Spatial Computing
Towards designing a right-brain type architecture

Christian Freksa
Spatial Cognition Research Center SFB/TR 8, University of Bremen, Germany
freksa@uni-bremen.de

Abstract
In an earlier paper on *Qualitative Spatial Reasoning* I suggested that spatial inference engines might provide the basis for general cognitive capabilities inside and outside the spatial domain. In the present contribution, I will follow up on this perspective and I will illustrate in which ways research in spatial cognition has progressed towards understanding spatial reasoning and spatial computing in a more literal sense. The chapter presents a progression of approaches to spatial reasoning from purely descriptive to increasingly spatially structured. It demonstrates how spatial structures are capable of replacing expensive computational processes. It discusses how these approaches could be developed and implemented in a way that may help us to better understand spatial abilities that are frequently attributed to the right-brain hemisphere in humans. The chapter concludes by suggesting that a suitable combination of abstract declarative representations and concrete spatio-temporal representations may be most effective for problem solving.

Spatial Problems

Let us consider examples of some common spatial problems we may be confronted with:

1. Given the triangle *ABC* with the coordinates $A = (1, 3)$, $B = (9, 2)$, $C = (6, 8)$; is $P = (8, 4)$ inside or outside the triangle *ABC*?
2. (How) can I get the piano into my living room?
3. How do I get from here to John's place?
4. Which is closer: from here to John or to Mary?
5. Is the tree (walkway, driveway) on my property or on your property?
6. 

Problem 1 is a classic high school geometry problem which can be solved abstractly with linear equations; the correct algebraic solution will locate *P* on the line *BC*; numeric solutions may place *P* inside or outside the triangle, depending on the number format and algorithm chosen; approximations to precise numeric values may cause slight deviations from the correct result. Problem 2 is a form of the classic *Piano Mover's Problem* in mathematics (Schwartz and Sharir 1983); although this problem can be represented geometrically, in practice it is rarely approached mathematically in the abstract representation domain but by trial and error in the physical problem domain.

Problem 3 cannot very well be presented in geometric terms; a graph structure that depicts the location 'here', John's place, and a traversable connection between them is more appropriate and often times preferable to a solution in the physical domain, particularly if John's place is far away. Problem 4 typically does not require the mathematically correct solution – which may take a long time to determine; a quickly provided estimate tends to be more helpful, in practice.

Problem 5 is another example where a formal approach may not be very helpful; but whereas in the Piano Mover's Problem the base information may be available in form of the piano's geometric dimensions, in the present example, we may have a legal document which specifies property boundaries in terms of geographic coordinates and a piece of property whose precise coordinates may be difficult to determine and therefore not known. Problem 6 also is related to the Piano Mover's Problem, but it is not specified in terms of numbers or language; it is a truly spatial problem presented physically to small children who will try to fit the small colored objects into the openings of the wooden cube and thus learn about spatial features like size and shape through physical processes by trial and error.

The examples illustrate that spatial problems may come in terms of numbers, language, or spatial configurations. Likewise, the solutions to spatial problems may be required in terms of numbers, language, or spatial configurations. The solution may or may not be needed in the same modality as the problem statement. The correct solution may not always be the best solution as quickly available sub-optimal solutions may be more useful in certain situations. In other words: we may need to transform problems and solutions between different modalities and the generation of the problem solution may take place in a variety of modalities (cp. Sloman 1985).

This observation raises the issue whether we need to transform spatial problems into geometric formalisms to enable computational solutions by means of sequential interpretation of classic computer languages; or whether we can find ways to process entire spatial configurations directly, as humans seem able to do (Shepard and Metzler 1971). I will dub the classic computer science approach as *left-brain computing,* as information processing in the left hemisphere of the brain is considered language-like sequential; I will dub the approach of processing entire spatial configuration as *right-brain computing,* as the right hemisphere of the brain is largely considered responsible for spatial knowledge processing in humans (Fischbach 1992).

In this chapter, I will first review progress in qualitative temporal and spatial reasoning; I will then discuss the notion of *conceptual neighborhood* and how we can exploit it for spatial computing; I will introduce tools for processing qualitative spatial relations; next I will address the transition from spatial relations to spatial configurations; finally I will demonstrate and analyze the notion of *spatial computing* as contrasted to *propositional computing*.

