

## Reading 3: Amplifiers

Most bioelectric signals are small and require amplification. Amplifiers are also used for interfacing sensors that sense body motions, temperature, and chemical concentrations. In addition to simple amplification, the amplifier may also modify the signal to produce frequency filtering or nonlinear effects. This chapter emphasizes the *operational amplifier* (op amp), which has revolutionized electronic circuit design. Most circuit design was formerly performed with discrete components, requiring laborious calculations, many components, and large expense. Now a 20-cent op amp, a few resistors, and knowledge of Ohm's law are all that is needed.

### 3.1 IDEAL OP AMPS

An *op amp* is a high-gain dc differential amplifier. It is normally used in circuits that have characteristics determined by external negative-feedback networks.

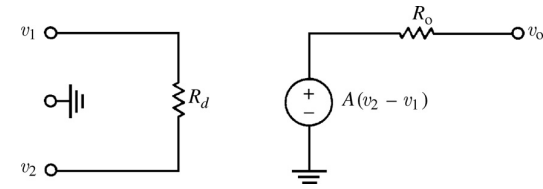
The best way to approach the design of a circuit that uses op amps is first to assume that the op amp is ideal. After the initial design, the circuit is checked to determine whether the nonideal characteristics of the op amp are important. If they are not, the design is complete; if they are, another design check is made, which may require additional components.

#### IDEAL CHARACTERISTICS

Figure 3.1 shows the equivalent circuit for a nonideal op amp. It is a dc differential amplifier, which means that any differential voltage,  $v_d = (v_2 - v_1)$ , is multiplied by the very high gain  $A$  to produce the output voltage  $v_o$ .

To simplify calculations, we assume the following characteristics for an ideal op amp:

1.  $A = \infty$  (gain is infinity)
2.  $v_o = 0$ , when  $v_1 = v_2$  (no offset voltage)



**Figure 3.1 Op-amp equivalent circuit** The two inputs are  $v_1$  and  $v_2$ . A differential voltage between them causes current flow through the differential resistance  $R_d$ . The differential voltage is multiplied by  $A$ , the gain of the op amp, to generate the output-voltage source. Any current flowing to the output terminal  $v_o$  must pass through the output resistance  $R_o$ .

3.  $R_d = \infty$  (input impedance is infinity)
4.  $R_o = 0$  (output impedance is zero)
5. Bandwidth  $= \infty$  (no frequency-response limitations) and no phase shift

Later in the chapter we shall examine the effect on the circuit of characteristics that are not ideal.

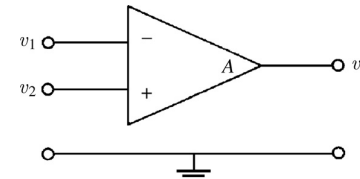
Figure 3.2 shows the op-amp circuit symbol, which includes two differential input terminals and one output terminal. All these voltages are measured with respect to the ground shown. Power supplies, usually  $\pm 15$  V, must be connected to terminals indicated on the manufacturer's specification sheet (Jung, 1986; Horowitz and Hill, 1989).

### TWO BASIC RULES

Throughout this chapter we shall use two basic rules (or input terminal restrictions) that are very helpful in designing op-amp circuits.

**RULE 1** When the op-amp output is in its linear range, the two input terminals are at the same voltage.

This is true because if the two input terminals were not at the same voltage, the differential input voltage would be multiplied by the infinite gain to yield an infinite output voltage. This is absurd; most op amps use a power supply of  $\pm 15$  V, so  $v_o$  is restricted to this range. Actually the op-amp specifications guarantee a



**Figure 3.2 Op-amp circuit symbol** A voltage at  $v_1$ , the inverting input, is greatly amplified and inverted to yield  $v_o$ . A voltage at  $v_2$ , the noninverting input, is greatly amplified to yield an in-phase output at  $v_o$ .

linear output range of only  $\pm 10$  V, although some saturate at about  $\pm 13$  V. A single supply is adequate with some op amps, such as the LM358 (Horowitz and Hill, 1989).

**RULE 2** No current flows into either input terminal of the op amp.

This is true because we assume that the input impedance is infinity, and no current flows into an infinite impedance. Even if the input impedance were finite, Rule 1 tells us that there is no voltage drop across  $R_i$ ; so therefore, no current flows.

## 3.2 INVERTING AMPLIFIERS

### CIRCUIT

Figure 3.3(a) shows the basic inverting-amplifier circuit. It is widely used in instrumentation. Note that a portion of  $v_o$  is fed back via  $R_f$  to the negative input of the op amp. This provides the inverting amplifier with the many advantages associated with the use of negative feedback—increased bandwidth, lower output impedance, and so forth. If  $v_o$  is ever fed back to the positive input of the op amp, examine the circuit carefully. Either there is a mistake, or the circuit is one of the rare ones in which a regenerative action is desired.

### EQUATION

Note that the positive input of the op amp is at 0 V. Therefore, by Rule 1, the negative input of the op amp is also at 0 V. Thus no matter what happens to the rest of the circuit, the negative input of the op amp remains at 0 V, a condition known as a *virtual ground*.

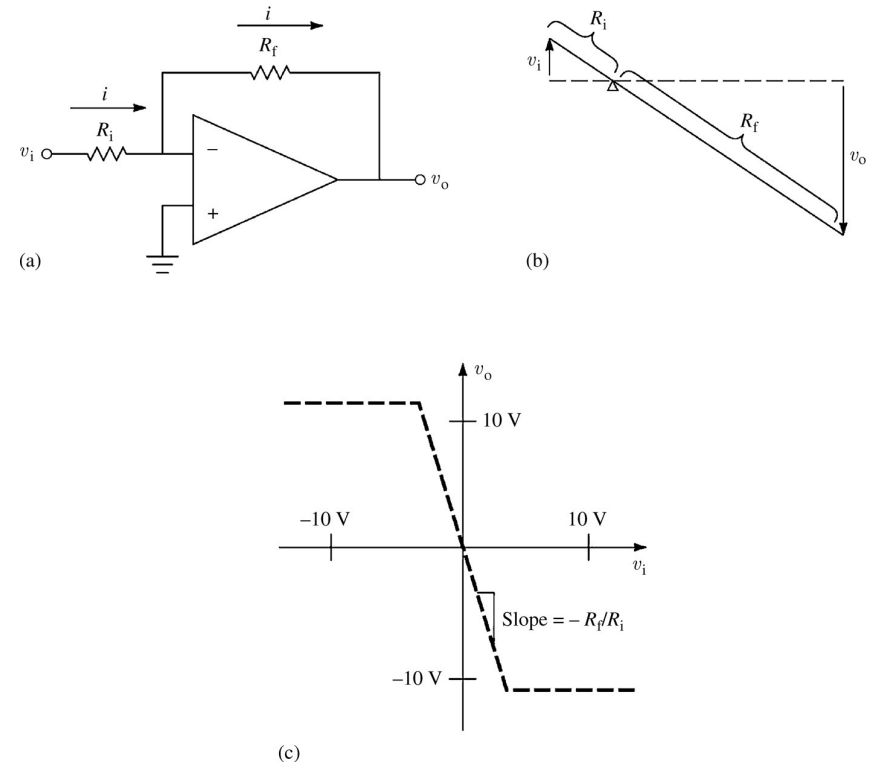
Because the right side of  $R_i$  is at 0 V and the left side is  $v_i$ , by Ohm's law the current  $i$  through  $R_i$  is  $i = v_i/R_i$ . By Rule 2, no current can enter the op amp; therefore  $i$  must also flow through  $R_f$ . This produces a voltage drop across  $R_f$  of  $iR_f$ . Because the left end of  $R_f$  is at 0 V, the right end must be

