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CS3343 Section 01
15 Oct 2017

CS3343 Analysis of Algorithms Fall 2017

Homework 4

Due 10/15/17 before 11:59pm (Central Time)

1. Heapsort (4 points)

Originally we stored our heap in an array. Consider instead storing our heap as a doubly linked list.

(1) (2 points) For a node i what are the new asymptotic run times for $left(i)$, $right(i)$, and $parent(i)$? Justify your answer. **Answers** :
   (a) $left(i)$ is $O(i)$ because the node locations in memory are unknown since it's a linked list. This restricts us from being able to jump straight to the left child.
   (b) $right(i)$ is $O(i)$ for the same reason $left(i)$ is $O(i)$
   (c) $parent(i)$ is $O(i/2)$ because we know the parent will come before the children in the linked list. However, $O(i/2)$ is still $O(i)$

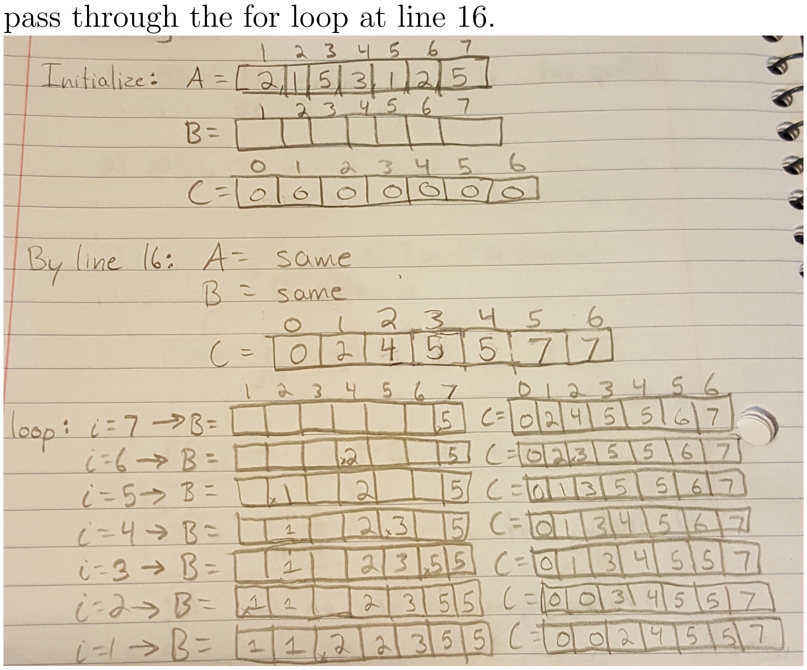
(2) (2 points) How does this affect the run times of findMax(), insert(key), extractMax()? Justify your answer.
   Answers :
   (a) findMAX() is $O(1)$ because it will always be the head node of our doubly linked list. In a linked list, the head node is always known.
   (b) insert(key) is $O(i^2)$ because we first have to insert the new key at the end of the linked list which will take i operations. Then we must heapify which will take another i^2 times going through it to reorder all the nodes.
   (c) extractMAX() is $O(i^2)$ Because we must traverse to the last element, swap it with the head node which is $i + 1$ times, then heapify which will require i^2 operations.

2. Counting Sort (4 points)

Algorithm 1 void countingSort(int $A[1 \ldots n]$, int k)

```
1:  //Precondition: The n values in A are all between 0 and k
2:  Let C[0...k] be a new array
3:  //We will store our sorted array in the array B.
4:  Let B[1...n] be a new array
5:  for i = 0 to k do
6:      C[i] = 0;
7:  end for
8:  for i = 1 to n do
9:      C[A[i]] = C[A[i]] + 1;
10: end for
11: //C[i] now contains the # of elements in A equal to i
12: for i = 1 to k do
13:     C[i] = C[i] + C[i - 1];
14: end for
15: //C[i] now contains the # of elements in A that are ≤ to i
16: for i = n down to 1 do
17:     B[C[A[i]]] = A[i];
18:     C[A[i]] = C[A[i]] - 1;
19: end for
20: return B;
```

(1) (2 points) Illustrate the operation of *countingSort*($\{2, 1, 5, 3, 1, 2, 5\}, 6$). Specifically, show the changes made to the arrays A, B, and C for each pass through the for loop at line 16.



(2) (2 points) Describe an algorithm that, given an unsorted array of n integers in the range 0 to k, preprocesses its input and then answers any query about how many of the n integers fall into a range a to b in $O(1)$ time. Your algorithm should use $O(n + k)$ preprocessing time.

(Hint: Look at the array C which is computed by the above code for inspiration)

Answer : By line 15 of the algoritm, the preprocessing time is at $O(n+k)$ which was given by the profesor. Also, at this point in the algorithm, the array $C = [0, 2, 4, 5, 5, 7, 7]$. By looking at this array, we can see the values at each $C[i]$ indicate how many numbers in the unsorted array are $\leq i$. Therefore, if we wanted to know how many integers are in the range between a and b in $O(1)$ time, we simply do:

```
if (a==0) then
    return C[b];
else
    return C[b] – C[a–1];
end if
```

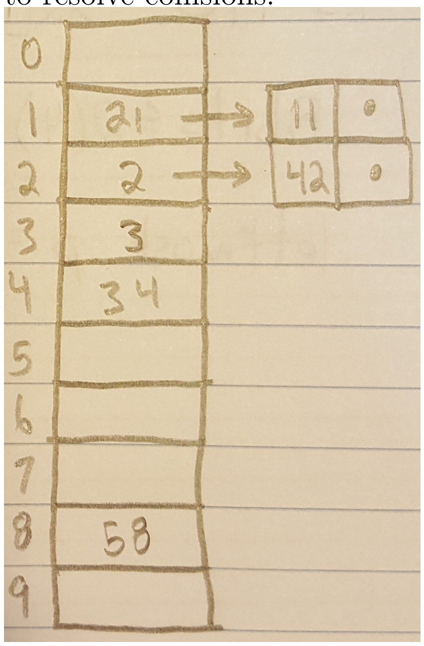
3. Hash Table (7 points)

(1) Consider inserting the keys $2, 21, 3, 58, 11, 42, 34$ into a hash table of length $m = 10$ with the hash function $h(k) = k \bmod 10$.

(a) (2 points) Illustrate the result of inserting these keys using linear probing to resolve collisions.

Index	**Key**
0	NULL
1	21
2	2
3	3
4	11
5	42
6	34
7	NULL
8	58
9	NULL

(b) (2 points) Illustrate the result of inserting these keys using chaining to resolve collisions.



(2) Consider inserting the keys $8, 5, 14$ into a hash table of length $m = 8$ with the hash function $h(k) = \lfloor m(kA - \lfloor kA \rfloor) \rfloor$ where $A = 0.625$.

(a) (2 points) Illustrate the result of inserting these keys.

Index	**Key**
0	8
1	5
2	NULL
3	NULL
4	NULL
5	NULL
6	14
7	NULL

(b) (1 point) Now compute the hash function of the key 14 using the implementation we described in our notes.

You can assume we have a word size $w = 4$. Since $m = 8 = 2^3$, $p = 3$. Since $A = 0.625 = 10/2^4 = 10/2^w$, $s = 10$.

(Hint: Compute ks and convert it to a binary number. This number will consist of $\leq 2w$ bits. Look at the rightmost w bits. Of those bits, convert the leftmost p bits back to an integer. This integer is your hash table slot.)

Answer:
$k = 14, s = 10, w = 4, p = 3$
$ks = (14)(10) = 140$
$140 = 10001100$ in binary which is $\leq 2w$ bits $\leq 2(4)$ bits ≤ 8 bits
Rightmost 4 bits $= 1100$
Leftmost $p = 3$ bits $= 110$
Binary $110 = 6$ which is the correct hash table slot for key 14