Brost, B. M., M. B. Hooten, and R. J. Small. Model-based clustering reveals patterns in central place use of a marine top predator.

Appendix S1. Model statement, posterior distribution, full-conditional distributions, and pseudocode detailing a Markov chain Monte Carlo algorithm to implement the model for haul-out site location estimation. We represented $\mathcal{S}$, the spatial domain of possible haul-out sites, as a 100-m resolution raster consisting of cells along the shoreline of Kodiak Island, and let the locations of potential haul-out sites ($\boldsymbol{\mu}_{ij}$) assume values corresponding to the centroids of cells in $\mathcal{S}$.

In the posterior and full-conditional distributions below, we use bracket notation to denote a probability distribution. For example, $[x]$ indicates the probability distribution of x. Similarly, $[x|y]$ indicates the probability distribution of x given the parameter y. The notation "$\cdot$" represents the data and other parameters in the model.

Model Statement

$$
\begin{aligned}
\mathbf{s}_{ic}(t) &\sim \begin{cases} \mathcal{N}\left(\boldsymbol{\mu}_i(t), \boldsymbol{\Sigma}_{ic}\right), & \text{with prob. } 0.5,\ y_i(t)=1 \\ \mathcal{N}\left(\boldsymbol{\mu}_i(t), \widetilde{\boldsymbol{\Sigma}}_{ic}\right), & \text{with prob. } 0.5,\ y_i(t)=1 \\ \mathcal{N}\left(\boldsymbol{\mu}_i(t), \tau_i^2\mathbf{I}+\boldsymbol{\Sigma}_{ic}\right), & \text{with prob. } 0.5,\ y_i(t)=0 \\ \mathcal{N}\left(\boldsymbol{\mu}_i(t), \tau_i^2\mathbf{I}+\widetilde{\boldsymbol{\Sigma}}_{ic}\right), & \text{with prob. } 0.5,\ y_i(t)=0 \end{cases} \\
\boldsymbol{\Sigma}_{ic} &= \sigma_{ic}^2 \begin{bmatrix} 1 & \rho_{ic}\sqrt{a_{ic}} \\ \rho_{ic}\sqrt{a_{ic}} & a_{ic} \end{bmatrix} \\
\widetilde{\boldsymbol{\Sigma}}_{ic} &= \sigma_{ic}^2 \begin{bmatrix} 1 & -\rho_{ic}\sqrt{a_{ic}} \\ -\rho_{ic}\sqrt{a_{ic}} & a_{ic} \end{bmatrix} \\
\boldsymbol{\mu}_i(t) &\sim \sum_{j=1}^{J} \pi_{ij}\delta_{\boldsymbol{\mu}_{ij}} \\
\pi_{ij} &= \eta_{ij}\prod_{l<j}\left(1-\eta_{il}\right) \\
\eta_{ij} &\sim \text{Beta}\left(1,\theta_i\right) \\
\boldsymbol{\mu}_{ij} &\sim f_{\mathcal{S}}\left(\mathbf{S}_i\right) \\
\theta_i &\sim \text{Gamma}\left(r_\theta, q_\theta\right) \\
\log\left(\tau_i\right) &\sim \mathcal{N}\left(\mu_\tau, \sigma_\tau^2\right) \\
\sigma_{ic} &\sim \text{Unif}\left(l_\sigma, u_\sigma\right) \\
a_{ic} &\sim \text{Unif}\left(l_a, u_a\right) \\
\rho_{ic} &\sim \text{Unif}\left(l_\rho, u_\rho\right),
\end{aligned}
$$

where J indicates the upper bound to the truncation approximation of the Dirichlet process (Sethuraman 1994, Ishwaran and James 2001), $\mathbf{S}_i = \{\mathbf{s}_{ic}(t), \forall t\}$ is a matrix of the observed telemetry locations for individual i, and $f_{\mathcal{S}}(\mathbf{S}_i)$ is the kernel density estimate of $\mathbf{S}_i$ truncated and normalized over $\mathcal{S}$.

Posterior Distribution

$$
\begin{aligned}
\left[\mathbf{M}_i, \boldsymbol{\eta}_i, \theta_i, \tau_i, \boldsymbol{\sigma}_i, \mathbf{a}_i, \boldsymbol{\rho}_i \mid \mathbf{S}_i, \mathbf{y}_i\right] \propto\ & \prod_{t\in\mathcal{T}}\prod_{j=1}^{J}\left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), y_i(t), \tau_i^2, \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}\right]\left[\boldsymbol{\mu}_i(t) \mid \boldsymbol{\mu}_{ij}, \eta_{ij}\right] \times \\
& \left[\boldsymbol{\mu}_{ij} \mid f_{\mathcal{S}}(\mathbf{S}_i)\right]\left[\eta_{ij} \mid \theta_i\right]\left[\theta_i\right]\left[\tau_i\right]\left[\boldsymbol{\sigma}_i\right]\left[\mathbf{a}_i\right]\left[\boldsymbol{\rho}_i\right],
\end{aligned}
$$

where $\mathbf{M}_i = \{\boldsymbol{\mu}_i(t), \forall t\}$ is a matrix of "functional" haul-out sites associated with the telemetry locations for individual i ($\mathbf{M}_i$ has the same dimensions as $\mathbf{S}_i$); $\boldsymbol{\eta}_i \equiv (\eta_{i1}, \ldots, \eta_{iJ})'$ is a vector of stick-breaking weights; $\mathbf{y}_i =$

$\{y_i(t), \forall t\}$ is a vector of behavioral (wet/dry) data for individual i; and $\boldsymbol{\sigma}_i \equiv (\sigma_{i3}, \sigma_{i2}, \sigma_{i1}, \sigma_{i0}, \sigma_{iA}, \sigma_{iB})'$, $\mathbf{a}_i \equiv (a_{i3}, a_{i2}, a_{i1}, a_{i0}, a_{iA}, a_{iB})'$, and $\boldsymbol{\rho} \equiv (\rho_{i3}, \rho_{i2}, \rho_{i1}, \rho_{i0}, \rho_{iA}, \rho_{iB})'$ are vectors of parameters that describe Argos telemetry location error.

