Device #1

$$y_{1ij} \sim \begin{cases} 0, & z_i = 0 & \text{— not occupied} \\ 0, & z_i = 1, \ u_{1ij} < 0 & \text{— occupied, not detected} \\ 1, & z_i = 1, \ u_{1ij} > 0 & \text{— occupied, detected} \end{cases}$$

Device #2

$$y_{2ij} \sim \begin{cases} 0, & z_i = 0, \\ 0, & z_i = 1, \ u_{2ij} < 0 \\ 1, & z_i = 1, \ u_{2ij} > 0 \end{cases}$$

$$z_i \sim \begin{cases} 0, & v_i < 0 \\ 1, & v_i > 0 \end{cases}$$

$$u_{1ij} \sim N(\underline{w}_{ij}' \underline{\alpha}_1, 1)$$

$$u_{2ij} \sim N(\underline{w}_{ij}' \underline{\alpha}_2, 1)$$

site and time-varying covariates on detection probability

$$V_i \sim N(\underline{x}_i' \underline{\beta}, 1)$$

$$\underline{\alpha}_1 \sim N(\underline{\mu}_{\alpha_1}, \Sigma_{\alpha_1})$$

$$\underline{\alpha}_2 \sim N(\underline{\mu}_{\alpha_2}, \Sigma_{\alpha_2})$$

$$\underline{\beta} \sim N(\mu_\beta, \Sigma_\beta)$$

$$z_{i1} \sim \text{Bern}(\psi_1)$$

$$z_{ij} \mid z_{i,j-1} \sim \text{Bern}(\pi(i,j)) \qquad j = 2, \ldots, J$$

$$\pi(i,j) = z_{i,j-1} \underbrace{\phi_{j-1}}_{\text{Probability of survival}} + [1 - z_{i,j-1}] \underbrace{\gamma_{j-1}}_{\text{local colonization probability}}$$

$$\text{logit}(\pi_{ij}) = a_j + b_j z_{i,j-1}$$

$\cancel{\boxed{y_{i\cdot\cdot} = 0}}$

$$[z_i \mid \cdot\,] \propto \prod_{j=1}^{J_{1i}} [y_{1ij} \mid z_i, u_{1ij}] \prod_{j=1}^{J_{2i}} [y_{2ij} \mid z_i, u_{2ij}]\, [z_i \mid v_i]$$

$$\propto \prod_{j=1}^{J_{1i}} 1_{\{y_{1ij}=0\}}^{1-z_i}\, \text{Bern}(y_{1ij} \mid p_{1ij})^{z_i} \prod_{j=1}^{J_{2i}} 1_{\{y_{2ij}=0\}}^{1-z_i}\, \text{Bern}(y_{2ij} \mid p_{2ij})^{z_i}$$

$$\times\ \psi_i^{z_i} (1-\psi_i)^{1-z_i}$$

$$\propto \underbrace{\prod_{j=1}^{J_{1i}} \left(p_{1ij}^{y_{1ij}} (1-p_{1ij})^{1-y_{1ij}}\right)^{z_i}}_{y_{1i\cdot}=0} \underbrace{\prod_{j=1}^{J_{2i}} \left(p_{2ij}^{y_{2ij}} (1-p_{1ij})^{1-y_{2ij}}\right)^{z_i}}_{y_{2i\cdot}=0}$$

$$\times\ \psi_i^{z_i} (1-\psi_i)^{1-z_i}$$

$$\propto \left(\psi_i \prod_{j=1}^{J_{1i}} (1-p_{1ij}) \prod_{j=1}^{J_{2i}} (1-p_{2ij})\right)^{z_i} (1-\psi_i)^{1-z_i}$$

$$[\beta|\cdot] \propto \prod_{i=1}^{n} \text{Bern}(z_i \mid \Phi(x_i'\beta))\, N(\beta \mid \mu_\beta, \Sigma_\beta)$$

$$\propto \prod_{j=1}^{J_{1i}} \left(p_{1ij}^{y_{1ij}} (1-p_{1ij})^{1-y_{1ij}}\right)^{z_i} \prod_{j=1}^{J_{2i}} \left(p_{2ij}^{y_{2ij}} (1-p_{2ij})^{1-y_{2ij}}\right)^{z_i} \psi_i^{z_i}$$

$$\times (1-\psi_i)^{1-z_i}\, 1_{\{y_{1ij}=0\}}^{1-z_i}\, 1_{\{y_{2ij}=0\}}^{1-z_i}$$

$$\propto \left(\psi_i \prod_{j=1}^{J_{1i}} \left(p_{1ij}^{y_{1ij}} (1-p_{1ij})^{1-y_{1ij}}\right) \prod_{j=1}^{J_{2i}} \left(p_{2ij}^{y_{2ij}} (1-p_{2ij})^{1-y_{2ij}}\right)\right)^{z_i}$$

$$\times \left((1-\psi_i)\left(\prod_{j=1} 1_{\{y_{1ij}=0\}}\right)\left(\prod_{j=1}^{J_{2i}} 1_{\{y_{2ij}=0\}}\right)\right)^{1-z_i}$$

$$y_{ij} \sim \begin{cases} 0 & , \ z_i = 0 \\ \text{Binom}(J, \underbrace{p_i}_{\text{detection probability}}) & , \ \underbrace{z_i = 1}_{\text{true occupancy}} \end{cases}$$

$$z_i \sim \text{Bern}(\psi_i)$$

$$\text{logit}(p) = w\alpha$$

$$\psi_i = \Phi(X\beta)$$

$$\underline{\alpha} \sim N(\mu_\alpha, \Sigma_\alpha)$$

$$\underline{\beta} \sim N(\mu_\beta, \Sigma_\beta)$$

$$\boxed{y_i=0} \quad [z_i \mid \cdot\,] \propto [y_i \mid \underline{\alpha}, z_i][z_i \mid \underline{\beta}]$$

$$\propto \left(\text{Binom}(y_i \mid J, \text{logit}^{-1}(\underline{w}_i\underline{\alpha}))\right)^{z_i} 1_{\{y_i=0\}}^{1-z_i} \text{Bern}(z_i \mid \Phi(\underline{x}'\underline{\beta}))$$

$$\propto \left(\left(1-\text{logit}^{-1}(\underline{w}_i'\underline{\alpha})\right)^J\right)^{z_i} \times \left(\Phi(\underline{x}_i'\underline{\beta})^{z_i}\right)\left(1-\Phi(\underline{x}_i'\underline{\beta})\right)^{1-z_i}$$

$$\propto \left(\Phi(\underline{x}_i'\beta)\left(1-\text{logit}^{-1}(\underline{w}_i'\underline{\alpha})\right)^J\right)^{z_i} \left(1-\Phi(\underline{x}_i'\beta)\right)^{1-z_i}$$

sum to 1

$$\text{Bern}\left(\frac{\Phi(\underline{x}_i'\beta)\left(1-\text{logit}^{-1}(\underline{w}_i'\underline{\alpha})\right)^J}{\Phi(\underline{x}_i'\beta)\left(1-\text{logit}^{-1}(\underline{w}_i'\underline{\alpha})\right)^J + \left(1-\Phi(\underline{x}_i'\beta)\right)}\right)$$