Reading 2:
Resistive and Capacitive Sensors

This chapter deals with basic mechanisms and principles of the sensors used in a number of medical instruments. A *transducer* is a device that converts energy from one form to another. A *sensor* converts a physical parameter to an electric output. An *actuator* converts an electric signal to a physical output. An electric output from the sensor is normally desirable because of the advantages it gives in further signal processing (Pallas-Areny and Webster, 2001). As we shall see in this chapter, there are many methods used to convert physiological events to electric signals. Dimensional changes may be measured by variations in resistance, inductance, capacitance, and piezoelectric effect. Thermistors and thermocouples are employed to measure body temperatures. Electromagnetic-radiation sensors include thermal and photon detectors. In our discussion of the design of medical instruments in the following chapters, we shall use the principles described in this chapter (Togawa *et al*., 1997).

2.1 DISPLACEMENT MEASUREMENTS

The physician and biomedical researcher are interested in measuring the size, shape, and position of the organs and tissues of the body. Variations in these parameters are important in discriminating normal from abnormal function. Displacement sensors can be used in both direct and indirect systems of measurement. Direct measurements of displacement are used to determine the change in diameter of blood vessels and the changes in volume and shape of cardiac chambers.

Indirect measurements of displacement are used to quantify movements of liquids through heart valves. An example is the movement of a microphone diaphragm that detects the movement of the heart indirectly and the resulting heart murmurs.

Here we will describe the following types of displacement-sensitive measurement methods: resistive, inductive, capacitive, and piezoelectric (Nyce, 2004).

2.2 RESISTIVE SENSORS

POTENTIOMETERS

Figure 2.1 shows three types of potentiometric devices for measuring displacement. The potentiometer shown in Figure 2.1(a) measures translational displacements from 2 to 500 mm. Rotational displacements ranging from 10° to more than 50° are detected as shown in Figure 2.1(b) and (c). The resistance elements (composed of wire-wound, carbon-film, metal-film, conducting-plastic, or ceramic material) may be excited by either dc or ac voltages. These potentiometers produce a linear output (within 0.01% of full scale) as a function of displacement, provided that the potentiometer is not electrically loaded.

The resolution of these potentiometers is a function of the construction. It is possible to achieve a continuous stepless conversion of resistance for low-resistance values up to 10 Ω by utilizing a straight piece of wire. For greater variations in resistance, from several ohms to several megohms, the resistance wire is wound on a mandrel or card. The variation in resistance is thereby not continuous, but rather stepwise, because the wiper moves from one turn of wire to the next. The fundamental limitation of the resolution is a function of the wire spacing, which may be as small as 20 μm. The frictional and inertial components of these potentiometers should be low in order to minimize dynamic distortion of the system.

STRAIN GAGES

When a fine wire (25 μm) is strained within its elastic limit, the wire's resistance changes because of changes in the diameter, length, and resistivity. The resulting strain gages may be used to measure extremely small displacements, on the order of nanometers. The following derivation shows how each of these parameters influences the resistance change. The basic equation for the

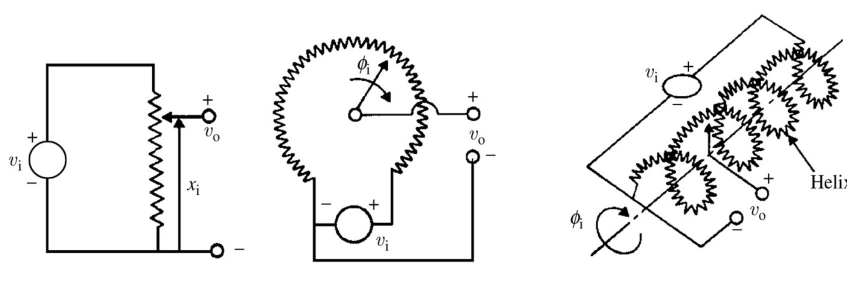

Figure 2.1 Three types of potentiometric devices for measuring displacements (a) Translational. (b) Single-turn, (c) Multiturn. (From *Measurement Systems: Application and Design*, by E. O. Doebelin. Copyright © 1990 by McGraw-Hill, Inc. Used with permission of McGraw-Hill Book Co.)