Full-Conditional Distributions

Location of "potential" haul-out sites ($\boldsymbol{\mu}_{ij}$):

$$
\begin{aligned}
[\boldsymbol{\mu}_{ij} \mid \cdot] \quad &\propto \quad \prod_{t \in \mathcal{T}} \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), y_i(t), \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}, \tau_i^2\right]^{1_{\{\boldsymbol{\mu}_i(t)=\boldsymbol{\mu}_{ij}\}}} [\boldsymbol{\mu}_{ij} \mid f_{\mathcal{S}}(\mathbf{S}_i)] \\
&\propto \prod_{\{t \in \mathcal{T}: \boldsymbol{\mu}_i(t)=\boldsymbol{\mu}_{ij}\}} \left\{\left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{y_i(t)} \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \tau_i^2\mathbf{I} + \boldsymbol{\Sigma}_{ic}, \tau_i^2\mathbf{I} + \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{1-y_i(t)}\right\} [\boldsymbol{\mu}_{ij} \mid f_{\mathcal{S}}(\mathbf{S}_i)] \\
&\propto \prod_{\{t \in \mathcal{T}: \boldsymbol{\mu}_i(t)=\boldsymbol{\mu}_{ij}\}} \left\{\left(0.5 \times \left(\mathcal{N}(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \boldsymbol{\Sigma}_{ic}) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \widetilde{\boldsymbol{\Sigma}}_{ic}\right)\right)\right)^{y_i(t)}\right. \\
&\qquad \left. \times \left(0.5 \times \left(\mathcal{N}(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \tau_i^2\mathbf{I} + \boldsymbol{\Sigma}_{ic}) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \tau_i^2\mathbf{I} + \widetilde{\boldsymbol{\Sigma}}_{ic}\right)\right)\right)^{1-y_i(t)}\right\} [\boldsymbol{\mu}_{ij} \mid f_{\mathcal{S}}(\mathbf{S}_i)]
\end{aligned}
$$

Note that the product is over all observed telemetry locations ($\mathbf{s}_{ic}(t)$) allocated to haul-out site $\boldsymbol{\mu}_{ij}$.

Stick-breaking weights (η_{ij}):

$$
\begin{aligned}
[\eta_{ij} \mid \cdot] \quad &\propto \quad \prod_{t \in \mathcal{T}} [\boldsymbol{\mu}_i(t) \mid \pi_{ij}]^{1_{\{\boldsymbol{\mu}_i(t)=\boldsymbol{\mu}_{ij}\}}} \prod_{l=j+1}^{J} \prod_{t \in \mathcal{T}} [\boldsymbol{\mu}_i(t) \mid \pi_{il}]^{1_{\{\boldsymbol{\mu}_i(t)=\boldsymbol{\mu}_{il}\}}} [\eta_{ij} \mid 1, \theta_i] \\
&\propto \prod_{t \in \mathcal{T}} \pi_{ij}^{1_{\{\boldsymbol{\mu}_i(t)=\boldsymbol{\mu}_{ij}\}}} \prod_{l=j+1}^{J} \prod_{t \in \mathcal{T}} \pi_{il}^{1_{\{\boldsymbol{\mu}_i(t)=\boldsymbol{\mu}_{il}\}}} \mathrm{Beta}(\eta_{ij} \mid 1, \theta_i) \\
&\propto \pi_{ij}^{\sum_{t \in \mathcal{T}} \left(1_{\{\boldsymbol{\mu}_i(t)=\boldsymbol{\mu}_{ij}\}}\right)} \prod_{l=j+1}^{J} \pi_{il}^{\sum_{t \in \mathcal{T}} \left(1_{\{\boldsymbol{\mu}_i(t)=\boldsymbol{\mu}_{il}\}}\right)} \eta_{ij}^{1-1} (1-\eta_{ij})^{\theta_i - 1} \\
&\propto \left(\eta_{ij} \prod_{l<j} (1-\eta_{il})\right)^{n_{ij}} \prod_{l=j+1}^{J} \left(\eta_{il} \prod_{m<l} (1-\eta_{il})\right)^{n_{il}} (1-\eta_{ij})^{\theta_i - 1} \\
&\propto {\eta_{ij}}^{n_{ij}} \prod_{l=j+1}^{J} \left(\prod_{m<l} (1-\eta_{il})\right)^{n_{il}} (1-\eta_{ij})^{\theta_i - 1} \\
&\propto {\eta_{ij}}^{n_{ij}} (1-\eta_{ij})^{\sum_{l=j+1}^{J} n_{il}} (1-\eta_{ij})^{\theta_i - 1} \\
&\propto {\eta_{ij}}^{n_{ij}} (1-\eta_{ij})^{\sum_{l=j+1}^{J} n_{il} + \theta_i - 1} \\
&= \mathrm{Beta}\left(n_{ij} + 1, \sum_{l=j+1}^{J} n_{il} + \theta_i\right),
\end{aligned}
$$

where $n_{ij} = \sum_{t \in \mathcal{T}} \left(1_{\{\boldsymbol{\mu}_i(t)=\boldsymbol{\mu}_{ij}\}}\right)$, i.e., the number of observed telemetry locations ($\mathbf{s}_{ic}(t)$) allocated to haul-out site $\boldsymbol{\mu}_{ij}$.

