# Chapter 2

## Basic Concepts of Electronics

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Medical instruments are widely used in clinical diagnosis, monitoring, therapy, and medical research. They provide a quick and precise means by which physicians can augment their five senses in diagnosing disease. These instruments contain electric components such as sensors, circuits, and integrated circuit (IC) chips. Modern electronics technology, which includes transistors, ICs, and computers, has revolutionized the design of medical instruments.

Biomedical engineers should have a fundamental understanding of their operations and a basic knowledge of their component electric and electronic systems. Using this knowledge provides a better understanding of the principles of various measurements, as well as developing new measurements and instruments.

Electrical engineering is too large a topic to cover completely in one chapter. Thus, this chapter presents some very basic concepts in several fields of electrical engineering. It discusses analog components such as resistors, capacitors, and inductors. It then goes on to basic circuit analysis, amplifiers, and filters. From this it moves to the digital domain, which includes converters, sampling theory, and digital signal processing. It then discusses the basic principles of microcomputers, programming languages, algorithms, database systems, display components, and recorders.

## 2.1 Electronic Components and Circuit Analysis

### 2.1.1 Current

An atom contains a nucleus surrounded by electrons. Most of the electrons are tightly bound to the atom, while some electrons in the outer orbits are loosely bound and can move from one atom to another. In conductors, there are many free electrons. This process of electron transfer occurs in random directions in materials.

Suppose there is an imaginary plane in a conductor (Figure 2.1(a)). The loosely bound outer orbital electrons continuously cross this plane. Due to the random direction of the electron movement, the number of electrons that cross the plane from left to right equals the number that cross from right to left. Thus, the net flow of electrons is zero.

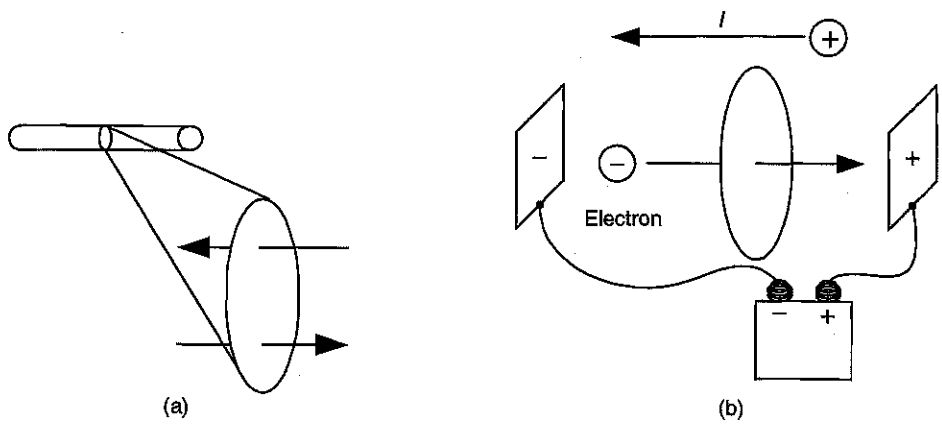


**Figure 2.1** Electric current within a conductor. (a) Random movement of electrons generates no current. (b) A net flow of electrons generated by an external force.

When an electric field is applied across a conductor, it causes a net flow of electrons in the conductor because the electrons are attracted to the positive side of the electric field (Figure 2.1(b)). The rate of flow of electrons through a region is called electric current.

If $\Delta Q$ is the amount of charge that passes perpendicularly through a surface with area $A$, in time interval $\Delta t$, the average current, $I_{av}$, is equal to the charge that passes through $A$, per unit of time.

$$I_{av} = \Delta Q/\Delta t \tag{2.1}$$

The rate at which current flows varies with time, as does charge. We therefore can define the instantaneous current, $I$, as the differential limit of Eq. (2.1).

$$I = dQ/dt \tag{2.2}$$

The unit of current is the ampere (A), which represents a net flow of 1 coulomb (1 C), of charge or $6.242 \times 10^{18}$ electrons across the plane per second (s).

The electron has a negative charge. When a negative charge moves in one direction, it yields the same result as a positive charge moving in the opposite direction. Conventionally, we define the direction of positive charge movement to be the direction of the electric current. Figure 2.1(b) shows that the direction of current is opposite to that of the flow of electrons.

Current can be generated in a circuit by a current source or by a voltage source and resistor in series.

### 2.1.2 Voltage and Potential

In moving a charge (+ or –) from point A to point B in an electric field, the potential energy of the charge changes. That change in energy is the work, $W$, done by the electric field on the charge. The amount of work is measured in Joules, J. If we let the electric potential, $V$, be equal to the potential energy, $U$, per unit charge, $q_0$, then we can define a potential difference, or voltage, as

$$\Delta V = V_B - V_A = \frac{W_{AB}}{q_0} \tag{2.3}$$

where $V_B$ and $V_A$ are the potentials at points B and A, respectively. The unit of potential is the volt (V), where 1 J/C = 1 V. If we choose the potential at infinity to be zero, the absolute potential of a point in an electric field can be defined as the total work per unit charge that has been done to move the charge from infinity to the point.

It is important that potential difference not be confused with difference in potential energy. The potential difference is *proportional* to the change in potential energy, where the two are related by $\Delta U = q_0 \Delta V$.

From Eq. (2.3), we can determine the work needed to move a charge from A to B if we know the potential difference between the two points. We are more interested in potential difference than the absolute potential for a single point. Notice that the potential is a property of the electric field, whether there is a charge in the field or not.

Voltage can be generated in a circuit by a voltage source or by a current source and resistor in parallel. Even though they exist, current sources are much less common than voltage sources in real circuits.

**Example 2.1** How much work is needed to move an electron (a charge of $1.6 \times 10^{-19}$ C) from 0 V to 4 V?

Rearranging Eq. (2.3)

$$W_{AB} = q_0 (V_B - V_A) = 1.6 \times 10^{-19}\ \text{C}(4\ \text{V} - 0\ \text{V}) = 6.4 \times 10^{-19}\ \text{J}$$

### 2.1.3 Resistors and Ohm's Law

When free electrons move in a conductor, they tend to bump into atoms. We call this property resistance and use a resistor as the electric component implementation. The unit of resistance is the Ohm ($\Omega$), in honor of Georg Simon Ohm, who discovered what is now known as Ohm's law. Ohm's law states that for many materials, when a potential difference is maintained across the conductor, the ratio of the current density $J$ (current per unit area) to the electric field that is producing the current, $E$, is a constant, $\sigma$, that is independent of the electric field. This constant is called the conductivity of the conductor.

$$J = \sigma E \tag{2.4}$$

It is useful to consider Ohm's law in the case of a straight wire with cross-sectional area $a$ and length $l$, as in Figure 2.2. $V = V_b - V_a$ is the potential difference maintained across the wire, which creates an electric field and current in the wire. The potential difference can be related to the electric field by the relationship

$$V = El \tag{2.5}$$

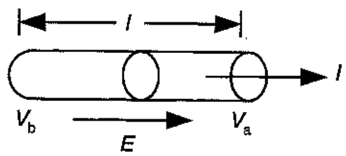


**Figure 2.2** A model of a straight wire of length $l$ and cross-sectional area $a$. A potential difference of $V_b - V_a$ is maintained across the conductor, setting up an electric field $E$. This electric field produces a current that is proportional to the potential difference.

