Multiscale occupancy model

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1 Description

A 3-level multiscale occupancy model.

2 Implementation

The file occ.multiscale.sim.R simulates data according to the model statement presented below, and occ.multiscale.mcmc.R contains the MCMC algorithm for parameter estimation.

3 Model Statement

Let y_{ijk} be a binary observation representing detection/non-detection, where $i = 1, \ldots, N$ indexes primary sample unit (e.g., individual or site), $j = 1, \ldots, J_i$ indexes subunits (i.e., a secondary unit nested within the primary sample unit), and $k = 1, \ldots K_{ij}$ indexes replicate observations within a subunit.

$$
\begin{aligned}
y_{ijk} &\sim \begin{cases} \text{Bern}\left(p_{ijk}\right), & a_{ij} = 1 \\ 0, & a_{ij} = 0 \end{cases} \\
a_{ij} &\sim \begin{cases} \text{Bern}\left(\theta_{ij}\right), & z_i = 1 \\ 0, & z_i = 0 \end{cases} \\
z_i &\sim \text{Bern}\left(\psi_i\right) \\
\psi_i &= \text{logit}^{-1}\left(\mathbf{x}_i'\boldsymbol{\beta}\right) \\
\theta_{ij} &= \text{logit}^{-1}\left(\mathbf{u}_{ij}'\boldsymbol{\gamma}\right) \\
p_{ijk} &= \text{logit}^{-1}\left(\mathbf{w}_{ijk}'\boldsymbol{\alpha}\right) \\
\boldsymbol{\beta} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\beta, \sigma_\beta^2\mathbf{I}\right) \\
\boldsymbol{\gamma} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\gamma, \sigma_\gamma^2\mathbf{I}\right) \\
\boldsymbol{\alpha} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\alpha, \sigma_\alpha^2\mathbf{I}\right)
\end{aligned}
$$

4 Full-conditional Distributions

4.1 Occupancy state (z_i)

$$
\begin{aligned}
[z_i \mid \cdot] &\propto \prod_{j=1}^{J_i} [a_{ij} \mid \theta_{ij}, z_i]\,[z_i] \\
&\propto \prod_{j=1}^{J_i} \left(\theta_{ij}^{a_{ij}}\left(1-\theta_{ij}\right)^{1-a_{ij}}\right)^{z_i}\left(1_{\{a_{ij}=0\}}^{1-z_i}\right)\psi_i^{z_i}\left(1-\psi_i\right)^{1-z_i} \\
&\propto \prod_{j=1}^{J_i} \left(\psi_i\theta_{ij}^{a_{ij}}\left(1-\theta_{ij}\right)^{1-a_{ij}}\right)^{z_i}\left(\left(1-\psi_i\right)1_{\{a_{ij}=0\}}\right)^{1-z_i} \\
&= \text{Bern}\left(\tilde{\psi}_i\right),
\end{aligned}
$$

where,

$$\tilde{\psi}_i = \frac{\psi_i \prod_{j=1}^{J_i} \theta_{ij}^{a_{ij}} \left(1-\theta_{ij}\right)^{1-a_{ij}}}{\psi_i \prod_{j=1}^{J_i} \theta_{ij}^{a_{ij}} \left(1-\theta_{ij}\right)^{1-a_{ij}} + \left(1-\psi_i\right) \prod_{j=1}^{J_i} 1_{\{a_{ij}=0\}}}.$$

4.2 "Use" state (a_{ij})

Note that the mixture specification for a_{ij} in the model statement above is equivalent to $a_{ij} \sim \text{Bern}\left(z_i\theta_{ij}\right)$. The update for a_{ij} relies on this alternative specification.