Dirichlet process concentration parameter (θ_i):

$$
\begin{aligned}
[\theta_i \mid \cdot] &\propto \prod_{j=1}^{J-1} [\eta_{ij} \mid 1, \theta_i] [\theta_i \mid r_\theta, q_\theta] \\
&\propto \prod_{j=1}^{J-1} \text{Beta}\left(\eta_{ij} \mid 1, \theta_i\right) \text{Gamma}\left(\theta_i \mid r_\theta, q_\theta\right) \\
&\propto \prod_{j=1}^{J-1} \frac{\Gamma\left(1+\theta_i\right)}{\Gamma\left(1\right)\Gamma\left(\theta_i\right)} \eta_{ij}^{1-1} \left(1-\eta_{ij}\right)^{\theta_i - 1} \theta_i^{r_\theta - 1} \exp\left\{-q_\theta \theta_i\right\} \\
&\propto \left(\frac{\theta_i \Gamma\left(\theta_i\right)}{\Gamma\left(1\right)\Gamma\left(\theta_i\right)}\right)^{J-1} \theta_i^{r_\theta - 1} \exp\left\{-q_\theta \theta_i + \log\left(\prod_{j=1}^{J-1} \left(1-\eta_{ij}\right)^{\theta_i - 1}\right)\right\} \\
&\propto \theta_i^{J-1+r_\theta-1} \exp\left\{-q_\theta \theta_i + \sum_{j=1}^{J-1} \left(\log\left(1-\eta_{ij}\right)^{\theta_i} \log\left(1-\eta_{ij}\right)^{-1}\right)\right\} \\
&\propto \theta_i^{J-1+r_\theta-1} \exp\left\{-q_\theta \theta_i + \sum_{j=1}^{J-1} \log\left(1-\eta_{ij}\right)^{\theta_i}\right\} \\
&\propto \theta_i^{J-1+r_\theta-1} \exp\left\{-q_\theta \theta_i + \theta_i \sum_{j=1}^{J-1} \log\left(1-\eta_{ij}\right)\right\} \\
&\propto \theta_i^{J-1+r_\theta-1} \exp\left\{-\theta_i \left(q_\theta - \sum_{j=1}^{J-1} \log\left(1-\eta_{ij}\right)\right)\right\} \\
&= \text{Gamma}\left(r_\theta + J - 1, q_\theta - \sum_{j=1}^{J-1} \log\left(1-\eta_{ij}\right)\right).
\end{aligned}
$$

Note that the product is over $j = 1, \ldots, J-1$ because $\eta_{iJ} = 1$ in the truncation approximation of a Dirichlet process (Sethuraman 1994, Ishwaran and James 2001).

Location of "functional" haul-out sites ($\boldsymbol{\mu}_i(t)$):

$$
\begin{aligned}
[\boldsymbol{\mu}_i(t) \mid \cdot] &\propto \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), y_i(t), \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}, \tau_i^2\right] [\boldsymbol{\mu}_i(t) \mid \boldsymbol{\pi}_i, \boldsymbol{\delta}_i] \\
&\propto \sum_{j=1}^{J} \pi_{ij} \delta_{\boldsymbol{\mu}_{ij}} \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, y_i(t), \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}, \tau_i^2\right],
\end{aligned}
$$

where $\boldsymbol{\pi}_i = (\pi_{i1}, \ldots, \pi_{iJ})$ and $\boldsymbol{\delta}_i = (\delta_{\mu_{i1}}, \ldots, \delta_{\mu_{iJ}})$. We introduce an indicator variable for the latent class status, $h_i(t) \in \{1, \ldots, J\}$, that assigns each telemetry location $\mathbf{s}_{ic}(t)$ to one of the potential haul-out sites $\boldsymbol{\mu}_{ij}$, for $j = 1, \ldots, J$. In other words, $\boldsymbol{\mu}_i(t) = \boldsymbol{\mu}_{i,h_i(t)}$. The update proceeds just as in multinomial sampling:

$$
\begin{aligned}
[h_i(t) \mid \cdot] \sim \text{Cat}\Bigg(& \frac{\pi_{i1}\left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{i1}, \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{y_i(t)} \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{i1}, \tau_i^2\mathbf{I} + \boldsymbol{\Sigma}_{ic}, \tau_i^2\mathbf{I} + \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{1-y_i(t)}}{\sum_{j=1}^{J} \pi_{ij}\left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{y_i(t)} \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \tau_i^2\mathbf{I} + \boldsymbol{\Sigma}_{ic}, \tau_i^2\mathbf{I} + \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{1-y_i(t)}}, \cdots, \\
& \frac{\pi_{iJ}\left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{iJ}, \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{y_i(t)} \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{iJ}, \tau_i^2\mathbf{I} + \boldsymbol{\Sigma}_{ic}, \tau_i^2\mathbf{I} + \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{1-y_i(t)}}{\sum_{j=1}^{J} \pi_{ij}\left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{y_i(t)} \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \tau_i^2\mathbf{I} + \boldsymbol{\Sigma}_{ic}, \tau_i^2\mathbf{I} + \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{1-y_i(t)}} \Bigg) \\
\sim \text{Cat}\left(\frac{a_{i1}}{b_i}, \cdots, \frac{a_{iJ}}{b_i}\right), &
\end{aligned}
$$

where $a_{ij} = \pi_{ij} \times \left(0.5 \times \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \boldsymbol{\Sigma}_{ic}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \widetilde{\boldsymbol{\Sigma}}_{ic}\right)\right)\right)^{y_i(t)} \times$
$\left(0.5 \times \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \tau_i^2\mathbf{I} + \boldsymbol{\Sigma}_{ic}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \tau_i^2\mathbf{I} + \widetilde{\boldsymbol{\Sigma}}_{ic}\right)\right)\right)^{1-y_i(t)}$
and $b_i = \sum_{j=1}^{J} \left\{\pi_{ij}\left(0.5 \times \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \boldsymbol{\Sigma}_{ic}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \widetilde{\boldsymbol{\Sigma}}_{ic}\right)\right)\right)^{y_i(t)} \times\right.$
$\left.\left(0.5 \times \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \tau_i^2\mathbf{I} + \boldsymbol{\Sigma}_{ic}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}, \tau_i^2\mathbf{I} + \widetilde{\boldsymbol{\Sigma}}_{ic}\right)\right)\right)^{1-y_i(t)}\right\}.$

Animal movement parameter (τ_i):

$$\begin{aligned}
[\tau_i \mid \cdot] &\propto \prod_{t \in \mathcal{T}} \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), y_i(t), \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}, \tau_i^2\right] [\tau_i \mid \mu_\tau, \sigma_\tau^2] \\
&\propto \prod_{t \in \mathcal{T}} \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}, \tau_i^2\right]^{1-y_i(t)} [\tau_i \mid \mu_\tau, \sigma_\tau^2] \\
&\propto \prod_{\{t \in \mathcal{T}: y_i(t)=0\}} 0.5 \times \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \tau_i^2\mathbf{I} + \boldsymbol{\Sigma}_{ic}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \tau_i^2\mathbf{I} + \widetilde{\boldsymbol{\Sigma}}_{ic}\right)\right) \times \\
&\qquad \mathcal{N}\left(\log(\tau_i) \mid \log(\mu_\tau), \sigma_\tau^2\right).
\end{aligned}$$

Note that the product is over all observed telemetry locations ($\mathbf{s}_{ic}(t)$) that are recorded when the individual is at-sea (i.e., $y_i(t) = 0$).

