



# Linear Programming Shipping Optimization

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## Background:



- World leader in the manufacture, sale, and service of loading dock equipment and more
- Headquarters in Milwaukee and Dubuque, IA with shipping and transshipping locations around the US and internationally.
- Parent facility to four industrial supply companies - RHEEC, RHAM, RHD, and DuctSox. RHEEC
- Rite Hite specializes in vinyl doors, industrial curtain walls and safety barriers.
- Provide products to corporation such as PepsiCo, 3M, and Cummins

# Shipment Cost Optimization: *Basic Model*

## ➤ Shipping Optimization – LINDO

- Minimize Total Shipping Cost
- Optimally schedule flow of goods within supply chain between manufacturing locations and distributor warehouses

## ➤ Model Assumptions

- Carrier costs are per full 40' x 8.5' truckload, or gross weight 20,000 lbs.
- Assume only one product line demanded throughout network

## ➤ Interpreting Solutions

- All optimal truckloads shipped between manufacturers and distributor locations are within the context of the entire system constraints
- Can't necessarily meet demands to distributor locations with cheapest route option

## LP Structure and Definitions

- $c_{ij}$ : Carrier cost coefficient
- $x_{ij}$ : Truck flow decision variable
- $m$ : # Manufacturing suppliers
- $n$ : # Distributor warehouses
- $a_i, b_j$ : Constraint RHS