Qualitative Temporal and Spatial Reasoning

The starting point for much of the research in qualitative temporal and spatial relations in the past twenty years was the paper *Maintaining knowledge about temporal intervals* by James Allen (1983) (Fig. 1), although the underlying insights had been published previously (Nicod 1924; Hamblin 1972).

The intriguing result of this research was that thirteen ‘qualitative’ relations could describe temporal relations between events uniquely and exhaustively. There was an expectation that the idea of qualitative relations could be extended to spatial objects that share the extendedness property of temporal intervals. Initially, researchers had in mind a single spatial calculus that would compute all-embracing spatial relations between objects based on information about spatial relations between other objects.

Relation	Symbol	Pictorial Example
before – after	< >	
equal	=	
meets – met by	m mi	
overlaps – overlapped by	o oi	
during – contains	d di	
starts – started by	s si	
finishes – finished by	f fi	

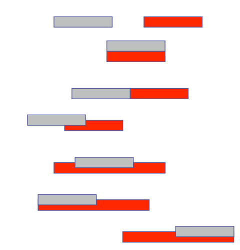


Fig. 1. The thirteen jointly exhaustive and mutually exclusive qualitative relations between two temporal intervals.

However, it became apparent soon that it would be more effective to develop specialized calculi that deal with individual aspects of space rather than a comprehensive spatial calculus that would integrate multiple aspects of space in a single formalism. For example, Allen’s interval calculus (Fig. 2) could be easily adapted to 1-dimensional directed space (Freksa 1991) or to three spatial dimensions individually (Guesgen 1989).

B r2 C / A r1 B	<	>	d	di	o	oi	m	mi	s	si	f	fi
"before" <	<	no info	< o m d s	<	<	< o m d s	<	< o m d s	<	<	< o m d s	<
"after" >	no info	>	> oi mi d f	>	> oi mi d f	>	> oi mi d f	>	> oi mi d f	>	>	>
"during" d	<	>	d	no info	< o m d s	> oi mi d f	<	>	d	> oi mi d f	d	< o m d s
"contains" di	< o m di fi	> oi di mi si	o oi dur con =	di	o di fi	oi di si	o di fi	oi di si	di fi o	di	di si oi	di
"overlaps" o	<	> oi di mi si	o d s	< o m di fi	< o m	o oi dur con =	<	oi di si	o	di fi o	d s o	< o m
"over-lapped-by" oi	< o m di fi	>	oi d f	> oi mi di si	o oi dur con =	> oi mi	o di fi	>	oi d f	oi > mi	oi	oi di si

Fig. 2. A part of the composition table for the qualitative temporal relations (without the ‘equals’ relation) from Allen (1983). In most cases, more than one relation may result from a composition. “no info” means that all 13 relations may result from a composition.

Conceptual Neighborhood

An important feature of physical time and space is that gradual changes result in small qualitative changes between the point relations involved. For example, in the transition from the *before* relation to the *meets* relation, only one of the four point relations between beginnings and endings of the two intervals changes: the relation between the ending of the first interval and the beginning of the second interval changes from *smaller than* to *equals*. Accordingly, perception and cognition of spatio-temporal configurations that result from small physical changes are closely related.

Furthermore, events in close temporal vicinity are related more easily to one another than events in different epochs. Similarly, nearby spatial locations are more easily related to one another than locations far apart – this insight is captured in the *First law of geography:* "Everything is related to everything else, but near things are more related than distant things" (Tobler 1971). The role of nearness extends from temporal and spatial neighborhood to the more abstract level of relations: certain relations are closer to one another than others; in fact, some relations are distinguished only by a single detail; these relations are called *conceptual neighbors* (Fig. 3).