Since the magnitude of the current density is related as $J = \sigma E$, and current density $J$ is defined as $I/a$, the potential difference can be rewritten as

$$V = (l/\sigma)J = (l/\sigma a)I \tag{2.6}$$

The quantity $(l/\sigma a)$ is defined as the resistance, $R$, of the conductor. The relationship can also be defined as a ratio of the potential difference to the current in the conductor

$$R = (l/\sigma a) = V/I \tag{2.7}$$

Experimental results show that the resistance, $R$, of a conductor can be rewritten as

$$R = \rho \frac{l}{a} \tag{2.8}$$

where $\rho$, the reciprocal of conductivity $\sigma$, is the resistivity ($\Omega \cdot$ m), $l$ is the length of the conductor in meters (m), and $a$ is the cross-sectional area of the conductor ($m^2$).

Ohm's law is written as

$$V = IR \tag{2.9}$$

where $V$ is the voltage across the resistor and $I$ is the current through the resistor.

Resistance depends on geometry as well as resistivity. Good electrical conductors have very low resistivity, where good insulators have very high resistivity. Resistivity is important in biomedical applications regarding not only electrical component resistances, but also resistances caused by body substances such as red blood cells, that may act as an insulator in counting procedures. Common carbon composition resistors used in electronic circuits have values between 10 Ω and 22 MΩ.

**Example 2.2** What is the resistance of a 12.0 cm long piece of copper wire, if the wire has a resistivity of $1.7 \times 10^{-8}\ \Omega \cdot \mathrm{m}$ and a radius of 0.321 mm?

The cross-sectional area of the wire is

$$a = \pi r^2 = \pi\left(0.321 \times 10^{-3}\ \mathrm{m}\right)^2 = 3.24 \times 10^{-7}\,\mathrm{m}^2$$

The resistance can be found using Eq. (2.8).

$$R = \rho \frac{l}{a} = 1.7 \times 10^{-8}\ \Omega \cdot \mathrm{m}\left(\frac{0.12\ \mathrm{m}}{3.24 \times 10^{-7}\ \mathrm{m}^2}\right) = 6.30 \times 10^{-3}\ \Omega$$

Most electric circuits make use of resistors to control the currents within the circuit. Resistors often have their resistance value in ohms color coded. This code serves as a quick guide to constructing electric circuits with the proper resistance. The first two colors give the first two digits in the resistance value. The third value, called the multiplier, represents the power of 10 that is multiplied by the first two digits. The last color is the tolerance value of the resistor, which is usually 5%, 10%, or 20% (Figure 2.3). In equation form, the value of a resistor's resistance can be calculated using the following

$$\mathrm{AB} \times 10^{\mathrm{C}} \pm \mathrm{D}\% \tag{2.10}$$

where A is the first color representing the tens digit, B is the second color representing the ones digit, C is the third color or multiplier, and D is the fourth color or tolerance. Table 2.1 gives the color code for each of these four categories, and their corresponding digit, multiplier, or tolerance value.

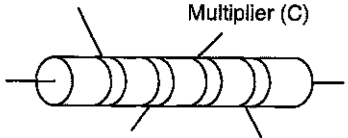


**Figure 2.3** The colored bands that are found on a resistor can be used to determine its resistance. The first and second bands of the resistor give the first two digits of the resistance, and the third band is the multiplier, which represents the power of ten of the resistance value. The final band indicates what tolerance value (in %) the resistor possesses. The resistance value written in equation form is $\mathrm{AB}\times 10^{\mathrm{C}} \pm \mathrm{D}\%$.

**Example 2.3** What is the resistance value of a resistor where the first band is orange, second band is blue, third band is yellow, and fourth band is gold?

Using Table 2.1, the first band, orange, represents the first digit, 3. The second band gives the second digit, 6. The third band is yellow, 4. The fourth band, or tolerance, is gold, 5%. Therefore, the resistance value is $AB \times 10^C \pm D\%$ or $36 \times 10^4 = 360$ kΩ, with a tolerance value of 5% or 18 kΩ.

**Table 2.1** The color code for resistors. Each color can indicate a first or second digit, a multiplier, or, in a few cases, a tolerance value.

| Color | Number | Tolerance (%) |
|---|---|---|
| Black | 0 | |
| Brown | 1 | |
| Red | 2 | |
| Orange | 3 | |
| Yellow | 4 | |
| Green | 5 | |
| Blue | 6 | |
| Violet | 7 | |
| Gray | 8 | |
| White | 9 | |
| Gold | −1 | 5% |
| Silver | −2 | 10% |
| Colorless | | 20% |

We define power, the rate at which work is done, as

$$P = VI \tag{2.11}$$

The unit of power is the watt (W). $V$ is the voltage between the two terminals of the resistor and $I$ is the current through the resistor.

Using Eq. (2.7) we can rewrite Eq. (2.11) as

$$P = \frac{V^2}{R} = I^2 R \tag{2.12}$$

Although power is most often used to describe the rate at which work can be done by mechanical objects, in electronics it represents the amount of energy dissipated by a component. The power lost as heat in a conductor, for example, is called joule heat and is referred to as a loss of $I^2R$.

**Example 2.4** Assume the chest is a cylinder 10 cm in diameter and 40 cm long with a resistivity of 0.8 Ω · m. For a voltage of 2 kV during defibrillation, calculate the current and power dissipated by the chest.

Calculate the current and power by using Eqs. (2.8), (2.9), and (2.11).

$$R = \rho \frac{l}{a} = 0.8 \frac{0.4}{\frac{\pi}{4} \times (0.1)^2} = 40.8\,\Omega$$

$$I = \frac{V}{R} = \frac{2000}{40.8} = 49\ \text{A}$$

$$P = VI = 2000 \times 49 = 98\ \text{kW}$$

### 2.1.4 Basic Circuit Analysis

An electric system generally contains many components such as voltage and current sources, resistors, capacitors, inductors, transistors, and others. Some IC chips, such as the Intel Pentium, contain more than one million components.

Electric systems composed of components are called networks, or circuits. We can analyze the performance of simple or complex circuits using one theorem and two laws of linear circuit analysis.

*Superposition theorem*: The current in an element of a linear network is the sum of the currents produced by each energy source.

In other words, if there is more than one source in the network, we can perform circuit analysis with one source at a time and then sum the results. The superposition theorem is useful because it simplifies the analysis of complex circuits.

One concept we need to introduce is the polarity of a voltage drop. Conventionally, if the direction of current flow in a circuit is known or assumed, then the direction of current flow across a circuit element is from + to –. In other words, by current flowing through an element, a voltage drop is created (as shown by Ohm's law) by that element with the polarity of + to – in the direction of current flow (Figure 2.4(a)).

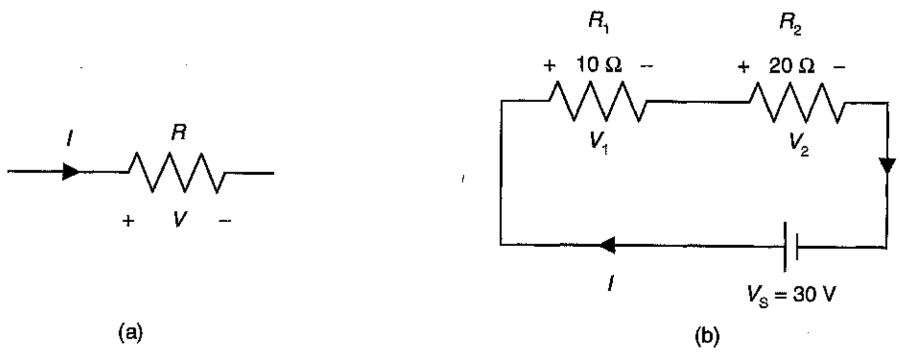


**Figure 2.4 (a)** The voltage drop created by an element has the polarity of + to – in the direction of current flow. (b) Kirchhoff's voltage law.

