N-mixture Model with Detection Modeled by Covariates

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Model implementation

The file N.mixture.sim.R simulates data according to the model statement presented below, and N.mixture.mcmc.R contains the MCMC algorithm for model fitting.

Model statement

Let y_{ij} be the j^{th} count of individuals at site i, for $j = 1, \ldots, J$ and $i = 1, \ldots, m$, and N_i be the true number of individuals at site i. Assuming the population is closed to mortality, recruitment, immigration, and emigration over the course of the J surveys conducted at any given site,

$$\begin{aligned}
y_{ij} &\sim \text{Binom}(N_i, p_{ij}) \\
N_i &\sim \text{Pois}(\lambda_i) \\
\lambda_i &\sim \text{Gamma}(r, q) \\
\text{logit}(p_{ij}) &\sim \mathbf{W}_{ij}\boldsymbol{\alpha} \\
\boldsymbol{\alpha} &\sim \text{N}(\boldsymbol{\mu}, \tau^2\mathbf{I}).
\end{aligned}$$

Note that this model allows λ to vary by site and p_{ij}, the detection probability, to be modeled as a function of covariates.

Posterior distribution

For a single site, i:

$$[N_i, \boldsymbol{\alpha}, \lambda_i | \mathbf{y}_i, \mathbf{W_i}] \propto \prod_{j=1}^{J} [y_{ij} | N_i, p_{ij}][N_i | \lambda_i][\lambda_i][\boldsymbol{\alpha}]$$

Full conditional distributions

Coefficients describing the effect of covariates on detection probability ($\boldsymbol{\alpha}$):

$$[\boldsymbol{\alpha} | \cdot] \quad \propto \quad \prod_{j=1}^{J} [y_{ij} | N_i, p_{ij}][\boldsymbol{\alpha}].$$

This full-conditional distribution does not have a known analytical form; therefore sample $\mathbf{p}$ using Metropolis-Hastings.

The true number of individuals (N_i):

$$\begin{aligned}
[N_i | \cdot] &\propto \prod_{j=1}^{J} [y_{ij} | N_i, p_{ij}][N_i | \lambda_i] \\
&\propto \prod_{j=1}^{J} \binom{N_i}{y_{ij}} p_{ij}^{y_{ij}} (1 - p_{ij})^{N_i - y_{ij}} \left(\frac{\lambda_i^{N_i} e^{-\lambda_i}}{N_i!} \right)
\end{aligned}$$

$$
\begin{aligned}
&\propto \prod_{j=1}^{J} \left(\frac{N_i!}{y_{ij}!(N_i - y_{ij})!} \right) p_{ij}^{y_{ij}} (1-p_{ij})^{N_i - y_{ij}} \left(\frac{\lambda_i^{N_i} e^{-\lambda_i}}{N_i!} \right) \\
&\propto \prod_{j=1}^{J} \frac{(1-p_{ij})^{N_i - y_{ij}} \lambda_i^{N_i}}{(N_i - y_{ij})!} \\
&\propto \prod_{j=1}^{J} \frac{(1-p_{ij})^{N_i - y_{ij}} \lambda_i^{N_i} \lambda_i^{-y_{ij}}}{(N_i - y_{ij})!} \\
&\propto \prod_{j=1}^{J} \frac{(\lambda_i(1-p_{ij}))^{\,N_i - y_{ij}}}{(N_i - y_{ij})!} e^{-\lambda_i(1-p_{ij})}.
\end{aligned}
$$

This full-conditional is a little strange because $[N_i - y_{ij}|\cdot] \propto \text{Pois}(\lambda_i(1-p_{ij}))$, which suggests there is one true abundance per replicate count at each site. This is in contrast to the case in which only one observation exists per site, i.e., $[N_i - y_i|\cdot] \propto \text{Pois}(\lambda_i(1-p_i))$. Given that $[N_i|\cdot]$ lacks a clear analytical solution, sample N_i using Metropolis-Hastings.

Rate of the process model (λ_i):

$$
\begin{aligned}
[\lambda_i|\cdot] &\propto [N_i|\lambda_i][\lambda_i] \\
&\propto \frac{\lambda_i^{N_i} e^{-\lambda_i}}{N_i!} \lambda_i^{r-1} e^{-\lambda_i q} \\
&\propto \lambda_i^{N_i} e^{-\lambda_i} \lambda_i^{r-1} e^{-\lambda_i q} \\
&\propto \lambda_i^{N_i + r - 1} e^{-\lambda_i(1+q)} \\
&= \text{Gamma}\left(N_i + r, 1 + q\right).
\end{aligned}
$$