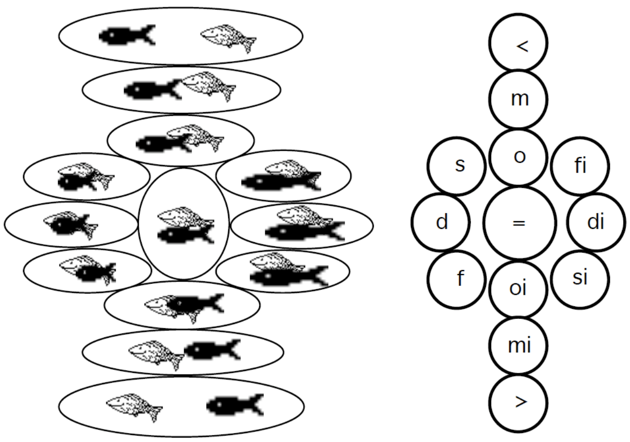


Fig. 3. (From Freksa 1991): Thirteen qualitative relations for one-dimensional directed space. The example compares the position of fishes in the horizontal dimension. The 13 relations are arranged by *conceptual neighborhood*.

The notion of *conceptual neighborhood* is closely connected to the notion of *spatial neighborhood:* Spatial neighbors can be defined as two locations, which are distinguished by a single detail, e.g. whose distance in the location graph is one (Fig. 4).

Structuring temporal and spatial relations by conceptual neighborhood enables numerous features for representing spatial knowledge and for spatial reasoning:

- Sets of neighboring relations can be lumped together to form *conceptual neighborhoods* and to define *coarse relations* (Freksa 1992a, b)*;*
- Conceptual neighborhoods define hierarchies for representing incomplete knowledge;
- Qualitative reasoning based on conceptual neighborhood allows for efficient non-disjunctive reasoning;
- Neighborhood-based incomplete knowledge can be easily augmented as additional knowledge is gained during successive reasoning;
- Coarse relations based on conceptual neighborhoods frequently exhibit a natural correspondence to everyday human concepts;
- Spatial and temporal inferences in qualitative reasoning typically result in conclusions that form conceptual neighborhoods;
- Reasoning that can be carried out on the basis of conceptual neighborhoods can reduce computation from exponential to polynomial complexity [Nebel & Bürckert];
- Conceptual neighborhoods can be formed on various levels of granularity.

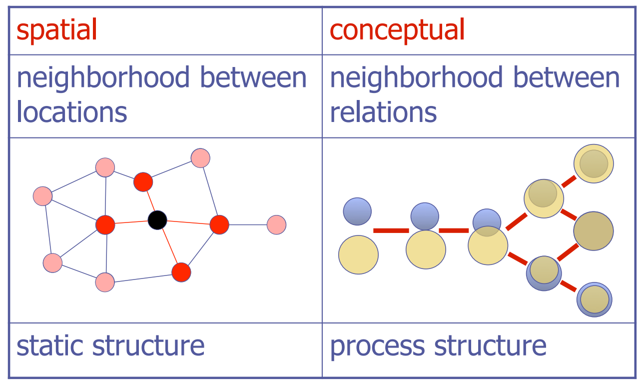


Fig. 4. Spatial and conceptual neighborhood: In the graph depicting spatial locations (left) nodes that are a single edge apart represent spatial neighbors. In the graph depicting spatial relations (right) relations that are a single qualitative criterion apart represent conceptual neighbors.

Neighborhood-Based Reasoning

One important feature of conceptual neighborhood-based abstraction is that *incomplete knowledge* can be conceptualized and represented as *coarse knowledge* (Fig. 5). By abstracting from missing or unnecessary details, reasoning can be carried out efficiently, avoiding computationally and conceptually problematic properties of disjunctive knowledge processing.

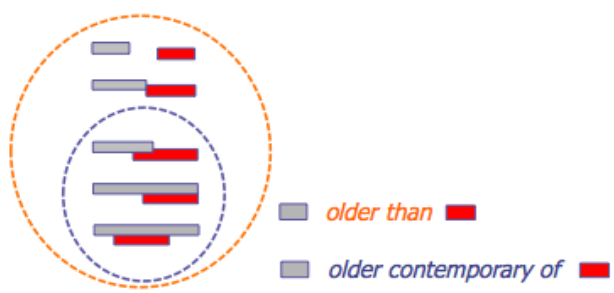


Fig. 5. Coarse temporal relations forming an abstraction hierarchy. The relation 'older contemporary of' corresponds to the conceptual neighborhood of the finer relations 'overlaps', 'finished by', and 'contains'. The even coarser relation 'older than' corresponds to a larger conceptual neighborhood that additionally includes the fine relations 'before' and 'meets'.

