# T:M DESIGN– a tool to maximize precision of the T:M ratio by optimally allocating tagged smolts between transported and in-river migrant groups

Richard A. Hinrichsen

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## Introduction

After 1975, when the federal hydropower system was in place on the Snake and Columbia rivers, the number of adult salmon returning to spawn declined, in part because of reduced survival of juvenile fish during their seaward migration (Raymond 1979, 1988). To aid juveniles during their seaward migration, smolts are collected at Snake River dams and transported (barged) downstream of Bonneville Dam (river km 234) and released (Ebel et al. 1973; Ebel 1980). An important question that emerges is "how effective is transporting juveniles at increasing survival to adulthood compared to allowing the juveniles to migrate in-river to the sea?" To answer this question, smolt-to-adult ratios (SARs) are calculated for these two groups and compared.

Calculation of the SARs is made possible by tagging smolts with passive integrated transponder (PIT) tags. Each tag has a unique numerical code, and those codes are detected as fish pass through detection systems installed in the juvenile bypass systems of most main-stem dams and in adult fish ladders at some of the mainstem dams. An SAR of a release group is simply the number of adult recoveries divided by number released. The SARs are calculated for transported fish and fish migrating in river.

The T:M ratio is the SAR for transported juveniles divided by SAR for the migrant juveniles. It is used to gauge the difference in survival (Muir et al. 2006). When a T:M ratio is greater than one, then transported juveniles have higher estimated smolt-to-adult survival than the migrant juveniles.

The goal of this statistical tool is to determine how to allocate a fixed number of smolts between transported and in-river migrant groups in order to maximize precision (minimize variance). To do this, we develop equations for estimating the log(T:M ratio) and its variance. We then use elementary calculus to determine the allocation of tagged smolts to transported and

in-river groups. Power of rejecting the null hypothesis that log(T:M ratio) is zero is then calculated for the optimal allocation of tagged smolts.

The website www.onefishtwofish.net contains a web-based tool that implements this analysis. The assumptions of the analysis are given in Table 1. The code for implementing this power analysis (Appendix A) was implemented in R, a system for statistical computation and graphics (Venerables et al. 2012).

Table 1.—Assumptions used in the power analysis.

| | |
|---|---|
| A1 | The number of tagged transported smolts and the number of tagged in-river migrant smolts are fixed. |
| A2 | The number of adult recoveries from each tagged group of juveniles is a random variable that follows a binomial distribution. |
| A3 | The probability that a smolt survives and is recovered as adult is a constant for a given release group. |
| A4 | The event that each individual fish is recovered as an adult is independent of the event that any other individual is recovered as an adult. |
| A5 | The probability that an adult is detected if it is alive (detection rate) is 100%. |
| A6 | The null hypothesis is that the true smolt-to-adult survival of transported fish is equal to that of migrant fish. |
| A7 | The estimator log(T:M ratio) is approximately normally distributed. |

## Methods

To derive equations for estimating the power to detect a difference in the smolt-to-adult survival of transported and in-river migrant groups, maximum likelihood theory is used (Mood et al. 1974).  This requires deriving variance equations for the estimators. I begin by deriving a formula for the variance of SAR and log(SAR). These formulas are then used to derive an estimate of the variance of the T:M ratio. Assume that the number of adult recoveries, $n$, follows a binomial distribution where the true SAR (denoted $\theta$) is the success probability and $N$, the

number of juvenile released, is fixed. Then, it is well known that the maximum likelihood estimator of the success probability is the SAR and the variance of the SAR is equal to

$$\mathrm{var}(SAR) = \theta(1-\theta)/N \tag{1}$$

The log of the survival estimate also has an asymptotic normal distribution, and Burnham et al. (1987) have shown that log (survival) is better approximated by a normal distribution than survival, especially when the coefficient of variation is large. Peterman (1981) examined ocean survival of three different salmon species and found that assuming a normal distribution for log(survival rate) was more consistent with the data than assuming a normal distribution for survival rate itself. Therefore, the power analysis is based on the log(survival) instead of the untransformed survival.

