Multiscale occupancy model that accommodates false detections

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1 Description

A 3-level multiscale occupancy model that accommodates false detections. A secondary "negative control" data set is used for estimating the probability of a false detection.

2 Implementation

The file occ.multiscale.fp.sim.R simulates data according to the model statement below, and occ.multiscale.fp.mcmc.R contains the MCMC algorithm for parameter estimation.

3 Model Statement

Let y_{ijk} be a binary observation representing a detection/non-detection, where $i = 1, \ldots, N$ indexes primary sample unit (e.g., individual or site), $j = 1, \ldots, J_i$ indexes subunits nested within the primary sample unit, and $k = 1, \ldots K_{ij}$ indexes replicate observations within a subunit. Also let v be the number of detections out of M trials obtained in the "negative control" dataset.

$$
\begin{aligned}
y_{ijk} &\sim \begin{cases} (1-\phi)\,\text{Bern}\left(p_{ijk}\right) + \phi 1_{\{y_{ijk}=1\}}, & a_{ij}=1 \\ \text{Bern}\left(\phi\right), & a_{ij}=0 \end{cases} \\
a_{ij} &\sim \begin{cases} \text{Bern}\left(\theta_{ij}\right), & z_i = 1 \\ 0, & z_i = 0 \end{cases} \\
z_i &\sim \text{Bern}\left(\psi_i\right) \\
\psi_i &= \text{logit}^{-1}\left(\mathbf{x}_i'\boldsymbol{\beta}\right) \\
\theta_{ij} &= \text{logit}^{-1}\left(\mathbf{u}_{ij}'\boldsymbol{\gamma}\right) \\
p_{ijk} &= \text{logit}^{-1}\left(\mathbf{w}_{ijk}'\boldsymbol{\alpha}\right) \\
\boldsymbol{\beta} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\beta, \sigma_\beta^2\mathbf{I}\right) \\
\boldsymbol{\gamma} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\gamma, \sigma_\gamma^2\mathbf{I}\right) \\
\boldsymbol{\alpha} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\alpha, \sigma_\alpha^2\mathbf{I}\right) \\
v &\sim \text{Binom}\left(M, \phi\right) \\
\phi &\sim \text{Beta}\left(a, b\right)
\end{aligned}
$$

4 Full-conditional Distributions

4.1 Occupancy state (z_i)

$$
\begin{aligned}
[z_i \mid \cdot] &\propto \prod_{j=1}^{J_i} [a_{ij} \mid \theta_{ij}, z_i]\,[z_i] \\
&\propto \prod_{j=1}^{J_i} \left(\theta_{ij}^{a_{ij}}\left(1-\theta_{ij}\right)^{1-a_{ij}}\right)^{z_i} \left(1_{\{a_{ij}=0\}}^{1-z_i}\right) \psi_i^{z_i}\left(1-\psi_i\right)^{1-z_i} \\
&\propto \prod_{j=1}^{J_i} \left(\psi_i\theta_{ij}^{a_{ij}}\left(1-\theta_{ij}\right)^{1-a_{ij}}\right)^{z_i} \left(\left(1-\psi_i\right)1_{\{a_{ij}=0\}}\right)^{1-z_i} \\
&= \text{Bern}\left(\tilde{\psi}_i\right),
\end{aligned}
$$

where,

$$\tilde{\psi}_i = \frac{\psi_i \prod_{j=1}^{J_i} \theta_{ij}^{a_{ij}} \left(1-\theta_{ij}\right)^{1-a_{ij}}}{\psi_i \prod_{j=1}^{J_i} \theta_{ij}^{a_{ij}} \left(1-\theta_{ij}\right)^{1-a_{ij}} + \left(1-\psi_i\right) \prod_{j=1}^{J_i} 1_{\{a_{ij}=0\}}}.$$

4.2 "Use" state (a_{ij})

Note that the mixture specification for a_{ij} in the model statement above is equivalent to $a_{ij} \sim \text{Bern}\left(z_i \theta_{ij}\right)$, an alternate specification that simplifies the update for a_{ij}.

$$\begin{aligned}
\left[a_{ij} \mid \cdot\right] &\propto \prod_{k=1}^{K_{ij}} \left[y_{ijk} \mid p_{ijk}, a_{ij}, \phi\right] \left[a_{ij}\right] \\
&\propto \prod_{k=1}^{K_{ij}} \left(\left(1-\phi\right) p_{ijk}^{y_{ijk}} \left(1-p_{ijk}\right)^{1-y_{ijk}} + \phi 1_{\{y_{ijk}=1\}}\right)^{a_{ij}} \left(\phi^{y_{ijk}} \left(1-\phi\right)^{1-y_{ijk}}\right)^{1-a_{ij}} \left(z_i\theta_{ij}\right)^{a_{ij}} \left(1-z_i\theta_{ij}\right)^{1-a_{ij}} \\
&\propto \prod_{k=1}^{K_{ij}} \left[\left(z_i\theta_{ij}\right) \left(\left(1-\phi\right) p_{ijk}^{y_{ijk}} \left(1-p_{ijk}\right)^{1-y_{ijk}} + \phi 1_{\{y_{ijk}=1\}}\right)\right]^{a_{ij}} \left(\left(1-z_i\theta_{ij}\right) \phi^{y_{ijk}} \left(1-\phi\right)^{1-y_{ijk}}\right)^{1-a_{ij}} \\
&= \text{Bern}\left(\tilde{\theta}_{ij}\right),
\end{aligned}$$