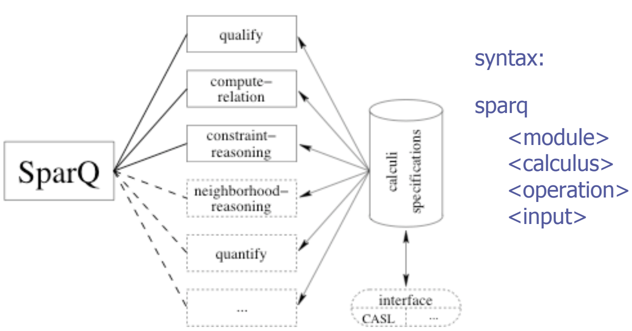
Coarse reasoning does not imply that inferences yield coarse knowledge only; conjunctions of partially overlapping coarse inferences based on imprecise or incomplete knowledge fragments from different sources result in precise or *fine* conclusions if the premises are appropriately chosen. With this property, the approach is suitable to model synergy of multimodal coarse knowledge sources that result in precise knowledge.

A Multitude of Specialized Calculi and SparQ

A considerable variety of spatial calculi have been developed over the past twenty years; these can be classified as

- Measurement calculi, e.g. Δ-Calculus (Zimmermann 1995))
- Topological calculi, e.g. 4-intersection calculus, 9-intersection calculus, RCC-5, RCC-8 (Egenhofer and Franzosa, 1991; Randell, Cohn et al.);
- Orientation calculi, e.g. point / line-based: DCC, FlipFlop, QTC, dipole or extended objects (Freksa 1992b; Ligozat 1993; Van de Weghe et al. 2005; Moratz et al. 2000);
- Position calculi, e.g. Ternary point configuration calculus (TPCC – Moratz et al. 2003).

To simplify the use of qualitative spatial calculi for specific reasoning tasks, various tools have been developed. The toolbox SparQ1 [Wallgruen et al. 2007] integrates numerous calculi for qualitative spatial reasoning and allows for adding arbitrary binary or ternary calculi through the specification of their base relations and their operations in list notation or through algebraic specification in metric space. SparQ employs functional list notation and allows for easy interfacing with other software through command lines or TCP/IP request. SparQ has a modular architecture and can easily extended by new



modules (Fig. 6).

Fig. 6. Modular SparQ architecture. Operations in different qualitative reasoning calculi can be evoked through standardized commands.

[1] www.sfbtr8.spatial-cognition.de/project/r3/sparq/

SparQ performs a number of operations that are helpful for dealing with spatial calculi:

- Qualify: quantitatively described configurations are translated into qualitative relations;
- Compute-relation: this operation generates a qualitative inference for a given calculus based on the premise relations and the calculus specification;
- Constraint-reasoning allows for the specification of an inference strategy on a given spatial configuration and returns scenarios that are consistent with the configuration; if the description of the scenario is inconsistent, SparQ informs about the inconsistency;
- Neighborhood-reasoning enables conceptually graceful constraint relaxation and yields semantically meaningful neighboring inferences;
- Quantify: this transformation is still in an experimental stage; the goal is to generate prototypical 'general' pictorial instances of abstract qualitative descriptions.

Although it is helpful to have a variety of calculi available in uniform specification and interface languages, there is still an issue about which calculus to use to solve a given task. Thus, there is a challenge to understand and describe spatial calculi on the meta-level in such a way that we can specify the given spatial configurations and the desired task solution in such a way that the available calculi can be automatically configured to solve the task.

From Spatial Relations to Spatial Configurations

Quantitative computation of spatial configurations by means of Euclidean geometry is well understood. For example, in planar geometry, we can compute all angles, heights, and the area of arbitrary triangles, if the lengths of the edges of the triangles are given by means of the formulae depicted in Fig. 7.