$$v_o = -iR_f = -v_i \frac{R_f}{R_i} \quad \text{or} \quad \frac{v_o}{v_i} = \frac{-R_f}{R_i} \quad (3.1)$$

Thus the circuit inverts, and the *inverting-amplifier* gain (not the op-amp gain) is given by the ratio of  $R_f$  to  $R_i$ .

### LEVER ANALOGY

Figure 3.3(b) shows an easy way to visualize the circuit's behavior. A lever is formed with arm lengths proportional to resistance values. Because the



**Figure 3.3** (a) An inverting amplifier. Current flowing through the input resistor  $R_i$  also flows through the feedback resistor  $R_f$ . (b) A lever with arm lengths proportional to resistance values enables the viewer to visualize the input–output characteristics easily. (c) The input–output plot shows a slope of  $-R_f/R_i$  in the central portion, but the output saturates at about  $\pm 13$  V.

negative input is at 0 V, the fulcrum is placed at 0 V, as shown. If  $R_f$  is three times  $R_i$ , as shown, any variation of  $v_i$  results in a three-times-bigger variation of  $v_o$ . The circuit in Figure 3.3(a) is a voltage-controlled current source (VCCS) for any load  $R_f$  (Jung, 1986). The current  $i$  through  $R_f$  is  $v_i/R_i$ , so  $v_i$  controls  $i$ . Current sources are useful in electrical impedance plethysmography for passing a fixed current through the body (Section 8.7).

### INPUT-OUTPUT CHARACTERISTIC

Figure 3.3(c) shows that the circuit is linear only over a limited range of  $v_i$ . When  $v_o$  exceeds about  $\pm 13$  V, it *saturates* (limits), and further increases in  $v_i$  produce no change in the output. The linear swing of  $v_o$  is about 4 V less than the difference in power-supply voltages. Although op amps usually have

power-supply voltages set at  $\pm 15$  V, reduced power-supply voltages may be used, with a corresponding reduction in the saturation voltages and the linear swing of  $v_o$ .

### SUMMING AMPLIFIER

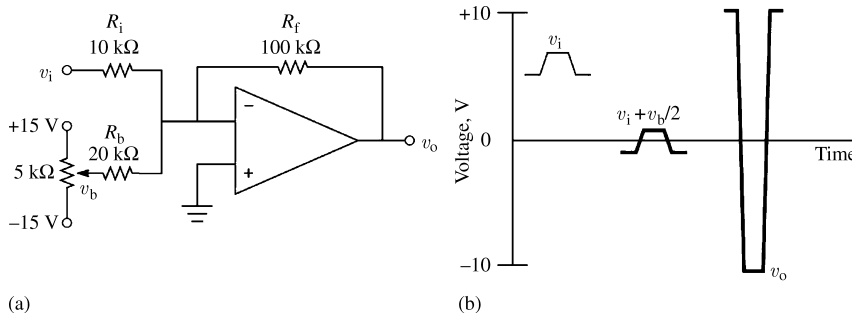
The inverting amplifier may be extended to form a circuit that yields the weighted sum of several input voltages. Each input voltage  $v_{i1}, v_{i2}, \dots, v_{ik}$  is connected to the negative input of the op amp by an individual resistor the conductance of which  $(1/R_{ik})$  is proportional to the desired weighting.

**EXAMPLE 3.1** The output of a biopotential preamplifier that measures the electro-oculogram (EOG) (Section 4.7) is an undesired dc voltage of  $\pm 5$  V due to electrode half-cell potentials (Section 5.1), with a desired signal of  $\pm 1$  V superimposed. Design a circuit that will balance the dc voltage to zero and provide a gain of  $-10$  for the desired signal without saturating the op amp.

**ANSWER** Figure E3.1(a) shows the design. We assume that  $v_b$ , the balancing voltage available from the  $5\text{ k}\Omega$  potentiometer, is  $\pm 10$  V. The undesired voltage at  $v_i = 5$  V. For  $v_o = 0$ , the current through  $R_f$  is zero. Therefore the sum of the currents through  $R_i$  and  $R_b$ , is zero.

$$\frac{v_i}{R_i} + \frac{v_b}{R_b} = 0$$

$$R_b = \frac{-R_i v_b}{v_i} = \frac{-10^4(-10)}{5} = 2 \times 10^4 \Omega$$



**Figure E3.1** (a) This circuit sums the input voltage  $v_i$  plus one-half of the balancing voltage  $v_b$ . Thus the output voltage  $v_o$  can be set to zero even when  $v_i$  has a nonzero dc component, (b) The three waveforms show  $v_i$ , the input voltage;  $(v_i + v_b)/2$ , the balanced-out voltage; and  $v_o$ , the amplified output voltage. If  $v_i$  were directly amplified, the op amp would saturate.

For a gain of  $-10$ , (3.1) requires  $R_f/R_i = 10$ , or  $R_f = 100\text{ k}\Omega$ . The circuit equation is

$$\begin{aligned} v_o &= -R_f \left( \frac{v_i}{R_i} + \frac{v_b}{R_b} \right) \\ v_o &= -10^5 \left( \frac{v_i}{10^4} + \frac{v_b}{2 \times 10^4} \right) \\ v_o &= -10 \left( v_i + \frac{v_b}{2} \right) \end{aligned}$$

The potentiometer can balance out any undesired voltage in the range  $\pm 5$  V, as shown by Figure E3.1(b). Here we have selected resistors of  $10\text{ k}\Omega$  to  $100\text{ k}\Omega$  from the common resistors used in electronic circuits that have values between  $10\text{ }\Omega$  and  $22\text{ M}\Omega$ .

### 3.3 NONINVERTING AMPLIFIERS

#### FOLLOWER

Figure 3.4(a) shows the circuit for a unity-gain follower. Because  $v_i$  exists at the positive input of the op amp, by Rule 1  $v_i$  must also exist at the negative input. But  $v_o$  is also connected to the negative input. Therefore  $v_o = v_i$ , or the output voltage follows the input voltage. At first glance it seems nothing is gained by using this circuit; the output is the same as the input. However, the circuit is very useful as a *buffer*, to prevent a high source resistance from being loaded down by a low-resistance load. By Rule 2, no current flows into the positive input, and therefore the source resistance in the external circuit is not loaded at all.

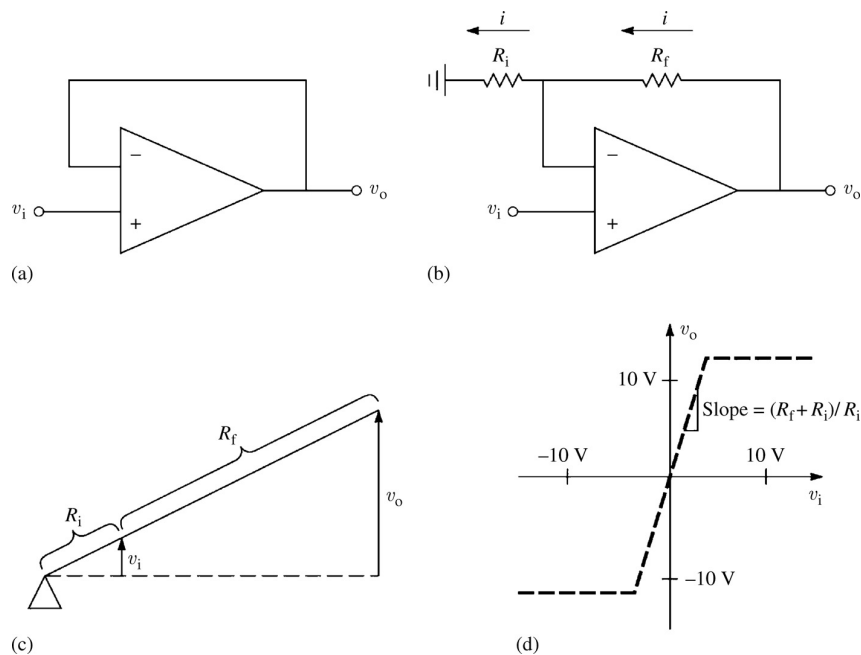
#### NONINVERTING AMPLIFIER

Figure 3.4(b) shows how the follower circuit can be modified to produce gain. By Rule 1,  $v_i$  appears at the negative input of the op amp. This causes current  $i = v_i/R_i$  to flow to ground. By Rule 2, none of  $i$  can come from the negative input; therefore all must flow through  $R_f$ . We can then calculate  $v_o = i(R_f + R_i)$  and solve for the gain.

$$\frac{v_o}{v_i} = \frac{i(R_f + R_i)}{iR_i} = \frac{R_f + R_i}{R_i} \quad (3.2)$$

We note that the circuit gain (not the op-amp gain) is positive, always greater than or equal to 1; and that if  $R_i = \infty$  (open circuit), the circuit reduces to Figure 3.4(a).

Figure 3.4(c) shows how a lever makes possible an easy visualization of the input–output characteristics. The fulcrum is placed at the left end, because  $R_i$  is grounded at the left end.  $v_i$  appears between the two resistors, so it provides an input at the central part of the diagram.  $v_o$  travels through an output excursion determined by the lever arms.



**Figure 3.4** (a) A follower,  $v_o = v_i$ . (b) A noninverting amplifier,  $v_i$  appears across  $R_i$ , producing a current through  $R_i$  that also flows through  $R_f$ . (c) A lever with arm lengths proportional to resistance values makes possible an easy visualization of input–output characteristics. (d) The input–output plot shows a positive slope of  $(R_f + R_i)/R_i$  in the central portion, but the output saturates at about  $\pm 13$  V.

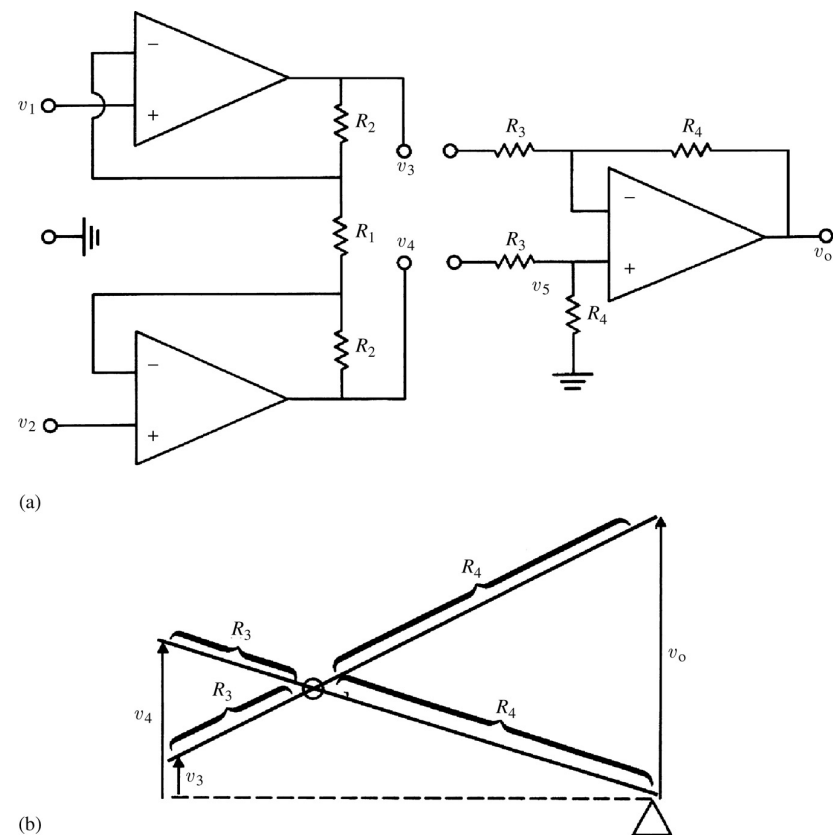
Figure 3.4(d), the input–output characteristic, shows that a one-op-amp circuit can have a positive amplifier gain. Again saturation is evident.