In a circuit, a loop is defined as a closed path containing circuit elements. Kirchhoff's voltage law (KVL) states

$$\sum V = 0 \tag{2.13}$$

or the sum of voltage drops around a loop is zero. In other words, starting at any point on the closed loop, we can use the voltage law to form an equation by adding each voltage drop across every element (resistors and sources). We can assume the direction of current, but if our calculations give a negative result then the actual current flows in the opposite direction. This law follows from conservation of energy. A charge that moves in a closed loop must gain as much energy as it loses if a potential can be found at each point in the circuit.

Figure 2.4(b) shows a circuit to which we can apply Kirchhoff's voltage law. If we sum the voltages counterclockwise around the loop we get

$$-V_s + V_1 + V_2 = 0$$

$$V_s = V_1 + V_2$$

Notice that the voltage source, $V_s$, has a polarity opposite that of the other voltage drops.

A resistor that is traversed in the direction of the current gives a drop in potential across the resistor equal to $-IR$, while a resistor that is traversed in the direction opposite of a current gives an increase in potential across the resistor equal to $+IR$. This is directly related to Eq. (2.9), where the direction of current flow across a resistor affects the potential change across that resistor. Also a voltage source that is traversed from the + to – terminal gives a drop in potential of $-V$, while a voltage source that is traversed from the – to + terminal gives a increase in potential of $+V$.

In circuit analysis, a node is a junction of two or more branches. Kirchhoff's current law (KCL) states that at any node

$$\sum I = 0 \tag{2.14}$$

or the sum of currents entering or leaving any node is zero, this follows the law of conservation of charge. In other words, the currents entering a node must equal the currents leaving the node (Figure 2.5).

When we use Kirchhoff's current law, we can arbitrarily label the current entering the node + and leaving the node –. The sum of the currents entering the node in Figure 2.5(b) is

$$+3 + 6 - I = 0$$

$$I = 9\text{ A}$$

Kirchhoff's voltage law and current law are basic laws for circuit analysis. There are two types of circuit analysis, each based on one of Kirchhoff's laws.

We assume unknown currents in the loops and set up the equations using Kirchhoff's voltage law and then solve these equations simultaneously.

We can use Figure 2.4(b) to illustrate loop analysis. We assume a current flows through the circuit in the direction already indicated in the figure. If we assumed the current in the other direction, our result would just be negative. Recalling Ohm's law, we have for the sum of the voltages through the clockwise loop

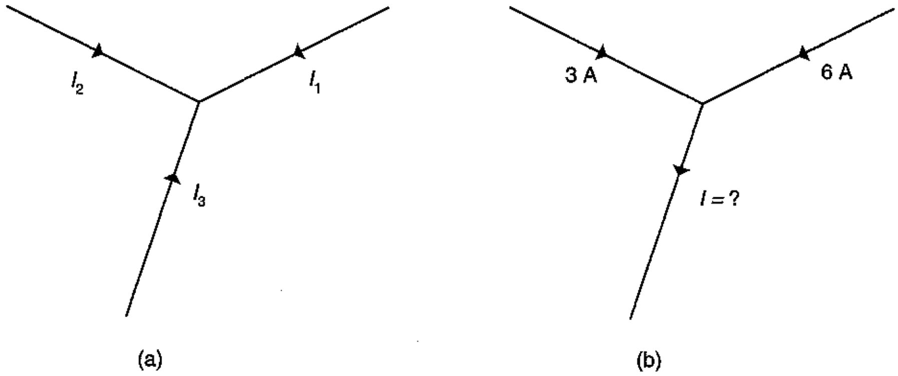


**Figure 2.5** (a) Kirchhoff's current law states that the sum of the currents entering a node is 0. (b) Two currents entering and one "negative entering," or leaving.

$$V_S = R_1 I + R_2 I$$

$$30 = 10I + 20I$$

$$I = 1\,\text{A}$$

Now that we know the current through the loop, we can find the voltage drop through either resistor using Ohm's law. In this example, the voltage drop for $R_1$ is 10 V.

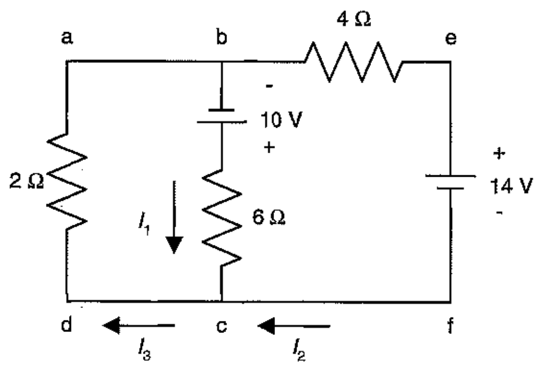


**Figure 2.6** Kirchhoff's current law for Example 2.5.

**Example 2.5** Find the currents $I_1$, $I_2$, and $I_3$ in the circuit shown in Figure 2.6.

Applying Kirchhoff's current law to the current junction at node c, we have

$$I_1 + I_2 = I_3 \tag{1}$$

All together there are three loops, *abcda, befcb*, and *aefda* (the outer loop) to which Kirchhoff's voltage law can be applied. Applying Kirchhoff's voltage law to the two inside loops in the clockwise direction, we have

$$\text{Loop } abcda\text{: } 10\,\text{V} - (6\,\Omega)I_1 - (2\,\Omega)I_3 = 0 \tag{2}$$

$$\text{Loop } befcb\text{: } -14\,\text{V} - 10\,\text{V} + (6\,\Omega)I_1 - (4\,\Omega)I_2 = 0 \tag{3}$$

Expressions (1), (2), and (3) represent three equations with three unknowns, and we can solve this system of three equations, first by substituting (1) into (2), giving

$$10 = 8I_1 + 2I_2 \tag{4}$$

Second by dividing each term in (3) by 2, we get

$$-12 = -3I_1 + 2I_2 \tag{5}$$

Finally, subtracting (5) from (4), $I_2$ is eliminated, giving

$$22 = 11I_1$$

$$I_1 = 2\,\text{A}$$

Substituting this value into (5), we find $I_2 = -3$ A., and substituting both these values in (1), we find $I_3 = -1$ A.

The other type of analysis is nodal analysis. We assume unknown voltages at each node and write the equations using Kirchhoff's current law. We solve these equations to yield the voltage at each node. Figure 2.7 shows an example using nodal analysis.

In Figure 2.7, we are given the voltage at A as 30 V and at B as 20 V. We assume the voltage at C is $V$. We apply Kirchhoff's current law at node C

$$\frac{30-V}{20} + \frac{20-V}{20} - \frac{V-0}{20} = \frac{50-3V}{20} = 0$$

$$V = 16.7\,\text{V}$$

When solving for voltages or currents in a circuit using the loop or nodal method, it is important to note that the number of independent equations needed is equal to the number of unknowns in the circuit problem. With an equal number of equations and unknowns, large numbers of unknowns can be solved using a matrix and linear algebra.

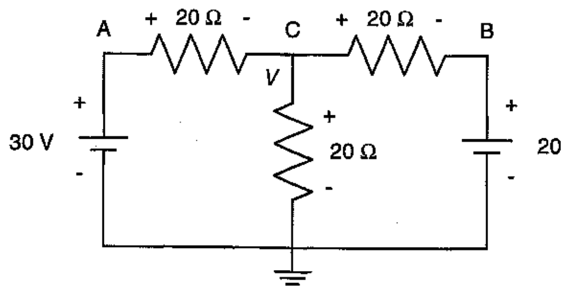


**Figure 2.7** Example of nodal analysis.

### 2.1.5 Attenuators

When we amplify the 1 mV electrocardiogram signal from the heart, the signal may be undesirably reduced (attenuated) by the input resistance of the amplifier. For example, Figure 2.8 shows that the resistance of the skin, $R_s$, may be 100 kΩ, and the input resistance, $R_i$ (input resistance of the oscilloscope used as an amplifier), is 1 MΩ.