Applying the Delta method to calculate the variance of a transformation (Seber 1982),

$$\mathrm{var}(\log(\hat{\theta})) \cong \mathrm{var}(\hat{\theta})/\theta^2 = \left(CV(\hat{\theta})\right)^2 \tag{2}$$

This formula is applied to the SAR to yield

$$\mathrm{var}(\log(SAR)) \cong \frac{(1-\theta)}{\theta N} \tag{3}$$

I next derive the estimate of the variance of a T:M ratio, which is the ratio of the SAR for transported fish divided the SAR for in-river migrating fish. Let $SAR_T$ represent the SAR of transported fish and let $SAR_M$ represent the SAR of in-river migrating fish. Then the T:M ratio is equal to

$$T:M\ ratio = SAR_T / SAR_M \tag{4}$$

Using the Delta method on a ratio of random variables, and assuming that the covariance between the SARs of the transported fish and in-river migrants is zero, the estimate of the variance of the T:M ratio is

$$\mathrm{var}(T:M\ ratio) = \mathrm{var}(SAR_T / SAR_M)$$ (5)

$$\cong \begin{bmatrix} \dfrac{1}{\theta_M} & -\dfrac{\theta_T}{\theta_M^2} \end{bmatrix} \begin{bmatrix} \mathrm{var}(SAR_T) & 0 \\ 0 & \mathrm{var}(SAR_M) \end{bmatrix} \begin{bmatrix} \dfrac{1}{\theta_M} \\ -\dfrac{\theta_T}{\theta_M^2} \end{bmatrix}$$

$$= \frac{\mathrm{var}(SAR_T)}{\theta_M^2} + \frac{\mathrm{var}(SAR_M)\theta_T^2}{\theta_M^4}$$

$$= \frac{\theta_T(1-\theta_T)}{\theta_M^2 N_T} + \frac{\theta_T^2 \theta_M(1-\theta_M)}{\theta_M^4 N_M}$$

$$= \left(\frac{\theta_T}{\theta_M}\right)^2 \left(\frac{(1-\theta_T)}{\theta_T N_T} + \frac{(1-\theta_M)}{\theta_M N_M}\right),$$

where is $N_T$ the number of transported juveniles, $\theta_T$ is the probability that a transported juvenile survives to be detected as an adult, $N_M$ is the number of juveniles that are in-river migrants, and $\theta_M$ is the probability that an in-river migrant juvenile survives to be detected as an adult. Applying the Delta method once again, this time to the log of the T:M ratio, yields

$$\mathrm{var}(\log(T:M\ ratio)) = \frac{(1-\theta_T)}{\theta_T N_T} + \frac{(1-\theta_M)}{\theta_M N_M}.$$ (6)

The goal is to minimize this variance with respect to the fraction of fish transported, holding the total number of tagged smolts ($N$) constant. Defining $f_T = N_T / N$, where $N = N_T + N_M$, it may be shown using elementary calculus, that for a fixed release size, the optimal fraction of smolts transported is

$$f_T^* = \frac{\sqrt{\dfrac{1-\theta_T}{\theta_T}}}{\sqrt{\dfrac{1-\theta_M}{\theta_M}} + \sqrt{\dfrac{1-\theta_T}{\theta_T}}} .$$

(7)

The optimal number of smolts allocated to the transported group is $N_T^* = f_T^* N$ and the optimal

number of in-river migrants is $N_M^* = (1 - f_T^*)N$.

We then use the optimal allocation in the variance formula in equation (6) to derive the statistical power, which represents the probability of rejecting the null hypothesis of "no treatment" effect when the true treatment effect is not zero. In this case, the treatment effect , defined as δ, is the (true) value of log(T:M ratio) that would be obtained if the sample size were infinite. The estimate of the treatment effect, denoted $\hat{\delta}$, is equal to the observed value of $\log(T : M \ ratio)$.

