HW 4 Questions

1. Use topological sort

Step 1. Organize the prerequisites as a directed graph:

$LA15 \to LA16$ $LA22 \to LA126$

$LA15 \to LA31$ $LA22 \to LA141$

$LA16 \to LA32$ $LA31 \to LA32$

$LA16 \to LA127$ $LA32 \to LA126$

$LA16 \to LA141$ $LA32 \to LA169$

Step 2. Start with cources without preregs. After taking a cource, remove its edges and repeat.

Sequence:

LA 15, LA 22, LA 16, LA 31, LA 31, LA 127, LA 141, LA 126, LA 169

2. Imagine a weighted graph $V = \{A, B, C, D\}$ with weights:

$A \to B$ weight: 4   Start vertex = A

$A \to C$ weight: 2

$C \to B$ weight: -5

$B \to D$ weight: 3

$C \to D$ weight: 2

Dijkstra's algorithm fails because it assumes that, once a distance is found, it cannot be improved upon.

For example:

- Start with A

  $dist(A) = \emptyset, dist(B) = \infty, dist(C) = \infty, dist(D) = \infty$

Since $A \to C$ has a weight of 2:

$dist(C) = 2$

Since $A \to B$ has a weight of 4:

$dist(B) = 4$

However, since $C \to B$ has a weight of -5:

$dist(B) = \underset{dist(C)}{2} + (-5) = -3$. Therefore, Dijkstra's algorithm is wrong

3. No. Prof. Amongus has proved that L is polynomial-time reducable to an NP-complete problem M. However, in order to prove P=NP, it must be shown that every NP (not just M) can be solved in polynomial time.

4. To show that this problem is NP-Complete, we must show that this problem is NP and NP-Hard

To set up this problem, have $C\{c_1, c_2, c_3, \cdots c_n\}$ for the different companies with $G = (C, E)$ as a graph where E contains an edge (c_i, c_j) if c_i and c_j are competitors. Also, have int k. Determine if it is possible to invite at least k companies such that no two invited companies are competitors.

Step 1. Show the problem is NP:

A "certificate" for the problem is a set of k companies selected for invitation.

Check the solution by checking that the set contains k vertices with no edges (no two selected companies compete)

$\therefore$ the problem is NP

Step 2. Show the problem is NP-Hard

Independent Set Problem:

Given graph $G = \{C, E\}$ and int k, determine if an Independent set of size k in G exists

Take an instance of this independent set

Treat the vertices of G as the company, C, and the edges as the "competitor" relationship, E.

The independent set corresponds exactly to a set of companies that can be invited without 2 competitors.

Since the problem is NP & NP-Hard, the decision version is NP-complete.