resistance R of a wire with resistivity ρ (ohm·meter), length L (meters), and cross-sectional area A (meter squared) is given by

$$R = \frac{\rho L}{A} \tag{2.1}$$

The differential change in R is found by taking the differential

$$dR = \frac{\rho dL}{A} - \rho A^{-2} L\, dA + L\frac{d\rho}{A} \tag{2.2}$$

We shall modify this expression so that it represents finite changes in the parameters and is also a function of standard mechanical coefficients. Thus dividing members of (2.2) by corresponding members of (2.1) and introducing incremental values, we get

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} - \frac{\Delta A}{A} + \frac{\Delta\rho}{\rho} \tag{2.3}$$

Poisson's ratio μ. relates the change in diameter ΔD to the change in length: $\Delta D/D = -\mu\,\Delta L/L$. Substituting this into the center term of (2.3) yields

$$\frac{\Delta R}{R} = \underbrace{(1+2\mu)\frac{\Delta L}{L}}_{\text{Dimensional effect}} + \underbrace{\frac{\Delta\rho}{\rho}}_{\text{Piezoresistive effect}} \tag{2.4}$$

Note that the change in resistance is a function of changes in dimension—length $(\Delta L/L)$ and area $(2\mu\,\Delta L/L)$—plus the change in resistivity due to strain-induced changes in the lattice structure of the material, $\Delta\rho/\rho$. The gage factor G, found by dividing (2.4) by $\Delta L/L$, is useful in comparing various strain-gage materials.

$$G = \frac{\Delta R/R}{\Delta L/L} = (1+2\mu) + \frac{\Delta\rho/\rho}{\Delta L/L} \tag{2.5}$$

Table 2.1 gives the gage factors and temperature coefficient of resistivity of various strain-gage materials. Note that the gage factor for semiconductor materials is approximately 50 to 70 times that of the metals.

Also note that the gage factor for metals is primarily a function of dimensional effects. For most metals, $\mu = 0.3$ and thus G is at least 1.6, whereas for semiconductors, the piezoresistive effect is dominant. The desirable feature of higher gage factors for semiconductor devices is offset by their greater resistivity–temperature coefficient.

Designs for instruments that use semiconductor materials must incorporate temperature compensation.

Strain gages can be classified as either unbonded or bonded. An unbonded strain-gage unit is shown in Figure 2.2(a). The four sets of strain-sensitive wires are connected to form a Wheatstone bridge, as shown in Figure 2.2(b). These wires are mounted under stress between the frame and the movable armature such that preload is greater than any expected external compressive load. This is necessary to avoid putting the wires in compression. This type of sensor may be used for converting blood pressure to diaphragm movement, to resistance change, then to an electric signal.

A bonded strain-gage element, consisting of a metallic wire, etched foil, vacuum-deposited film, or semiconductor bar, is cemented to the strained surface. Figure 2.3 shows typical bonded strain gages. The deviation from linearity is approximately 1%. One method of temperature compensation for the natural temperature sensitivity of bonded strain gages involves using a second strain gage as a dummy element that is also exposed to the temperature variation, but not to strain. When possible, the four-arm bridge shown in Figure 2.2 should be used, because it not only provides temperature compensation but also yields four times greater output if all four arms contain active gages. Four bonded metal strain gages can be used on cantilever beams to measure bite force in dental research (Dechow, 2006).

Table 2.1 Properties of Strain-Gage Materials

Material	Composition (%)	Gage Factor	Temperature Coefficient of Resistivity ($°C^{-1}$–10^{-5})
Constantan (advance)	Ni_{45}, Cu_{55}	2.1	±2
Isoelastic	Ni_{36}, Cr_8 $(Mn, Si, Mo)_4$ Fe_{52}	3.52 to 3.6	+17
Karma	Ni_{74}, Cr_{20}, Fe_3 Cu_3	2.1	+2
Manganin	Cu_{84}, Mn_{12}, Ni_4	0.3 to 0.47	±2
Alloy 479	Pt_{92},W_8	3.6 to 4.4	+24
Nickel	Pure	−12 to −20	670
Nichrome V	Ni_{80}, Cr_{20}	2.1 to 2.63	10
Silicon	(p type)	100 to 170	70 to 700
Silicon	(n type)	−100 to −140	70 to 700
Germanium	(p type)	102	
Germanium	(n type)	−150	

SOURCE: From R. S. C. Cobbold, *Transducers for Biomedical Measurements*, 1974, John Wiley & Sons, Inc.. Used with permission of John Wiley & Sons, Inc., New York.

