Quiz 5

Solutions

Please write all answers on these pages.

1. Consider the task of classifying days. The following data was collected on several different days, along with whether people rated the day as "nice" or "not nice". **Temperature** can be either "warm" or "cold", **Precipitation** can be either "low" or "high", and **Foggy?** can either be "yes" or "no".

Day	***Temperature***	***Precipitation***	***Foggy*?**	**Rating** (Class)
D1	warm	low	no	nice
D2	warm	high	yes	not nice
D3	cold	low	no	nice
D4	warm	high	no	nice
D5	warm	low	yes	not nice
D6	cold	low	yes	not nice
D7	cold	high	yes	not nice

The entropy of this data set as a whole is 0.98.

(a) What is the information gain of the feature ***Foggy***?

Entropy (Foggy) = 0. Thus, InformationGain(Foggy) = 0.98.

(b) Which of the following is true?

InformationGain(**Temperature**) < InformationGain(**Precipitation**)

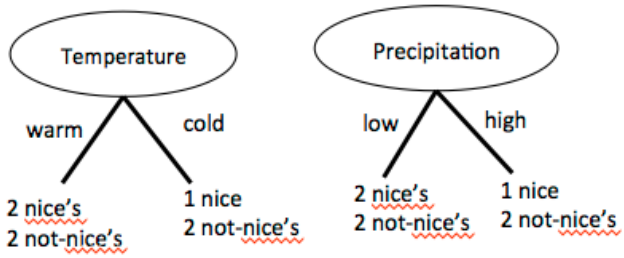
InformationGain(**Temperature**) > InformationGain(**Precipitation**)

InformationGain(**Temperature**) = InformationGain(**Precipitation**)

Temperature and Precipitation have equal information gain.

Explain your answer.

They have the same entropy, as shown below:



2. State Bayes' rule and show how it is derived from the definition of conditional probability.

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

Derivation

$$P(h|D) = \frac{P(h,D)}{P(D)}$$

$$P(D|h) = \frac{P(h,D)}{P(h)}$$

$$P(h,D) = P(D|h)P(h) \ = \ P(h|D)P(D)$$

Thus,

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

3. Sixty percent of women in the US are currently married. Ninety percent of married women wear wedding rings. One percent of unmarried women wear wedding rings. Your new neighbor Julia does not wear a wedding ring. What is the probability that she is married? Show your work. (If you don't have a calculator, just give the answer as a numerical expression.)

Let h_1 = "Julia is married", and h_2 = "Julia is not married".

D = "Julia does not wear a wedding ring".

$P(h_1 \mid D) = P(D \mid h_1)\, P(h_1) \,/\, P(D)$

$P(h_1) = .6$, $P(h_2) = .4$

$P(D \mid h_1) = .1$ (i.e., probability she does not wear a ring given she is married)

$P(D \mid h_2) = .99$ (i.e., probability she does not wears a ring given she is unmarried)

$P(D) = P(D \mid h_1)\, P(h_1) + P(D \mid h_2)\, P(h_2)$

$$P(h_1|D) = \frac{(.1)(.6)}{(.1)(.6) + (.99)(.4)} = \frac{.06}{.06 + .396} = \frac{.06}{.456} = .13$$

4. **Optional extra credit!** You are given two coins, one of which is a fair coin and one of which is a biased coin. The fair coin has a 50% chance of coming up heads, and the biased coin has a 60% chance of coming up heads.

You choose one of these coins at random, and flip it four times. It comes up heads every time. What is the probability that this coin is the fair one? Show your work. (If you don't have a calculator, just give the answer as a numerical expression.)

Let h_1 = "I chose the fair coin", and h_2 = "I chose the biased coin".

D = "Four heads in a row".

$P(h_1 \mid D) = P(D \mid h_1)\, P(h_1) \,/\, P(D)$

$P(h_1) = .5$, $P(h_2) = .5$ (i.e., you chose one of the coins at random)

$P(D \mid h_1) = (.5)^4 = .0625$

$P(D \mid h_2) = (.6)^4 = .1296$

$$P(h_1|D) = \frac{(.5)(.0625)}{(.5)(.0625) + (.5)(.1296)} \approx .33$$