Longitudinal telemetry measurement error (σ_{ic}):

$$\begin{aligned}
[\sigma_{ic} \mid \cdot] &\propto \prod_{t \in \mathcal{T}} \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), y_i(t), \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}, \tau_i^2\right]^{1_{\{\mathbf{s}_{ic}(t): t \in \mathcal{T}_c\}}} [\sigma_{ic} \mid l_\sigma, u_\sigma] \\
&\propto \prod_{t \in \mathcal{T}_c} \left\{\left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{y_i(t)} \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \tau_i^2\mathbf{I} + \boldsymbol{\Sigma}_{ic}, \tau_i^2\mathbf{I} + \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{1-y_i(t)}\right\} [\sigma_{ic} \mid l_\sigma, u_\sigma] \\
&\propto \prod_{t \in \mathcal{T}_c} \left\{\left(0.5 \times \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \boldsymbol{\Sigma}_{ic}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \widetilde{\boldsymbol{\Sigma}}_{ic}\right)\right)\right)^{y_i(t)} \times\right. \\
&\qquad \left.\left(0.5 \times \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \tau_i^2\mathbf{I} + \boldsymbol{\Sigma}_{ic}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \tau_i^2\mathbf{I} + \widetilde{\boldsymbol{\Sigma}}_{ic}\right)\right)\right)^{1-y_i(t)}\right\} \times \\
&\qquad \mathrm{Unif}(\sigma_{ic} \mid l_\sigma, u_\sigma).
\end{aligned}$$

Note that $\mathcal{T}_c$ defines the times at which telemetry locations in Argos location class c were recorded. In other words, the product is over all observed telemetry locations ($\mathbf{s}_{ic}(t)$) in Argos location quality class c.

Adjustment for latitudinal telemetry measurement error (a_{ic}):

$$\begin{aligned}
[a_{ic} \mid \cdot] &\propto \prod_{t \in \mathcal{T}} \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), y_i(t), \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}, \tau_i^2\right]^{1_{\{\mathbf{s}_{ic}(t): t \in \mathcal{T}_c\}}} [a_{ic} \mid l_a, u_a] \\
&\propto \prod_{t \in \mathcal{T}_c} \left\{\left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{y_i(t)} \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \tau_i^2\mathbf{I} + \boldsymbol{\Sigma}_{ic}, \tau_i^2\mathbf{I} + \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{1-y_i(t)}\right\} [a_{ic} \mid l_a, u_a] \\
&\propto \prod_{t \in \mathcal{T}_c} \left\{\left(0.5 \times \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \boldsymbol{\Sigma}_{ic}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \widetilde{\boldsymbol{\Sigma}}_{ic}\right)\right)\right)^{y_i(t)} \times\right. \\
&\qquad \left.\left(0.5 \times \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \tau_i^2\mathbf{I} + \boldsymbol{\Sigma}_{ic}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \tau_i^2\mathbf{I} + \widetilde{\boldsymbol{\Sigma}}_{ic}\right)\right)\right)^{1-y_i(t)}\right\} \times \\
&\qquad \mathrm{Unif}(a_{ic} \mid l_a, u_a).
\end{aligned}$$

Note that $\mathcal{T}_c$ defines the times at which telemetry locations in Argos location class c were recorded. In other words, the product is over all observed telemetry locations ($\mathbf{s}_{ic}(t)$) in Argos location quality class c.

Correlation between longitudinal and latitudinal telemetry measurement error (ρ_{ic}):

$$
\begin{aligned}
[\rho_{ic} \mid \cdot] &\propto \prod_{t \in \mathcal{T}} \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), y_i(t), \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}, \tau_i^2\right]^{1_{\{\mathbf{s}_{ic}(t): t \in \mathcal{T}_c\}}} [\rho_{ic} \mid l_\rho, u_\rho] \\
&\propto \prod_{t \in \mathcal{T}_c} \left\{\left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \boldsymbol{\Sigma}_{ic}, \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{y_i(t)} \left[\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \tau_i^2\mathbf{I} + \boldsymbol{\Sigma}_{ic}, \tau_i^2\mathbf{I} + \widetilde{\boldsymbol{\Sigma}}_{ic}\right]^{1-y_i(t)}\right\} [\rho_{ic} \mid l_\rho, u_\rho] \\
&\propto \prod_{t \in \mathcal{T}_c} \left\{\left(0.5 \times \left(\mathcal{N}(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \boldsymbol{\Sigma}_{ic}) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \widetilde{\boldsymbol{\Sigma}}_{ic}\right)\right)\right)^{y_i(t)} \times \right. \\
&\qquad \left. \left(0.5 \times \left(\mathcal{N}(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \tau_i^2\mathbf{I} + \boldsymbol{\Sigma}_{ic}) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t), \tau_i^2\mathbf{I} + \widetilde{\boldsymbol{\Sigma}}_{ic}\right)\right)\right)^{1-y_i(t)}\right\} \times \\
&\qquad \mathrm{Unif}(\rho_{ic} \mid l_\rho, u_\rho).
\end{aligned}
$$

Note that $\mathcal{T}_c$ defines the times at which telemetry locations in Argos location class c were recorded. In other words, the product is over all observed telemetry locations ($\mathbf{s}_{ic}(t)$) in Argos location quality class c.