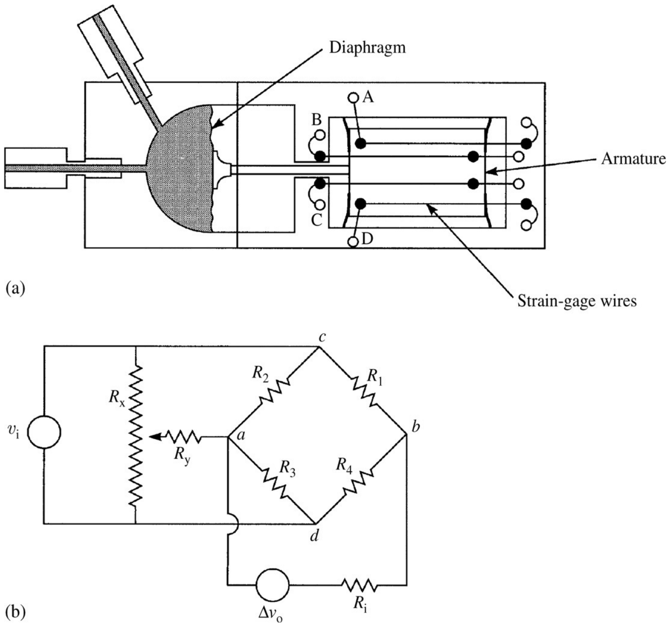

Figure 2.2 (a) Unbonded strain-gage pressure sensor. The diaphragm is directly coupled by an armature to an unbonded strain-gage system. With increasing pressure, the strain on gage pair B and C is increased, while that on gage pair A and D is decreased. (b) Wheatstone bridge with four active elements: $R_1 = \mathrm{B}$, $R_2 = \mathrm{A}$, $R_3 = \mathrm{D}$, and $R_4 = \mathrm{C}$ when the unbonded strain gage is connected for translational motion. Resistor R_y and potentiometer R_x are used to initially balance the bridge, V_i is the applied voltage, and Δv_o is the output voltage on a voltmeter or similar device with an internal resistance of R_i.

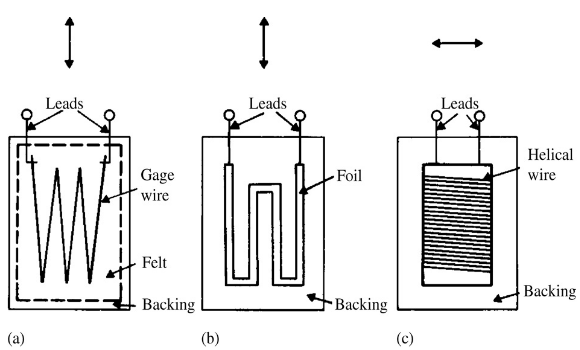

Figure 2.3 **Typical bonded strain-gage units** (a) resistance-wire type, (b) foil type, (c) helical-wire type. Arrows above units show direction of maximal sensitivity to strain. [Parts (a) and (b) are modified from *Instrumentation in Scientific Research*, K. S. Lion. Copyright © 1959 by McGraw-Hill, Inc. Used with permission of McGraw-Hill Book Co.]

2.3 BRIDGE CIRCUITS

The Wheatstone bridge circuit is ideal for measuring small changes in resistance. Figure 2.2(b) shows a Wheatstone bridge with an applied dc voltage of v_i and a readout meter Δv_o with internal resistance R_i. It can be shown by the voltage-divider approach that Δv_o is zero—that is, the bridge is balanced—when $R_1/R_2 = R_4/R_3$.

Resistance-type sensors may be connected in one or more arms of a bridge circuit. The variation in resistance can be detected by measuring Δv_o with a differential amplifier feeding an analog-to-digital converter (ADC), which feeds a computer.