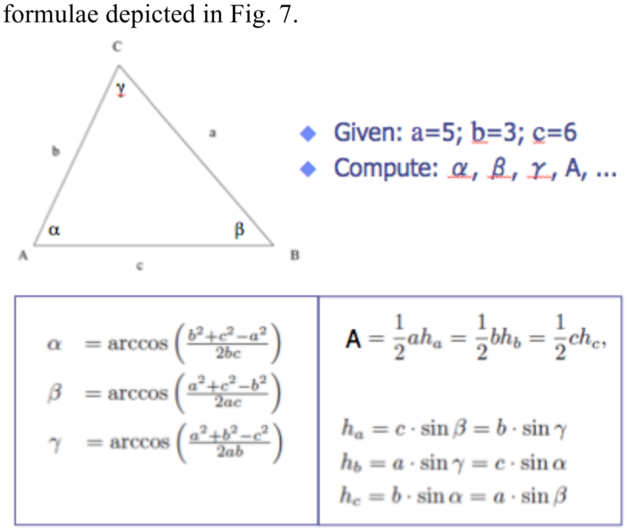


Fig. 7. Abstraction of geometric relations in the Euclidean plane.

These formulae are valid for planar spatial configurations independently of position, orientation, scale, or other influences. The reason for this is that in comparison to many other domains, spatial relations in the physical environment conform to strict internal laws that are not affected by contextual influences from other modalities. In other words: only few constraints need to be specified and all spatial relations are determined.

The principle is well known from constructive geometry. For example, on a flat sheet of paper, we can construct exactly two triangles from the specification of three edges, provided the specified lengths conform to the triangle inequality. In this construction, compass and ruler are capable of qualitative representation and they exhibit certain abstraction capabilities: the compass represents a length equal to some given length and can apply this length abstracting from location and orientation. Similarly, the ruler represents a distance and can apply it to any pair of points, independently of orientation and location.

Preserving Spatio-Temporal Structure

Although the formal abstraction shown in Fig. 8 is capable of generating arbitrary spatial relations through abstract computation, the abstraction mechanism does not preserve spatial structure in the way neighborhood-based representations preserve the domain structure. Structure-preserving representations have the advantage that essentially the same operations can be applied to the representation as to the represented domain. For example, on a geographic map we can navigate much like in the geographic environment with the advantage that we much more easily can maintain an overview and that we do not need to cover large distances.

As a consequence, structure-preserving representations are advantageous at least for those situations in which humans use the representations; this is the case for assistance systems, for example, where spatial and temporal representations are employed as human / machine interfaces. Humans can carry out zooming operations by moving towards or away from the representation medium; at the same time they can perform refining and coarsening operations; they can perform perspective transformations by looking at the medium from different perspectives; they can aggregate and partition spatial regions by making use of the natural neighborhood structures; they can move across the medium much like in the represented domain and they can experience spatial and conceptual transitions while doing so; structure-preserving media also may support shape transformation operations in similar ways as in the represented domain.

Are there further reasons for exploring structure-preserving representations besides the convenience for human users? I believe so. The operations described in the previous paragraph are important operations not only to be carried out by humans, but for spatial and temporal structures, in general; thus, structure-preserving representations also may be advantageous for machine processing. We will come back to this consideration in the next section.

Structure-preserving representations exploit structural correspondences between the representation medium and the represented domain. Geometric / diagrammatic constructions on a piece of paper may serve as structure-preserving representations of space, since flat paper provides the universal spatial structure that guarantees the correctness of trigonometric relations in a planar domain. Fig. 8 depicts universal correspondences between geometric functions in plane spatial structures.

