Stat 567
HW # 2

4. Suppose a die is rolled twice. What are the possible values that the following random variables can take on?
   (a) The maximum value to appear in the two rolls.
   (b) The minimum value to appear in the two rolls.
   (c) The sum of the two rolls.
   (d) The value of the first roll minus the value of the second roll.

5. If the die in Exercise 4 is assumed fair, calculate the probabilities associated with the random variables in (i)–(iv).

16. An airline knows that 5 percent of the people making reservations on a certain flight will not show up. Consequently, their policy is to sell 52 tickets for a flight that can hold only 50 passengers. What is the probability that there will be a seat available for every passenger who shows up?

32. If you buy a lottery ticket in 50 lotteries, in each of which your chance of winning a prize is $\frac{1}{100}$, what is the (approximate) probability that you will win a prize (a) at least once, (b) exactly once, (c) at least twice?

33. Let X be a random variable with probability density

$$f(x) = \begin{cases} c(1 - x^2), & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

   (a) What is the value of c?
   (b) What is the cumulative distribution function of X?

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4,5 Roll 2 dice

a) $X = \max$

X	$p(x)$
1	1/36
2	3/36
3	5/36
4	7/36
5	9/36
6	11/36

b) $Y = \min$

Y	$p(y)$
1	11/36
2	9/36
3	7/36
4	5/36
5	3/36
6	1/36

c) Z = sum

Z	$p(z)$
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36

d) $W = 1^{st} - 2^{nd}$

w	$p(w)$
-5	1/36
-4	2/36
-3	3/36
-2	4/36
-1	5/36
0	6/36
1	5/36
2	4/36
3	3/36
4	2/36
5	1/36

16. let X = # who show up. $X \sim Bino(n = 52, p = .95)$

$$P(X \le 50) = 1 - [p(51) + p(52)] = 1 - \left[\binom{52}{51}(.95)^{51}(.05) + \binom{52}{52}(.95)^{52}\right]$$

$$= 1 - \left[52(.95)^{51}(.05) + (.95)^{52}\right] = .7405$$

32. $X \sim \text{Bino}(n=50, p=.01)$

a) $P(X \geq 1) = 1 - p(0) = 1 - \binom{50}{0}(.01)^0(.99)^{50}$

$= 1 - .99^{50} = \underline{.3950}$

b) $P(X=1) = p(1) = \binom{50}{1}(.01)^1(.99)^{49} = 50(.01)(.99)^{49}$

$= \underline{.3056}$

c) $P(X \geq 2) = 1 - [p(0) + p(1)] = 1 - [.6050 + .3056]$

$= \underline{.0894}$

<u>OR</u>, approximat with Poisson($\lambda = np = .5$)

a) $1 - p(0) = 1 - \dfrac{e^{-.5} .5^0}{0!} = 1 - e^{-.5} = .3935$

b) $p(1) = \dfrac{e^{-.5} .5^1}{1!} = .5e^{-.5} = .3033$

c) $1 - [.6065 + .3033] = .0902$

33. $f(x) = c(1-x^2) \qquad -1 < x < 1$

a) $1 = \int_{-1}^{1} c(1-x^2)dx = c\left[x - \frac{x^3}{3}\right]_{-1}^{1} = c\left[\left(1-\frac{1}{3}\right) - \left(-1+\frac{1}{3}\right)\right]$

$= c\left[2 - \frac{2}{3}\right] = \frac{4c}{3} \quad \therefore \underline{c = \frac{3}{4}}$

b) $\int_{-1}^{x} \frac{3}{4}(1-t^2)dt = \frac{3}{4}\left[t - \frac{t^3}{3}\right]_{-1}^{x} = \frac{3}{4}\left[x - \frac{x^3}{3} - \left(-1+\frac{1}{3}\right)\right] = \frac{3}{4}\left[x - \frac{x^3}{3} + \frac{2}{3}\right], \quad -1 < x < 1$