Name Solutions

**Quiz 4**

Please write all answers on these pages.

1. State and prove Bayes Theorem, using the definition of conditional probability. You may use pictures to illustrate your proof. (2 points)

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

Proof:

$$P(X|Y) \overset{\text{by definition}}{=} \frac{P(X \wedge Y)}{P(Y)}$$

$$P(Y|X) = \frac{P(X \wedge Y)}{P(X)}$$

2. It has been estimated that .05 percent of the US population has HIV. There is a test for HIV: If a person has HIV, the test has a 98% chance of being positive. If a person does not have HIV, the test has a 3% chance of being positive. Tom has tested positive. Assuming these values, what is the probability that he has HIV? Show your work. (3 points)

**(Remember that you need to multiply percentages by .01 to turn them into probabilities.)**

$h_1 = HIV$

$h_2 = \neg HIV$

$D =$ test is positive (+)

$P(HIV) = .0005 \qquad P(\neg HIV) = .9995$

$P(+|HIV) = .98$

$P(+|\neg HIV) = .03$

$$P(HIV|+) = \frac{P(+|HIV)\,P(HIV)}{P(+)} = \frac{(.98)(.0005)}{(.98)(.0005) + (.9995)(.03)} = 0.016$$

3. Let A, B, and Z be random variables. What does it mean for A and B to be conditionally independent, given Z? (1 point)

$$P(A, B|Z) = P(A|Z)\,P(B|Z)$$

4. Consider the following training set, in which each example has four binary attributes and a binary class.

| Example | Attribute1 | Attribute2 | Attribute3 | Attribute4 | Class |
|---|---|---|---|---|---|
| $\mathbf{x}_1$ | 1 | 1 | 1 | 1 | +1 |
| $\mathbf{x}_2$ | 1 | 1 | 0 | 1 | +1 |
| $\mathbf{x}_3$ | 0 | 1 | 1 | 0 | +1 |
| $\mathbf{x}_4$ | 1 | 0 | 0 | 1 | +1 |
| $\mathbf{x}_5$ | 1 | 0 | 0 | 0 | +1 |
| $\mathbf{x}_6$ | 1 | 0 | 1 | 0 | −1 |
| $\mathbf{x}_7$ | 0 | 1 | 0 | 0 | −1 |
| $\mathbf{x}_8$ | 0 | 0 | 1 | 0 | −1 |

**(a)** How would a naïve Bayes classifier trained on this training set classify the following new example? Show your work. (No smoothing of probabilities is needed.) (3 points)

| | | | | | |
|---|---|---|---|---|---|
| $\mathbf{x}_9$ | 1 | 1 | 0 | 0 | ? |

For class +:

$P(A_1=1|+)\ P(A_2=1|+)\ P(A_3=0|+)\ P(A_4=0|+)\ P(+)$

$= (4/5)(3/5)(3/5)(2/5)(5/8) = .072$

For class −:

$P(A_1=1|-)\ P(A_2=1|-)\ P(A_3=0|-)\ P(A_4=0|-)\ P(-)$

$= (1/3)(1/3)(1/3)(1)(3/8) = .014$

class is +1

**(b)** Give the probability P(Attribute4 = 1 | class = −1), after applying Laplace (add-one) smoothing. (1 point)

Original probability: $\frac{0}{3}$

Laplace smoothed prob.: add 1 to numerator, 2 to denominator (because attributes are binary)

$\frac{1}{5}$