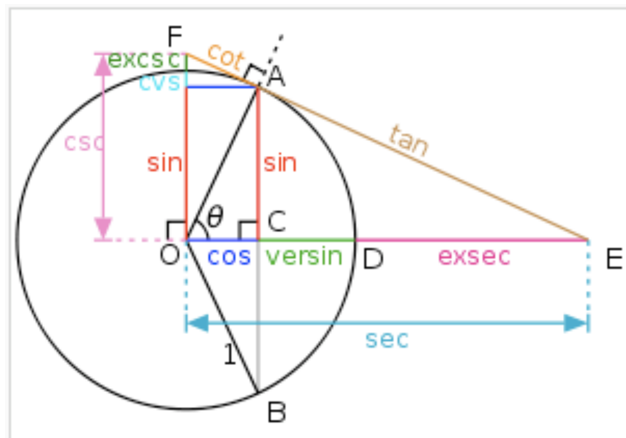


Fig. 8. Spatial construction of geometric functions. The graph depicts interdependencies of geometric relations. All trigonometric functions of an angle Q can be constructed geometrically in terms of a unit circle centered at O.

Computation by diagrammatic construction is a form of *analogical reasoning* [c.f. Gentner 1983]: the basis for establishing analogies is given through the universal spatial interdependencies that justify the comparison between the source domain and the target domain; the analogies usually concern the abstraction from specific values in the domain. Nevertheless, geometric constructions are sequential constructions that are most easily described by classical algorithms and procedures.

Space as Computer

In his book *Rechnender Raum* (Computing Cosmos / Calculating Space) (Zuse 1969), the computer pioneer Konrad Zuse discussed the issue of structure correspondence between computational representations and the physical domain. He addressed the issue on the microlevel of discrete vs. continuous structures, maintaining that discrete representations only approximate continuous structures and mimic random deviations rather than replicating the physical laws of quantum mechanics.

We want to discuss the idea of structure correspondence on the macro-level of spatial configurations and carry the notion of diagrammatic construction one step further, in this section. Suppose we apply three line segments to a flat surface as shown in Figure 9.

What do we see in this figure? We can easily identify nine additional line segments of specific lengths, three line intersections at specific locations, twelve specific pairwise identical angles, one triangle with a specific area, and numerous relations between those entities.

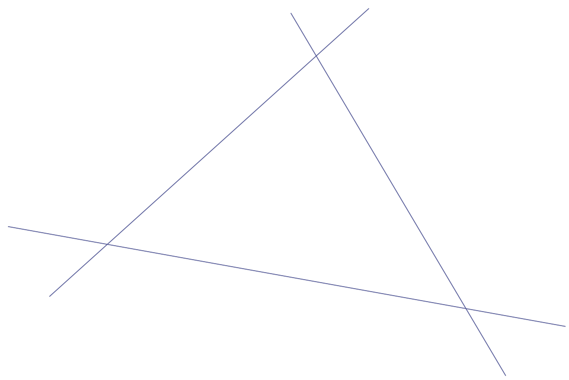


Fig. 9. Three line segments are applied to a spatially structured domain. Numerous specific entities and relations are established through the interaction of these lines and the constraints of the domain: nine new line segments, twelve angles, a triangle, its area, etc.

Where did all these entities and relations come from as we only placed three simple straight lines onto the surface? One way to answer this question is: The flat surface *computed* these entities and relations according to the laws of geometry. This would be the type of answer we would give if we would give a computer the line equations and the procedures to generate the mentioned entities and relations. What is the difference between the computer approach and the flat paper approach?

The computer algorithm encodes knowledge about the spatial structure of the surface that enables its interpreter to reconstruct in a sequential procedure step-by-step certain abstractions of its spatial structure that are constrained by abstract representations of the lines and their relationships. On the other hand, the flat surface itself and its spatial structure relate directly and instantly to the lines and generate the entities and relations without computational procedure by means of the inherent structural properties. If we are interested in the new entities and their relationships, we merely need to read them off the surface. The different approaches to generating spatial inferences are shown in Figure 10.

The formal procedural approach to computing spatial relations is shown in the upper part of the figure; the approach that applies spatial structures directly and instantaneously is shown in the bottom part of the figure. The intrinsically spatial and the formal representation can be transformed into one another; this allows applying (intrinsically) spatial computation to formal specifications and formal procedural approaches to intrinsically spatial representations; similarly, the results of either approach can be exhibited either in a formal or in a spatial representation.