$$TC = \sum_{i=1}^n \sum_{j=1}^m c_{ij} x_{ij}$$

The objective function is thus

$$\text{Min } Z = TC = \sum_{i=1}^n \sum_{j=1}^m c_{ij} x_{ij}$$

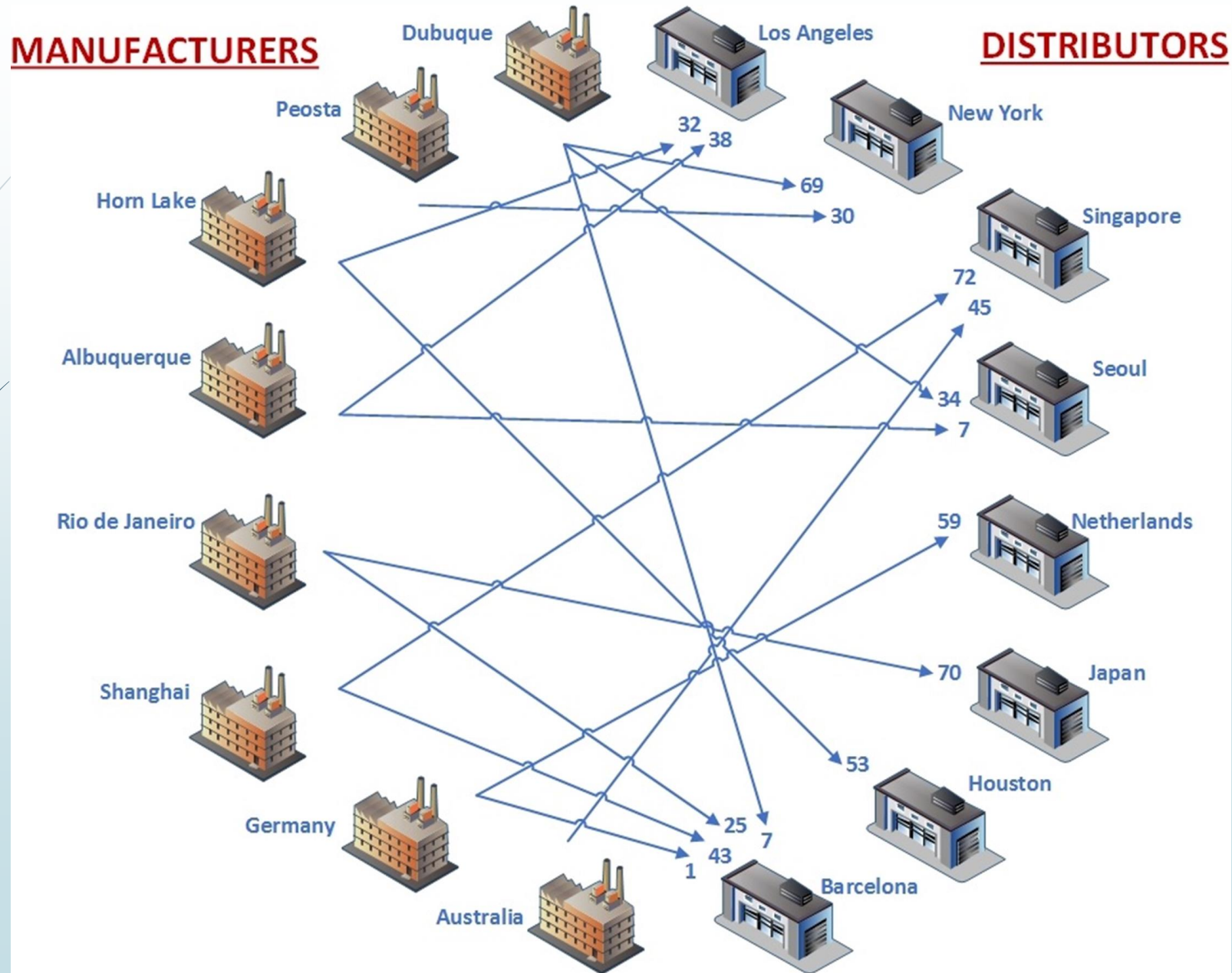
Manufacturing supply constraint

$$\sum_{j=1}^n x_{ij} = a_i \text{ for } 1 \leq i \leq m$$

