False positive occupancy model with latent 'false positive' indicator variables

Brian M. Brost / 12 June 2017 / Vetted: NO

1 Description

An occupancy model that accommodates false negative and false positive errors. A secondary "negative control" data set is used for estimating the probability of a false positive error.

2 Implementation

The file occ.fp.sim.R simulates data according to the model statement below, and occ.fp.latent.var.mcmc.R contains the MCMC algorithm for parameter estimation.

3 Model Statement

Let y_{ij} be a binary observation representing a detection/non-detection, where $i = 1, \ldots, N$ indexes sample units and $j = 1, \ldots, J_i$ indexes replicate observations within a sample unit. Also let v be the number of detections out of M trials obtained in the "negative control" dataset. Let z_i denote the latent occupancy state and v_{ij} represent the latent 'false positive' state variable.

$$
\begin{aligned}
y_{ij} &\sim \begin{cases} \text{Bern}\left(p_{ij}\right), & v_{ij} = 0, z_i = 1 \\ 1, & v_{ij} = 1, z_i = 1 \\ 0, & v_{ij} = 0, z_i = 0 \\ 1 & v_{ij} = 1, z_i = 0 \end{cases} \\
v_{ij} &\sim \text{Bern}\left(\phi\right) \\
z_i &\sim \text{Bern}\left(\psi_i\right) \\
\psi_i &= \text{logit}^{-1}\left(\mathbf{x}_i'\boldsymbol{\beta}\right) \\
p_{ij} &= \text{logit}^{-1}\left(\mathbf{w}_{ij}'\boldsymbol{\alpha}\right) \\
\boldsymbol{\beta} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\beta, \sigma_\beta^2\mathbf{I}\right) \\
\boldsymbol{\alpha} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\alpha, \sigma_\alpha^2\mathbf{I}\right) \\
v &\sim \text{Binom}\left(M, \phi\right) \\
\phi &\sim \text{Beta}\left(a, b\right)
\end{aligned}
$$

4 Full-conditional Distributions

4.1 Occupancy state (z_i)

$$
\begin{aligned}
[z_i \mid \cdot] &\propto \prod_{j=1}^{J_i} [y_{ij} \mid p_{ij}, v_{ij}, z_i] \, [z_i \mid \psi_i] \\
&\propto \prod_{j=1}^{J_i} \left(\text{Bern}\left(y_{ij} \mid p_{ij}\right)^{1-v_{ij}} \right)^{z_i} \left(1_{\{y_{ij}=1\}}^{v_{ij}} \right)^{z_i} \left(1_{\{y_{ij}=0\}}^{1-v_{ij}} \right)^{1-z_i} \left(1_{\{y_{ij}=1\}}^{v_{ij}} \right)^{1-z_i} \text{Bern}\left(z_i \mid \psi_i\right) \\
&\propto \prod_{j=1}^{J_i} \left(1_{\{y_{ij}=1\}}^{v_{ij}} \left(p_{ij}^{y_{ij}} \left(1 - p_{ij}\right)^{1-y_{ij}} \right)^{1-v_{ij}} \right)^{z_i} \left(1_{\{y_{ij}=0\}}^{1-v_{ij}} 1_{\{y_{ij}=1\}}^{v_{ij}} \right)^{1-z_i} \psi_i^{z_i} \left(1 - \psi_i\right)^{1-z_i} \\
&\propto \prod_{j=1}^{J_i} \left(\psi_i 1_{\{y_{ij}=1\}}^{v_{ij}} \left(p_{ij}^{y_{ij}} \left(1 - p_{ij}\right)^{1-y_{ij}} \right)^{1-v_{ij}} \right)^{z_i} \left(\left(1 - \psi_i\right) 1_{\{y_{ij}=0\}}^{1-v_{ij}} 1_{\{y_{ij}=1\}}^{v_{ij}} \right)^{1-z_i} \\
&= \text{Bern}\left(\tilde{\psi}_i\right),
\end{aligned}
$$

where,

$$
\tilde{\psi}_i = \frac{\psi_i \prod_{j=1}^{J_i} \left(1_{\{y_{ij}=1\}}^{v_{ij}} \right) \left(p_{ij}^{y_{ij}} \left(1 - p_{ij}\right)^{1-y_{ij}} \right)^{1-v_{ij}}}{\psi_i \prod_{j=1}^{J_i} \left(1_{\{y_{ij}=1\}}^{v_{ij}} \right) \left(p_{ij}^{y_{ij}} \left(1 - p_{ij}\right)^{1-y_{ij}} \right)^{1-v_{ij}} + \left(1 - \psi_i\right) \prod_{j=1}^{J_i} \left(1_{\{y_{ij}=0\}}^{1-v_{ij}} 1_{\{y_{ij}=1\}}^{v_{ij}} \right)}.
$$