Assume that all values of resistance of the bridge are initially equal to R_0 and that $R_0 \ll R_1$. An increase in resistance, ΔR, of all resistances still results in a balanced bridge. However, if R_1 and R_3 increase by ΔR, and R_2 and R_4 decrease by ΔR, then

$$\Delta v_o = \frac{\Delta R}{R_0} v_i \tag{2.6}$$

Because of the symmetry a similar expression results if R_2 and R_4 increase by ΔR and R_1 and R_3 decrease by ΔR. Note that (2.6), for the four-active-arm bridge, shows that Δv_o is linearly related to ΔR. A nonlinearity in $\Delta R/R_0$ is present even when $R_0/R_1 = 0$.

It is common practice to incorporate a balancing scheme in the bridge circuit [see Figure 2.2(b)]. Resistor R_y and potentiometer R_x are used to change the initial resistance of one or more arms. This arrangement brings the bridge into balance so that zero voltage output results from "zero" (or *base-level*) input of the measured parameter.

To minimize loading effects, R_x is approximately 10 times the resistance of the bridge leg, and R_y limits the maximal adjustment. Strain-gage applications normally use a value of $R_y = 25$ times the resistance of the bridge leg. Alternating-current (ac) balancing circuits are more complicated because a reactive as well as a resistive imbalance must be compensated.

2.4 INDUCTIVE SENSORS

An inductance L can be used to measure displacement by varying any three of the coil parameters:

$$L = n^2 G \mu \tag{2.7}$$

where

n = number of turns of coil
G = geometric form factor
μ = effective permeability of the medium

Each of these parameters can be changed by mechanical means.

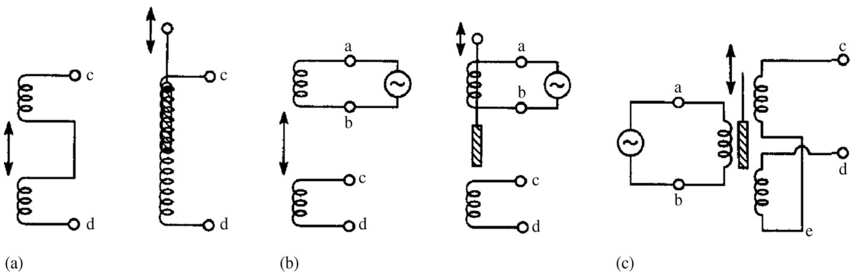

Figure 2.7 Inductive displacement sensors (a) self-inductance, (b) mutual inductance, (c) differential transformer.

Figure 2.7 shows (a) self-inductance, (b) mutual-inductance, and (c) differential transformer types of inductive displacement sensors. It is usually possible to convert a mutual-inductance system into a self-inductance system by series of parallel connections of the coils. Note in Figure 2.7 that the mutual-inductance device (b) becomes a self-inductance device (a) when terminals *b–c* are connected.

An inductive sensor has an advantage in not being affected by the dielectric properties of its environment. However, it may be affected by external magnetic fields due to the proximity of magnetic materials.

The variable-inductance method employing a single displaceable core is shown in Figure 2.7(a). This device works on the principle that alterations in the self-inductance of a coil may be produced by changing the geometric form factor or the movement of a magnetic core within the coil. The change in inductance for this device is not linearly related to displacement. The fact that these devices have low power requirements and produce large variations in inductance makes them attractive for radiotelemetry applications.

The mutual-inductance sensor employs two separate coils and uses the variation in their mutual magnetic coupling to measure displacement [Figure 2.7(b)]. Cobbold (1974) describes the application of these devices in measuring cardiac dimensions, monitoring infant respiration, and ascertaining arterial diameters.

Van Citters (1966) provides a good description of applications of mutual inductance transformers in measuring changes in dimension of internal organs (kidney, major blood vessels, and left ventricle). The induced voltage in the secondary coil is a function of the geometry of the coils (separation and axial alignment), the number of primary and secondary turns, and the frequency and amplitude of the excitation voltage. The induced voltage in the secondary coil is a nonlinear function of the separation of the coils. In order to maximize the output signal, a frequency is selected that causes the secondary coil (tuned circuit) to be in resonance. The output voltage is detected with standard demodulator and amplifier circuits.