MCMC Algorithm for Parameter Estimation

One can implement a MCMC algorithm to estimate the parameters of the observation and process models using the sequence of steps outlined below. Proposal distributions for all parameters with non-conjugate full-conditional distributions (i.e., $\boldsymbol{\mu}_{ij}$, τ_i, σ_{ic}, a_{ic}, and ρ_{ic}) are assumed to be symmetric and updates proceed using Metropolis sampling; therefore, the proposal distribution is not factored into the associated ratios as in Metropolis-Hastings sampling. Also note that normalizing constants cancel in the Metropolis ratios and thus may be omitted for clarity.

1. Define initial values for: $\boldsymbol{\mu}_{ij}^{(0)}$ and $\pi_{ij}^{(0)}$ for $j = 1, \ldots, J$; $\theta_i{}^{(0)}$; $\tau_i^{(0)}$; and $\sigma_{ic}^{(0)}$, $a_{ic}^{(0)}$, and $\rho_{ic}^{(0)}$ for $c =$ 3, 2, 1, 0, A, and B.

2. For each Argos location quality class, let

$$
\boldsymbol{\Sigma}_{ic}^{(0)} = \left(\sigma_{ic}^{(0)}\right)^2 \begin{bmatrix} 1 & \rho_{ic}^{(0)}\sqrt{a_{ic}^{(0)}} \\ \rho_{ic}^{(0)}\sqrt{a_{ic}^{(0)}} & a_{ic}^{(0)} \end{bmatrix}
$$

and

$$
\begin{aligned}
\widetilde{\boldsymbol{\Sigma}}_{ic}^{(0)} &= \left(\sigma_{ic}^{(0)}\right)^2 \begin{bmatrix} 1 & -\rho_{ic}^{(0)}\sqrt{a_{ic}^{(0)}} \\ -\rho_{ic}^{(0)}\sqrt{a_{ic}^{(0)}} & a_{ic}^{(0)} \end{bmatrix} \\
&= \mathbf{H}\boldsymbol{\Sigma}_{ic}^{(0)}\mathbf{H}',
\end{aligned}
$$

where

$$
\mathbf{H} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.
$$

Also let

$$
\mathbf{Q}_{ic}^{(0)} = \boldsymbol{\Sigma}_{ic}^{(0)} + \left(\tau_i^{(0)}\right)^2 \mathbf{I}
$$

and

$$
\begin{aligned}
\widetilde{\mathbf{Q}}_{ic}^{(0)} &= \widetilde{\boldsymbol{\Sigma}}_{ic}^{(0)} + \left(\tau_i^{(0)}\right)^2 \mathbf{I} \\
&= \mathbf{H}\mathbf{Q}_{ic}^{(0)}\mathbf{H}'.
\end{aligned}
$$

3. Set $k = 1$.

4. Update the spatial process model parameters (i.e., $h_i(t)$, η_{ij}, θ_i, and $\boldsymbol{\mu}_{ij}$).

   (a) Sample $h_i(t)^{(k)}$:

   $$\left[h_i(t)^{(k)} \mid \cdot\right] \quad \sim \quad \text{Cat}\left(\frac{a_{ij}}{b_i}, \ldots, \frac{a_{iJ}}{b_i}\right),$$

   where $a_{ij} = \pi_{ij}^{(k-1)} \times \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(k-1)}, \boldsymbol{\Sigma}_{ic}^{(k-1)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(k-1)}, \widetilde{\boldsymbol{\Sigma}}_{ic}^{(k-1)}\right)\right)^{y_i(t)} \times$ $\left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(k-1)}, \mathbf{Q}_{ic}^{(k-1)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(k-1)}, \mathbf{H}\mathbf{Q}_{ic}^{(k-1)}\mathbf{H}'\right)\right)^{1-y_i(t)}$ and $b_i = \sum_{j=1}^{J}\left\{\pi_{ij}^{(k-1)}\left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(k-1)}, \boldsymbol{\Sigma}_{ic}^{(k-1)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(k-1)}, \widetilde{\boldsymbol{\Sigma}}_{ic}^{(k-1)}\right)\right)^{y_i(t)} \times\right.$ $\left.\left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(k-1)}, \mathbf{Q}_{ic}^{(k-1)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(k-1)}, \mathbf{H}\mathbf{Q}_{ic}^{(k-1)}\mathbf{H}'\right)\right)^{1-y_i(t)}\right\}$.

   (b) Tabulate cluster membership for $j = 1, \ldots, J$:

   $$n_{ij}^{(k)} = \sum_{t \in \mathcal{T}} 1_{\left\{h_i(t)^{(k)} = j\right\}}.$$

   In other words, $n_{ij}^{(k)}$ denotes the number of observed telemetry locations ($\mathbf{s}_{ic}(t)$) allocated to haul-out site $\boldsymbol{\mu}_{ij}^{(k-1)}$.

   (c) Update $\eta_{ij}^{(k-1)}$, for $j = 1, \ldots, J-1$, using a Gibbs step:

   $$\left[\eta_{ij}^{(k)} \mid \cdot\right] \quad \sim \quad \text{Beta}\left(1 + n_{ij}^{(k)}, \theta_i^{(k-1)} + \sum_{l=j+1}^{J} n_{il}^{(k)}\right).$$

   Set $\eta_{iJ}^{(k)} = 1$.

   (d) Update $\pi_{ij}^{(k-1)}$, for $j = 1, \ldots J$, which is calculated as:

   $$\pi_{ij}^{(k)} = \eta_{ij}^{(k)} \prod_{l<j}\left(1 - \eta_{il}^{(k)}\right).$$

   Letting $\eta_{iJ}^{(k)} = 1$ ensures $\sum_{j=1}^{J} \pi_{ij}^{(k)} = 1$.