Power is a function of the true treatment effect, the probability of a type I error (usually called the significance level and denoted by $\alpha$), and the standard error of the estimator. By maximum likelihood theory, the estimator of the treatment effect is asymptotically normally distributed. Therefore, power is approximately

$$\Pi(\delta) = \Phi(-z_{\alpha/2} - \delta/se(\hat{\delta})) + 1 - \Phi(z_{\alpha/2} - \delta/se(\hat{\delta})) ,$$

(8)

where $\Phi(z)$ is the cumulative distribution function of a standard normal random variable, and $z_{\alpha/2}$ is the critical value such that $\alpha/2$ probability of a standard normal distribution lies to the right of the value $z_{\alpha/2}$, $\delta$ is the true value of the treatment effect, and $se(\hat{\delta})$ is the square root of

the variance of $\hat{\delta}$, which may be calculated from equation (6). Experimenters usually choose a design that will deliver a power of 0.80 or greater.

*Example* —As an example, I estimate the optimal allocation and power when the true T:M ratio takes on the values of 0.50, 0.75, 1.25 and 1.50, and the true SAR of the transported group is varied from 0.003 to 0.03. This is within the range of SARs observed for juveniles transported Lower Granite Dam (river km 695) on the Snake River. I ask the question, "Given an optimal allocation between transported and in-river groups, how many total juveniles must be tagged in order to reliably detect a significant difference in the SARs of transported and in-river migrants?" To answer this question I set the significance level at 0.05 and seek the release size necessary to deliver a power of 0.80. The optimal allocation of fish to transported and in-river migrant groups is also presented.