The linear variable differential transformer (LVDT) is widely used in physiological research and clinical medicine to measure pressure, displacement, and force (Kesavan and Reddy, 2006). As shown in Figure 2.7(c), the LVDT is composed of a primary coil (terminals a–b) and two secondary coils (c–e and d–e) connected in series. The coupling between these two coils is changed by the motion of a high-permeability alloy slug between them. The two secondary coils are connected in opposition in order to achieve a wider region of linearity.

The primary coil is sinusoidally excited, with a frequency between 60 Hz and 20 kHz. The alternating magnetic field induces nearly equal voltages v_{ce} and v_{de} in the secondary coils. The output voltage $v_{cd} = v_{ce} - v_{de}$. When the slug is symmetrically placed, the two secondary voltages are equal and the output signal is zero.

Linear variable differential transformer characteristics include linearity over a large range, a change of phase by 180° when the core passes through the center position, and saturation on the ends. Specifications of commercially available LVDTs include sensitivities on the order of 0.5 to 2 mV for a displacement of 0.01 mm/V of primary voltage, full-scale displacement of 0.1 to 250 mm, and linearity of ±0.25%. Sensitivity for LVDTs is much higher than that for strain gages.

A disadvantage of the LVDT is that it requires more complex signal-processing instrumentation. Figure 2.8 shows that essentially the same

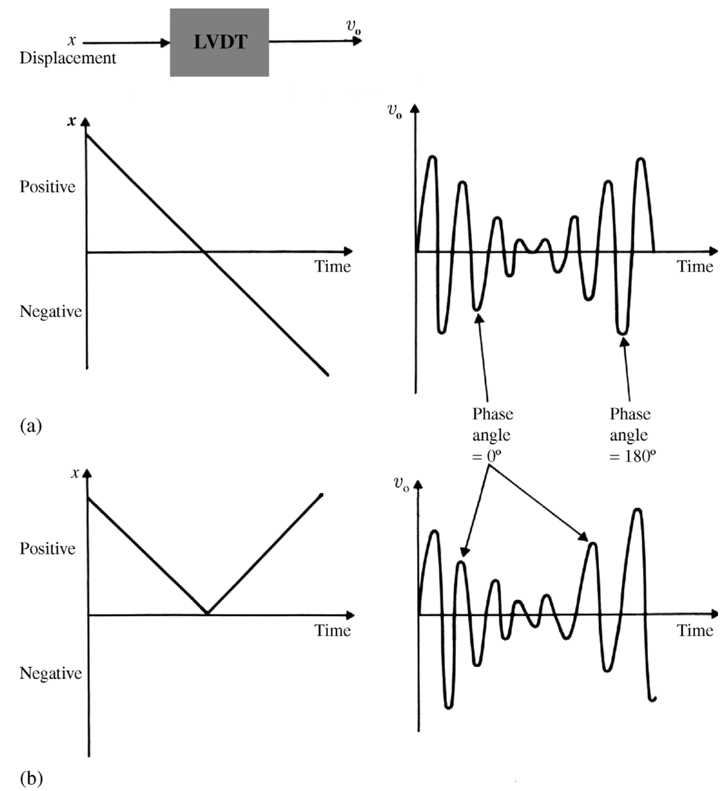

Figure 2.8 (a) As x moves through the null position, the phase changes 180°, while the magnitude of v_o is proportional to the magnitude of x. (b) An ordinary rectifier demodulator cannot distinguish between (a) and (b), so a phase-sensitive demodulator is required.

magnitude of output voltage results from two very different input displacements. The direction of displacement may be determined by using the fact that there is a 180° phase shift when the core passes through the null position. A phase-sensitive demodulator is used to determine the direction of displacement. Figure 3.17 shows a ring-demodulator system that could be used with the LVDT.