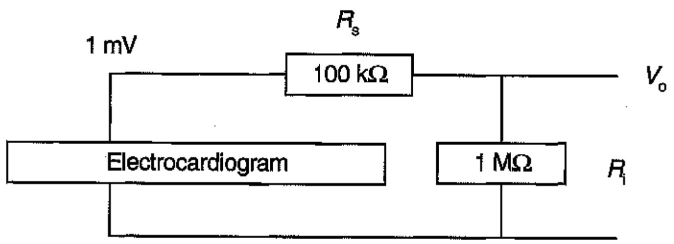


**Figure 2.8** The 1 mV signal from the electrocardiogram is attenuated by the resistive divider formed by the 100 kΩ skin resistance and the 1 MΩ input resistance of the oscilloscope.

The current through the resistive divider is

$$i = \frac{V}{R} = \frac{1\,\text{mV}}{100\,\text{k}\Omega + 1\,\text{M}\Omega} = 0.91\,\text{nA}$$

The output voltage $V_o$ is

$$V_o = iR_i = (0.91\,\text{nA})(1\,\text{M}\Omega) = 0.91\,\text{mV}$$

Thus the input voltage (ECG) has been undesirably attenuated by an amount that we can calculate from the voltage divider equation

$$\frac{V_o}{V_i} = \frac{R_i}{R_s + R_i} \tag{2.15}$$

$$\frac{V_o}{V_i} = \frac{R_i}{R_s + R_i} = \frac{1\,\mathrm{M\Omega}}{100\,\mathrm{k\Omega} + 1\,\mathrm{M\Omega}} = 0.91$$

Figure 2.8 is a typical example of voltage divider. With two resistors in series, the output voltage is part of the input voltage. By adjusting the values of $R_i$ and $R_s$, one can flexibly obtain any percentage of the input voltage.

A potentiometer is a three-terminal resistor with an adjustable sliding contact that functions as an adjustable voltage divider or attenuator. Figure 2.9 shows that if the slider is at the top, $v_o = v_i$. If the slider is at the bottom, $v_o = 0$. If the slider is in the middle, $v_o = 0.5v_o$. Potentiometers are usually circular with a rotatable shaft that can be turned by hand or a screwdriver. The potentiometer is useful to provide a variable gain for an amplifier or for the volume control on a radio. Alternatively, a potentiometer can be used as a two-terminal variable resistor by using the variable resistance between the slider and only one end of the potentiometer.

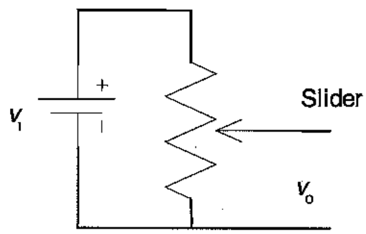


**Figure 2.9** A potentiometer is a three-terminal resistor with an adjustable sliding contact shown by the arrow. The input signal $v_i$ is attenuated by the potentiometer to yield an adjustable smaller voltage $v_o$.

### 2.1.6 Common Electrical Instruments

#### *Galvanometer*

The galvanometer is a main component that is used in creating ammeters (devices that measure current in a circuit) and voltmeters (devices that measure potential difference in a circuit), as the galvanometer can be used in conjunction with other circuit elements to measure current or potential differences of an electronic signal.

A common type of galvanometer consists of a coil of wire mounted in such a way that it is free to deflect or rotate in the presence of a magnetic field provided by a permanent magnet. The main principal behind the galvanometer makes use of a torque that acts on the loop of current when a magnetic field is applied. The torque is proportional to the amount of current that passes through the galvanometer, such that the larger the current, the greater amount of deflection or rotation of the coiled wire.

Typically, an off-the-shelf galvanometer is not directly used as an ammeter since it has a large resistance (about 60 Ω) that would considerably reduce the amount of current in the circuit in which the galvanometer is placed. Also, the fact that the galvanometer gives a full-scale deflection for low currents (1 mA or less) makes it unusable for currents greater in magnitude. If a galvanometer is placed in parallel with a *shunt resistor*, $R_p$, with a relatively small resistance value compared with that of the galvanometer, the device can be used effectively as an ammeter. Most of the current that is measured will pass through this resistor (Figure 2.10(a)).

If an external resistor, $R_s$, is placed in series with the galvanometer, such that its resistance value is relatively larger than that of the galvanometer, the device can be used as a voltmeter. This ensures that the potential drop across the galvanometer doesn't alter the voltage in the circuit to which it is connected (Figure 2.10(b)).

Older ammeters and voltmeters that used galvanometers with moving coils have been largely replaced by digital multimeters with no moving parts.

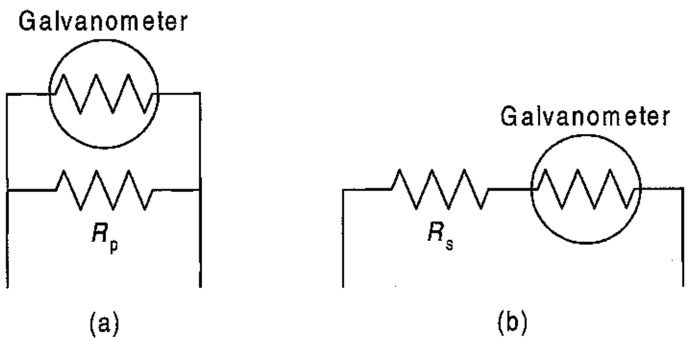


**Figure 2.10** (a) When a shunt resistor, $R_p$, is placed in parallel with a galvanometer, the device can be used as an ammeter. (b) When a resistor, $R_s$, is connected in series with the galvanometer, it can be used as a voltmeter.

### *Wheatstone bridge*

Often in biomedical instruments, unknown resistances or changes in resistance are measured within a circuit. Many times, an electric element known as Wheatstone bridge is used to measure these unknown resistances. The Wheatstone bridge has several applications and is widely used in electronics.

One application of the Wheatstone bridge consists of a common source of electric current (such as a battery) and a galvanometer (or other device that measures current) that connects two parallel branches, containing four resistors, three of which are known. One parallel branch contains one known resistance ($R_1$) and an unknown resistance ($R_x$) as shown in Figure 2.11. The other parallel branch contains resistors of known resistances, $R_2$ and $R_3$.

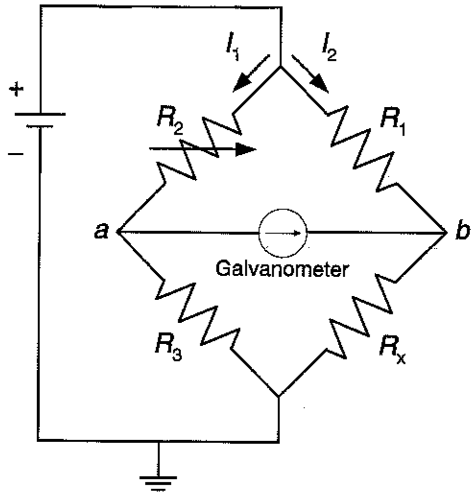


**Figure 2.11** A circuit diagram for a Wheatstone bridge. The circuit is often used to measure an unknown resistance $R_x$, when the three other resistances are known. When the bridge is balanced, no current passes from node *a* to node *b*.

In order to determine the resistance of the unknown resistor, the resistances of the other three are adjusted and balanced until the current passing through the decreases to zero. Since the voltage potentials at *a* and *b* are equal, the potential difference across $R_2$ must equal $R_1$. Similarly, the potential difference across $R_3$ must also equal that across $R_x$. With this we have

$$I_1R_2 = I_2R_1 \tag{1}$$

$$I_1R_3 = I_2R_x \tag{2}$$

Dividing (2) by (1), we can eliminate the current, and solve for $R_x$

$$R_x = \frac{R_1R_3}{R_2} \tag{2.16}$$

From Eq. (2.16), we can calculate the value of the unknown resistor.