The results of this *a priori* power analysis are given in Figures 1 and 2. Notice that when the SAR of the transported group is 0.003, then N = 50,000 is not sufficient to achieve a power of 0.80. When the SAR of the transported group is 0.012, the N=50,000 is sufficient to achieve a power 0.80 when the true T:M ratio is 0.50, 0.75, or 1.50. The important result here is that as the SAR increases, so does power. Assuming a fixed T:M ration, when SARs increase, the required number of tagged smolts needed to achieve a power of 0.80 decreases. The optimal fraction of fish allocated to the transport group decreases with T:M ratio. This occurs because a larger fraction of fish should always be allocated to the group with smallest SAR (Figure 2).
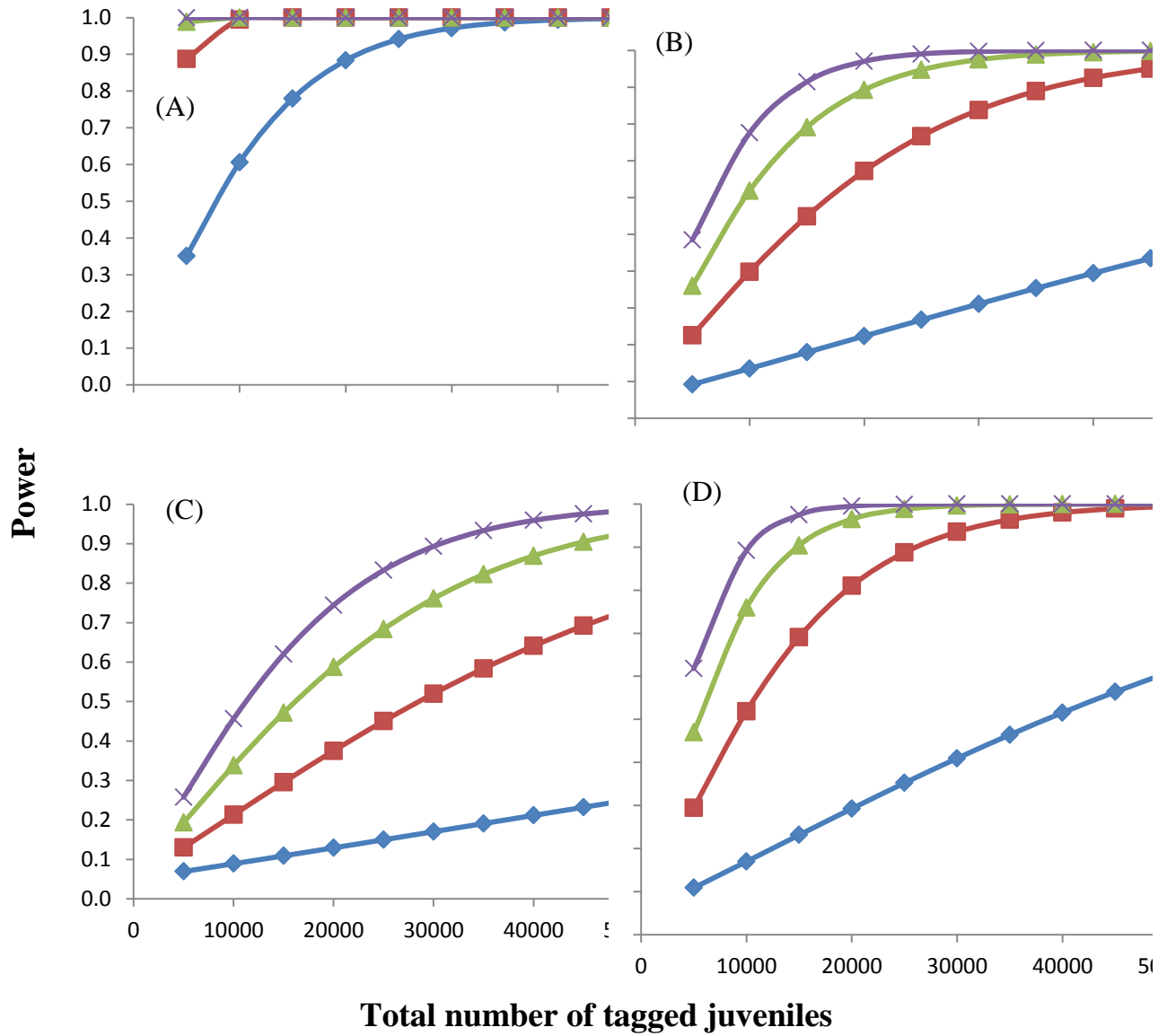
Figure 1.— Power to detect a log(T:M ratio) that is significantly different from zero using the optimal allocation between transported and in-river migrant groups. The true T:M ratio was assumed to take on the values (A) 0.50 (B) 0.75 (C) 1.25 and (D) 1.50. The values of true $SAR_T$ were 0.003 (diamonds), 0.012 (squares), 0.021 (triangles) and 0.030 (Xs).
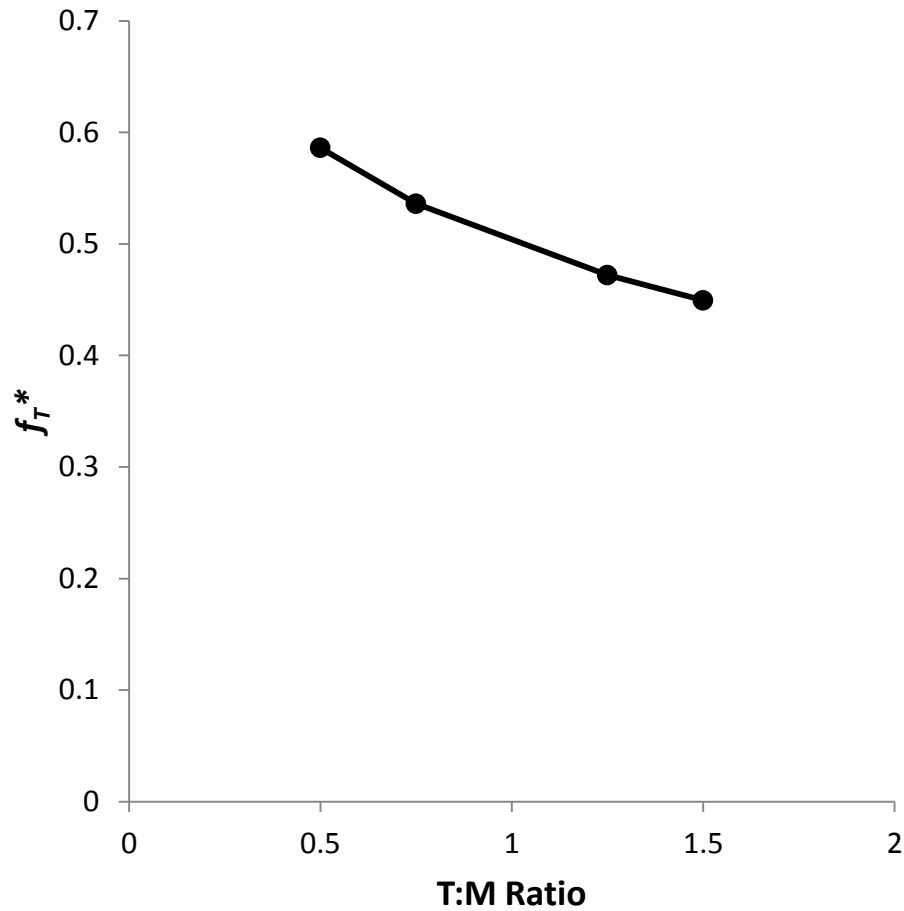
Figure 2.— Optimal allocation between transported and in-river migrant groups. The true SAR for the transported group was assumed to be 0.003. Notice that the optimal fraction of fish allocated to the transported group ($f_T^*$) decreases with T:M Ratio. That is, a larger fraction of the smolts should always be allocated to the group with smaller SAR ratio.