### 3.4 DIFFERENTIAL AMPLIFIERS

#### ONE-OP-AMP DIFFERENTIAL AMPLIFIER

The right side of Figure 3.5(a) shows a simple one-op-amp differential amplifier. Current flows from  $v_4$  through  $R_3$  and  $R_4$  to ground. By Rule 2, no current flows into the positive input of the op amp. Hence  $R_3$  and  $R_4$  act as a simple voltage-divider attenuator, which is unaffected by having the op amp attached or by any other changes in the circuit. The voltages in this part of the circuit are visualized in Figure 3.5(b) by the single lever that is attached to the fulcrum (ground).

By Rule 1, whatever voltage appears at the positive input also appears at the negative input. Once this voltage is fixed, the top half of the circuit



**Figure 3.5** (a) The right side shows a one-op-amp differential amplifier, but it has low input impedance. The left side shows how two additional op amps can provide high input impedance and gain. (b) For the one-op-amp differential amplifier, two levers with arm lengths proportional to resistance values make possible an easy visualization of input–output characteristics.

behaves like an inverting amplifier. For example, if  $v_4$  is 0 V, the positive input of the op amp is 0 V and the  $v_3$ – $v_o$  circuit behaves exactly like an inverting amplifier. For other values of  $v_4$ , an inverting relation is obtained about some voltage intermediate between  $v_4$  and 0 V. The relationship can be visualized in Figure 3.5(b) by noting that the two levers behave like a pair of scissors. The thumb and finger holes are  $v_4$  and  $v_3$ , and the points are at  $v_o$  and 0 V.

We solve for the gain by finding  $v_5$ .

$$v_5 = \frac{v_4 R_4}{R_3 + R_4} \quad (3.3)$$

Then, solving for the current in the top half, we get

$$i = \frac{v_3 - v_5}{R_3} = \frac{v_5 - v_o}{R_4} \quad (3.4)$$

Substituting (3.3) into (3.4) yields

$$v_o = \frac{(v_4 - v_3)R_4}{R_3} \quad (3.5)$$

This is the equation for a differential amplifier. If the two inputs are hooked together and driven by a common source, with respect to ground, then the *common-mode voltage*  $v_c$  is  $v_3 = v_4$ . Equation (3.5) shows that the ideal output is 0. The differential amplifier-circuit (not op-amp) *common-mode gain*  $G_c$  is 0. In Figure 3.5(b), imagine the scissors to be closed. No matter how the inputs are varied,  $v_o = 0$ .

If on the other hand  $v_3 \neq v_4$ , then the differential voltage  $(v_4 - v_3)$  produces an amplifier-circuit (not op-amp) *differential gain*  $G_d$  that from (3.5) is equal to  $R_4/R_3$ . This result can be visualized in Figure 3.5(b) by noting that as the scissors open,  $v_o$  is geometrically related to  $(v_4 - v_3)$  in the same ratio as the lever arms,  $R_4/R_3$ .

No differential amplifier perfectly rejects the common-mode voltage. To quantify this imperfection, we use the term *common-mode rejection ratio* (CMRR), which is defined as

$$\text{CMRR} = \frac{G_d}{G_c} \quad (3.6)$$

This factor may be lower than 100 for some oscilloscope differential amplifiers and higher than 10,000 for a high-quality biopotential amplifier.

**EXAMPLE 3.2** A blood-pressure sensor uses a four-active-arm Wheatstone strain gage bridge excited with dc. At full scale, each arm changes resistance by  $\pm 0.3\%$ . Design an amplifier that will provide a full-scale output over the op amp's full range of linear operation. Use the minimal number of components.

**ANSWER** From (2.6),  $\Delta v_o = v_i \Delta R / R = 5 \text{ V}(0.003) = 0.015 \text{ V}$ . Gain =  $20 / 0.015 = 1333$ . Assume  $R = 120 \Omega$ . Then the Thevenin source impedance =  $60 \Omega$ . Use this to replace  $R_3$  of Figure 3.5(a) right side. Then  $R_4 = R_3(\text{gain}) = 60 \Omega(1333) = 80 \text{ k}\Omega$ .

### THREE-OP-AMP DIFFERENTIAL AMPLIFIER

The one-op-amp differential amplifier is quite satisfactory for low-resistance sources, such as strain-gage Wheatstone bridges (Section 2.3). But the input

resistance is too low for high-resistance sources. Our first recourse is to add the simple follower shown in Figure 3.4(a) to each input. This provides the required buffering. Because this solution uses two additional op amps, we can also obtain gain from these buffering amplifiers by using a noninverting amplifier, as shown in Figure 3.4(b). However, this solution amplifies the common-mode voltage, as well as the differential voltage, so there is no improvement in CMRR.

A superior solution is achieved by hooking together the two  $R_i$ 's of the noninverting amplifiers and eliminating the connection to ground. The result is shown on the left side of Figure 3.5(a). To examine the effects of common-mode voltage, assume that  $v_1 = v_2$ . By Rule 1,  $v_1$  appears at both negative inputs to the op amps. This places the same voltage at both ends of  $R_1$ . Hence current through  $R_1$  is 0. By Rule 2, no current can flow from the op-amp inputs. Hence the current through both  $R_2$ 's is 0, so  $v_1$  appears at both op-amp outputs and the  $G_c$  is 1.

To examine the effects when  $v_1 \neq v_2$ , we note that  $v_1 - v_2$  appears across  $R_1$ . This causes a current to flow through  $R_1$  that also flows through the resistor string  $R_2, R_1, R_2$ . Hence the output voltage

$$v_3 - v_4 = i(R_2 + R_1 + R_2)$$

whereas the input voltage

$$v_1 - v_2 = iR_1$$

The differential gain is then

$$G_d = \frac{v_3 - v_4}{v_1 - v_2} = \frac{2R_2 + R_1}{R_1} \quad (3.7)$$

Since the  $G_c$  is 1, the CMRR is equal to the  $G_d$ , which is usually much greater than 1. When the left and right halves of Figure 3.5(a) are combined, the resulting three-op-amp amplifier circuit is frequently called an *instrumentation amplifier*. It has high input impedance, a high CMRR, and a gain that can be changed by adjusting  $R_1$ . This circuit finds wide use in measuring biopotentials (Section 6.7), because it rejects the large 60 Hz common-mode voltage that exists on the body.