The fact that a Wheatstone bridge is valuable for measuring small changes of a resistance makes it also suitable to measure the resistance change in a device called a *strain gage*. This device, commonly used in biomedical instruments to measure experimental stresses, often consists of a thin wire matrix attached to a flexible plastic backing and glued to the stretched metal. Stresses are measured by detecting changes in resistance of the strain gage as it bends. The resistance measurement is made with the strain gage as one or more elements in the Wheatstone bridge. Strain gages diffused into the diaphragm of a silicon integrated circuit blood pressure sensor are formed into a Wheatstone bridge.

### 2.1.7 Capacitors

From Ohm's law we know the voltage across a resistor is related to the current through the resistor by the resistance value. Now we investigate two elements with different relationships between voltage and current.

Conventionally, we use capital letters to denote variables that do not change with time and small letters to denote variables that change with time.

A capacitor is a two-terminal element in which the current, $i$, is proportional to the change of voltage with respect to time, $dv/dt$, across the element (Figure 2.12), or

$$i = C\frac{dv}{dt} \tag{2.19}$$

where $C$ is capacitance and is measured in farads (F). One farad is quite large, so in practice we often see μF ($10^{-6}$ F), nF ($10^{-9}$ F), and pF ($10^{-12}$ F). Common capacitor values are 10 pF to 1 μF. Capacitors are commonly used in a variety of electric circuits and are a main component of electronic filtering systems (introduced in 2.3) in biomedical instruments.

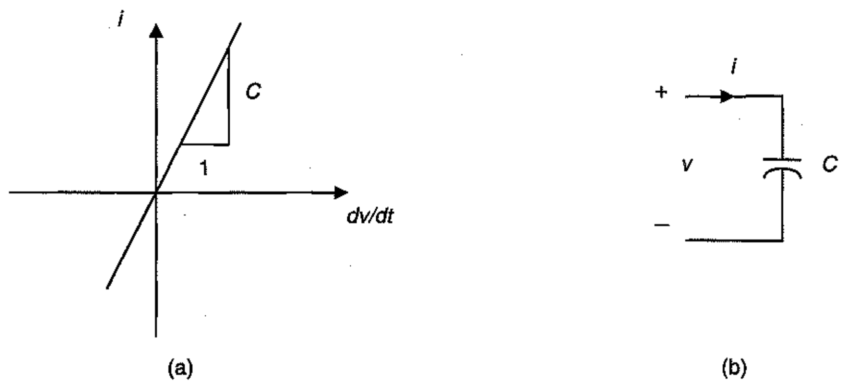


**Figure 2.12** (a) Capacitor current changes as the derivative of the voltage. (b) Symbol of the capacitor.

We can also represent $v$ as a function of $i$ and $t$

$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i dt \tag{2.20}$$

where $t_0$ and $t$ are the beginning and ending times over which we observe the current, $v(t_0)$ is the initial voltage across the capacitor. We usually choose a $t_0$ of zero.

A capacitor usually consists of two conductors that are separated by an insulator. The capacitance of a device depends on its geometry and the dielectric material. A parallel–plate capacitor consists of two parallel plates of equal area $A$, separated by a distance $d$ (Figure 2.13). One plate has a charge $+Q$ and the other $-Q$, where the charge per unit area on either plate is $\sigma = Q/A$. If the charged plates are very close together, compared to their length and width, we can assume that the electric field is uniform between the plates and zero everywhere else.

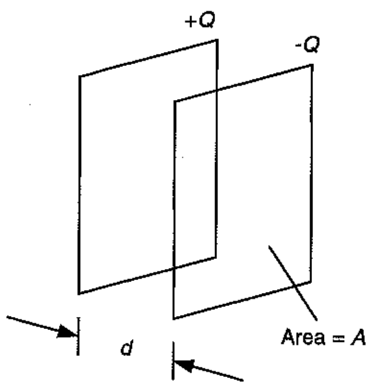


**Figure 2.13** Diagram of a parallel–plate capacitor. The component consists of two parallel plates of area $A$ separated by a distance $d$. When charged, the plates carry equal charges of opposite sign.

We can then see that the electric field between the plates is

$$E = \sigma/\varepsilon_0 = Q/\varepsilon_0 A \tag{2.21}$$

where $\varepsilon_0$ is the permittivity of free space (8.85 pF/m). Since the potential difference between the plates is equal to $Ed$, we find that the capacitance is

$$C = \varepsilon_0 A/d \tag{2.22}$$

Capacitances can be found in many different configurations within a circuit. Two capacitances, or several capacitances, can be reduced to one capacitance value by simple algebraic expressions depending on the situation. For the series capacitances in Figure 2.14(a), the equivalent capacitance, $C_e$, can be calculated by

$$C_e = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \frac{C_1 C_2}{C_1 + C_2} \qquad \text{2 capacitors}$$

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \Lambda\,\Lambda \qquad \text{multiple capacitors} \tag{2.23}$$

For the parallel capacitances in Figure 2.14(b), the equivalent capacitance, $C_e$, can be calculated by

$$C_e = C_1 + C_2 \qquad \text{2 capacitors}$$

$$C_e = C_1 + C_2 + C_3 + \Lambda\,\Lambda \qquad \text{multiple capacitors} \tag{2.24}$$

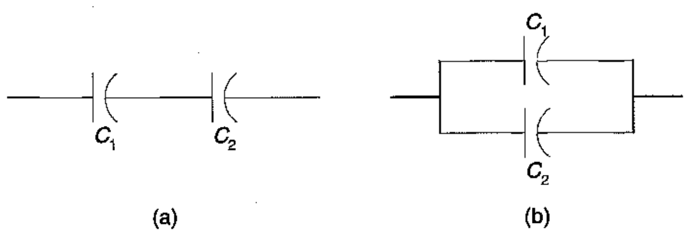


**Figure 2.14** (a) A series combination of two capacitors. (b) A parallel combination of two capacitors.

### 2.1.8 Inductors

An inductor is a two-terminal element in which the voltage, $v$, is proportional to the change of current with respect to time, $di/dt$, across the element (Figure 2.15), or

$$v = L\frac{di}{dt} \tag{2.25}$$

where $L$ is the inductance. Its unit is the henry (H). Like the farad, this unit is large so we use more convenient units such as mH ($10^{-3}$ H) and μH ($10^{-6}$ H). This is usually taken to be the defining equation for the inductance of any inductor, regardless of its shape, size, or material characteristics. Just as resistance is a measure of opposition to a current, inductance is a measure of the opposition to any change in current. The inductance of an element often depends on its geometry.