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# **Acknowledgements**

# Appendix A. R code used to estimate the optimal allocation of tagged juveniles between transported and in-river migrant groups.

```
#R-code used to estimate the optimal allocation of tagged smolts between transported
#and in-river groups.

#inputs
#N -- the total number of tagged smolts
#SART -- SAR of transported juveniles
#SARM -- SAR of in-river juvenile migrants

tmdesign<-function(N=1000,SART=.01 ,SARM=.02,alpha=0.05){
if(N<=0){
  warning("The number of juveniles released must exceed zero")
  return(NULL)}
if((SART>1)|(SARM<0)){
  warning("The SARs are survival rates and so must be less than one")
  return(NULL)}
if((SARM>1)|(SARM<0)){
  warning("The SARs are survival rates and so must be less than one")
  return(NULL)}
s1<-sqrt((1-SART)/SART)
s2<-sqrt((1-SARM)/SARM)
ft<-s1/(s1+s2)
NT<-ft*N
NM<-N-NT
delta<-log(SART/SARM)
q<-qnorm(1-alpha/2)
tmvar<-(1-SART)/(SART*NT)+(1-SARM)/(SARM*NM)
se<-sqrt(tmvar)
power<-(1-pnorm(q*se,mean=delta,sd=se))+pnorm(-q*se,mean=delta,sd=se)
return(list(N=N,SART=SART, SARM=SARM ,alpha=alpha, NT=NT, NM=NM,
delta=delta,se=se,cv=se/delta,power=power))
}
#outputs
#NT -- the optimal number of tagged transported juveniles
#NM – the optimal number of tagged in-river juvenile migrants
#delta is the true log(SART/SARM)
#se is the standard error of the estimate
#cv is the CV of the estimate
#power is the probability of rejecting the null hypothesis of delta=0.
```