# False positive occupancy model with marginalized likelihood

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## 1 Description

An occupancy model that accommodates false negative and false positive errors. A secondary "negative control" data set is used for estimating the probability of a false positive error.

## 2 Implementation

The file occ.fp.sim.R simulates data according to the model statement below, and occ.fp.marginal.lik.mcmc.R contains the MCMC algorithm for parameter estimation.

## 3 Model Statement

Let $y_{ij}$ be a binary observation representing a detection/non-detection, where $i = 1, \ldots, N$ indexes sample units and $j = 1, \ldots, J_i$ indexes replicate observations within a sample unit. Also let $v$ be the number of detections out of $M$ trials obtained in the "negative control" dataset.

$$
\begin{aligned}
y_{ij} &\sim \begin{cases} (1-\phi)\,\text{Bern}\left(p_{ij}\right) + \phi 1_{\{y_{ij}=1\}}, & z_i = 1 \\ \text{Bern}\left(\phi\right), & z_i = 0 \end{cases} \\
z_i &\sim \text{Bern}\left(\psi_i\right) \\
\psi_i &= \text{logit}^{-1}\left(\mathbf{x}_i'\boldsymbol{\beta}\right) \\
p_{ij} &= \text{logit}^{-1}\left(\mathbf{w}_{ij}'\boldsymbol{\alpha}\right) \\
\boldsymbol{\beta} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\beta, \sigma_\beta^2\mathbf{I}\right) \\
\boldsymbol{\alpha} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\alpha, \sigma_\alpha^2\mathbf{I}\right) \\
v &\sim \text{Binom}\left(M, \phi\right) \\
\phi &\sim \text{Beta}\left(a, b\right)
\end{aligned}
$$

## 4 Full-conditional Distributions

### 4.1 *Occupancy state* ($z_i$)

$$
\begin{aligned}
[z_i \mid \cdot] &\propto \prod_{j=1}^{J_i} [y_{ij} \mid p_{ij}, \phi, z_i]\,[z_i \mid \psi_i] \\
&\propto \prod_{j=1}^{J_i} \left((1-\phi)\,p_{ij}^{y_{ij}}\left(1-p_{ij}\right)^{1-y_{ij}} + \phi 1_{\{y_{ij}=1\}}\right)^{z_i} \left(\phi^{y_{ij}}\left(1-\phi\right)^{1-y_{ij}}\right)^{1-z_i} \psi_i^{z_i}\left(1-\psi_i\right)^{1-z_i} \\
&\propto \prod_{j=1}^{J_i} \left[\psi_i\left((1-\phi)\,p_{ij}^{y_{ij}}\left(1-p_{ij}\right)^{1-y_{ij}} + \phi 1_{\{y_{ij}=1\}}\right)\right]^{z_i} \left(\left(1-\psi_i\right)\phi^{y_{ij}}\left(1-\phi\right)^{1-y_{ij}}\right)^{1-z_i} \\
&= \text{Bern}\left(\tilde{\psi}_i\right),
\end{aligned}
$$

where,

$$
\tilde{\psi}_i = \frac{\psi_i \prod_{j=1}^{J_i}\left((1-\phi)\,p_{ij}^{y_{ij}}\left(1-p_{ij}\right)^{1-y_{ij}} + \phi 1_{\{y_{ij}=1\}}\right)}{\psi_i \prod_{j=1}^{J_i}\left((1-\phi)\,p_{ij}^{y_{ij}}\left(1-p_{ij}\right)^{1-y_{ij}} + \phi 1_{\{y_{ij}=1\}}\right) + \left(1-\psi_i\right)\prod_{j=1}^{J_i}\left(\phi^{y_{ij}}\left(1-\phi\right)^{1-y_{ij}}\right)}.
$$

### 4.2 *Occupancy coefficients* ($\boldsymbol{\beta}$)

$$
\begin{aligned}
[\boldsymbol{\beta} \mid \cdot] &\propto \prod_{i=1}^{N} [z_i \mid \psi_i] \, [\boldsymbol{\beta}] \\
&\propto \prod_{i=1}^{N} \text{Bern} \left( z_i \mid \psi_i \right) \mathcal{N} \left( \boldsymbol{\beta} \mid \boldsymbol{\mu}_\beta, \sigma_\beta^2 \mathbf{I} \right).
\end{aligned}
$$

The update for $\boldsymbol{\beta}$ proceeds using Metropolis-Hastings.

### 4.3 *Detection coefficients* ($\boldsymbol{\alpha}$)

$$
\begin{aligned}
[\boldsymbol{\alpha} \mid \cdot] &\propto \prod_{i=1}^{N} \prod_{j=1}^{J_i} [y_{ij} \mid p_{ij}, \phi, z_i] \, [\boldsymbol{\alpha}] \\
&\propto \prod_{i=1}^{N} \prod_{j=1}^{J_i} \left( (1-\phi) \, \text{Bern} \left( y_{ij} \mid p_{ij} \right) + \phi 1_{\{y_{ij}=1\}} \right)^{z_i} \mathcal{N} \left( \boldsymbol{\alpha} \mid \boldsymbol{\mu}_\alpha, \sigma_\alpha^2 \mathbf{I} \right).
\end{aligned}
$$

The update for $\boldsymbol{\alpha}$ proceeds using Metropolis-Hastings. Note that the product over $i$ and $j$ only includes instances of $i$ and $j$ such that $z_i = 1$.

### 4.4 *Probability of a false detection* ($\phi$)

$$
\begin{aligned}
[\phi \mid \cdot] &\propto [v \mid M, \phi] \, [\phi] \\
&\propto \text{Binom} \left( v \mid M, \phi \right) \text{Beta} \left( a, b \right) \\
&\propto \phi^v \left( 1 - \phi \right)^{M-v} \phi^{a-1} (1-\phi)^{b-1} \\
&\propto \phi^{v+a-1} \left( 1 - \phi \right)^{M-v+b-1} \\
&= \text{Beta} \left( v + a, M - v + b \right)
\end{aligned}
$$