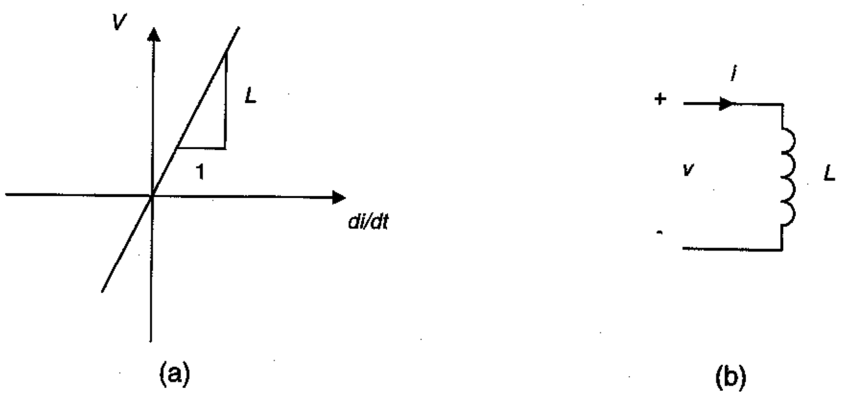


**Figure 2.15** (a) Inductor voltage changes as the derivative of the current. (b) Symbol of the inductor.

We can also represent $i$ in terms of $v$ and $t$

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v dt \tag{2.26}$$

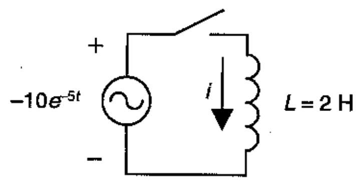


**Figure 2.16** Simple inductor circuit.

**Example 2.6** In Figure 2.16, an inductor is connected, at time $t = 0$ by a switch, to a voltage source whose signal can be described as $-10e^{-5t}$ V. Derive an equation that describes the current through the inductor as a function of time.

Using Eq. (2.26) we can solve for $i(t)$. There is no current through the inductor at $t = 0$, so $i(t_0) = 0$.

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v dt$$

$$i(t) = 0 + \frac{1}{2\,\mathrm{H}} \int_0^t -10e^{-5t}\ \mathrm{V}\ dt = \frac{1}{2}\left(\frac{-10}{-5}\right) e^{-5t}$$

$$i(t) = e^{-5t}\ \mathrm{A}$$

For multiple series inductors, the equivalent inductance $L_e$, can be calculated by

$$L_e = L_1 + L_2 + L_3 + \Lambda\,\Lambda \tag{2.27}$$

For multiple parallel inductors, $L_e$ can be calculated by

$$\frac{1}{L_e} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \Lambda\,\Lambda \tag{2.28}$$

### 2.1.9 First-Order Systems

Consider the simple resistor and capacitor ($RC$) circuit shown in Figure 2.17(a).

Suppose that at time $t = 0$ the voltage across the capacitor is $v_C(0)$. For $v_C(t)$ when $t \geq 0$, we have

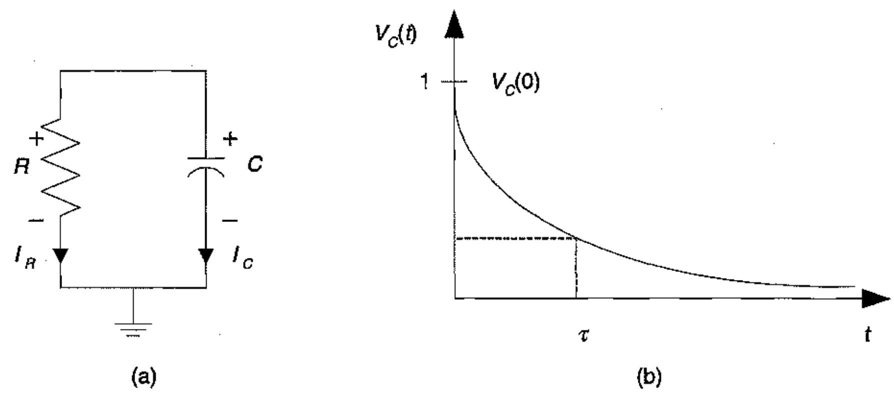


**Figure 2.17** (a) Simple $RC$ circuit with $v(0)$ on capacitor at time $t = 0$. (b) Normalized voltage across the capacitor for $t \geq 0$ (normalized means the largest value is 1).

$$v_R = v_C \ \text{ and } i_R + i_C = 0 \quad \Rightarrow \quad \frac{v_C}{R} + C\frac{dv_C}{dt} = 0 \tag{2.29}$$

Upon rearranging, we get

$$\frac{dv_C}{dt} + \frac{1}{RC} v_C = 0 \tag{2.30}$$

This equation contains the variable $v_C$ and its first derivative $dv_C/dt$. This is a first-order differential equation. Many complex systems can be reduced to multiple first-order systems, which are more easily solved than high-order systems.

The solution of this differential equation, assuming at $t = 0$, $v_C(t) = v_C(0)$, is

$$v_C(t) = v_C(0) e^{-\frac{t}{RC}} \tag{2.31}$$

Figure 2.17(b) shows $v_C(t)$ versus $t$ for $t \geq 0$. Also, we have

$$i_C(t) = -i_R(t) = \frac{v_C(t)}{R} = -\frac{v_C(0)}{R} e^{-\frac{t}{RC}} \tag{2.32}$$

From these results we can see that voltage and current in the circuit decrease as time increases and approach zero as time goes to infinity. The voltage and current in this circuit can be described by equations of the form

$$f(t) = Ke^{-\frac{t}{\tau}} \tag{2.33}$$

where $K$ and $\tau$ are constants. The constant $\tau$ is called the time constant of the first-order system. In this case, $\tau = RC$. When time $t$ reaches $\tau$, $v_C(t = \tau)$ is $v_C(0)e^{-1}$. When $t = 3\tau$, the value of $v_C(t)$ is down to only 5% of the value of $v_C(0)$. We can assume the value of $v_C(t)$ is practically zero after 5 time constants.

In the circuit shown in Figure 2.17(a), there was no independent source (or input), only an initial state of the circuit. For this reason, we call the voltage and current we obtained the natural response.

**Example 2.7** Consider the circuit in Figure 2.17(a). Suppose $R = 1$ kΩ, $C = 40$ μF and the initial voltage across the capacitor at $t = 0$ is 10 V. How long will it take for the capacitor to discharge to half of its original voltage?

This simple $RC$ circuit can be solved with a first-order differential equation. $v_C(0) = 10$ V, so half of that voltage would give us $v_C(t) = 5$ V. Using Eq. (2.31) and algebra we can solve for $t$.

$$v_C(t) = v_C(0) e^{-\frac{t}{RC}}$$

$$\ln\left(\frac{v_C(t)}{v_C(0)}\right) = -\frac{t}{RC}$$

$$t = -RC\ln\left(\frac{v_C(t)}{v_C(0)}\right) = -(1\,\mathrm{k\Omega})(40\,\mu\mathrm{F})\ln(0.5) = 27.7\text{ ms}$$

Now let us consider the case of nonzero input using the circuit shown in Figure 2.18(a). From $t = -\infty$ to just before $t = 0$ ($t = 0^-$), the switch has been open. Therefore, no current has been flowing and no charge is in the capacitor. At $t = 0$, we close the switch which causes current to flow and charge to accumulate in the capacitor.

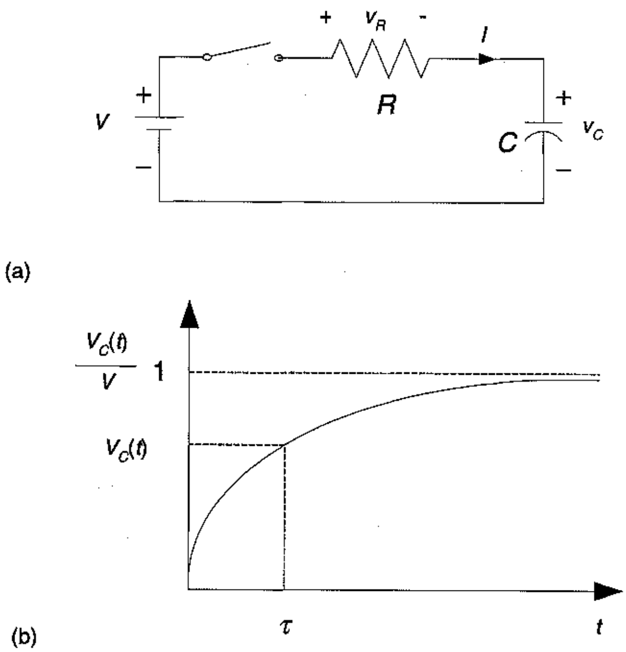


**Figure 2.18** (a) Series *RC* circuit with voltage step input at time 0. (b) Normalized voltage across the capacitor.

Using KVL, the equation for this current is

$$V = C\frac{dv_C}{dt} + \frac{v_C}{R} \quad \text{for } t \geq 0$$

or

$$\frac{dv_C}{dt} + \frac{1}{RC}v_C = \frac{V}{RC} \quad \text{for } t \geq 0 \tag{2.34}$$

