Classical occupancy model

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1 Description

A classical occupancy model that relies on replication to accommodate false negative errors.

2 Implementation

The file occ.sim.R simulates data according to the model statement below, and occ.mcmc.R contains the MCMC algorithm for parameter estimation.

3 Model Statement

Let y_{ij} be a binary observation representing a detection/non-detection, where $i = 1, \ldots, N$ indexes sample units and $j = 1, \ldots, J_i$ indexes replicate observations within a sample unit.

$$\begin{aligned}
y_{ij} &\sim \begin{cases} \text{Bern}\left(p_{ij}\right), & z_i = 1 \\ 0, & z_i = 0 \end{cases} \\
z_i &\sim \text{Bern}\left(\psi_i\right) \\
\psi_i &= \text{logit}^{-1}\left(\mathbf{x}_i'\boldsymbol{\beta}\right) \\
p_{ij} &= \text{logit}^{-1}\left(\mathbf{w}_{ij}'\boldsymbol{\alpha}\right) \\
\boldsymbol{\beta} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\beta, \sigma_\beta^2\mathbf{I}\right) \\
\boldsymbol{\alpha} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\alpha, \sigma_\alpha^2\mathbf{I}\right)
\end{aligned}$$

4 Full-conditional Distributions

4.1 *Occupancy state* (z_i)

$$\begin{aligned}
\left[z_i \mid \cdot\right] &\propto \prod_{j=1}^{J_i} \left[y_{ij} \mid p_{ij}, z_i\right]\left[z_i \mid \psi_i\right] \\
&\propto \prod_{j=1}^{J_i} \text{Bern}\left(y_{ij} \mid p_{ij}\right)^{z_i} 1_{\{y_{ij}=0\}}^{1-z_i} \text{Bern}\left(z_i \mid \psi_i\right) \\
&\propto \prod_{j=1}^{J_i} \left(p_{ij}^{y_{ij}}\left(1-p_{ij}\right)^{1-y_{ij}}\right)^{z_i}\left(1_{\{y_{ij}=0\}}^{1-z_i}\right)\psi_i^{z_i}\left(1-\psi_i\right)^{1-z_i} \\
&\propto \prod_{j=1}^{J_i} \left(\psi_i p_{ij}^{y_{ij}}\left(1-p_{ij}\right)^{1-y_{ij}}\right)^{z_i}\left(1_{\{y_{ij}=0\}}\left(1-\psi_i\right)\right)^{1-z_i} \\
&= \text{Bern}\left(\tilde{\psi}_i\right),
\end{aligned}$$

where,

$$\tilde{\psi}_i = \frac{\psi_i \prod_{j=1}^{J_i}\left(p_{ij}^{y_{ij}}\left(1-p_{ij}\right)^{1-y_{ij}}\right)}{\psi_i \prod_{j=1}^{J_i}\left(p_{ij}^{y_{ij}}\left(1-p_{ij}\right)^{1-y_{ij}}\right) + \left(1-\psi_i\right)\prod_{j=1}^{J_i}\left(1_{\{y_{ij}=0\}}\right)}.$$

4.2 *Occupancy coefficients ($\boldsymbol{\beta}$)*

$$
\begin{aligned}
[\boldsymbol{\beta} \mid \cdot] &\propto \prod_{i=1}^{N} [z_i \mid \psi_i] [\boldsymbol{\beta}] \\
&\propto \prod_{i=1}^{N} \mathrm{Bern}\left(z_i \mid \psi_i\right) \mathcal{N}\left(\boldsymbol{\beta} \mid \boldsymbol{\mu}_\beta, \sigma_\beta^2 \mathbf{I}\right).
\end{aligned}
$$

The update for $\boldsymbol{\beta}$ proceeds using Metropolis-Hastings.

4.3 *Detection coefficients ($\boldsymbol{\alpha}$)*

$$
\begin{aligned}
[\boldsymbol{\alpha} \mid \cdot] &\propto \prod_{i=1}^{N} \prod_{j=1}^{J_i} [y_{ij} \mid p_{ij}, z_i] [\boldsymbol{\alpha}] \\
&\propto \prod_{i=1}^{N} \prod_{j=1}^{J_i} \mathrm{Bern}\left(y_{ij} \mid p_{ij}\right)^{z_i} \mathcal{N}\left(\boldsymbol{\alpha} \mid \boldsymbol{\mu}_\alpha, \sigma_\alpha^2 \mathbf{I}\right).
\end{aligned}
$$

The update for $\boldsymbol{\alpha}$ proceeds using Metropolis-Hastings. Note that the product over i and j only includes instances of i and j such that $z_i = 1$.