Distributor demand constraint

$$\sum_{i=1}^m x_{ij} = b_j \text{ for } 1 \leq j \leq n$$

# Results: Optimal Flow Network Diagram



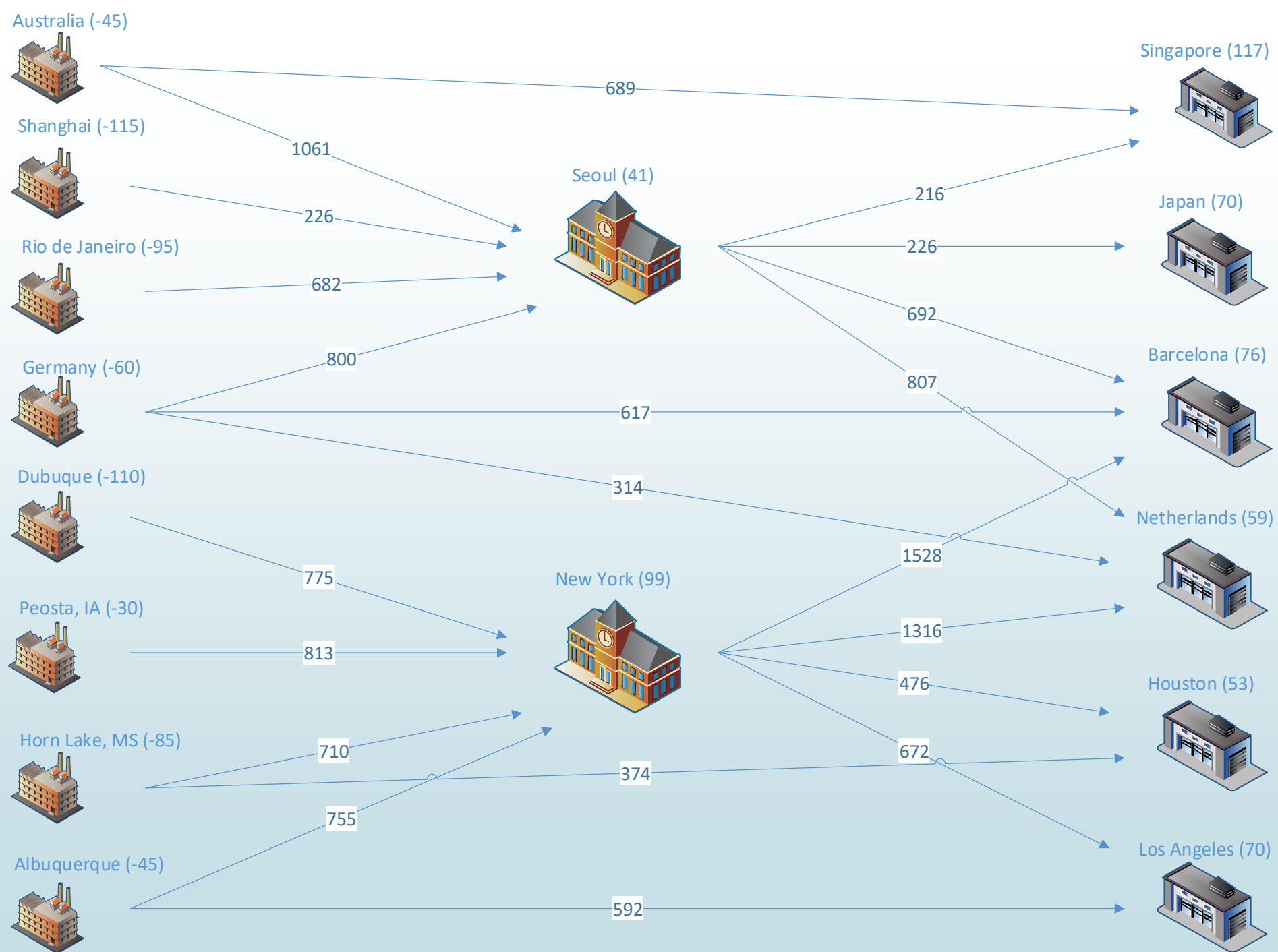
## Results: *LINDO* Output & Sensitivity Analysis

