Inhomogeneous Poisson Point Process Model for 2-Dimensional (Spatial) Data

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Description

An inhomogeneous Poisson point process model for spatial locations.

Implementation

The file spatial.ppp.sim.R simulates data according to the model statement presented below, and spatial.ppp.mcmc.R contains the MCMC algorithm for model fitting.

Model statement

Let $\mathbf{s}(t) = (s_x(t), s_y(t))'$, for $t \in \mathcal{T}$, be observed spatial locations. Also let $\mathbf{x}(\mathbf{s}(t))$ be a vector of covariates associated with the location $\mathbf{s}(t)$ for which inference is desired, and the vector $\boldsymbol{\beta}$ be the corresponding coefficients.

$$\begin{aligned} \mathbf{s}(t) &\sim \frac{\exp\left(\mathbf{x}(\mathbf{s}(t))'\boldsymbol{\beta}\right)}{\int \exp\left(\mathbf{x}(\mathbf{s})'\boldsymbol{\beta}\right) d\mathbf{s}} \\ \boldsymbol{\beta} &\sim \mathcal{N}(\mathbf{0}, \sigma_\beta^2 \mathbf{I}) \end{aligned}$$

Full conditional distributions

Regression coefficients ($\boldsymbol{\beta}$):

$$\begin{aligned} [\boldsymbol{\beta}|\cdot] &\propto \prod_{t \in \mathcal{T}} [\mathbf{s}(t)|\boldsymbol{\beta}][\boldsymbol{\beta}] \\ &\propto \prod_{t \in \mathcal{T}} \left(\frac{\exp\left(\mathbf{x}(\mathbf{s}(t))'\boldsymbol{\beta}\right)}{\int \exp\left(\mathbf{x}(\mathbf{s})'\boldsymbol{\beta}\right) d\mathbf{s}} \right) \mathcal{N}(\boldsymbol{\beta}|\mathbf{0}, \sigma_\beta^2 \mathbf{I}). \end{aligned}$$

The update for $\boldsymbol{\beta}$ proceeds using Metropolis-Hastings.