Multiscale Community Occupancy Model

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1 Description

A 3-level multiscale occupancy model for multiple species.

2 Implementation

The file occ.community.multiscale.sim.R simulates data according to the model statement below, and occ.community.multiscale
contains the MCMC algorithm for parameter estimation.

3 Model Statement

Let y_{ijkt} represent the number of detections for species i $(i = 1, \ldots, n)$ at site k $(k = 1, \ldots, M_j)$ in unit j $(j = 1, \ldots, R)$ during sampling period t $(t = 1, \ldots, T_{jk})$. Note that sites are nested within units (or regions). Also, it is unnecessary to survey sites repeatedly through time and $T_{jk} = 1$ is okay for some or all j and k.

$$
\begin{aligned}
y_{ijkt} &\sim \begin{cases} \text{Binom}\left(J_{jkt}, p_{ijkt}\right), & a_{ijkt} = 1 \\ 0, & a_{ijkt} = 0 \end{cases} \\
a_{ijkt} &\sim \begin{cases} 0, & z_{ijt} = 0 \\ 0, & z_{ijt} = 1 \text{ and } v_{\gamma,ijkt} \leq 0 \\ 1, & z_{ijt} = 1 \text{ and } v_{\gamma,ijkt} > 0 \end{cases} \\
z_{ijt} &\sim \begin{cases} 0, & v_{\beta,ijt} \leq 1 \\ 1, & v_{\beta,ijt} > 0 \end{cases} \\
v_{\beta,ijt} &\sim \mathcal{N}\left(\mathbf{x}'_{jt}\boldsymbol{\beta}_i, 1\right) \\
v_{\gamma,ijkt} &\sim \mathcal{N}\left(\mathbf{u}'_{jkt}\boldsymbol{\gamma}_i, 1\right) \\
p_{ijkt} &= \text{logit}^{-1}\left(\mathbf{w}'_{jkt}\boldsymbol{\alpha}_i\right) \\
\boldsymbol{\beta}_i &\sim \mathcal{N}\left(\boldsymbol{\mu}_\beta, \sigma^2_\beta \mathbf{I}\right) \\
\boldsymbol{\gamma}_i &\sim \mathcal{N}\left(\boldsymbol{\mu}_\gamma, \sigma^2_\gamma \mathbf{I}\right) \\
\boldsymbol{\alpha}_i &\sim \mathcal{N}\left(\boldsymbol{\mu}_\alpha, \sigma^2_\alpha \mathbf{I}\right) \\
\boldsymbol{\mu}_\beta &\sim \mathcal{N}\left(\mathbf{0}, \sigma^2_{\mu_\beta} \mathbf{I}\right) \\
\boldsymbol{\mu}_\gamma &\sim \mathcal{N}\left(\mathbf{0}, \sigma^2_{\mu_\gamma} \mathbf{I}\right) \\
\boldsymbol{\mu}_\alpha &\sim \mathcal{N}\left(\mathbf{0}, \sigma^2_{\mu_\alpha} \mathbf{I}\right) \\
\sigma^2_\beta &\sim \text{IG}\left(r, q\right) \\
\sigma^2_\gamma &\sim \text{IG}\left(r, q\right) \\
\sigma^2_\alpha &\sim \text{IG}\left(r, q\right)
\end{aligned}
$$

4 Full-conditional Distributions

4.1 Occupancy state (z_{ijt})

$$
\begin{aligned}
[z_{ijt} \mid \cdot] &\propto \prod_{k=1}^{M_j} [a_{ijkt} \mid v_{\gamma,ijkt}, z_{ijt}] [z_{ijt} \mid v_{\beta,ijt}] \\
&\propto \prod_{k=1}^{M_j} \text{Bern}\left(a_{ijkt} \mid v_{\gamma,ijkt}\right)^{z_{ijt}} 1_{\{a_{ijkt}=0\}}^{1-z_{ijt}} \text{Bern}\left(z_{ijt} \mid v_{\beta,ijt}\right) \\
&\propto \prod_{k=1}^{M_j} \left(\theta_{ijkt}^{a_{ijkt}} \left(1-\theta_{ijkt}\right)^{1-a_{ijkt}}\right)^{z_{ijkt}} \left(1_{\{a_{ijkt}=0\}}^{1-z_{ijt}}\right) \psi_{ijt}^{z_{ijt}} \left(1-\psi_{ijt}\right)^{1-z_{ijt}} \\
&\propto \left(\psi_{ijt} \prod_{k=1}^{M_j} \theta_{ijkt}^{a_{ijkt}} \left(1-\theta_{ijkt}\right)^{1-a_{ijkt}}\right)^{z_{ijt}} \left(\left(1-\psi_{ijt}\right) \prod_{k=1}^{M_j} 1_{\{a_{ijkt}=0\}}\right)^{1-z_{ijt}} \\
&= \text{Bern}\left(\tilde{\psi}_{ijt}\right),
\end{aligned}
$$

where

$$
\tilde{\psi}_{ijt} = \frac{\psi_{ijt} \prod_{k=1}^{M_j} \theta_{ijkt}^{a_{ijkt}} \left(1-\theta_{ijkt}\right)^{1-a_{ijkt}}}{\psi_{ijt} \prod_{k=1}^{M_j} \theta_{ijkt}^{a_{ijkt}} \left(1-\theta_{ijkt}\right)^{1-a_{ijkt}} + \left(1-\psi_{ijt}\right) \prod_{k=1}^{M_j} 1_{\{a_{ijkt}=0\}}},
$$

$\psi_{ijt} = \Phi\left(\mathbf{x}'_{jt}\boldsymbol{\beta}_i\right)$, and $\theta_{ijkt} = \Phi\left(\mathbf{u}'_{jkt}\boldsymbol{\gamma}_i\right)$.

4.2 Use state (a_{ijkt})

Note that the mixture specification for a_{ijkt} in the model statement above is equivalent to $a_{ijkt} \sim \text{Bern}\left(z_{ijt}\theta_{ijkt}\right)$, an alternate specification that simplifies the update for a_{ijkt}.

$$
\begin{aligned}
[a_{ijkt} \mid \cdot] &\propto [y_{ijkt} \mid p_{ijkt}, a_{ijkt}] [a_{ijkt} \mid v_{\gamma,ijkt}, z_{ijt}] \\
&\propto \text{Binom}\left(y_{ijkt} \mid J_{jkt}, p_{ijkt}\right)^{a_{ijkt}} \left(1_{\{y_{ijkt}=0\}}^{1-a_{ijkt}}\right) \text{Bern}\left(a_{ijkt} \mid v_{\gamma,ijkt}\right)^{z_{ijt}} \\
&\propto \left(p_{ijkt}^{y_{ijkt}} \left(1-p_{ijkt}\right)^{J_{jkt}-y_{ijkt}}\right)^{a_{ijkt}} \left(1_{\{y_{ijkt}=0\}}^{1-a_{ijkt}}\right) \left(z_{ijt}\theta_{ijkt}\right)^{a_{ijkt}} \left(1-z_{ijt}\theta_{ijkt}\right)^{1-a_{ijkt}} \\
&\propto \left(\left(z_{ijt}\theta_{ijkt}\right) p_{ijkt}^{y_{ijkt}} \left(1-p_{ijkt}\right)^{J_{jkt}-y_{ijkt}}\right)^{a_{ijkt}} \left(\left(1-z_{ijt}\theta_{ijkt}\right) 1_{\{y_{ijkt}=0\}}\right)^{1-a_{ijkt}} \\
&= \text{Bern}\left(\tilde{\theta}_{ijkt}\right),
\end{aligned}
$$

where

$$
\tilde{\theta}_{ijkt} = \frac{z_{ijt}\theta_{ijkt} p_{ijkt}^{y_{ijkt}} \left(1-p_{ijkt}\right)^{J_{jkt}-y_{ijkt}}}{z_{ijt}\theta_{ijkt} p_{ijkt}^{y_{ijkt}} \left(1-p_{ijkt}\right)^{J_{jkt}-y_{ijkt}} + \left(1-z_{ijt}\theta_{ijkt}\right) 1_{\{y_{ijkt}=0\}}}
$$

and $\theta_{ijkt} = \Phi\left(\mathbf{u}'_{jkt}\boldsymbol{\gamma}_i\right)$.

4.3 Occupancy state auxiliary variable ($v_{\beta,ijt}$)

$$
\begin{aligned}
[v_{\beta,ijt} \mid \cdot] &\propto [z_{ijt} \mid v_{\beta,ijt}] [v_{\beta,ijt} \mid \boldsymbol{\beta}_i] \\
&\propto \left(1_{\{z_{ijt}=0\}} 1_{\{v_{\beta,ijt} \leq 0\}} + 1_{\{z_{ijt}=1\}} 1_{\{v_{\beta,ijt} > 0\}}\right) \times \mathcal{N}\left(v_{\beta,ijt} \mid \mathbf{x}'_{jt}\boldsymbol{\beta}_i, 1\right) \\
&= \begin{cases} \mathcal{TN}\left(\mathbf{x}'_{jt}\boldsymbol{\beta}_i, 1\right)_{-\infty}^{0}, & z_{ijt} = 0 \\ \mathcal{TN}\left(\mathbf{x}'_{jt}\boldsymbol{\beta}_i, 1\right)_{0}^{\infty}, & z_{ijt} = 1 \end{cases}
\end{aligned}
$$

4.4 Use state auxiliary variable ($v_{\gamma,ijkt}$)

$$
\begin{aligned}
[v_{\gamma,ijkt} \mid \cdot] &\propto [a_{ijkt} \mid v_{\gamma,ijkt}, z_{ijt}] [v_{\gamma,ijkt} \mid \boldsymbol{\gamma}_i] \\
&\propto \left(1_{\{a_{ijkt}=0\}} 1_{\{v_{\gamma,ijkt} \le 0\}} + 1_{\{a_{ijkt}=1\}} 1_{\{v_{\gamma,ijkt} > 0\}}\right)^{z_{ijt}} \times \mathcal{N}\left(v_{\gamma,ijkt} \mid \mathbf{u}'_{jkt}\boldsymbol{\gamma}_i, 1\right) \\
&= \begin{cases} \mathcal{TN}\left(\mathbf{u}'_{jkt}\boldsymbol{\gamma}_i, 1\right)_{-\infty}^{0}, & z_{ijt} = 1 \text{ and } a_{ijkt} = 0 \\ \mathcal{TN}\left(\mathbf{u}'_{jkt}\boldsymbol{\gamma}_i, 1\right)_{0}^{\infty}, & z_{ijt} = 1 \text{ and } a_{ijkt} = 1 \end{cases}
\end{aligned}
$$

4.5 Occupancy coefficients ($\boldsymbol{\beta}_i$)

$$
\begin{aligned}
[\boldsymbol{\beta}_i \mid \cdot] &\propto [\mathbf{v}_{\beta,i} \mid \boldsymbol{\beta}_i] \left[\boldsymbol{\beta}_i \mid \boldsymbol{\mu}_\beta, \sigma_\beta^2\right] \\
&\propto \mathcal{N}\left(\mathbf{v}_{\beta,i} \mid \mathbf{X}\boldsymbol{\beta}_i, \mathbf{1}\right) \mathcal{N}\left(\boldsymbol{\beta}_i \mid \boldsymbol{\mu}_\beta, \sigma_\beta^2 \mathbf{I}\right) \\
&\propto \exp\left\{-\frac{1}{2}\left(\mathbf{v}_{\beta,i} - \mathbf{X}\boldsymbol{\beta}_i\right)' \left(\mathbf{v}_{\beta,i} - \mathbf{X}\boldsymbol{\beta}_i\right)\right\} \exp\left\{-\frac{1}{2}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}_\beta\right)' \left(\sigma_\beta^2\mathbf{I}\right)^{-1} \left(\boldsymbol{\beta}_i - \boldsymbol{\mu}_\beta\right)\right\} \\
&\propto \exp\left\{-\frac{1}{2}\left(-2\mathbf{v}'_{\beta,i}\mathbf{X}\boldsymbol{\beta}_i + \boldsymbol{\beta}'_i\mathbf{X}'\mathbf{X}\boldsymbol{\beta}_i\right)\right\} \exp\left\{-\frac{1}{2}\left(-2\left(\boldsymbol{\mu}'_\beta\left(\sigma_\beta^2\mathbf{I}\right)^{-1}\right)\boldsymbol{\beta}_i + \boldsymbol{\beta}'_i\left(\sigma_\beta^2\mathbf{I}\right)^{-1}\boldsymbol{\beta}_i\right)\right\} \\
&\propto \exp\left\{-\frac{1}{2}\left(-2\left(\mathbf{v}'_{\beta,i}\mathbf{X} + \boldsymbol{\mu}'_\beta\left(\sigma_\beta^2\mathbf{I}\right)^{-1}\right)\boldsymbol{\beta}_i + \boldsymbol{\beta}'_i\left(\mathbf{X}'\mathbf{X} + \left(\sigma_\beta^2\mathbf{I}\right)^{-1}\right)\boldsymbol{\beta}_i\right)\right\} \\
&= \mathcal{N}(\mathbf{A}^{-1}\mathbf{b}, \mathbf{A}^{-1}),
\end{aligned}
$$

where $\mathbf{A} = \mathbf{X}'\mathbf{X} + \left(\sigma_\beta^2\mathbf{I}\right)^{-1}$, $\mathbf{b}' = \mathbf{v}'_{\beta,i}\mathbf{X} + \boldsymbol{\mu}'_\beta\left(\sigma_\beta^2\mathbf{I}\right)^{-1}$, $\mathbf{v}'_{\beta,i} = \{v_{\beta,ijt}, \forall j, t\}$, and $\mathbf{X} = \left\{\mathbf{x}'_{jt}, \forall j, t\right\}$.

4.6 Use coefficients ($\boldsymbol{\gamma}_i$)

$$
\begin{aligned}
[\boldsymbol{\gamma}_i \mid \cdot] &\propto [\mathbf{v}_{\gamma,i} \mid \boldsymbol{\gamma}_i, z_{ijt}] \left[\boldsymbol{\gamma}_i \mid \boldsymbol{\mu}_\gamma, \sigma_\gamma^2\right] \\
&\propto \mathcal{N}\left(\mathbf{v}_{\gamma,i} \mid \mathbf{U}\boldsymbol{\gamma}_i, \mathbf{1}\right)^{z_{ijt}} \mathcal{N}\left(\boldsymbol{\gamma}_i \mid \boldsymbol{\mu}_\gamma, \sigma_\gamma^2\mathbf{I}\right) \\
&= \mathcal{N}(\mathbf{A}^{-1}\mathbf{b}, \mathbf{A}^{-1}),
\end{aligned}
$$

where $\mathbf{A} = \mathbf{U}'\mathbf{U} + \left(\sigma_\gamma^2\mathbf{I}\right)^{-1}$, $\mathbf{b}' = \mathbf{v}'_{\gamma,i}\mathbf{U} + \boldsymbol{\mu}'_\gamma\left(\sigma_\gamma^2\mathbf{I}\right)^{-1}$, $\mathbf{v}'_{\gamma,i} = \{v_{\gamma,ijkt}, \forall j, k, t : z_{ijt} = 1\}$, and $\mathbf{U} = \left\{\mathbf{u}'_{jkt}, \forall j, k, t : z_{ijt} = 1\right\}$. Note that the full condition for $\boldsymbol{\gamma}_i$ only includes instances of j, k, and t such that $z_{ijt} = 1$.

4.7 Detection coefficients ($\boldsymbol{\alpha}_i$)

$$
\begin{aligned}
[\boldsymbol{\alpha}_i \mid \cdot] &\propto \prod_{j=1}^{R}\prod_{k=1}^{M_j}\prod_{t=1}^{T_{jk}} [y_{ijkt} \mid p_{ijkt}, a_{ijkt}] \left[\boldsymbol{\alpha}_i \mid \boldsymbol{\mu}_\alpha, \sigma_\alpha^2\right] \\
&\propto \prod_{j=1}^{R}\prod_{k=1}^{M_j}\prod_{t=1}^{T_{jk}} \text{Binom}\left(y_{ijkt} \mid J_{jkt}, p_{ijkt}\right)^{a_{ijkt}} \mathcal{N}\left(\boldsymbol{\alpha}_i \mid \boldsymbol{\mu}_\alpha, \sigma_\alpha^2\mathbf{I}\right).
\end{aligned}
$$

The update for $\boldsymbol{\alpha}_i$ proceeds using Metropolis-Hastings. Note that the product over j, k, and t only includes instances of j, k , and t such that $a_{ijkt} = 1$.

4.8 Mean of occupancy coefficients ($\boldsymbol{\mu}_\beta$)

$$
\begin{aligned}
[\boldsymbol{\mu}_\beta \mid \cdot] &\propto \prod_{i=1}^{n} [\boldsymbol{\beta}_i \mid \boldsymbol{\mu}_\beta, \sigma_\beta^2] \, [\boldsymbol{\mu}_\beta] \\
&\propto \prod_{i=1}^{n} \mathcal{N}\left(\boldsymbol{\beta}_i \mid \boldsymbol{\mu}_\beta, \sigma_\beta^2 \mathbf{I}\right) \mathcal{N}\left(\boldsymbol{\mu}_\beta \mid \mathbf{0}, \sigma_{\mu_\beta}^2 \mathbf{I}\right) \\
&\propto \exp\left\{\sum_{i=1}^{n}\left(-\frac{1}{2}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}_\beta\right)'\left(\sigma_\beta^2 \mathbf{I}\right)^{-1}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}_\beta\right)\right)\right\} \exp\left\{-\frac{1}{2}\left(\boldsymbol{\mu}_\beta - \mathbf{0}\right)'\left(\sigma_{\mu_\beta}^2 \mathbf{I}\right)^{-1}\left(\boldsymbol{\mu}_\beta - \mathbf{0}\right)\right\} \\
&\propto \exp\left\{-\frac{1}{2}\left(-2\left(\sum_{i=1}^{n} \boldsymbol{\beta}_i'\left(\sigma_\beta^2 \mathbf{I}\right)^{-1}\right)\boldsymbol{\mu}_\beta + \boldsymbol{\mu}_\beta'\left(n\left(\sigma_\beta^2 \mathbf{I}\right)^{-1}\right)\boldsymbol{\mu}_\beta\right)\right\} \exp\left\{-\frac{1}{2}\left(\boldsymbol{\mu}_\beta'\left(\sigma_{\mu_\beta}^2 \mathbf{I}\right)^{-1}\boldsymbol{\mu}_\beta\right)\right\} \\
&\propto \exp\left\{-\frac{1}{2}\left(-2\left(\sum_{i=1}^{n} \boldsymbol{\beta}_i'\left(\sigma_\beta^2 \mathbf{I}\right)^{-1}\right)\boldsymbol{\mu}_\beta + \boldsymbol{\mu}_\beta'\left(n\left(\sigma_\beta^2 \mathbf{I}\right)^{-1} + \left(\sigma_{\mu_\beta}^2 \mathbf{I}\right)^{-1}\right)\boldsymbol{\mu}_\beta\right)\right\} \\
&= \mathcal{N}(\mathbf{A}^{-1}\mathbf{b}, \mathbf{A}^{-1}),
\end{aligned}
$$

where $\mathbf{A} = n\left(\sigma_\beta^2 \mathbf{I}\right)^{-1} + \left(\sigma_{\mu_\beta}^2 \mathbf{I}\right)^{-1}$, $\mathbf{b}' = \boldsymbol{\beta}'\left(\sigma_\beta^2 \mathbf{I}\right)^{-1}$, and $\boldsymbol{\beta}$ is the vector sum $\sum_{i=1}^{n} \boldsymbol{\beta}_i$.

4.9 Mean of use coefficients ($\boldsymbol{\mu}_\gamma$)

$$
\begin{aligned}
[\boldsymbol{\mu}_\gamma \mid \cdot] &\propto \prod_{i=1}^{n} [\boldsymbol{\gamma}_i \mid \boldsymbol{\mu}_\gamma, \sigma_\gamma^2] \, [\boldsymbol{\mu}_\gamma] \\
&\propto \prod_{i=1}^{n} \mathcal{N}\left(\boldsymbol{\gamma}_i \mid \boldsymbol{\mu}_\gamma, \sigma_\gamma^2 \mathbf{I}\right) \mathcal{N}\left(\boldsymbol{\mu}_\gamma \mid \mathbf{0}, \sigma_{\mu_\gamma}^2 \mathbf{I}\right) \\
&= \mathcal{N}(\mathbf{A}^{-1}\mathbf{b}, \mathbf{A}^{-1}),
\end{aligned}
$$

where $\mathbf{A} = n\left(\sigma_\gamma^2 \mathbf{I}\right)^{-1} + \left(\sigma_{\mu_\gamma}^2 \mathbf{I}\right)^{-1}$, $\mathbf{b}' = \boldsymbol{\gamma}'\left(\sigma_\gamma^2 \mathbf{I}\right)^{-1}$, and γ is the vector sum $\sum_{i=1}^{n} \boldsymbol{\gamma}_i$.

4.10 Mean of detection coefficients ($\boldsymbol{\mu}_\alpha$)

$$
\begin{aligned}
[\boldsymbol{\mu}_\alpha \mid \cdot] &\propto \prod_{i=1}^{n} [\boldsymbol{\alpha}_i \mid \boldsymbol{\mu}_\alpha, \sigma_\alpha^2] \, [\boldsymbol{\mu}_\alpha] \\
&\propto \prod_{i=1}^{n} \mathcal{N}\left(\boldsymbol{\alpha}_i \mid \boldsymbol{\mu}_\alpha, \sigma_\alpha^2 \mathbf{I}\right) \mathcal{N}\left(\boldsymbol{\mu}_\alpha \mid \mathbf{0}, \sigma_{\mu_\alpha}^2 \mathbf{I}\right) \\
&= \mathcal{N}(\mathbf{A}^{-1}\mathbf{b}, \mathbf{A}^{-1}),
\end{aligned}
$$

where $\mathbf{A} = n\left(\sigma_\alpha^2 \mathbf{I}\right)^{-1} + \left(\sigma_{\mu_\alpha}^2 \mathbf{I}\right)^{-1}$, $\mathbf{b}' = \boldsymbol{\alpha}'\left(\sigma_\alpha^2 \mathbf{I}\right)^{-1}$, and $\boldsymbol{\alpha}$ is the vector sum $\sum_{i=1}^{n} \boldsymbol{\alpha}_i$.