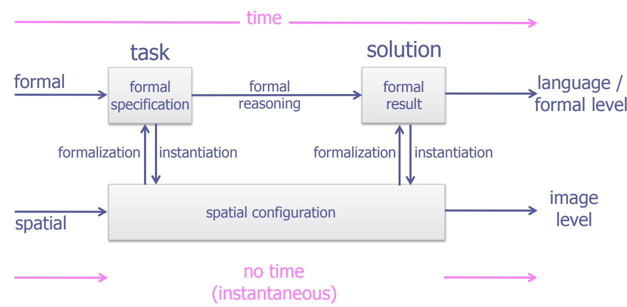


Fig. 10. Two approaches to generating spatial entities and relations: in the upper part of the figure, a classical sequential computational approach transforms a formal specification of a spatial configuration by means of formal reasoning into a result in terms of a formal language. In the bottom part of the figure the configuration is applied directly to a spatial structure; the spatial structure manifests additional entities and relations that can be read off by perceptual processes. Transformations between the intrinsically spatial structures and their formalizations are possible at various stages.

Basic Entities of Cognitive Processing

In geometry, the spatial world can be described in terms of infinitesimally small points; lines are viewed as infinite sets of points that conform to certain constraints, etc. In contrast, in cognition, basic entities usually are not infinitesimally small points; instead, they may be entire physical objects like books or chairs. The basic entities carry meaning related to their use and function and we perceive and conceptualize them in their entirety even if certain details are not accessible to our perception. It is known that we can apply simple mental operations, e.g. mental rotation, to simple spatial objects at once.

The cognitive apparatus appears to be flexible as to which level in a huge hierarchy of part-whole relations to select as 'basic level' [cp. Rosch, 1978]; the cognitive apparatus also appears to be able to focus either on the relation between an object and a configuration of objects, or, alternatively, on the relation between an object an its parts. Both transitions involve cognitive effort, while the mere consideration of the basic level appears almost effortless.

Concrete versus Abstract Computation

The approach presented in this paper follows up on the considerations presented at the Las Navas Advanced Study Institute on *Cognitive and Linguistic Aspects of Geographic Space* twenty years ago (Freksa 1991): abstractions are extremely useful for computation and for understanding general principles; however, it may be highly advantageous to maintain essential structural properties, rather than abstracting from them and recreating them by formal reconstruction (cp. Fig. 12). Abstraction is an excellent preparation for reasoning *about* certain features and structures and to generalize; but using features and structures does not require abstraction (cp. Furbach et al. 1985).

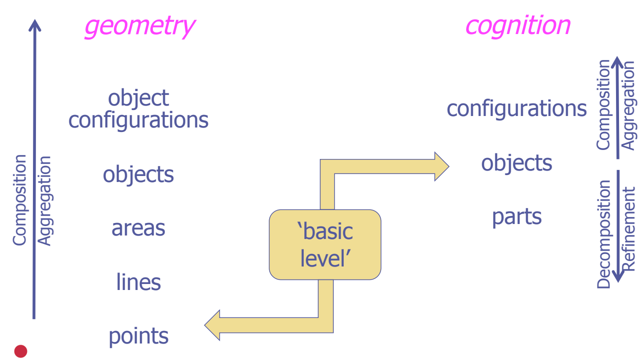


Fig. 11. Two ways to conceptualize physical objects. In geometry (left side), we build arbitrarily complex structures from atomic point entities. In cognition, the basic entities are complex, meaningful entities; through cognitive effort, basic entities can be decomposed into more elementary entities or aggregated into more complex configurations.

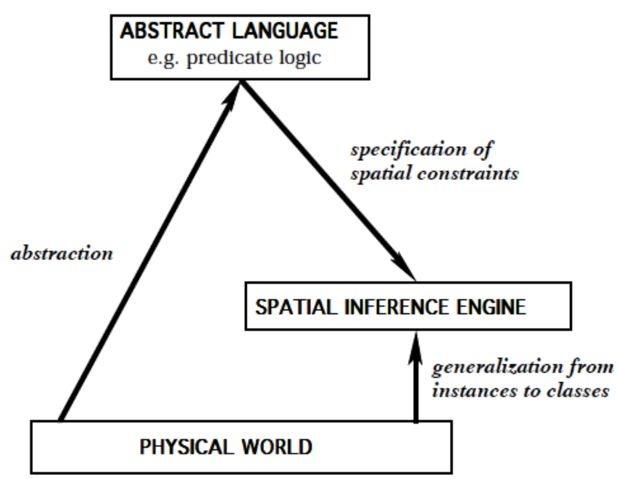


Fig. 12. Spatial inference engines can be constructed by constraining very abstract formalization languages or by generalizing over the physical world. (from Freksa, 1991).

