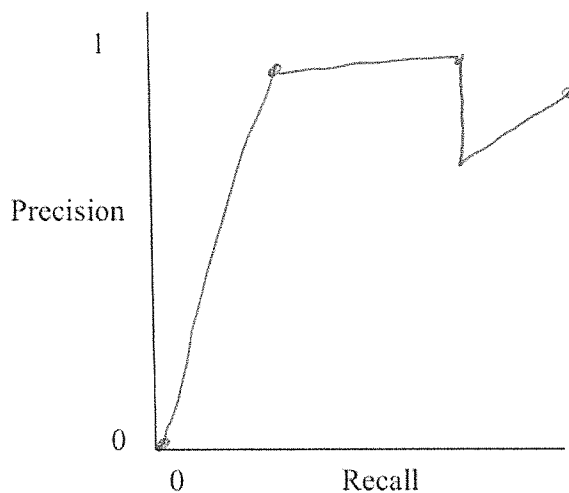
(b) Sketch the ROC curve for these points: