

# Appendix S1. Updating of latent encounter history frequencies in **multimark**

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1 The feasible set of latent encounter histories ( $\mathbf{Y}$ ) is explored in **multimark** using  
2 an extension of the MCMC algorithms proposed by Bonner & Holmberg (2013)  
3 and McClintock *et al.* (2013, 2014) that were originally conceived for a different  
4 application by Link *et al.* (2010). The new algorithm conditions on the observed  
5 data (thus reducing the dimension of the problem) and only proposes updates with  
6 non-negative latent encounter history frequencies,  $\mathbf{f} = (f_1, f_2, \dots, f_{5^T})$ . Let  $\mathbf{r}$  denote  
7 the set of  $4^T - 2^{T+1} + 1$  indices for latent encounter histories that spawn  $>1$  observed  
8 history, and let  $f_{j(1)}$  and  $f_{j(2)}$  denote the corresponding frequencies for type 1 and  
9 type 2 histories that arise from latent encounter history  $j \in \mathbf{r}$ . Referring back to  
10 Table 2 with  $T = 2$ ,  $\mathbf{r} = \{4, 8, 9, 12, 14, 16, 17, 18, 19\}$  and one potential update would  
11 involve frequencies for latent history ‘31’ ( $f_{17}$ ) and its progeny ‘11’ ( $f_{17(1)} = f_7$ ) and  
12 ‘20’ ( $f_{17(2)} = f_{11}$ ).

13 When conditioning on the observed encounter histories, the size of the prob-  
 14 lem is typically greatly reduced because many of the potential latent histories and  
 15 corresponding moves are not permissable. For example, with  $T = 2$ , if encounter  
 16 history ‘11’ was never observed, then  $f_7 = f_9 = f_{17} = f_{19} = 0$  can be ignored and  
 17  $j \in \{9, 17, 19\}$  can be removed from  $\mathbf{r}$  for subsequent computations.

18 Starting from a permissible  $\mathbf{f}$  conditional on the observed encounter histories, the  
 19 algorithm proceeds as follows:

- 20 1. Randomly draw a latent encounter history index  $r \in \mathbf{r}^*$ , where  $\mathbf{r}^*$  is the subset  
 21 of  $\mathbf{r}$  with corresponding frequencies that satisfy  $\min(f_1, f_j) + \min(f_{j(1)}, f_{j(2)}) > 0$   
 22 for  $j \in \mathbf{r}$ .
- 23 2. Randomly draw  $c_r$  from the integer set  $\{-\min(f_1, f_r), \dots, -1, 1, \dots, \min(f_{r(1)}, f_{r(2)})\}$ .
- 24 3. Propose  $f_r^* = f_r + c_r$ ,  $f_{r(1)}^* = f_{r(1)} - c_r$ , and  $f_{r(2)}^* = f_{r(2)} - c_r$ .
- 25 4. Apportion  $\mathbf{f}^*$  to individuals following McClintock *et al.* (2014), and accept  
 26 proposed move based on the Metropolis-Hastings ratio described therein [pp.  
 27 2470-2472, steps 9(b)-9(c)].

28 Note that because **multimark** uses “semi-complete” data likelihoods that con-  
 29 dition on the number of unique individuals encountered at least once ( $n$ ), the di-  
 30 mension of the data-augmented encounter histories ( $M$ ) described in McClintock  
 31 *et al.* (2014) is determined by the number of observed encounter histories (i.e.,  
 32  $M = n_1 + n_2 + n_{known}$ ), such that  $f_1 = M - n$  and  $n = \sum_{j=2}^{5^T} f_j$ . Any addi-  
 33 tional constraints, such as those resulting from encounter histories being designated  
 34 as known with certainty using the *known* argument in *processdata()*, are accounted  
 35 for by simple modifications to steps 1-2.

## 36 **References**

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