4.11 Variance of occupancy coefficients (σ^2_β)

$$\begin{aligned}
\left[\sigma^2_\beta \mid \cdot\right] &\propto \prod_{i=1}^{n} \left[\boldsymbol{\beta}_i \mid \boldsymbol{\mu}_\beta, \sigma^2_\beta\right] \left[\sigma^2_\beta\right] \\
&\propto \prod_{i=1}^{n} \mathcal{N}\left(\boldsymbol{\beta}_i \mid \boldsymbol{\mu}_\beta, \sigma^2_\beta \mathbf{I}\right) \mathrm{IG}\left(\sigma^2_\beta \mid r, q\right) \\
&\propto \prod_{i=1}^{n} \left|\sigma^2_\beta \mathbf{I}\right|^{-1/2} \exp\left\{-\frac{1}{2}\left(\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}_\beta\right)' \left(\sigma^2_\beta \mathbf{I}\right)^{-1} \left(\boldsymbol{\beta}_i - \boldsymbol{\mu}_\beta\right)\right)\right\} \left(\sigma^2_\beta\right)^{-(q+1)} \exp\left\{-\frac{1}{r\sigma^2_\beta}\right\} \\
&\propto \left(\sigma^2_\beta\right)^{-(qX \times n)/2} \exp\left\{\sum_{i=1}^{n}\left(-\frac{1}{2\sigma^2_\beta}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}_\beta\right)' \left(\boldsymbol{\beta}_i - \boldsymbol{\mu}_\beta\right)\right)\right\} \left(\sigma^2_\beta\right)^{-(q+1)} \exp\left\{-\frac{1}{r\sigma^2_\beta}\right\} \\
&\propto \left(\sigma^2_\beta\right)^{-((qX \times n)/2+q+1)} \exp\left\{-\frac{1}{\sigma^2_\beta}\left(\frac{\sum_{i=1}^{n}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}_\beta\right)' \left(\boldsymbol{\beta}_i - \boldsymbol{\mu}_\beta\right)}{2} + \frac{1}{r}\right)\right\} \\
&= \mathrm{IG}\left(\left(\frac{\sum_{i=1}^{n}\left(\boldsymbol{\beta}_i - \boldsymbol{\mu}_\beta\right)' \left(\boldsymbol{\beta}_i - \boldsymbol{\mu}_\beta\right)}{2} + \frac{1}{r}\right)^{-1}, \frac{qX \times n}{2} + q\right),
\end{aligned}$$

where qX is the column dimension of $\mathbf{X}$ (or length of $\boldsymbol{\beta}_i$).

4.12 Variance of use coefficients (σ^2_γ)

$$\begin{aligned}
\left[\sigma^2_\gamma \mid \cdot\right] &\propto \prod_{i=1}^{n} \left[\boldsymbol{\gamma}_i \mid \boldsymbol{\mu}_\gamma, \sigma^2_\gamma\right] \left[\sigma^2_\gamma\right] \\
&\propto \prod_{i=1}^{n} \mathcal{N}\left(\boldsymbol{\gamma}_i \mid \boldsymbol{\mu}_\gamma, \sigma^2_\gamma \mathbf{I}\right) \mathrm{IG}\left(\sigma^2_\gamma \mid r, q\right) \\
&= \mathrm{IG}\left(\left(\frac{\sum_{i=1}^{n}\left(\boldsymbol{\gamma}_i - \boldsymbol{\mu}_\gamma\right)' \left(\boldsymbol{\gamma}_i - \boldsymbol{\mu}_\gamma\right)}{2} + \frac{1}{r}\right)^{-1}, \frac{qU \times n}{2} + q\right),
\end{aligned}$$

where qU is the column dimension of $\mathbf{U}$ (or length of $\boldsymbol{\gamma}_i$).

4.13 Variance of detection coefficients (σ^2_α)

$$\begin{aligned}
\left[\sigma^2_\alpha \mid \cdot\right] &\propto \prod_{i=1}^{n} \left[\boldsymbol{\alpha}_i \mid \boldsymbol{\mu}_\alpha, \sigma^2_\alpha\right] \left[\sigma^2_\alpha\right] \\
&\propto \prod_{i=1}^{n} \mathcal{N}\left(\boldsymbol{\alpha}_i \mid \boldsymbol{\mu}_\alpha, \sigma^2_\alpha \mathbf{I}\right) \mathrm{IG}\left(\sigma^2_\alpha \mid r, q\right) \\
&= \mathrm{IG}\left(\left(\frac{\sum_{i=1}^{n}\left(\boldsymbol{\alpha}_i - \boldsymbol{\mu}_\alpha\right)' \left(\boldsymbol{\alpha}_i - \boldsymbol{\mu}_\alpha\right)}{2} + \frac{1}{r}\right)^{-1}, \frac{qW \times n}{2} + q\right),
\end{aligned}$$

where qW is the column dimension of $\mathbf{W}$ (or length of $\boldsymbol{\alpha}_i$).