### 3.9 DIFFERENTIATORS

Interchanging the integrator's  $R$  and  $C$  yields the differentiator shown in Figure 3.11. The current through a capacitor is given by

$$i = C \frac{dv}{dt} \quad (3.12)$$

If  $dv_i/dt$  is positive,  $i$  flows through  $R$  in a direction such that it yields a negative  $v_o$ . Thus

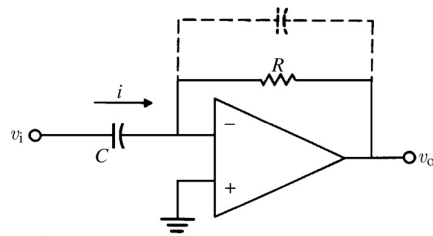
$$v_o = -RC \frac{dv_i}{dt} \quad (3.13)$$

The frequency response of a differentiator is given by the ratio of feedback to input impedance.

$$\begin{aligned} \frac{V_o(j\omega)}{V_i(j\omega)} &= -\frac{Z_f}{Z_i} = -\frac{R}{1/j\omega C} \\ &= -j\omega RC = -j\omega\tau \end{aligned} \quad (3.14)$$

Equation (3.14) shows that the circuit gain increases as  $f$  increases and that it is equal to unity when  $\omega\tau = 1$ . Figure 3.10 shows the frequency response.

Unless specific preventive steps are taken, the circuit tends to oscillate. The output also tends to be noisy, because the circuit emphasizes high frequencies. A differentiator followed by a comparator is useful for detecting



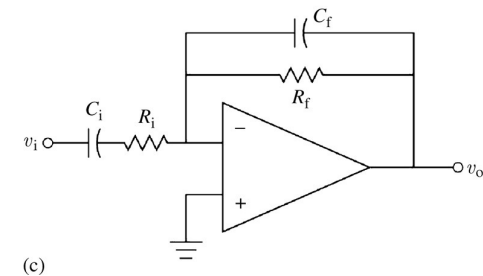
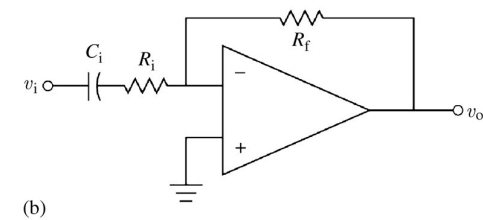
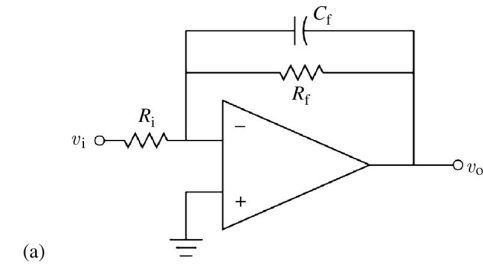
**Figure 3.11 A differentiator** The dashed lines indicate that a small capacitor must usually be added across the feedback resistor to prevent oscillation.

an event the slope of which exceeds a given value—for example, detection of the R wave in an electrocardiogram.

### 3.10 ACTIVE FILTERS

#### LOW-PASS FILTER

Figure 1.9(a) shows a low-pass filter that is useful for attenuating high-frequency noise. A low-pass active filter can be obtained by using the one-op-amp circuit shown in Figure 3.12(a). The advantages of this circuit are that



**Figure 3.12 Active filters** (a) A low-pass filter attenuates high frequencies. (b) A high-pass filter attenuates low frequencies and blocks dc. (c) A bandpass filter attenuates both low and high frequencies.

it is capable of gain and that it has a very low output impedance. The frequency response is given by the ratio of feedback to input impedance.

$$\begin{aligned}\frac{V_o(j\omega)}{V_i(j\omega)} &= -\frac{Z_f}{Z_i} = -\frac{(R_f/j\omega C_f)}{[(1/j\omega C_f) + R_i]} \\ &= -\frac{R_f}{(1 + j\omega R_i C_f)R_i} = -\frac{R_f}{R_i} \frac{1}{1 + j\omega\tau}\end{aligned}\quad (3.15)$$

where  $\tau = R_i C_f$ . Note that (3.15) has the same form as (1.23). Figure 3.10 shows the frequency response, which is similar to that shown in Figure 1.8(d). For  $\omega \ll 1/\tau$ , the circuit behaves as an inverting amplifier (Figure 3.3), because the impedance of  $C_f$  is large compared with  $R_f$ . For  $\omega \gg 1/\tau$ , the circuit behaves as an integrator (Figure 3.9), because  $C_f$  is the dominant feedback impedance. The *corner frequency*  $f_c$ , which is defined by the intersection of the two asymptotes shown, is given by the relation  $\omega\tau = 2\pi f_c\tau = 1$ . When a designer wishes to limit the frequency of a wide-bandwidth amplifier, it is not necessary to add a separate stage, as shown in Figure 3.12(a), but only to add the correct size  $C_f$  to the existing wide-band amplifier.

## HIGH-PASS FILTER

Figure 3.12(b) shows a one-op-amp high-pass filter. Such a circuit is useful for amplifying a small ac voltage that rides on top of a large dc voltage, because  $C_i$  blocks the dc. The frequency-response equation is

$$\begin{aligned}\frac{V_o(j\omega)}{V_i(j\omega)} &= -\frac{Z_f}{Z_i} = -\frac{R_f}{1/j\omega C_i + R_i} \\ &= -\frac{j\omega R_f C_i}{1 + j\omega C_i R_i} = -\frac{R_f}{R_i} \frac{j\omega\tau}{1 + j\omega\tau}\end{aligned}\quad (3.16)$$

where  $\tau = R_i C_i$ . Figure 3.10 shows the frequency response. For  $\omega \ll 1/\tau$ , the circuit behaves as a differentiator (Figure 3.11), because  $C_i$  is the dominant input impedance. For  $\omega \gg 1/\tau$ , the circuit behaves as an inverting amplifier, because the impedance of  $R_i$  is large compared with that of  $C_i$ . The corner frequency  $f_c$ , which is defined by the intersection of the two asymptotes shown, is given by the relation  $\omega\tau = 2\pi f_c\tau = 1$ .

## BANDPASS FILTER

A series combination of the low-pass filter and the high-pass filter results in a *bandpass filter*, which amplifies frequencies over a desired range and attenuates higher and lower frequencies. Figure 3.12(c) shows that the bandpass function can be achieved with a one-op-amp circuit. Figure 3.10 shows the frequency response. The corner frequencies are defined by the same relations as those for the low-pass and the high-pass filters. This circuit is useful for

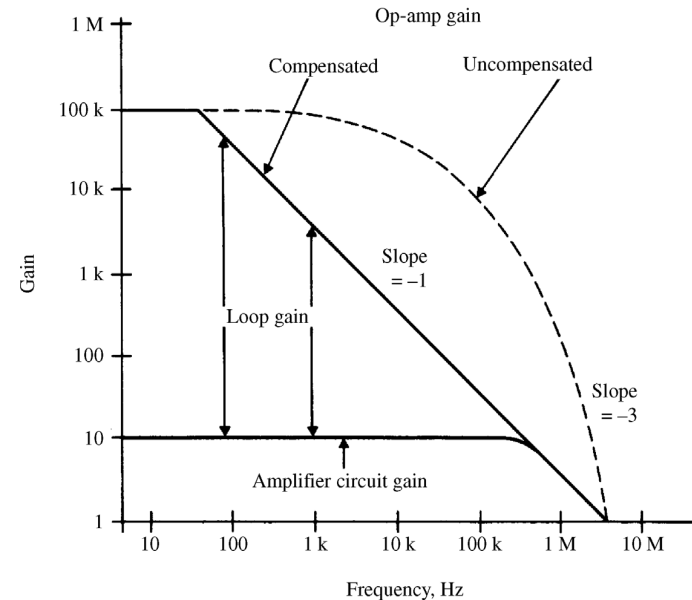
amplifying a certain band of frequencies, such as those required for recording heart sounds or the electrocardiogram.

## 3.11 FREQUENCY RESPONSE

Up until now, we have found it useful to consider the op amp as ideal. Now we shall examine the effects of several nonideal characteristics, starting with that of frequency response.

### OPEN-LOOP GAIN

Because the op amp requires very high gain, it has several stages. Each of these stages has stray or junction capacitance that limits its high-frequency response in the same way that a simple  $RC$  low-pass filter reduces high-frequency gain. At high frequencies, each stage has a  $-1$  slope on a log-log plot of gain versus frequency, and each has a  $-90^\circ$  phase shift. Thus a three-stage op amp, such as type 709, reaches a slope of  $-3$ , as shown by the dashed curve in Figure 3.13.



**Figure 3.13 Op-amp frequency characteristics** Early op amps (such as the 709) were uncompensated, had a gain greater than 1 when the phase shift was equal to  $-180^\circ$ , and therefore oscillated unless compensation was added externally. A popular op amp, the 411, is compensated internally; so for a gain greater than 1, the phase shift is limited to  $-90^\circ$ . When feedback resistors are added to build an amplifier circuit, the loop gain on this log-log plot is the difference between the op-amp gain and the amplifier-circuit gain.

The phase shift reaches  $-270^\circ$ , which is quite satisfactory for a comparator, because feedback is not employed. For an amplifier, if the gain is greater than 1 when the phase shift is equal to  $-180^\circ$  (the closed-loop condition for oscillation), there is undesirable oscillation.

## COMPENSATION

Adding an external capacitor to the terminals indicated on the specification sheet moves one of the  $RC$  filter corner frequencies to a very low frequency. This compensates the uncompensated op amp, resulting in a slope of  $-1$  and a maximal phase shift of  $-90^\circ$ . This is done with an internal capacitor in the 411, resulting in the solid curve shown in Figure 3.13. This op amp does not oscillate for any amplifier we have described. This op amp has very high dc gain, but the gain is progressively reduced at higher frequencies, until it is only 1 at 4 MHz.

## CLOSED-LOOP GAIN

It might appear that the op amp has very poor frequency response, because its gain is reduced for frequencies above 40 Hz. However, an amplifier circuit is never built using the op-amp open loop, so we shall therefore discuss only the circuit closed-loop response. For example, if we build an amplifier circuit with a gain of 10, as shown in Figure 3.13, the frequency response is flat up to 400 kHz and is reduced above that frequency only because the amplifier-circuit gain can never exceed the op-amp gain. We find this an advantage of using negative feedback, in that the frequency response is greatly extended.

## LOOP GAIN

The loop gain for an amplifier circuit is obtained by breaking the feedback loop at any point in the loop, injecting a signal, and measuring the gain around the loop. For example, in a unity-gain follower [Figure 3.4(a)] we break the feedback loop and then the injected signal enters the negative input, after which it is amplified by the op-amp gain. Therefore, the loop gain equals the op-amp gain. To measure loop gain in an inverting amplifier with a gain of  $-1$  [Figure 3.3(a)], assume that the amplifier-circuit input is grounded. The injected signal is divided by 2 by the attenuator formed of  $R_f$  and  $R_i$ , and is then amplified by the op-amp gain. Thus the loop gain is equal to (op-amp gain)/2.

Figure 3.13 shows the loop-gain concept for a noninverting amplifier. The amplifier-circuit gain is 10. On the log-log plot, the difference between the op-amp gain and the amplifier-circuit gain is the loop gain. At low frequencies, the loop gain is high and the closed-loop amplifier-circuit characteristics are determined by the feedback resistors. At high frequencies, the loop gain is low and the amplifier-circuit characteristics follow the op-amp characteristics. High loop gain is good for accuracy and stability, because the feedback resistors can be made much more stable than the op-amp characteristics.

## GAIN-BANDWIDTH PRODUCT

The gain-bandwidth product of the op amp is equal to the product of gain and bandwidth at a particular frequency. Thus in Figure 3.13 the unity-gain-bandwidth product is 4 MHz, a typical value for op amps. Note that along the entire curve with a slope of  $-1$ , the gain-bandwidth product is still constant, at 4 MHz. Thus, for any amplifier circuit, we can obtain its bandwidth by dividing the gain-bandwidth product by the amplifier-circuit gain. For higher-frequency applications, op amps such as the OP-37E are available with gain-bandwidth products of 60 MHz.

## SLEW RATE

Small-signal response follows the amplifier-circuit frequency response predicted by Figure 3.13. For large signals there is an additional limitation. When rapid changes in output are demanded, the capacitor added for compensation must be charged up from an internal source that has limited current capability  $I_{\max}$ . The change in voltage across the capacitor is then limited,  $dv/dt = I_{\max}/C$ , and  $dv_o/dt$  is limited to a maximal slew rate (15 V/ $\mu$ s for the 411). If this slew rate  $S_r$  is exceeded by a large-amplitude, high-frequency sine wave, distortion occurs. Thus there is a limitation on the sine-wave *full-power response*, or maximal frequency for rated output,

$$f_p = \frac{S_r}{2\pi V_{or}} \quad (3.17)$$

where  $V_{or}$  is the rated output voltage (usually 10 V). If the slew rate is too slow for fast switching of a comparator, an uncompensated op amp can be used, because comparators do not contain the negative-feedback path that may cause oscillations.

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## 3.12 OFFSET VOLTAGE

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Another nonideal characteristic is that of offset voltage. The two op-amp inputs drive the bases of transistors, and the base-to-emitter voltage drop may be slightly different for each. Thus, so that we can obtain  $v_o = 0$ , the voltage ( $v_1 - v_2$ ) must be a few millivolts. This offset voltage is usually not important when  $v_i$  is 1 to  $-10$  V. But when  $v_i$  is on the order of millivolts, as when amplifying the output from thermocouples or strain gages, the offset voltage must be considered.

## NULLING

The offset voltage may be reduced to zero by adding an external nulling pot to the terminals indicated on the specification sheet. Adjustment of this pot



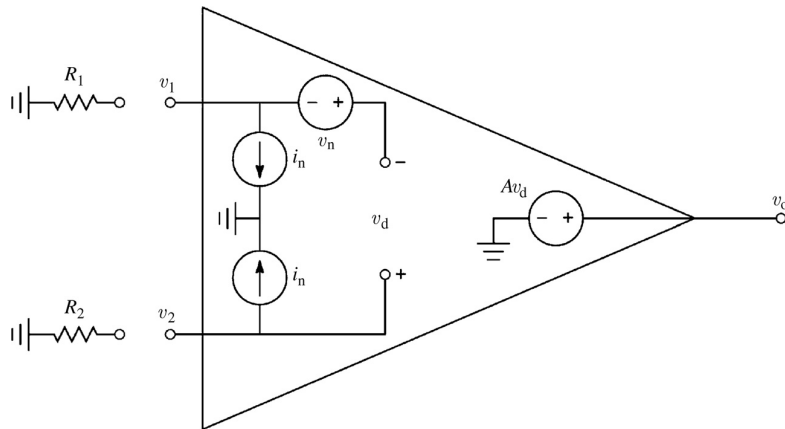
increases emitter current through one of the input transistors and lowers it through the other. This alters the base-to-emitter voltage of the two transistors until the offset voltage is reduced to zero.

## DRIFT

Even though the offset voltage may be set to 0 at 25 °C, it does not remain there if temperature is not constant. Temperature changes that affect the base-to-emitter voltages may be due to either environmental changes or to variations in the dissipation of power in the chip that result from fluctuating output voltage. The effects of temperature may be specified as a maximal offset voltage change in volts per degree Celsius or a maximal offset voltage change over a given temperature range, say –25 °C to +85 °C. If the drift of an inexpensive op amp is too high for a given application, tighter specifications (0.1  $\mu\text{V}/^\circ\text{C}$ ) are available with temperature-controlled chips. An alternative technique modulates the dc as in chopper-stabilized and varactor op amps (Tobey *et al.*, 1971).

## NOISE

All semiconductor junctions generate noise, which limits the detection of small signals. Op amps have transistor input junctions, which generate both noise-voltage sources and noise-current sources. These can be modeled as shown in Figure 3.14. For low source impedances, only the noise voltage  $v_n$  is important; it is large compared with the  $i_n R$  drop caused by the current noise  $i_n$ . The noise is random, but the amplitude varies with frequency. For example, at low



**Figure 3.14** Noise sources in an op amp The noise-voltage source  $v_n$  is in series with the input and cannot be reduced. The noise added by the noise-current sources in can be minimized by using small external resistances.

frequencies the noise power density varies as  $1/f$  (flicker noise), so a large amount of noise is present at low frequencies. At the midfrequencies, the noise is lower and can be specified in root-mean-square (rms) units of  $\text{V}\cdot\text{Hz}^{-1/2}$ . In addition, some silicon planar-diffused bipolar integrated-circuit op amps exhibit bursts of noise, called *popcorn noise* (Wait *et al.*, 1975).

## 3.13 BIAS CURRENT

Because the two op-amp inputs drive transistors, base or gate current must flow all the time to keep the transistors turned on. This is called *bias current*, which for the 411 is about 200 pA. This bias current must flow through the feedback network. It causes errors proportional to feedback-element resistances. To minimize these errors, small feedback resistors, such as those with resistances of 10 k $\Omega$ , are normally used. Smaller values should be used only after a check to determine that the current flowing through the feedback resistor, plus the current flowing through all load resistors, does not exceed the op-amp output current rating (20 mA for the 411).

## DIFFERENTIAL BIAS CURRENT

The difference between the two input bias currents is much smaller than either of the bias currents alone. A degree of cancellation of the effects of bias current can be achieved by having each bias current flow through the same equivalent resistance. This is accomplished for the inverting amplifier and the noninverting amplifier by adding, in series with the positive input, a compensation resistor the value of which is equal to the parallel combination of  $R_i$  and  $R_f$ . There still is an error, but it is now determined by the difference in bias current.

## DRIFT

The input bias currents are transistor base or gate currents, so they are temperature sensitive, because transistor gain varies with temperature. However, the changes in gain of the two transistors tend to track together, so the additional compensation resistor that we have described minimizes the problem.