The solution of the equation is

$$v_C = V - Ve^{-\frac{t}{RC}} \tag{2.35}$$

The voltage across the resistor, $v_R$, and current, $i$, are

$$\begin{aligned} v_R &= Ve^{-\frac{t}{RC}} \\ i &= \frac{V}{R}e^{-\frac{t}{RC}} \end{aligned} \tag{2.36}$$

The voltage $v_C(t)$ is shown in Figure 2.18(b). The voltage across the resistor decreases exponentially and the voltage across the capacitor increases exponentially. From the definition of a capacitor, we know the voltage across it cannot change abruptly or the current will become infinite. When the switch is closed, the voltage on the capacitor is still zero and the voltage drop across the resistor is $V$, the source voltage. As time increases, the capacitor becomes charged and as time approaches infinity, all the source voltage is across the capacitor while the voltage drop across the resistor is zero.

**Example 2.8** Consider the circuit in Figure 2.18(a). Suppose that $V = 12$ V, $R = 1$ kΩ and $C = 10$ μF. At time $t = 0$ the switch closes. Sketch the voltage across the capacitor versus time. How long will it take for the capacitor to become 90% of the voltage source?

Using Eq. (2.35):

$$v_C = V - Ve^{-\frac{t}{RC}}$$

$$v_C = 12 - 12e^{-\frac{t}{0.01}}\ \text{V}$$

90% of the voltage source is 10.8 V. Substituting this value in for $v_C$ in the above equation and solving for $t$ yields a time of 23 ms with a time constant of 10 ms.

We can also use a resistor and an inductor to make a first-order circuit where $\tau = L/R$.

### 2.1.10 Frequency

Sinusoidal waves are widely used in electric circuits. Figure 2.20 shows two sinusoidal waveforms, which can be represented as

$$A\sin(2\pi ft+\theta) = A\sin(\omega t+\theta) = A\sin\left(\frac{2\pi}{T}t+\theta\right) \tag{2.37}$$

where: $A$ = amplitude of sine wave
$f$ = frequency of sine wave in hertz (Hz)
$\omega = 2\pi f$ = angular frequency of sine wave (radians per second)
$\theta$ = phase angle of sine wave (radians)
$T = 1/f$ = period (seconds)

Note: The sine function lags 90° in phase behind the cosine function.

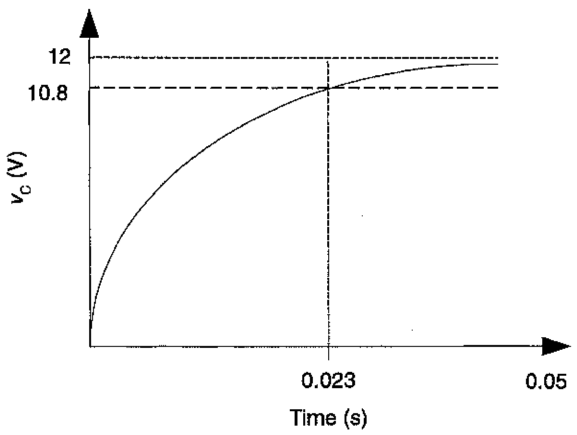


**Figure 2.19** Plot of $v_C$ for Example 2.8.

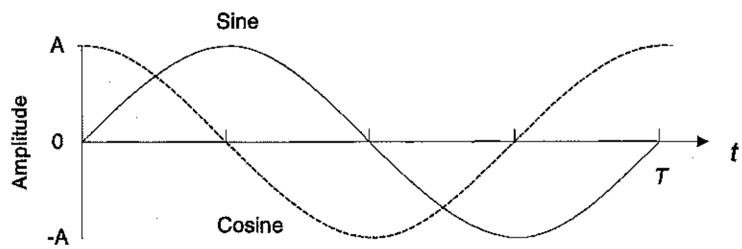


**Figure 2.20** One period, $T$, of the sine and cosine waveforms.

**Example 2.9** Figures 2.21 and 2.22 show a comparison of a sinusoidal waveform with different frequency and different phase angle.

In Figure 2.21 the solid line shows the function $y = \sin(\omega t)$, the dashed line shows the function with the frequency doubled, $y' = \sin(2\omega t)$. By doubling the frequency the period is reduced by half by $T = 1/f$.

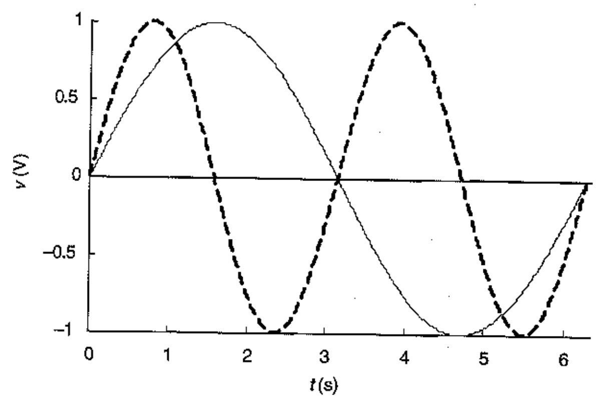


**Figure 2.21** Sinusoidal waveforms with different frequencies.

In Figure 2.22 the solid line again shows the function $y = \sin(\omega t)$, the dashed line shows the function with a phase shift of 180° ($\pi$ in radians), $y' = \sin(\omega t - \pi)$. The minus sign in front of the phase angle shifts this waveform to the right and can be said that it is leading the original waveform. Also, with a phase shift of 180°, the two waveforms are said to be completely out of phase.

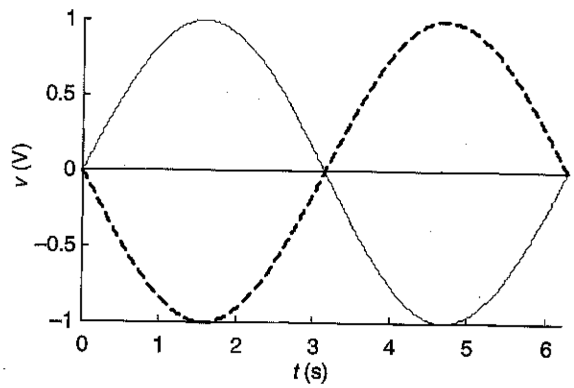


**Figure 2.22** Sinusoidal waveforms with 0° phase angle (solid) and 180° phase angle (dashed).

When a voltage or current source is a sinusoidal wave, Kirchhoff's laws still apply in differential form to include capacitors and inductors. Since sinusoidal sources change with time and its differential is still sinusoidal, a more efficient way to solve the equations is to use complex sinusoidal forms.

For example, we can write $A\cos(\omega t + \phi)$ in complex sinusoidal form as

$$Ae^{j(\omega t+\phi)} = Ae^{j\phi}e^{j\omega t} \tag{2.38}$$

This representation comes from Euler's identity and has the same amplitude, frequency, and phase angle as the noncomplex form. Let us denote the complex number

$$\mathbf{A} = Ae^{j\phi} \tag{2.39}$$

by

$$\mathbf{A} = A\angle\phi \tag{2.40}$$

which is known as a phasor, with bold type meaning it is a vector. Equation 2.40 can be read as: "The vector **A** has a magnitude of $A$ and an angle of $\phi$."