   (e) Update $\theta_i^{(k-1)}$ using a Gibbs step:

   $$\left[\theta_i^{(k)} \mid \cdot\right] \quad \sim \quad \text{Gamma}\left(r_\theta + J - 1, q_\theta - \sum_{j=1}^{J-1} \log\left(1 - \eta_{ij}^{(k)}\right)\right).$$

   (f) Update $\boldsymbol{\mu}_{ij}^{(k-1)}$, for each j such that $n_{ij}^{(k)} > 0$, using Metropolis sampling. Sample $\boldsymbol{\mu}_{ij}{}^{(*)}$ from a proposal distribution $\left[\boldsymbol{\mu}_{ij}^{(*)} | \boldsymbol{\mu}_{ij}^{(k-1)}\right]$. Depending on the nature of $\mathcal{S}$ (e.g., linear support like a coastline), proposals generated from $\mathcal{N}\left(\boldsymbol{\mu}_{ij}^{(*)} \mid \boldsymbol{\mu}_{ij}^{(k-1)}, \tau_\mu^2 \mathbf{I}\right)$, where τ_μ^2 is a tuning parameter, may rarely occur in $\mathcal{S}$. Therefore, sample all possible locations $\mathbf{M} \in \mathcal{S}$ with probability proportional to $\mathcal{N}\left(\mathbf{M} \mid \boldsymbol{\mu}_{ij}^{(k-1)}, \tau_{\mu}^2\mathbf{I}\right)$, thus guaranteeing $\boldsymbol{\mu}_{ij}^{(*)} \in \mathcal{S}$. Calculate the Metropolis ratio as

$$
\begin{aligned}
r_{\mu} \quad = \quad & \left(\frac{\prod_{\left\{t\in\mathcal{T}:h_i(t)^{(k)}=j\right\}} \left\{ \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(*)}, \boldsymbol{\Sigma}_{ic}^{(k-1)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(*)}, \mathbf{H}\boldsymbol{\Sigma}_{ic}^{(k-1)}\mathbf{H}'\right)\right)^{y_i(t)}\right.}{\prod_{\left\{t\in\mathcal{T}:h_i(t)^{(k)}=j\right\}} \left\{ \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(k-1)}, \boldsymbol{\Sigma}_{ic}^{(k-1)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(k-1)}, \mathbf{H}\boldsymbol{\Sigma}_{ic}^{(k-1)}\mathbf{H}'\right)\right)^{y_i(t)}\right.} \right. \\
& \left. \times \frac{\left.\left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(*)}, \mathbf{Q}_{ic}^{(k-1)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(*)}, \mathbf{H}\mathbf{Q}_{ic}^{(k-1)}\mathbf{H}'\right)\right)^{1-y_i(t)}\right\}}{\left.\left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(k-1)}, \mathbf{Q}_{ic}^{(k-1)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_{ij}^{(k-1)}, \mathbf{H}\mathbf{Q}_{ic}^{(k-1)}\mathbf{H}'\right)\right)^{1-y_i(t)}\right\}} \right) \\
& \times \left(\frac{\left[\boldsymbol{\mu}_{ij}^{(*)} | f_{\mathcal{S}}(\mathbf{S}_i)\right]}{\left[\boldsymbol{\mu}_{ij}^{(k-1)} | f_{\mathcal{S}}(\mathbf{S}_i)\right]} \right).
\end{aligned}
$$

Note that the product is over all observed telemetry locations ($\mathbf{s}_{ic}(t)$) that are allocated to haul-out site $\boldsymbol{\mu}_{ij}$ (i.e., $t \in \mathcal{T}$ such that $h_i(t)^{(k)} = j$). If $r_{\mu} > u$, where $u \sim$ Uniform(0,1), let $\boldsymbol{\mu}_{ij}^{(k)} = \boldsymbol{\mu}_{ij}^{(*)}$. Otherwise, let $\boldsymbol{\mu}_{ij}^{(k)} = \boldsymbol{\mu}_{ij}^{(k-1)}$ if $r_{\mu} < u$, or if $\boldsymbol{\mu}_{ij}^{(*)} \notin \mathcal{S}$.

(g) For each j such that $n_{ij}^{(k)} = 0$ (i.e., potential haul-out sites $\boldsymbol{\mu}_{ij}^{(k-1)}$ with zero membership), sample ${\boldsymbol{\mu}_{ij}}^{(k)}$ from the prior $\left[\boldsymbol{\mu}_{ij}^{(k)} | f_{\mathcal{S}}(\mathbf{S}_i)\right]$. As in Step 4(f), sample all possible locations $\mathbf{M} \in \mathcal{S}$ with probability proportional to $f_{\mathcal{S}}(\mathbf{S}_i)$ to ensure $\boldsymbol{\mu}_{ij}^{(k)} \in \mathcal{S}$.

(h) Use $h_i(t)^{(k)}$ to map the location of haul-out sites $\boldsymbol{\mu}_{ij}^{(k)}$, for $j = 1, \ldots, J$, to telemetry locations $\mathbf{s}_{ic}(t)$, for times $t \in \mathcal{T}$:

$$\boldsymbol{\mu}_i(t)^{(k)} = \boldsymbol{\mu}_{i,h_i(t)^{(k)}}^{(k)}.$$

5. Update $\tau_i^{(k-1)}$ using Metropolis sampling. Sample $\tau_i^{(*)}$ from a proposal distribution $\left[\tau_i^{(*)} | \tau_i^{(k-1)}\right]$ (e.g., $\mathcal{N}\left(\tau_i^{(*)} | \tau_i^{(k-1)}, \tau_{\tau}^2\mathbf{I}\right)$, where τ_{τ}^2 is a tuning parameter). If $\tau_i^{(*)} \geq 0$, let

$$\mathbf{Q}_{ic}^{(*)} = \boldsymbol{\Sigma}_{ic}^{(k-1)} + \left(\tau_i^{(*)}\right)^2 \mathbf{I}$$

for $c = 3$, 2, 1, 0, A, and B. Calculate the Metropolis ratio as

$$
\begin{aligned}
r_{\tau} \quad = \quad & \left(\frac{\prod_{\{t\in\mathcal{T}:y_i(t)=0\}} \left\{\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{Q}_{ic}^{(*)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{H}\mathbf{Q}_{ic}^{(*)}\mathbf{H}'\right)\right\}}{\prod_{\{t\in\mathcal{T}:y_i(t)=0\}} \left\{\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{Q}_{ic}^{(k-1)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{H}\mathbf{Q}_{ic}^{(k-1)}\mathbf{H}\right)\right\}} \right) \\
& \times \left(\frac{\mathcal{N}\left(\log\left(\tau_i^{(*)}\right) \mid \log(\mu_{\tau}), \sigma_{\tau}^2\right)}{\mathcal{N}\left(\log\left(\tau_i^{(k-1)}\right) \mid \log(\mu_{\tau}), \sigma_{\tau}^2\right)} \right).
\end{aligned}
$$

