N-mixture Model with Detection Modeled by Covariates and Random Effect on Detection Probability

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Model implementation

The file N.mixture.random.p.sim.R simulates data according to the model statement presented below, and N.mixture.random.p.mcmc.R contains the MCMC algorithm for model fitting.

Model statement

Let y_{ij} be the j^{th} count of individuals at site i, for $j = 1, \ldots, J$ and $i = 1, \ldots, m$, and N_i be the true number of individuals at site i. Assuming the population is closed to mortality, recruitment, immigration, and emigration over the course of the J surveys conducted at any given site,

$$
\begin{aligned}
y_{ij} &\sim \text{Binom}(N_i, p_{ij}) \\
N_i &\sim \text{Pois}(\lambda_i) \\
\text{logit}(p_{ij}) &\sim \mathbf{W}_{ij}\boldsymbol{\alpha} + \epsilon_{ij} \\
\epsilon_{it} &\sim \text{N}(0, \zeta^2) \\
\lambda_i &\sim \text{Gamma}(r, q) \\
\boldsymbol{\alpha} &\sim \text{N}(\boldsymbol{\mu}, \tau^2\mathbf{I}) \\
\zeta &\sim \text{Unif}(a, b).
\end{aligned}
$$

Note that this model allows λ to vary by site and p_{ij}, the detection probability, to be modeled as a function of covariates.

Posterior distribution

For a single site, i:

$$[N_i, \boldsymbol{\alpha}, \lambda_i, \boldsymbol{\epsilon}, \zeta | \mathbf{y}_i, \mathbf{W_i}] \propto \prod_{j=1}^{J} [y_{ij}|N_i, p_{ij}][N_i|\lambda_i][\epsilon_{ij}][\lambda_i][\boldsymbol{\alpha}][\zeta]$$

Full conditional distributions

Coefficients describing the effect of covariates on detection probability ($\boldsymbol{\alpha}$):

$$[\boldsymbol{\alpha}|\cdot] \propto \prod_{j=1}^{J} [y_{ij}|N_i, p_{ij}][\boldsymbol{\alpha}].$$

This full-conditional distribution does not have a known analytical form; therefore sample $\mathbf{p}$ using Metropolis-Hastings.

The true number of individuals (N_i):

$$
\begin{aligned}
[N_i|\cdot] &\propto \prod_{j=1}^{J}[y_{ij}|N_i, p_{ij}][N_i|\lambda_i] \\
&\propto \prod_{j=1}^{J}\left(\begin{array}{c} N_i \\ y_{ij} \end{array}\right) p_{ij}^{y_{ij}}(1-p_{ij})^{N_i-y_{ij}}\left(\frac{\lambda_i^{N_i}e^{-\lambda_i}}{N_i!}\right) \\
&\propto \prod_{j=1}^{J}\left(\frac{N_i!}{y_{ij}!(N_i-y_{ij})!}\right) p_{ij}^{y_{ij}}(1-p_{ij})^{N_i-y_{ij}}\left(\frac{\lambda_i^{N_i}e^{-\lambda_i}}{N_i!}\right) \\
&\propto \prod_{j=1}^{J}\frac{(1-p_{ij})^{N_i-y_{ij}}\lambda_i^{N_i}}{(N_i-y_{ij})!} \\
&\propto \prod_{j=1}^{J}\frac{(1-p_{ij})^{N_i-y_{ij}}\lambda_i^{N_i}\lambda_i^{-y_{ij}}}{(N_i-y_{ij})!} \\
&\propto \prod_{j=1}^{J}\frac{(\lambda_i(1-p_{ij}))^{N_i-y_{ij}}}{(N_i-y_{ij})!}e^{-\lambda_i(1-p_{ij})}.
\end{aligned}
$$

This full-conditional is a little strange because $[N_i - y_{ij}|\cdot] \propto \text{Pois}(\lambda_i(1-p_{ij}))$, which suggests there is one true abundance per replicate count at each site. This is in contrast to the case in which only one observation exists per site, i.e., $[N_i - y_i|\cdot] \propto \text{Pois}(\lambda_i(1-p_i))$. Given that $[N_i|\cdot]$ lacks a clear analytical solution, sample N_i using Metropolis-Hastings.

Rate of the process model (λ_i):

$$
\begin{aligned}
[\lambda_i|\cdot] &\propto [N_i|\lambda_i][\lambda_i] \\
&\propto \frac{\lambda_i^{N_i}e^{-\lambda_i}}{N_i!}\lambda_i^{r-1}e^{-\lambda_i q} \\
&\propto \lambda_i^{N_i}e^{-\lambda_i}\lambda_i^{r-1}e^{-\lambda_i q} \\
&\propto \lambda_i^{N_i+r-1}e^{-\lambda_i(1+q)} \\
&= \text{Gamma}\left(N_i+r, 1+q\right).
\end{aligned}
$$

Random effect on detection probability (ϵ_{ijt}):

$$
[\epsilon_{ij}|\cdot] \propto [y_{ij}|N_t, p_{ij}][\epsilon_{ij}]
$$

This full-conditional lacks an analytical solution; therefore, sample ϵ_{it} using Metropolis-Hastings.

Standard deviation of random effect (ζ):

$$
[\zeta|\cdot] \propto \prod_{j=1}^{J}\prod_{i=1}^{m}[\epsilon_{ij}|\zeta][\zeta].
$$

This full-conditional lacks an analytical solution; therefore, sample ζ using Metropolis-Hastings.