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1 The Limiting MPC's

For $m_t > 0$ we can define $\mathrm{e}_t(m_t) = \mathrm{c}_t(m_t)/m_t$ and $\mathrm{a}_t(m_t) = m_t - \mathrm{c}_t(m_t)$ and the Euler equation (6) can be rewritten

$$\begin{aligned}\mathrm{e}_t(m_t)^{-\rho} &= \beta \mathsf{R}\,\mathbb{E}_t\left[\left(\mathrm{e}_{t+1}(m_{t+1})\left(\frac{\overbrace{\mathsf{R}\mathrm{a}_t(m_t)+\mathbf{\Phi}_{t+1}\boldsymbol{\xi}_{t+1}}^{=m_{t+1}\mathbf{\Phi}_{t+1}}}{m_t}\right)\right)^{-\rho}\right] \\ &= \quad (1-\wp)\beta\mathsf{R}m_t^{\rho}\,\mathbb{E}_t\left[\left(\mathrm{e}_{t+1}(m_{t+1})m_{t+1}\mathbf{\Phi}_{t+1}\right)^{-\rho}\mid \boldsymbol{\xi}_{t+1}>0\right] \\ &+ \wp\beta\mathsf{R}^{1-\rho}\,\mathbb{E}_t\left[\left(\mathrm{e}_{t+1}(\mathcal{R}_{t+1}\mathrm{a}_t(m_t))\frac{m_t-\mathrm{c}_t(m_t)}{m_t}\right)^{-\rho}\mid \boldsymbol{\xi}_{t+1}=0\right].\end{aligned}$$

Consider the first conditional expectation in (6), recalling that if $\boldsymbol{\xi}_{t+1} > 0$ then $\boldsymbol{\xi}_{t+1} \equiv \boldsymbol{\theta}_{t+1}/(1-\wp)$. Since $\lim_{m\downarrow 0}\mathrm{a}_t(m) = 0$, $\mathbb{E}_t[(\mathrm{e}_{t+1}(m_{t+1})m_{t+1}\mathbf{\Phi}_{t+1})^{-\rho} \mid \boldsymbol{\xi}_{t+1} > 0]$ is contained within bounds defined by $(\mathrm{e}_{t+1}(\underline{\boldsymbol{\theta}}/(1-\wp))\mathbf{\Phi}\underline{\boldsymbol{\Psi}\boldsymbol{\theta}}/(1-\wp))^{-\rho}$ and $(\mathrm{e}_{t+1}(\bar{\boldsymbol{\theta}}/(1-\wp))\mathbf{\Phi}\bar{\boldsymbol{\Psi}}\bar{\boldsymbol{\theta}}/(1-\wp))^{-\rho}$ both of which are finite numbers, implying that the whole term multiplied by $(1-\wp)$ goes to zero as m_t^{ρ} goes to zero. As $m_t \downarrow 0$ the expectation in the other term goes to $\bar{\kappa}_{t+1}^{-\rho}(1-\bar{\kappa}_t)^{-\rho}$. (This follows from the strict concavity and differentiability of the consumption function.) It follows that the limiting $\bar{\kappa}_t$ satisfies $\bar{\kappa}_t^{-\rho} = \beta\wp\mathsf{R}^{1-\rho}\bar{\kappa}_{t+1}^{-\rho}(1-\bar{\kappa}_t)^{-\rho}$. Exponentiating by ρ, we can conclude that

$$\bar{\kappa}_t = \wp^{-1/\rho}(\beta\mathsf{R})^{-1/\rho}\mathsf{R}(1-\bar{\kappa}_t)\bar{\kappa}_{t+1}$$

$$\underbrace{\wp^{1/\rho}\overbrace{\mathsf{R}^{-1}(\beta\mathsf{R})^{1/\rho}}^{\text{Þ}_{\mathsf{R}}}}_{\equiv\wp^{1/\rho}\text{Þ}_{\mathsf{R}}}\bar{\kappa}_t = (1-\bar{\kappa}_t)\bar{\kappa}_{t+1}$$

which yields a useful recursive formula for the maximal marginal propensity to consume:

$$\begin{aligned}(\wp^{1/\rho}\text{Þ}_{\mathsf{R}}\bar{\kappa}_t)^{-1} &= (1-\bar{\kappa}_t)^{-1}\bar{\kappa}_{t+1}^{-1} \\ \bar{\kappa}_t^{-1}(1-\bar{\kappa}_t) &= \wp^{1/\rho}\text{Þ}_{\mathsf{R}}\bar{\kappa}_{t+1}^{-1} \\ \bar{\kappa}_t^{-1} &= 1+\wp^{1/\rho}\text{Þ}_{\mathsf{R}}\bar{\kappa}_{t+1}^{-1}.\end{aligned}$$

As noted in the main text, we need the WRIC (32) for this to be a convergent sequence:

$$0 \le \wp^{1/\rho}\text{Þ}_{\mathsf{R}} < 1, \tag{1}$$

Since $\bar{\kappa}_T = 1$, iterating (1) backward to infinity (because we are interested in the limiting consumption function) we obtain:

$$\lim_{n\to\infty}\bar{\kappa}_{T-n} = \bar{\kappa} \equiv 1-\wp^{1/\rho}\text{Þ}_{\mathsf{R}} \tag{2}$$

and we will therefore call $\bar{\kappa}$ the 'limiting maximal MPC.'

The minimal MPC's are obtained by considering the case where $m_t \uparrow \infty$. If the so that $(\{\underline{\kappa}_{T-n}^{-1}\})_{n=0}^{\infty}$ is also an increasing convergent sequence, and we define

$$\underline{\kappa}^{-1} \equiv \lim_{n \uparrow \infty} \kappa_{T-n}^{-1} \tag{4}$$

as the limiting (inverse) marginal MPC. If the RIC does *not* hold, then $\lim_{n\to\infty} \underline{\kappa}_{T-n}^{-1} = \infty$ and so the limiting MPC is $\underline{\kappa} = 0$.

For the purpose of constructing the limiting perfect foresight consumption function, it is useful further to note that the PDV of consumption is given by

$$c_t \underbrace{\left(1 + \text{Þ}_{\mathsf{R}} + \text{Þ}_{\mathsf{R}}^2 + \cdots\right)}_{=1+\text{Þ}_{\mathsf{R}}(1+\text{Þ}_{\mathsf{R}}\underline{\kappa}_{t+2}^{-1})\ldots} = c_t \underline{\kappa}_{T-n}^{-1}.$$

which, combined with the intertemporal budget constraint, yields the usual formula for the perfect foresight consumption function:

$$c_t = (b_t + h_t)\underline{\kappa}_t \tag{5}$$