Note that the product is over all $t \in \mathcal{T}$ such that $y_i(t) = 0$. If $r_{\tau} > u$, where $u \sim$ Uniform(0,1), let $\tau_i^{(k)} = \tau_i^{(*)}$ and $\mathbf{Q}_{ic}^{(k)} = \mathbf{Q}_{ic}^{(*)}$. Otherwise, let $\tau_i^{(k)} = \tau_i^{(k-1)}$ and $\mathbf{Q}_{ic}^{(k)} = \mathbf{Q}_{ic}^{(k-1)}$ if $r_{\tau} < u$, or if $\tau_i^{(*)} < 0$.

6. For each Argos location quality class $c = 3, 2, 1, 0, A,$ and B, update the observation model parameters related to telemetry measurement error (i.e., $\boldsymbol{\sigma}_i$, $\mathbf{a}_i$, and $\boldsymbol{\rho}_i$).

(a) Let $\mathcal{T}_c$ define the times at which telemetry locations in Argos location class c were recorded.

(b) Update $\sigma_{ic}^{(k-1)}$ using Metropolis sampling. Sample $\sigma_{ic}^{(*)}$ from a proposal distribution $\left[\sigma_{ic}^{(*)} | \sigma_{ic}^{(k-1)}\right]$ (e.g., $\mathrm{N}\left(\sigma_{ic}^{(*)} | \sigma_{ic}^{(k-1)}, \tau_{\sigma}^2\right)$, where τ_{σ}^2 is a tuning parameter). If $\sigma_{ic}^{(*)} \in [l_{\sigma}, u_{\sigma}]$, let

$$\boldsymbol{\Sigma}_{ic}^{(*)} = \left(\sigma_{ic}^{(*)}\right)^2 \begin{bmatrix} 1 & \rho_{ic}^{(k-1)}\sqrt{a_{ic}^{(k-1)}} \\ \rho_{ic}^{(k-1)}\sqrt{a_{ic}^{(k-1)}} & a_{ic}^{(k-1)} \end{bmatrix}$$

and

$$\mathbf{Q}_{ic}^{(*)} = \boldsymbol{\Sigma}_{ic}^{(*)} + \left(\tau_i^{(k)}\right)^2 \mathbf{I}.$$

Calculate the Metropolis ratio as

$$\begin{aligned} r_\sigma &= \left(\frac{\prod_{t\in\mathcal{T}_c} \left\{ \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \boldsymbol{\Sigma}_{ic}^{(*)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{H}\boldsymbol{\Sigma}_{ic}^{(*)}\mathbf{H}'\right)\right)^{y_i(t)} \right.}{\prod_{t\in\mathcal{T}_c} \left\{ \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \boldsymbol{\Sigma}_{ic}^{(k-1)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{H}\boldsymbol{\Sigma}_{ic}^{(k-1)}\mathbf{H}'\right)\right)^{y_i(t)} \right.} \right. \\ &\qquad \left. \times \frac{\left. \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{Q}_{ic}^{(*)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{H}\mathbf{Q}_{ic}^{(*)}\mathbf{H}'\right)\right)^{1-y_i(t)} \right\}}{\left. \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{Q}_{ic}^{(k)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{H}\mathbf{Q}_{ic}^{(k)}\mathbf{H}'\right)\right)^{1-y_i(t)} \right\}} \right). \end{aligned}$$

If $r_\sigma > u$, where $u \sim$ Uniform(0,1), let $\sigma_{ic}^{(k)} = \sigma_{ic}^{(*)}$, $\boldsymbol{\Sigma}_{ic}^{(k)} = \boldsymbol{\Sigma}_{ic}^{(*)}$, and $\mathbf{Q}_{ic}^{(k)} = \mathbf{Q}_{ic}^{(*)}$. Otherwise, let $\sigma_{ic}^{(k)} = \sigma_{ic}^{(k-1)}$, $\boldsymbol{\Sigma}_{ic}^{(k)} = \boldsymbol{\Sigma}_{ic}^{(k-1)}$, and $\mathbf{Q}_{ic}^{(k)} = \mathbf{Q}_{ic}^{(k)} = \boldsymbol{\Sigma}_{ic}^{(k-1)} + \left(\tau_i^{(k)}\right)^2 \mathbf{I}$ if $r_\sigma < u$, or if $\sigma_{ic}^{(*)} \notin [l_\sigma, u_\sigma]$.

(c) Update $a_{ic}^{(k-1)}$ using Metropolis sampling. Sample $a_{ic}^{(*)}$ from a proposal distribution $\left[a_{ic}^{(*)} | a_{ic}^{(k-1)}\right]$ (e.g., $\mathrm{N}\left(a_{ic}^{(*)} | a_{ic}^{(k-1)}, \tau_a^2\right)$, where τ_a^2 is a tuning parameter). If $a_{ic}^{(*)} \in [l_a, u_a]$, let

$$\boldsymbol{\Sigma}_{ic}^{(*)} = \left(\sigma_{ic}^{(k)}\right)^2 \begin{bmatrix} 1 & \rho_{ic}^{(k-1)}\sqrt{a_{ic}^{(*)}} \\ \rho_{ic}^{(k-1)}\sqrt{a_{ic}^{(*)}} & a_{ic}^{(*)} \end{bmatrix}$$

and

$$\mathbf{Q}_{ic}^{(*)} = \boldsymbol{\Sigma}_{ic}^{(*)} + \left(\tau_i^{(k)}\right)^2 \mathbf{I}.$$