$$\begin{aligned}
[a_{ij} \mid \cdot] &\propto \prod_{k=1}^{K_{ij}} [y_{ijk} \mid p_{ijk}, a_{ij}] \, [a_{ij}] \\
&\propto \prod_{k=1}^{K_{ij}} \left(p_{ijk}^{y_{ijk}} \left(1-p_{ijk}\right)^{1-y_{ijk}}\right)^{a_{ij}} \left(1_{\{y_{ijk}=0\}}^{1-a_{ij}}\right) \left(z_i\theta_{ij}\right)^{a_{ij}} \left(1-z_i\theta_{ij}\right)^{1-a_{ij}} \\
&\propto \prod_{k=1}^{K_{ij}} \left(\left(z_i\theta_{ij}\right) p_{ijk}^{y_{ijk}} \left(1-p_{ijk}\right)^{1-y_{ijk}}\right)^{a_{ij}} \left(\left(1-z_i\theta_{ij}\right) 1_{\{y_{ijk}=0\}}\right)^{1-a_{ij}} \\
&= \text{Bern}\left(\tilde{\theta}_{ij}\right),
\end{aligned}$$

where,

$$\tilde{\theta}_{ij} = \frac{z_i\theta_{ij} \prod_{k=1}^{K_{ij}} p_{ijk}^{y_{ijk}} \left(1-p_{ijk}\right)^{1-y_{ijk}}}{z_i\theta_{ij} \prod_{k=1}^{K_{ij}} p_{ijk}^{y_{ijk}} \left(1-p_{ijk}\right)^{1-y_{ijk}} + \left(1-z_i\theta_{ij}\right) \prod_{k=1}^{K_{ij}} 1_{\{y_{ijk}=0\}}}.$$

4.3 Regression coefficients affecting occupancy probability ($\boldsymbol{\beta}$)

$$\begin{aligned}
[\boldsymbol{\beta} \mid \cdot] &\propto \prod_{i=1}^{N} [z_i \mid \psi_i] \, [\boldsymbol{\beta}] \\
&\propto \prod_{i=1}^{N} \text{Bern}\left(z_i \mid \psi_i\right) \mathcal{N}\left(\boldsymbol{\beta} \mid \boldsymbol{\mu}_\beta, \sigma_\beta^2 \mathbf{I}\right).
\end{aligned}$$

The update for $\boldsymbol{\beta}$ proceeds using Metropolis-Hastings.

4.4 Regression coefficients affecting probability of use ($\boldsymbol{\gamma}$)

$$\begin{aligned}
[\boldsymbol{\gamma} \mid \cdot] &\propto \prod_{i=1}^{N} \prod_{j=1}^{J_i} [a_{ij} \mid \theta_{ij}, z_i] \, [\boldsymbol{\gamma}] \\
&\propto \prod_{i=1}^{N} \prod_{j=1}^{J_i} \text{Bern}\left(a_{ij} \mid \theta_{ij}\right)^{z_i} \mathcal{N}\left(\boldsymbol{\gamma} \mid \boldsymbol{\mu}_\gamma, \sigma_\gamma^2 \mathbf{I}\right).
\end{aligned}$$

The update for $\boldsymbol{\gamma}$ proceeds using Metropolis-Hastings. Note that, in effect, the product over i only includes instances of i such that $z_i = 1$.

4.5 Regression coefficients affecting detection probability ($\boldsymbol{\alpha}$)

$$\begin{aligned}
[\boldsymbol{\alpha} \mid \cdot] &\propto \prod_{i=1}^{N} \prod_{j=1}^{J_i} \prod_{k=1}^{K_{ij}} [y_{ijk} \mid p_{ijk}, a_{ij}] \, [\boldsymbol{\alpha}] \\
&\propto \prod_{i=1}^{N} \prod_{j=1}^{J_i} \prod_{k=1}^{K_{ij}} \text{Bern}\left(y_{ijk} \mid p_{ijk}\right)^{a_{ij}} \mathcal{N}\left(\boldsymbol{\alpha} \mid \boldsymbol{\mu}_\alpha, \sigma_\alpha^2 \mathbf{I}\right).
\end{aligned}$$

The update for $\boldsymbol{\alpha}$ proceeds using Metropolis-Hastings. Note that, in effect, the product over i and j only includes instances of i and j such that $a_{ij} = 1$.