Universidad de Valparaíso
CHILE

Filtrado espacial

Unidad 2

BME423 · Procesamiento de imágenes médicas

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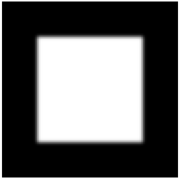
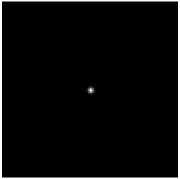
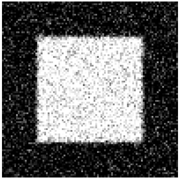
Convolución

La operación convolución es útil para eliminar ruido.

$$\underbrace{\hat{g}(x,y)}_{\text{imagen filtrada}} = \underbrace{w(x,y)}_{\text{filtro}} * \underbrace{g(x,y)}_{\text{imagen medida}}$$







Convolución

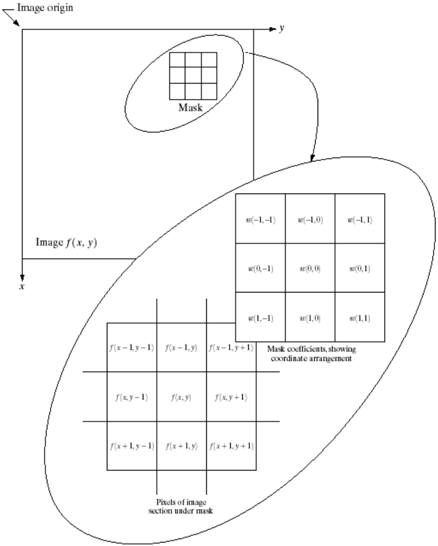
Considere una imagen $f(x, y)$ de $M \times N$ y un kernel $w(s, t)$ de $m \times n$.

La convolución está dada por:

$$\hat{g}(x, y) = w * f = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)$$

con $a = (m - 1)/2$ y $b = (n - 1)/2$

Convolución



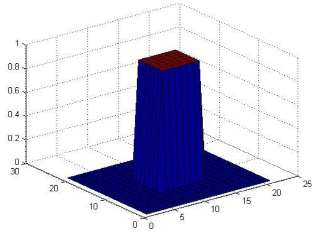
Filtrado - suavizado

Box and weighted average filters.

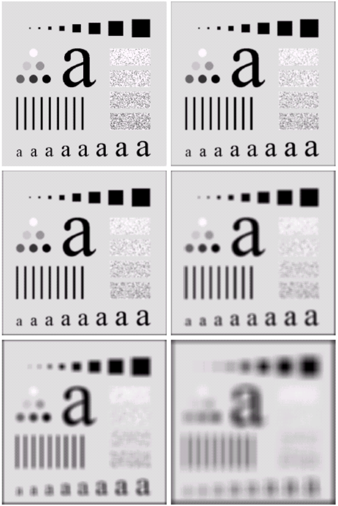
$$\frac{1}{9} \times \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

a b

FIGURE 3.34 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to the sum of the values of its coefficients, as is required to compute an average.



Filtrado - suavizado

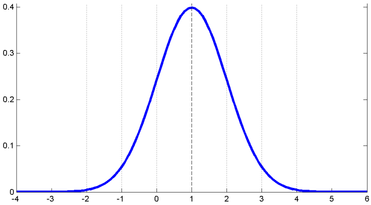


- Máscara de suavizado de n: 3, 5, 9, 15 y 35.
- Tamaños de los cuadrados negros: 3, 5, 9, 15, 25, 35, 45, 55 (separación de 25 píxeles).
- Las letras van desde 10 hasta 24 ptos. en incrementos de 2.
- Las barras verticales tienen un ancho de 5 píxeles, 100 de alto y una separación entre barras de 20 píxeles.
- El diámetro de los círculos es de 25 píxeles, y están separados por 15 píxeles. Sus niveles de gris varían de 0-100% en incrementos de un 20%.

Filtrado - suavizado

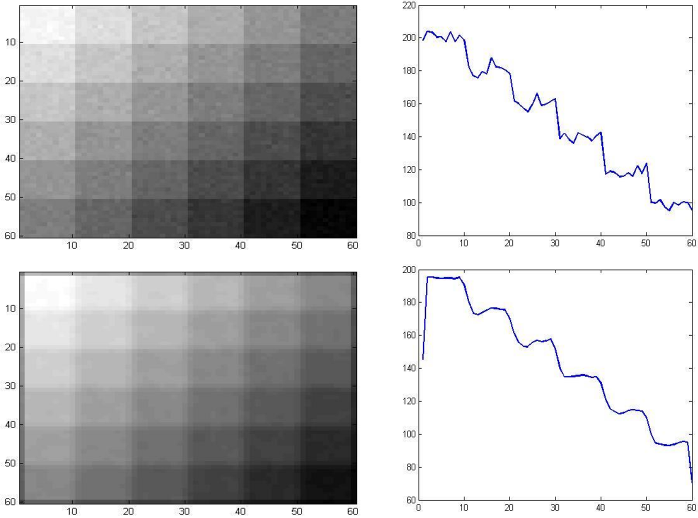
Kernel gaussiano

$$h_g(x,y) = e^{\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)} \qquad h(x,y) = \frac{h_g(x,y)}{\sum_x \sum_y h_g(x,y)}$$

