Multiscale occupancy model that accommodates false detections

Brian M. Brost / 02 November 2016 / Vetted: NO

1 Description

A 3-level multiscale occupancy model that accommodates false detections. A secondary “negative control” data set is used for estimating the probability of a false detection.

2 Implementation

The file occ.multiscale.fp.sim.R simulates data according to the model statement presented below, and occ.multiscale.fp.mcmc.R contains the MCMC algorithm for parameter estimation.

3 Model Statement

Let y_{ijk} be a binary observation representing a detection/non-detection, where $i = 1, \ldots, N$ indexes primary sample unit (e.g., individual or site), $j = 1, \ldots, J_i$ indexes subunits (i.e., a secondary unit nested within the primary sample unit), and $k = 1, \ldots K_{ij}$ indexes replicate observations within a subunit. Also let v be the number of detections out of n “trials” obtained in the “negative control” dataset.

$$
\begin{aligned}
y_{ijk} &\sim \begin{cases} (1-\phi)\,\text{Bern}\left(p_{ijk}\right) + \phi 1_{\{y_{ijk}=1\}}, & a_{ij} = 1 \\ \text{Bern}\left(\phi\right), & a_{ij} = 0 \end{cases} \\
a_{ij} &\sim \begin{cases} \text{Bern}\left(\theta_{ij}\right), & z_i = 1 \\ 0, & z_i = 0 \end{cases} \\
z_i &\sim \text{Bern}\left(\psi_i\right) \\
\psi_i &= \text{logit}^{-1}\left(\mathbf{x}_i'\boldsymbol{\beta}\right) \\
\theta_{ij} &= \text{logit}^{-1}\left(\mathbf{u}_{ij}'\boldsymbol{\gamma}\right) \\
p_{ijk} &= \text{logit}^{-1}\left(\mathbf{w}_{ijk}'\boldsymbol{\alpha}\right) \\
\boldsymbol{\beta} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\beta, \sigma_\beta^2\mathbf{I}\right) \\
\boldsymbol{\gamma} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\gamma, \sigma_\gamma^2\mathbf{I}\right) \\
\boldsymbol{\alpha} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\alpha, \sigma_\alpha^2\mathbf{I}\right) \\
v &\sim \text{Binom}\left(M, \phi\right) \\
\phi &\sim \text{Beta}\left(a, b\right)
\end{aligned}
$$

4 Full-conditional Distributions

4.1 Occupancy state (z_i)

$$
\begin{aligned}
[z_i \mid \cdot] &\propto \prod_{j=1}^{J_i} [a_{ij} \mid \theta_{ij}, z_i] [z_i] \\
&\propto \prod_{j=1}^{J_i} \left(\theta_{ij}^{a_{ij}} (1-\theta_{ij})^{1-a_{ij}}\right)^{z_i} \left(1_{\{a_{ij}=0\}}^{1-z_i}\right) \psi_i^{z_i} (1-\psi_i)^{1-z_i} \\
&\propto \prod_{j=1}^{J_i} \left(\psi_i \theta_{ij}^{a_{ij}} (1-\theta_{ij})^{1-a_{ij}}\right)^{z_i} \left((1-\psi_i) 1_{\{a_{ij}=0\}}\right)^{1-z_i} \\
&= \text{Bern}\left(\tilde{\psi}_i\right),
\end{aligned}
$$

where,

$$
\tilde{\psi}_i = \frac{\psi_i \prod_{j=1}^{J_i} \theta_{ij}^{a_{ij}} (1-\theta_{ij})^{1-a_{ij}}}{\psi_i \prod_{j=1}^{J_i} \theta_{ij}^{a_{ij}} (1-\theta_{ij})^{1-a_{ij}} + (1-\psi_i) \prod_{j=1}^{J_i} 1_{\{a_{ij}=0\}}}.
$$

4.2 "Use" state (a_{ij})

Note that the mixture specification for a_{ij} in the model statement above is equivalent to $a_{ij} \sim \text{Bern}(z_i \theta_{ij})$. The update for a_{ij} relies on this alternative specification.