where,

$$\tilde{\theta}_{ij} = \frac{z_i\theta_{ij} \prod_{k=1}^{K_{ij}} \left[\left(1-\phi\right) p_{ijk}^{y_{ijk}} \left(1-p_{ijk}\right)^{1-y_{ijk}} + \phi 1_{\{y_{ijk}=1\}}\right]}{z_i\theta_{ij} \prod_{k=1}^{K_{ij}} \left[\left(1-\phi\right) p_{ijk}^{y_{ijk}} \left(1-p_{ijk}\right)^{1-y_{ijk}} + \phi 1_{\{y_{ijk}=1\}}\right] + \left(1-z_i\theta_{ij}\right) \prod_{k=1}^{K_{ij}} \phi^{y_{ijk}} \left(1-\phi\right)^{1-y_{ijk}}}.$$

4.3 Regression coefficients affecting occupancy probability ($\boldsymbol{\beta}$)

$$\begin{aligned}
\left[\boldsymbol{\beta} \mid \cdot\right] &\propto \prod_{i=1}^{N} \left[z_i \mid \psi_i\right] \left[\boldsymbol{\beta}\right] \\
&\propto \prod_{i=1}^{N} \text{Bern}\left(z_i \mid \psi_i\right) \mathcal{N}\left(\boldsymbol{\beta} \mid \boldsymbol{\mu}_\beta, \sigma_\beta^2 \mathbf{I}\right).
\end{aligned}$$

The update for $\boldsymbol{\beta}$ proceeds using Metropolis-Hastings.

4.4 Regression coefficients affecting probability of use ($\boldsymbol{\gamma}$)

$$\begin{aligned}
\left[\boldsymbol{\gamma} \mid \cdot\right] &\propto \prod_{i=1}^{N} \prod_{j=1}^{J_i} \left[a_{ij} \mid \theta_{ij}, z_i\right] \left[\boldsymbol{\gamma}\right] \\
&\propto \prod_{i=1}^{N} \prod_{j=1}^{J_i} \text{Bern}\left(a_{ij} \mid \theta_{ij}\right)^{z_i} \mathcal{N}\left(\boldsymbol{\gamma} \mid \boldsymbol{\mu}_\gamma, \sigma_\gamma^2 \mathbf{I}\right).
\end{aligned}$$

The update for $\boldsymbol{\gamma}$ proceeds using Metropolis-Hastings. Note that the product over i only includes instances of i such that $z_i = 1$.

4.5 Regression coefficients affecting detection probability ($\boldsymbol{\alpha}$)

$$\begin{aligned}
\left[\boldsymbol{\alpha} \mid \cdot\right] &\propto \prod_{i=1}^{N} \prod_{j=1}^{J_i} \prod_{k=1}^{K_{ij}} \left[y_{ijk} \mid p_{ijk}, a_{ij}, \phi\right] \left[\boldsymbol{\alpha}\right] \\
&\propto \prod_{i=1}^{N} \prod_{j=1}^{J_i} \prod_{k=1}^{K_{ij}} \left[\left(1-\phi\right) p_{ijk}^{y_{ijk}} \left(1-p_{ijk}\right)^{1-y_{ijk}} + \phi 1_{\{y_{ijk}=1\}}\right]^{a_{ij}} \mathcal{N}\left(\boldsymbol{\alpha} \mid \boldsymbol{\mu}_\alpha, \sigma_\alpha^2 \mathbf{I}\right).
\end{aligned}$$

The update for $\boldsymbol{\alpha}$ proceeds using Metropolis-Hastings. Note that the product over i and j only includes instances of i and j such that $a_{ij} = 1$.

4.6 Probability of a false detection (ϕ)

$$
\begin{aligned}
[\phi \mid \cdot] &\propto [v \mid M, \phi]\,[\phi] \\
&\propto \text{Binom}\,(v \mid M, \phi)\,\text{Beta}\,(a, b) \\
&\propto \phi^{v}\,(1-\phi)^{M-v}\,\phi^{a-1}(1-\phi)^{b-1} \\
&\propto \phi^{v+a-1}\,(1-\phi)^{M-v+b-1} \\
&= \text{Beta}\,(v+a, M-v+b)
\end{aligned}
$$