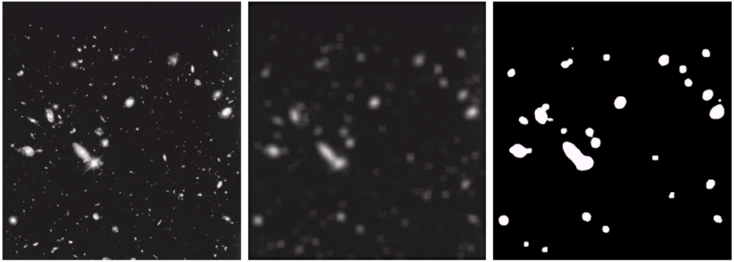


0.0751	0.1238	0.0751
0.1238	0.2042	0.1238
0.0751	0.1238	0.0751

Filtrado - suavizado



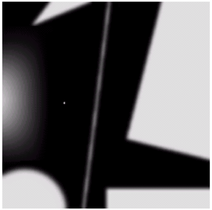
Filtrado - suavizado

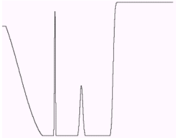


a b c

FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

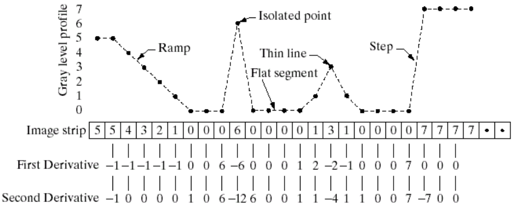
Filtrado - derivadas y detalles





$$\partial f(x,y)/\partial x = f(x+1,y) - f(x,y)$$

$$\partial^2 f(x,y)/\partial x^2 = f(x+1,y) + f(x-1,y) - 2f(x,y)$$



Filtrado - Laplaciano ∇_f^2

Rosenfeld y Kak, 1982

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$\partial f(x,y)/\partial x = f(x+1,y) - f(x,y)$

$\partial f(x,y)/\partial y = f(x,y+1) - f(x,y)$

$\partial^2 f(x,y)/\partial x^2 = f(x+1,y) + f(x-1,y) - 2f(x,y)$

$\partial^2 f(x,y)/\partial y^2 = f(x,y+1) + f(x,y-1) - 2f(x,y)$

Se puede implementar como una convolución

$$\nabla^2 f = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

Filtrado - Laplaciano ∇_f^2

$$\nabla^2 f = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

(a)

0	1	0
1	–4	1
0	1	0

(b)

1	1	1
1	–8	1
1	1	1

(c)

0	–1	0
–1	4	–1
0	–1	0

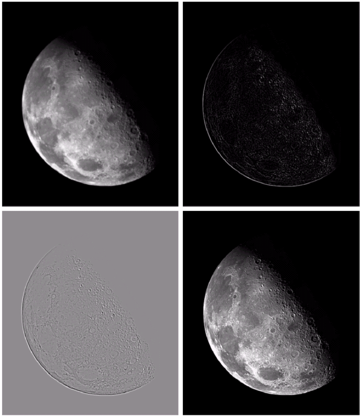
(d)

–1	–1	–1
–1	8	–1
–1	–1	–1

FIGURE 3.39 (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

Filtrado - agudizado

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \text{Si el coef del centro de } \nabla_f^2 \text{ es negativo} \\ f(x,y) + \nabla^2 f(x,y) & \text{Si el coef del centro de } \nabla_f^2 \text{ es positivo} \end{cases}$$



a b
c d

FIGURE 3.40 (a) Image of the North Pole of the moon. (b) Laplacian-filtered image. (c) Laplacian image scaled for display purposes. (d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)

Filtrado - agudizado

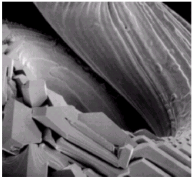
$$g(x,y) = f(x,y) - \nabla^2 f$$

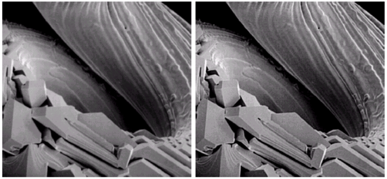
$$g(x,y) = f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)]$$

$$g(x,y) = 5f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)]$$

0	–1	0
–1	5	–1
0	–1	0

–1	–1	–1
–1	9	–1
–1	–1	–1





a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Filtrado - agudizado

Unsharp Masking:

$$f_s(x,y) = f(x,y) - \bar{f}(x,y)$$

donde $f_s(x,y)$ es la imagen agudizada (sharpened) dada por la sustracción de la imagen y su versión suavizada