2.5 CAPACITIVE SENSORS

The capacitance between two parallel plates of area A separated by distance x is

$$C = \varepsilon_0 \varepsilon_r \frac{A}{x} \tag{2.8}$$

where ε_0 is the dielectric constant of free space (Appendix A.1) and ε_r is the relative dielectric constant of the insulator (1.0 for air) (Bowman and Meindl, 1988). In principle it is possible to monitor displacement by changing any of the three parameters ε_r, A, or x. However, the method that is easiest to implement and that is most commonly used is to change the separation between the plates.

The sensitivity K of a capacitive sensor to changes in plate separation Δx is found by differentiating (2.8).

$$K = \frac{\Delta C}{\Delta x} = -\varepsilon_0 \varepsilon_r \frac{A}{x^2} \tag{2.9}$$

Note that the sensitivity increases as the plate separation decreases.

By substituting (2.8) into (2.9), we can develop an expression showing that the percent change in C about any neutral point is equal to the per-unit change in x for small displacements. Thus

$$\frac{dC}{dx} = \frac{-C}{x} \tag{2.10}$$

or

$$\frac{dC}{C} = \frac{-dx}{x} \tag{2.11}$$

The capacitance microphone shown in Figure 2.9 is an excellent example of a relatively simple method for detecting variation in capacitance (Doebelin, 1990; Cobbold, 1974). This is a dc-excited circuit, so no current flows when the capacitor is stationary (with separation x_0), and thus $v_1 = E$. A change in

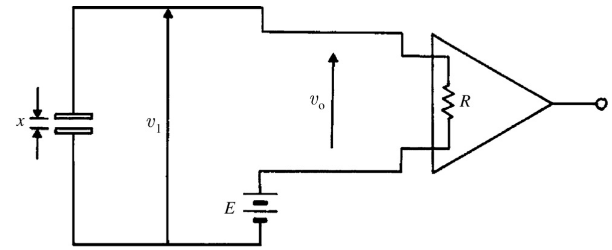

Figure 2.9 Capacitance sensor for measuring dynamic displacement changes.

position $\Delta x = x_1 - x_0$ produces a voltage $v_o = v_1 - E$. The output voltage V_o is related to x_1 by

$$\frac{V_o(j\omega)}{X_1(j\omega)} = \frac{(E/x_0)\, j\omega\tau}{j\omega\tau + 1} \tag{2.12}$$

where $\tau = RC = R\varepsilon_0\varepsilon_r A/x_0$.

Typically, R is 1 MΩ or higher, and thus the readout device must have a high (10 MΩ or higher) input impedance.

For $\omega\tau \gg 1$, $V_0(j\omega)/X_1(j\omega) \cong E/x_0$, which is a constant. However, the response drops off for low frequencies, and it is zero when $\omega = 0$. Thus (2.12) describes a high-pass filter. This frequency response is quite adequate for a microphone that does not measure sound pressures at frequencies below 20 Hz. However, it is inadequate for measuring most physiological variables because of their low-frequency components.

Compliant plastics of different dielectric constants may be placed between foil layers to form a capacitive mat to be placed on a bed. Patient movement generates charge, which is amplified and filtered to display respiratory movements from the lungs and ballistographic movements from the heart (Alihanka *et al,* 1982).

A capacitance sensor can be fabricated from layers of mica insulators sandwiched between corrugated metal layers. Applied pressure flattens the corrugations and moves the metallic plates closer to each other, thus increasing the capacitance. The sensor is not damaged by large overloads, because flattening of the corrugations does not cause the metal to yield. The sensor measures the pressure between the foot and the shoe (Patel *et al.*, 1989). Tsoukalas *et al.* (2006) describe micromachined silicon capacitive sensors and their electronic interfaces.

EXAMPLE 2.1 For a 1 cm^2 capacitance sensor, R is 100 MΩ. Calculate x, the plate spacing required to pass sound frequencies above 20 Hz.

ANSWER From the corner frequency, $C = 1/2\pi fR = 1/(2\pi 20 \times 10^8) =$ 80 pF From (2.8) we can calculate x given the value of C.

$$x = \frac{\varepsilon_0 \varepsilon_r A}{C} = \frac{(8.854 \times 10^{-12})(1 \times 10^{-4})}{80 \times 10^{-12}} = 1.11 \times 10^5 \text{ m} = 11.1\ \mu\text{m}$$