One-dimensional "haul-out" model

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Model implementation

The file haulout.1d.sim.R simulates data according to the model statement presented below, and haulout.1d.mcmc.R contains the MCMC algorithm for model fitting.

Model statement

Let $\mathcal{S}$ be the support of the movement process and $\tilde{S}$ bethe support of haul-out sites. Here, both are defined in 1-dimensional space along a line. Let $\tilde{\mathcal{S}}$ exist over the interval $[x_1, x_2]$ and $\mathcal{S}$ exist over the interval $[x_1, x_3]$. Note that $\mathcal{S}$ and $\tilde{S}$ overlap, and that $\mathcal{S} \cap \tilde{S} = \tilde{\mathcal{S}}$. The domain defined by $\mathcal{S}$ therefore represents at-sea locations or locations of the individual while milling adjacent to the haul-out site. Also note that $\bar{\mathcal{S}}$, the complement of $\mathcal{S}$, represents inaccessible locations (i.e., land). Let s_t and μ_t, for $t = 1, \ldots, T$, be observed and true locations respectively.

$$
\begin{aligned}
s_t &\sim \mathrm{N}(\mu_t, \sigma^2) \\
\mu_t &\sim \begin{cases} \mathrm{Unif}(\tilde{\mathcal{S}}), & z_t = 1 \\ \mathrm{Unif}(\mathcal{S}), & z_t = 0 \end{cases} \\
z_t &\sim \mathrm{Bern}(p) \\
p &\sim \mathrm{Beta}(\alpha, \beta) \\
\sigma^2 &\sim \mathrm{IG}(r, q)
\end{aligned}
$$

Full conditional distributions

True locations (μ_t):

$$
\begin{aligned}
[\mu_t|\cdot] &\propto [s_t|\mu_t, \sigma^2][\mu_t|z_t, \mathcal{S}, \tilde{\mathcal{S}}] \\
&\propto [s_t|\mu_t, \sigma^2][\mu_t|\tilde{\mathcal{S}}]^{z_t}[\mu_t|\mathcal{S}]^{1-z_t}.
\end{aligned}
$$

For $z_t = 1$,

$$
\begin{aligned}
[\mu_t|\cdot] &\propto [s_t|\mu_t, \sigma^2][\mu_t|\tilde{\mathcal{S}}]^{z_t} \\
&\propto [s_t|\mu_t, \sigma^2] \\
&\propto \exp\left\{-\frac{1}{2\sigma^2}(s_t - \mu_t)^2\right\} \\
&\propto \exp\left\{-\frac{1}{2\sigma^2}(s_t^2 - 2s_t\mu_t + \mu_t^2)\right\} \\
&\propto \exp\left\{-\frac{1}{2\sigma^2}(2s_t\mu_t + \mu_t^2)\right\} \\
&\propto \exp\left\{-\frac{1}{2}\left(\frac{2s_t\mu_t}{\sigma^2} + \mu_t^2\left(\frac{1}{\sigma^2}\right)\right)\right\} \\
&\propto \mathrm{N}(A^{-1}b, A^{-1})
\end{aligned}
$$

where $A = 1/\sigma^2$ and $b = s_t/\sigma^2$; therefore, $[\mu_t|\cdot] = \mathrm{N}(s_t, \sigma^2)$ for $z_t = 1$. Note that proposed values for μ_t not in $\tilde{\mathcal{S}}$ are rejected. For $z_t = 0$,

$$[\mu_t|\cdot] = \mathrm{N}(s_t, \sigma^2).$$

In this case, note that proposed values for μ_t not in $\mathcal{S}$ are rejected.

Haul-out indicator variable (z_t):

$$[z_t|\cdot] \quad \propto \quad [\mu_t|\tilde{\mathcal{S}}]^{z_t}[\mu_t|\mathcal{S}[z_t|p].$$

For all $\mu_t \notin \tilde{\mathcal{S}}$, let $z_t = 0$. For all $\mu_t \in \tilde{\mathcal{S}}$, sample z_t from

$$\begin{aligned} [z_t|\cdot] &\propto [\mu_t|\tilde{\mathcal{S}}]^{z_t}[\mu_t|\mathcal{S}]^{1-z_t}[z_t|p] \\ &= \text{Bern}(\tilde{p}), \end{aligned}$$

where

$$\begin{aligned} \tilde{p} &= \frac{p[\mu_t|\tilde{\mathcal{S}}]}{p[\mu_t|\tilde{\mathcal{S}}] + (1-p)[\mu_t|\mathcal{S}]} \\ &= \frac{p(x_2 - x_1)^{-1}}{p(x_2 - x_1)^{-1} + (1-p)(x_3 - x_1)^{-1}}. \end{aligned}$$

Probability of being hauled-out (p):

$$\begin{aligned} [p|\cdot] &\propto \prod_{t=1}^{T}[z_t|p][p] \\ &\propto \prod_{t=1}^{T} p^{z_t}(1-p)^{1-z_t} p^{\alpha-1}(1-p)^{\beta-1} \\ &\propto p^{\sum_{t=1}^{T} z_t}(1-p)^{\sum_{t=1}^{T}(1-z_t)} p^{\alpha-1}(1-p)^{\beta-1} \\ &= \text{Beta}\left(\sum_{t=1}^{T} z_t + \alpha, \sum_{t=1}^{T}(1-z_t) + \beta\right) \end{aligned}$$

Error in the observation process (σ^2):

$$\begin{aligned} [\sigma^2|\cdot] &\propto \prod_{t=1}^{T}[s_t|\mu_t, \sigma^2][\sigma^2] \\ &\propto \prod_{t=1}^{T}(\sigma^2)^{-(T/2+q+1)} \exp\left\{-\frac{1}{\sigma^2}\left(\frac{\sum_{t=1}^{T}(s_t-\mu_t)^2}{2} + \frac{1}{r}\right)\right\} \\ &= \text{IG}\left(\left(\frac{\sum_{t=1}^{T}(s_t-\mu_t)^2}{2} + \frac{1}{r}\right)^{-1}, \frac{T}{2} + q\right). \end{aligned}$$

Note that the current version of haulout.1d.mcmc.R contains code for the conjugate update of σ^2 presented above, but this code is currently 'commented' out. Instead, error is modeled as $[\sigma|\cdot] \sim \text{Unif}(a, b)$, and the update for σ proceeds using Metropolis-Hastings.