Calculate the Metropolis ratio as

$$\begin{aligned} r_a &= \left(\frac{\prod_{t\in\mathcal{T}_c} \left\{ \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \boldsymbol{\Sigma}_{ic}^{(*)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{H}\boldsymbol{\Sigma}_{ic}^{(*)}\mathbf{H}'\right)\right)^{y_i(t)} \right.}{\prod_{t\in\mathcal{T}_c} \left\{ \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \boldsymbol{\Sigma}_{ic}^{(k)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{H}\boldsymbol{\Sigma}_{ic}^{(k)}\mathbf{H}'\right)\right)^{y_i(t)} \right.} \right. \\ &\qquad \left. \times \frac{\left. \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{Q}_{ic}^{(*)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{H}\mathbf{Q}_{ic}^{(*)}\mathbf{H}'\right)\right)^{1-y_i(t)} \right\}}{\left. \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{Q}_{ic}^{(k)}\right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{H}\mathbf{Q}_{ic}^{(k)}\mathbf{H}'\right)\right)^{1-y_i(t)} \right\}} \right). \end{aligned}$$

If $r_a > u$, where $u \sim$ Uniform(0,1), let $a_{ic}^{(k)} = a_{ic}^{(*)}$, $\boldsymbol{\Sigma}_{ic}^{(k)} = \boldsymbol{\Sigma}_{ic}^{(*)}$, and $\mathbf{Q}_{ic}^{(k)} = \mathbf{Q}_{ic}^{(*)}$. Otherwise, let $a_{ic}^{(k)} = a_{ic}^{(k-1)}$, $\boldsymbol{\Sigma}_{ic}^{(k)} = \boldsymbol{\Sigma}_{ic}^{(k)}$, and $\mathbf{Q}_{ic}^{(k)} = \mathbf{Q}_{ic}^{(k)}$ if $r_a < u$, or if $a_{ic}^{(*)} \notin [l_a, u_a]$.

(d) Update $\rho_{ic}^{(k-1)}$ using Metropolis sampling. Sample $\rho_{ic}^{(*)}$ from a proposal distribution $\left[\rho_{ic}^{(*)} | \rho_{ic}^{(k-1)}\right]$ (e.g., $\mathrm{N}\left(\rho_{ic}^{(*)} | \rho_{ic}^{(k-1)}, \tau_\rho^2\right)$, where τ_ρ^2 is a tuning parameter). If $\rho_{ic}^{(*)} \in [l_\rho, u_\rho]$, let

$$\boldsymbol{\Sigma}_{ic}^{(*)} = \left(\sigma_{ic}^{(k)}\right)^2 \begin{bmatrix} 1 & \rho_{ic}^{(*)}\sqrt{a_{ic}^{(k)}} \\ \rho_{ic}^{(*)}\sqrt{a_{ic}^{(k)}} & a_{ic}^{(k)} \end{bmatrix}$$

and

$$\mathbf{Q}_{ic}^{(*)} = \boldsymbol{\Sigma}_{ic}^{(*)} + \left(\tau_i^{(k)}\right)^2 \mathbf{I}.$$

Calculate the Metropolis ratio as

$$
\begin{aligned}
r_\rho &= \left(\frac{\prod_{t \in \mathcal{T}_c} \left\{ \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \boldsymbol{\Sigma}_{ic}^{(*)} \right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{H}\boldsymbol{\Sigma}_{ic}^{(*)}\mathbf{H}' \right) \right)^{y_i(t)} \right.}{\prod_{t \in \mathcal{T}_c} \left\{ \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \boldsymbol{\Sigma}_{ic}^{(k)} \right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{H}\boldsymbol{\Sigma}_{ic}^{(k)}\mathbf{H}' \right) \right)^{y_i(t)} \right.} \right. \\
&\qquad \left. \times \frac{\left. \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{Q}_{ic}^{(*)} \right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{H}\mathbf{Q}_{ic}^{(*)}\mathbf{H}' \right) \right)^{1-y_i(t)} \right\}}{\left. \left(\mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{Q}_{ic}^{(k)} \right) + \mathcal{N}\left(\mathbf{s}_{ic}(t) \mid \boldsymbol{\mu}_i(t)^{(k)}, \mathbf{H}\mathbf{Q}_{ic}^{(k)}\mathbf{H}' \right) \right)^{1-y_i(t)} \right\}} \right).
\end{aligned}
$$

If $r_\rho > u$, where $u \sim$ Uniform(0,1), let $\rho_{ic}^{(k)} = \rho_{ic}^{(*)}$, $\boldsymbol{\Sigma}_{ic}^{(k)} = \boldsymbol{\Sigma}_{ic}^{(*)}$, and $\mathbf{Q}_{ic}^{(k)} = \mathbf{Q}_{ic}^{(*)}$. Otherwise, let $\rho_{ic}^{(k)} = \rho_{ic}^{(k-1)}$, $\boldsymbol{\Sigma}_{ic}^{(k)} = \boldsymbol{\Sigma}_{ic}^{(k)}$, and $\mathbf{Q}_{ic}^{(k)} = \mathbf{Q}_{ic}^{(k)}$ if $r_\rho < u$, or if $\rho_{ic}^{(*)} \notin [l_\rho, u_\rho]$.

(e) Repeat Steps 6(a) through 6(d) for each error class c.

7. Save $\boldsymbol{\mu}_i(t)^{(k)}$ for $t \in \mathcal{T}$; $\theta_i^{(k)}$; $\tau_i^{(k)}$; π_{ij} for $j = 1, \ldots, J$; and $\sigma_{ic}^{(k)}$, $a_{ic}^{(0)}$, and $\rho_{ic}^{(0)}$ for $c = 3$, 2, 1, 0, A, and B.
8. Set $k = k + 1$ and return to Step 4. The algorithm is iterated by repeating Steps 4 through 7 until a sufficiently large sample has been obtained from which to approximate the posterior distribution.

References

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