Una generalización de Unsharp Masking es High-Boost Filtering:

$$f_{\mathrm{hb}}(x,y) = Af(x,y) - \bar{f}(x,y)$$

donde $A \geq 1$

Filtrado - agudizado

La ecuación del filtro High-Boost puede ser expresada mediante:

$$f_{\mathrm{hb}}(x,y) = (A-1)f(x,y) + f(x,y) - \bar{f}(x,y)$$

donde $A \geq 1$. Teniendo en cuenta que:

$$f_s(x,y) = f(x,y) - \bar{f}(x,y)$$

se obtiene:

$$f_{\mathrm{hb}}(x,y) = (A-1)f(x,y) + f_s(x,y)$$

Si se emplea el Laplaciano para obtener $f_s(x,y)$ se tiene:

$$f_{\mathrm{hb}}(x,y) = \begin{cases} Af(x,y) - \nabla^2 f(x,y) & \text{Si el coef del centro de } \nabla_f^2 \text{ es negativo} \\ Af(x,y) + \nabla^2 f(x,y) & \text{Si el coef del centro de } \nabla_f^2 \text{ es positivo} \end{cases}$$

Filtrado - agudizado

Note que si A = 1, $f_{hb}(x,y)$ corresponde al agudizado Laplaciano estándar

Si A >> 1, el efecto sharpening se vuelve menos significativo

0	−1	0
−1	$A + 4$	−1
0	−1	0

−1	−1	−1
−1	$A + 8$	−1
−1	−1	−1

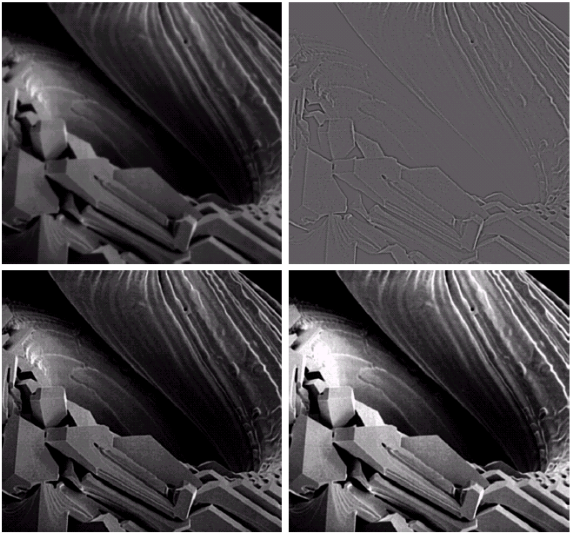
a b

FIGURE 3.42 The high-boost filtering technique can be implemented with either one of these masks, with $A \geq 1$.

Filtrado - agudizado

a b
c d

FIGURE 3.43 (a) Same as Fig. 3.41(c), but darker. (a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$. (c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.



Filtrado - Gradiente

El vector gradiente de una imagen en las coordenadas (x,y) está dado por:

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

La magnitud es:

$$\begin{aligned} \nabla f &= \operatorname{mag}(\nabla \mathbf{f}) \\ &= \left[G_x^2 + G_y^2\right]^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2} \end{aligned}$$

$$\nabla f \approx |G_x| + |G_y|$$

Filtrado - Gradiente

Varios autores han propuesto máscaras para obtener el gradiente de una imagen vía convolución:

La aproximación más simple plantea:

$$G_x = (z_8 - z_5);\ G_y = (z_6 - z_5)$$

Roberts (1965): $G_x = (z_9 - z_5);\ G_y = (z_8 - z_6)$

Entonces:

$$\nabla f = [(z_9 - z_5)^2 + (z_8 - z_6)^2]^{1/2} \quad \text{ó} \quad \nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

−1	0
0	1

0	−1
1	0

Roberts cross-gradient operators.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Filtrado - Gradiente

Sobel:

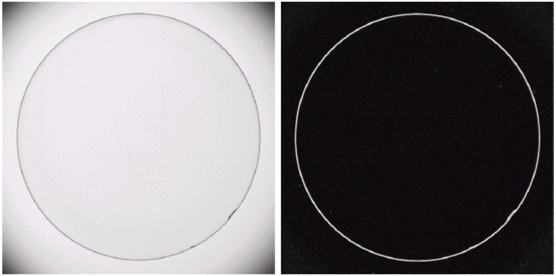
$$\nabla f \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Filtrado - Gradiente

Sobel operators



a b

FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

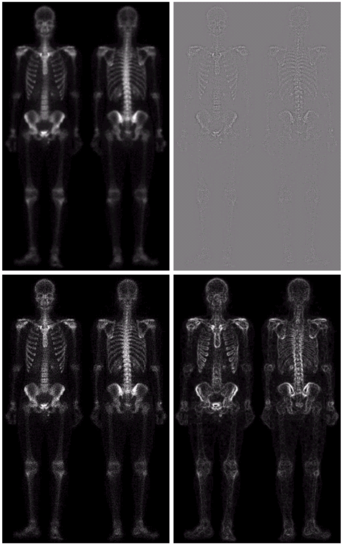
Filtrado - Gradiente

Filtro agudizador para la detección de bordes

Filtrado - efectos combinados

a b
c d

FIGURE 3.46 (a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a).



Filtrado - efectos combinados

e	f
g	h

FIGURE 3.46 *(Continued)* (e) Sobel image smoothed with a 5 × 5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)