Let us consider a resistor. If the current and voltage of the resistor are all complex sinusoids, say

$$\begin{aligned} i &= Ie^{j(\omega t+\theta)} \\ v &= Ve^{j(\omega t+\phi)} \end{aligned} \tag{2.41}$$

Then, by Ohm's law

$$v = Ri$$

$$Ve^{j(\omega t+\theta)} = RIe^{j(\omega t+\phi)}$$

$$Ve^{j\theta} = RIe^{j\phi} \text{ or } V\angle\theta = RI\angle\phi \tag{2.42}$$

Eq. (2.42) shows that $\theta = \phi$, which means the voltage and the current are in phase across the resistor (Figure 2.23(a)). We can rewrite this as the phasor

$$\mathbf{V} = R\mathbf{I} \tag{2.43}$$

As to an inductor, the voltage across it is given by

$$v = L\frac{di}{dt} \tag{2.44}$$

The $V$–$I$ relationship can be written as

$$Ve^{j\theta} = j\omega LIe^{j\phi} \text{ or } V\angle\theta = j\omega LI\angle\phi \tag{2.45}$$

Therefore,

$$\mathbf{V} = j\omega L\mathbf{I} \tag{2.46}$$

and $\theta = \phi + 90°$, which shows that current lags the voltage by 90° (Figure 2.23(b)).

Using the methods presented in Eq. (2.44) through (2.46) yields

$$\mathbf{V} = \frac{1}{j\omega C}\mathbf{I} \tag{2.47}$$

and $\theta = \phi - 90°$, which shows that current leads the voltage by 90° (Figure 2.23(c)).

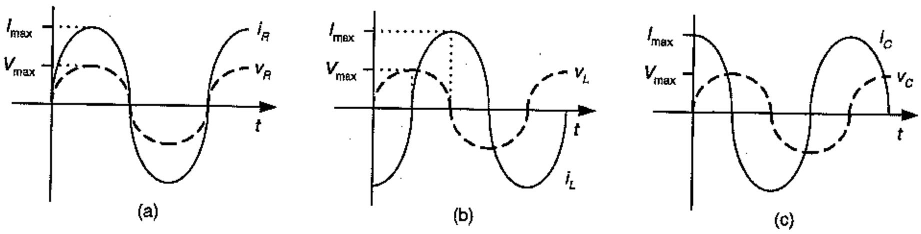


**Figure 2.23** Plots of the current and voltage as a function of time. (a) With a resistor both the current and the voltage vary as sin($\omega t$), the current is in phase with the voltage, meaning that when the current is at a maximum, the voltage is also. (b) For an inductor, the current lags behind the voltage 90°. (c) For a capacitor, the current leads the voltage by 90°.

From the equations above, we can see a general equation in phasor notation for the resistor, capacitor, and inductor. That equation is

$$\mathbf{V} = \mathbf{ZI} \tag{2.48}$$

where **Z** is the impedance of the element. For the three elements,

$$\mathbf{Z}_R = R \qquad \mathbf{Z}_C = \frac{1}{j\omega C} \qquad \mathbf{Z}_L = j\omega L \tag{2.49}$$

With impedance form of Eq. (2.49) and the method introduced in Section 2.1.4, the relationship of voltage and current is expressed in algebraic form instead of differential form. Phasor equations simplify the circuit analysis and can be used by computers to solve the problem.

Note that the impedance of capacitors and inductors changes with frequency. Therefore, the performance of the circuit will also change with frequency. This change in circuit performance is called the frequency response of the circuit.

**Example 2.10** The current through a 50 mH inductor is $100\angle 0°$ mA. If $\omega = 1000$ rad/s, what is the voltage across the inductor?

Using the phasor equation in Eq. (2.46), we can solve for **V**.

$$\mathbf{V} = j\omega L\mathbf{I}$$

$$\mathbf{V} = j1000(0.050)(100\angle 0° \text{ mA}) = 5\angle 90° \text{ V}$$

### 2.1.11 Series and Parallel Impedances

Now that the impedances for the three passive circuit elements are defined (Eq. 2.49), it is time to study two types of configurations for them in a circuit. Figure 2.24(a) shows two impedances in series. These two impedances, and in general infinitely many impedances, can be combined into one equivalent impedance. For the series impedances in Figure 2.24(a), the equivalent impedance, $Z_e$, can be calculated by

$$Z_e = Z_1 + Z_2 \tag{2.50a}$$

For multiple impedances in series, the equivalent impedance $Z_e$ is

$$Z_e = Z_1 + Z_2 + Z_3 + \Lambda\,\Lambda \tag{2.50b}$$

For the parallel impedances in Figure 2.24(b), the equivalent impedance, $Z_e$, can be calculated by

$$Z_e = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} = \frac{Z_1 Z_2}{Z_1 + Z_2} \tag{2.51a}$$

For multiple impedances in parallel, the equivalent impedance $Z_e$ is

$$\frac{1}{Z_e} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \Lambda\,\Lambda \tag{2.51b}$$

A shorthand notation for indicating parallel circuit elements as in Eq. (2.51) is to use a double vertical bar between the parallel elements, such as $Z_e = Z_1 \| Z_2$.

Using these definitions, it is possible to reduce large complicated circuits down to (ideally) one or two impedances and a voltage source. Figure 2.24(c) shows the equivalent impedance of the combined series or combined parallel circuit.

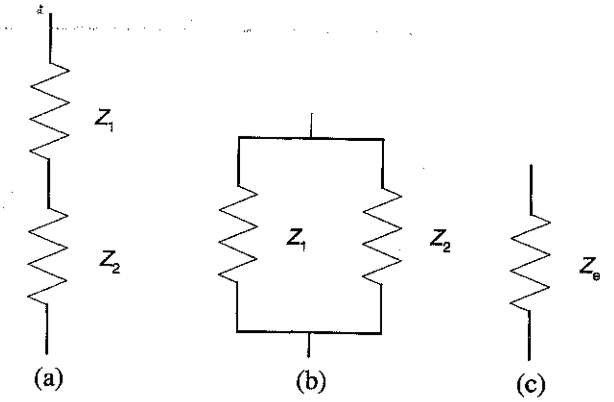


**Figure 2.24** (a) Series circuit. (b) Parallel circuit. (c) Single impedance equivalent.

### 2.1.12 Electrical Safety

Because safety is paramount in considering biomedical devices in the real world setting, great emphasis is placed on currents and voltages that exist in an apparatus and what dangers these may pose to patients and operators who use the device.

Electric shock may result in fatal burns and can cause muscles and even the heart to malfunction. Often the degree of damage depends on the magnitude of current that is applied, how long the current acts, and through which point on the body the current passes. Typically, skin currents of 5 mA or less may cause a light sensation of shock, but usually do no damage. Currents larger than 10 mA tend to cause the muscles to contract to the point a person is unable to let go of a live wire. If currents of 100 mA pass through the body, even for a few seconds, respiratory muscles can become paralyzed and breathing stops. The heart can go into ventricular fibrillation, which is fatal (macroshock). If an electrode or catheter is within the heart, current must not exceed 10 μA because the current density is high (microshock). Contact with live wires or voltages above 24 V is not recommended.

Often in biomedical engineering, currents and voltage outputs must be amplified, attenuated, or filtered safely in order to achieve the best possible electronic signal with minimal harm to the patient or operator. In the following sections, we will explore ways in which this can be done.

## 2.2 Amplifiers

Most bioelectric signals have a very small magnitude (on the order of millivolts or microvolts) and therefore require amplification so that users can process them. This section emphasizes the operational amplifier (op amp) for use in amplifier design.