Conclusions and Outlook

Let me come back to the spatial problems that I used in the beginning of the chapter to introduce various perspectives on spatial challenges. A main message of this exercise is that spatial problems consist of more than solving equations. First of all, a spatial problem needs to be perceived as one. Second, it needs to be represented as one. Here we have lots of options, as there are many ways to conceive of space and of representing space. For example, space may be conceived of as empty space "what is there if nothing is there" or the space spanned by physical objects. Space can be described in terms of a multitude of reference systems as becomes evident if we look at the multitude of spatial representation systems and calculi we can develop. All these representations have benefits and disadvantages, depending on the problems we want to tackle or the situations we want to describe.

Nevertheless, spatial structures – and to a somewhat lesser extent – temporal structures appear to play special roles in everyday actions and problem solving. Many other dimensions seem to dominate our lives: monetary values, quality assessments, efficiency criteria, social structures, etc. – but do they play comparable roles with respect to cognitive representation and processing? I do not think so. I propose that this has to do with the fact, that internal representations may be amodal, but they cannot be "a-structural". In other words: Cognitive representations and processes depend on a spatio-temporal substrate; without such a substrate, they cannot exist. But they may not depend on a specific spatio-temporal substrate: a multitude of structures may do the job – in some cases better, in some cases worse. Different abstractions from physical space may be advantageous in different situations.

Space and time provide fundamental structures for many tasks cognitive agents must perform and for many aspects of the world that they can reason about. Maintaining these structures as a foundation simplifies many cognitive tasks tremendously, including perceiving, memorizing, retrieving, reasoning, and acting. This is well known from everyday experiences as using geographic maps for wayfinding. For other domains it is helpful to create spatially structured foundations to support and simplify orientation; for example, spatial structure is the basis for diagrams that help us reason about many domains.

A conceptually simple implementation of a truly spatial computer could be a robot system that manipulates physical objects in a spatial domain and perceives and represents these objects, the configurations constructed from these objects, and the parts of the objects as well as their relations from various orientations and perspectives. A more sophisticated approach would involve the construction of a (visuo-?) spatial working memory whose basic entities are entire objects, rather than their constituents. Spatial operations like translation, rotation, and distortion would globally modify configurations. Perception operators extract qualitative spatial relations from these representations. The development of this implementation can be guided by our knowledge about working memory capabilities and limitations as well as by our knowledge about spatial representations in the human brain.

A Final Note

Although we talk about spatial cognition, spatial reasoning, and spatial computing, we frequently fail to characterize the type of solution to spatial problems that we want to achieve. However, our repertoire of approaches yields results on different levels of sophistication: some approaches yield solutions to spatial problems, others yield some sort of explanations along with the solutions, like ‘this is the only solution’ or ‘this is one of possibly several solutions’ or ‘these are all solutions.

Why is sophistication an issue? For highly abstract, formal approaches, the quality of a solution is not obvious. Formal proofs and/or explanations are required to characterize the type of solution. In the more concrete, spatially structured solutions, the solutions are more easily perceptible, more obvious – proofs may not be required; on the other hand: can we be *sure,* that we found the best solutions, the only solution, all solutions? This is a debate that reminds of the discussion on the validity of constructive geometry to find solutions or to prove their correctness.

Apparently, there are different domains in which we can ground our knowledge: perceptual experience about spatial and temporal environments and formal logics that does not require empirical justification. Both domains are important for human experience and human reasoning. It does not make much sense to say one is superior over the other; they are two rather different realms. They become particularly powerful when they are engaged jointly: one to carry out spatio-temporal actions and the other to reason about them and to explain what’s going on in an overarching theory.

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