## NOISE

Figure 3.14 shows how variations in bias current contribute to overall noise. The noise currents flow through the external equivalent resistances so that the total rms noise voltage is

$$v \cong \{[v_n^2 + (i_n R_1)^2 + (i_n R_2)^2 + 4kTR_1 + 4kTR_2]BW\}^{1/2} \quad (3.18)$$

where

$R_1$  and  $R_2$  = equivalent source resistances

$v_n$  = mean value of the rms noise voltage, in  $\text{V} \cdot \text{Hz}^{-1/2}$ ,  
across the frequency range of interest

$i_n$  = mean value of the rms noise current, in  $\text{A} \cdot \text{Hz}^{-1/2}$ ,  
across the frequency range of interest

$\kappa$  = Boltzmann's constant (Appendix)

$T$  = temperature, K

BW = noise bandwidth, Hz

The specification sheet provides values of  $v_n$  and  $i_n$  (sometimes  $v_n^2$  and  $i_n^2$ ), thus making it possible to compare different op amps. If the source resistances are 10 k $\Omega$ , bipolar-transistor op amps yield the lowest noise. For larger source resistances, low-input-current amplifiers such as the field-effect transistor (FET) input stage are best because of their lower current noise. Ary (1977) presents design factors and performance specifications for a low-noise amplifier.

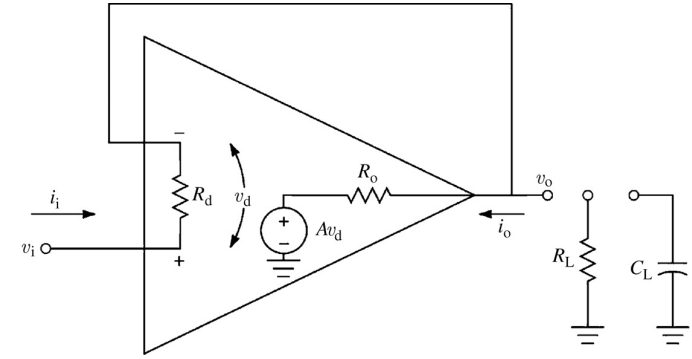
For ac amplifiers, the lowest noise is obtained by calculating the characteristic noise resistance  $R_n = v_n/i_n$  and setting it equal to the equivalent source resistance  $R_2$  (for the noninverting amplifier). This is accomplished by inserting a transformer with turns ratio 1 :  $N$ , where  $N = (R_n/R_2)^{1/2}$ , between the source and the op amp (Jung, 1986).

### 3.14 INPUT AND OUTPUT RESISTANCE

#### INPUT RESISTANCE

The op-amp differential-input resistance  $R_d$  is shown in Figures 3.1 and 3.15. For the FET-input 411, it is 1 T $\Omega$ , whereas for BJT-input op amps, it is about 2 M $\Omega$ , which is comparable to the value of some feedback resistors used. However, we shall see that its value is usually not important because of the benefits of feedback. Consider the follower shown in Figure 3.15. In order to calculate the amplifier-circuit input resistance  $R_{ai}$ , assume a change in input voltage  $v_i$ . Because this is a follower,

$$\begin{aligned} \Delta v_o &= A \Delta v_d = A(\Delta v_i - \Delta v_o) \\ &= \frac{A \Delta v_i}{A + 1} \\ \Delta i_i &= \frac{\Delta v_d}{R_d} = \frac{\Delta v_i - \Delta v_o}{R_d} = \frac{\Delta v_i}{(A + 1)R_d} \\ R_{ai} &= \frac{\Delta v_i}{\Delta i_i} = (A + 1)R_d \cong AR_d \end{aligned} \quad (3.19)$$



**Figure 3.15** The amplifier input impedance is much higher than the op-amp input impedance  $R_d$ . The amplifier output impedance is much smaller than the op-amp output impedance  $R_o$ .

Thus the amplifier-circuit input resistance  $R_{ai}$  is about  $(10^5) \times (2 \text{ M}\Omega) = 200 \text{ G}\Omega$ . This value cannot be achieved in practice, because surface leakage paths in the op-amp socket lower it considerably. In general, all noninverting amplifiers have a very high input resistance, which is equal to  $R_d$  times the loop gain. This is not to say that very large source resistances can be used, because the bias current usually causes much larger problems than the amplifier-circuit input impedance. For large source resistances, FET op amps such as the 411 are helpful.

The input resistance of an inverting amplifier is easy to determine. Because the negative input of the op amp is a virtual ground,

$$R_{ai} = \frac{\Delta v_i}{\Delta i_i} = R_i \quad (3.20)$$

Thus the amplifier-circuit input resistance  $R_{ai}$  is equal to  $R_i$ , the input resistor. Because  $R_i$  is usually a small value, the inverting amplifier has small input resistance.

#### OUTPUT RESISTANCE

The op-amp output resistance  $R_o$  is shown in Figures 3.1 and 3.15. It is about 40  $\Omega$  for the typical op amp, which may seem large for some applications. However, its value is usually not important because of the benefits of feedback. Consider the follower shown in Figure 3.15. In order to calculate the amplifier-circuit output resistance  $R_{ao}$ , assume that load resistor  $R_L$  is attached to the output, causing a change in output current  $\Delta i_o$ . Because  $i_o$  flows through  $R_o$ , there is an additional voltage drop  $\Delta i_o R_o$ .

$$\begin{aligned} -\Delta v_d &= \Delta v_o = A \Delta v_d + \Delta i_o R_o = -A \Delta v_o + \Delta i_o R_o \\ (A + 1) \Delta v_o &= \Delta i_o R_o \\ R_{ao} &= \frac{\Delta v_o}{\Delta i_o} = \frac{R_o}{A + 1} \cong R_o / A \end{aligned} \quad (3.21)$$

Thus the amplifier-circuit output resistance  $R_{ao}$  is about  $40/10^5 = 0.0004 \Omega$ , a value negligible in most circuits. In general, all noninverting and inverting amplifiers have an output resistance that is equal to  $R_o$  divided by the loop gain. This is not to say that very small load resistances can be driven by the output. If  $R_L$  shown in Figure 3.15 is smaller than  $500 \Omega$ , the op amp saturates internally, because the maximal current output for a typical op amp is 20 mA. This maximal current output must also be considered when driving large capacitances  $C_L$  at a high slew rate. Then the output current

$$i_o = C_L \frac{dv_o}{dt} \quad (3.22)$$

The  $R_o$ – $C_L$  combination also acts as a low-pass filter, which introduces additional phase shift around the loop and can cause oscillation. The cure is to add a small resistor between  $v_o$  and  $C_L$ , thus isolating  $C_L$  from the feedback loop.

To achieve larger current outputs, the *current booster* is used. An ordinary op amp drives high-power transistors (on heat sinks if required). Then we can use the entire circuit as an op amp by connecting terminals  $v_1$ ,  $v_2$ , and  $v_o$  to external feedback networks. This places the booster section within the feedback loop and keeps distortion low.