$$
\begin{aligned}
[a_{ij} \mid \cdot] &\propto \prod_{k=1}^{K_{ij}} [y_{ijk} \mid p_{ijk}, a_{ij}, \phi] [a_{ij}] \\
&\propto \prod_{k=1}^{K_{ij}} \left((1-\phi) p_{ijk}^{y_{ijk}} (1-p_{ijk})^{1-y_{ijk}} + \phi 1_{\{y_{ijk}=1\}}\right)^{a_{ij}} \left(\phi^{y_{ijk}} (1-\phi)^{1-y_{ijk}}\right) (z_i \theta_{ij})^{a_{ij}} (1-z_i \theta_{ij})^{1-a_{ij}} \\
&\propto \prod_{k=1}^{K_{ij}} \left[(z_i \theta_{ij}) \left((1-\phi) p_{ijk}^{y_{ijk}} (1-p_{ijk})^{1-y_{ijk}} + \phi 1_{\{y_{ijk}=1\}}\right)\right]^{a_{ij}} \left((1-z_i \theta_{ij}) \phi^{y_{ijk}} (1-\phi)^{1-y_{ijk}}\right)^{1-a_{ij}} \\
&= \text{Bern}\left(\tilde{\theta}_{ij}\right),
\end{aligned}
$$

where,

$$
\tilde{\theta}_{ij} = \frac{z_i \theta_{ij} \prod_{k=1}^{K_{ij}} \left[(1-\phi) p_{ijk}^{y_{ijk}} (1-p_{ijk})^{1-y_{ijk}} + \phi 1_{\{y_{ijk}=1\}}\right]}{z_i \theta_{ij} \prod_{k=1}^{K_{ij}} \left[(1-\phi) p_{ijk}^{y_{ijk}} (1-p_{ijk})^{1-y_{ijk}} + \phi 1_{\{y_{ijk}=1\}}\right] + (1-z_i \theta_{ij}) \prod_{k=1}^{K_{ij}} \phi^{y_{ijk}} (1-\phi)^{1-y_{ijk}}}.
$$

4.3 Regression coefficients affecting occupancy probability ($\boldsymbol{\beta}$)

$$
\begin{aligned}
[\boldsymbol{\beta} \mid \cdot] &\propto \prod_{i=1}^{N} [z_i \mid \psi_i] [\boldsymbol{\beta}] \\
&\propto \prod_{i=1}^{N} \text{Bern}(z_i \mid \psi_i) \mathcal{N}\left(\boldsymbol{\beta} \mid \boldsymbol{\mu}_\beta, \sigma_\beta^2 \mathbf{I}\right).
\end{aligned}
$$

The update for $\boldsymbol{\beta}$ proceeds using Metropolis-Hastings.

4.4 Regression coefficients affecting probability of use ($\boldsymbol{\gamma}$)

$$
\begin{aligned}
[\boldsymbol{\gamma} \mid \cdot] &\propto \prod_{i=1}^{N}\prod_{j=1}^{J_i}[a_{ij} \mid \theta_{ij}, z_i]\,[\boldsymbol{\gamma}] \\
&\propto \prod_{i=1}^{N}\prod_{j=1}^{J_i}\mathrm{Bern}\left(a_{ij} \mid \theta_{ij}\right)^{z_i}\mathcal{N}\left(\boldsymbol{\gamma} \mid \boldsymbol{\mu}_{\gamma}, \sigma_{\gamma}^2\mathbf{I}\right).
\end{aligned}
$$

The update for $\boldsymbol{\gamma}$ proceeds using Metropolis-Hastings. Note that, in effect, the product over i only includes instances of i such that $z_i = 1$.

4.5 Regression coefficients affecting detection probability ($\boldsymbol{\alpha}$)

$$
\begin{aligned}
[\boldsymbol{\alpha} \mid \cdot] &\propto \prod_{i=1}^{N}\prod_{j=1}^{J_i}\prod_{k=1}^{K_{ij}}[y_{ijk} \mid p_{ijk}, a_{ij}]\,[\boldsymbol{\alpha}] \\
&\propto \prod_{i=1}^{N}\prod_{j=1}^{J_i}\prod_{k=1}^{K_{ij}}\left[(1-\phi)\,p_{ijk}^{y_{ijk}}\left(1-p_{ijk}\right)^{1-y_{ijk}} + \phi 1_{\{y_{ijk}=1\}}\right]^{a_{ij}}\mathcal{N}\left(\boldsymbol{\alpha} \mid \boldsymbol{\mu}_{\alpha}, \sigma_{\alpha}^2\mathbf{I}\right).
\end{aligned}
$$

The update for $\boldsymbol{\alpha}$ proceeds using Metropolis-Hastings. Note that, in effect, the product over i and j only includes instances of i and j such that $a_{ij} = 1$.

4.6 Probability of a false detection (ϕ)

$$
\begin{aligned}
[\phi \mid \cdot] &\propto [v \mid M, \phi]\,[\phi] \\
&\propto \mathrm{Binom}\left(v \mid M, \phi\right)\mathrm{Beta}\left(a, b\right) \\
&\propto \phi^{v}\left(1-\phi\right)^{M-v}\phi^{a-1}(1-\phi)^{b-1} \\
&\propto \phi^{v+a-1}\left(1-\phi\right)^{M-v+b-1} \\
&= \mathrm{Beta}\left(v+a, M-v+b\right)
\end{aligned}
$$