4.2 False positive state (v_{ij})

$$
\begin{aligned}
[v_{ij} \mid \cdot] &\propto [y_{ij} \mid p_{ij}, v_{ij}, z_i] \, [v_{ij} \mid \phi] \\
&\propto \left(\text{Bern}\left(y_{ij} \mid p_{ij}\right)^{1-v_{ij}} \right)^{z_i} \left(1_{\{y_{ij}=1\}}^{v_{ij}} \right)^{z_i} \left(1_{\{y_{ij}=0\}}^{1-v_{ij}} \right)^{1-z_i} \left(1_{\{y_{ij}=1\}}^{v_{ij}} \right)^{1-z_i} \text{Bern}\left(v_{ij} \mid \phi\right) \\
&\propto \left(\text{Bern}\left(y_{ij} \mid p_{ij}\right)^{z_i} \right)^{1-v_{ij}} \left(1_{\{y_{ij}=1\}}^{z_i} \right)^{v_{ij}} \left(1_{\{y_{ij}=0\}}^{1-z_i} \right)^{1-v_{ij}} \left(1_{\{y_{ij}=1\}}^{1-z_i} \right)^{v_{ij}} \text{Bern}\left(v_{ij} \mid \phi\right) \\
&\propto \left(\left(p_{ij}^{y_{ij}} \left(1 - p_{ij}\right)^{1-y_{ij}} \right)^{z_i} \right)^{1-v_{ij}} \left(1_{\{y_{ij}=1\}}^{z_i} \right)^{v_{ij}} \left(1_{\{y_{ij}=0\}}^{1-z_i} \right)^{1-v_{ij}} \left(1_{\{y_{ij}=1\}}^{1-z_i} \right)^{v_{ij}} \phi^{v_{ij}} \left(1 - \phi\right)^{1-v_{ij}} \\
&\quad \left(\phi 1_{\{y_{ij}=1\}}^{z_i} 1_{\{y_{ij}=1\}}^{1-z_i} \right)^{v_{ij}} \left(\left(1 - \phi\right) \left(p_{ij}^{y_{ij}} \left(1 - p_{ij}\right)^{1-y_{ij}} \right)^{z_i} 1_{\{y_{ij}=0\}}^{1-z_i} \right)^{1-v_{ij}} \\
&= \text{Bern}\left(\tilde{\phi}\right),
\end{aligned}
$$

where,

$$
\tilde{\psi}_i = \frac{\phi 1_{\{y_{ij}=1\}}^{z_i} 1_{\{y_{ij}=1\}}^{1-z_i}}{\phi 1_{\{y_{ij}=1\}}^{z_i} 1_{\{y_{ij}=1\}}^{1-z_i} + \left(1 - \phi\right) \left(p_{ij}^{y_{ij}} \left(1 - p_{ij}\right)^{1-y_{ij}} \right)^{z_i} 1_{\{y_{ij}=0\}}^{1-z_i}}.
$$

4.3 Occupancy coefficients ($\boldsymbol{\beta}$)

$$
\begin{aligned}
[\boldsymbol{\beta} \mid \cdot] &\propto \prod_{i=1}^{N} [z_i \mid \psi_i] \, [\boldsymbol{\beta}] \\
&\propto \prod_{i=1}^{N} \text{Bern}\left(z_i \mid \psi_i\right) \mathcal{N}\left(\boldsymbol{\beta} \mid \boldsymbol{\mu}_\beta, \sigma_\beta^2 \mathbf{I}\right).
\end{aligned}
$$

The update for $\boldsymbol{\beta}$ proceeds using Metropolis-Hastings.

4.4 Detection coefficients ($\boldsymbol{\alpha}$)

$$
\begin{aligned}
[\boldsymbol{\alpha} \mid \cdot] &\propto \prod_{i=1}^{N}\prod_{j=1}^{J_i} [y_{ij} \mid p_{ij}, v_{ij}, z_i]\,[\boldsymbol{\alpha}] \\
&\propto \prod_{i=1}^{N}\prod_{j=1}^{J_i} \left(\mathrm{Bern}\left(y_{ij} \mid p_{ij}\right)^{1-v_{ij}}\right)^{z_i} \mathcal{N}\left(\boldsymbol{\alpha} \mid \boldsymbol{\mu}_{\alpha}, \sigma_{\alpha}^2 \mathbf{I}\right).
\end{aligned}
$$

The update for $\boldsymbol{\alpha}$ proceeds using Metropolis-Hastings. Note that the product over i and j only includes instances of i and j such that $z_i = 1$ and $v_{ij} = 0$.

4.5 Probability of a false detection (ϕ)

$$
\begin{aligned}
[\phi \mid \cdot] &\propto [v \mid M, \phi]\,[\phi] \\
&\propto \mathrm{Binom}\left(v \mid M, \phi\right) \mathrm{Beta}\left(a, b\right) \\
&\propto \phi^{v}\left(1-\phi\right)^{M-v} \phi^{a-1}(1-\phi)^{b-1} \\
&\propto \phi^{v+a-1}\left(1-\phi\right)^{M-v+b-1} \\
&= \mathrm{Beta}\left(v+a, M-v+b\right)
\end{aligned}
$$