- *MINIMUM TOTAL ROUTE COST* = \$435,483.60
- *HIGHLIGHT*: Barcelona warehouse needs 76 truckloads/ mo.

Manufacturing Supplier	Truck Flow	Carrier Cost	Allow. Cost Increase	Allow. Cost Decrease
Germany	1	\$617	1036	50
Shanghai	43	\$696	326	60
Rio de Janeiro	25	\$1535	61	55
Dubuque, IA	7	\$2801	0	61

- Optimal flow from Shanghai to Barcelona is 43 truckloads at \$696 each, with allowable increase of \$326 or decrease of \$60 for solution to still remain optimal (while all other data remaining fixed)

# Add Trans-shipment Nodes: New York & Seoul





## Add Cost Levels: 3 Carrier Options –

- “Blue Carrier”, “Orange Carrier”, and “White Carrier”
- **Adjust Objective Equation:** 3 Carrier Cost Levels of 21 Route Options = 63 Decision Variables
- **Adjust Constraints:** Flow Balancing Rules into and out of each node
  - Truckloads into a node are positive
  - Truckloads out of a node are negative
  - Set equal to production quantity for manufacturers
  - Set equal to demand quantity for distributors



# Results

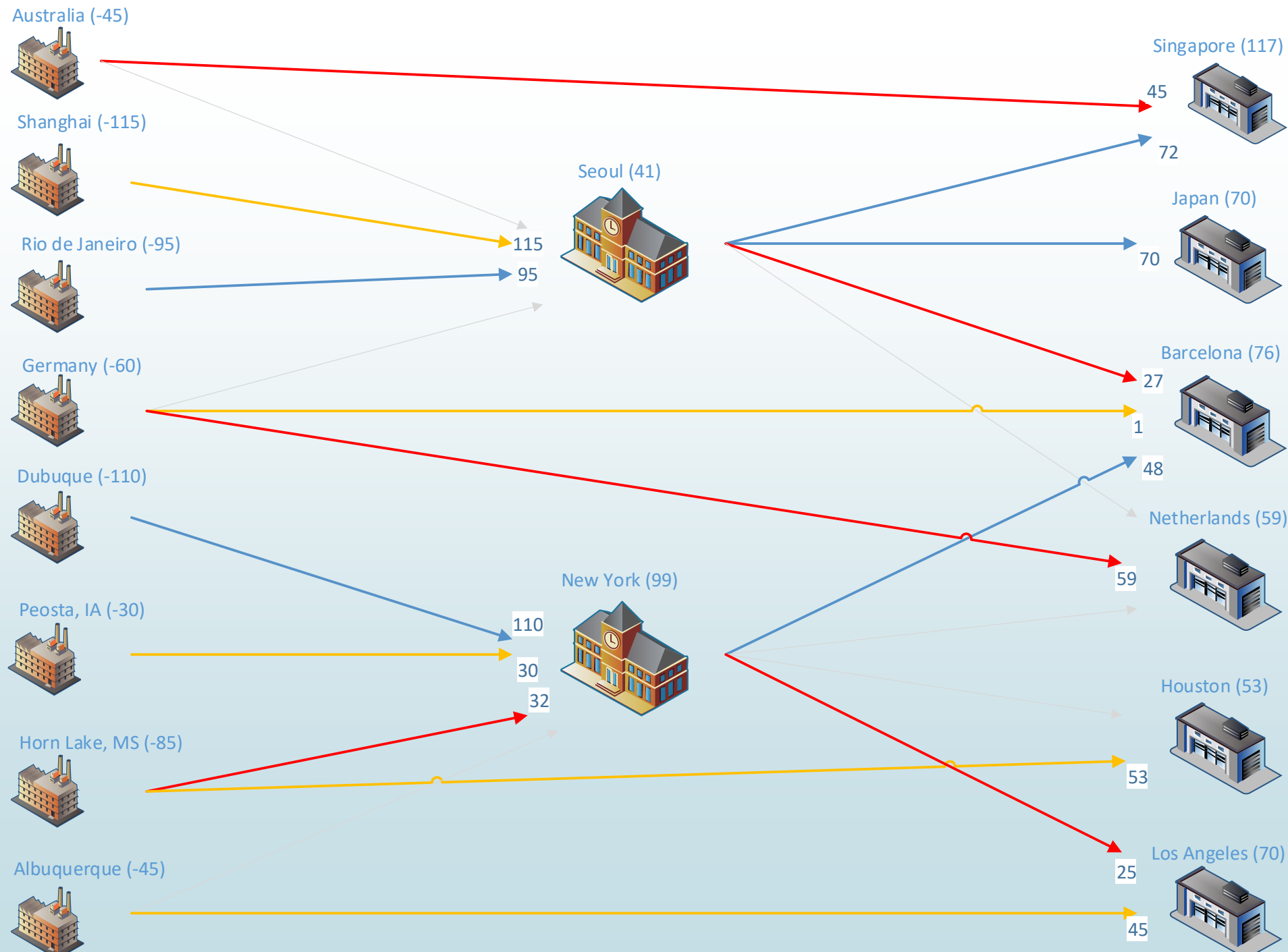
## KEY

**Blue Line =**  
**Blue Carrier**

**Orange Line =**  
**Orange Carrier**

**Red Line =**  
**White Carrier**

**Grey Line =**  
**Unused Route**



## Compare Results: Choose Minimum Cost

Carrier Option Level	Minimum Total Cost	Number of Routes Used
Mixed Carrier's	\$411,605	14 out of 63
Blue Carrier	\$416,277	15 out of 21
Orange Carrier	\$427,919	15 out of 21
White Carrier	\$426,217	15 out of 21

- **Savings of \$4,672 / month (\$56,064 / year) using Mixed Carrier's**
- **Potentially Less Routes to Manage with Mixed Carrier**



Questions?

# Appendix

<i>Costs (international based on 40' container flat rate, domestic + Canada based on natl avg shipping rates)</i>								
<b>From/ To</b>	Los Angeles	New York	Singapore	Seoul	Randstad	Osaka	Houston	Barcelona
Dubuque, IA	\$ 1,499.87	\$ 775.00	\$ 2,385.00	\$ 1,887.00	\$ 2,547.00	\$ 2,060.00	\$ 903.90	\$ 2,801.00
Peosta, IA	\$ 1,492.71	\$ 813.19	\$ 2,423.19	\$ 1,925.19	\$ 2,585.19	\$ 2,098.19	\$ 896.74	\$ 2,839.19
Horn Lake, MS	\$ 1,161.62	\$ 710.45	\$ 2,320.45	\$ 1,822.45	\$ 2,482.45	\$ 1,995.45	\$ 374.16	\$ 2,736.45
Albuquerque, NM	\$ 591.89	\$ 1,494.70	\$ 1,826.89	\$ 1,170.89	\$ 3,275.89	\$ 1,343.89	\$ 660.07	\$ 3,343.89
Rio de Janeiro, Brazil	\$ 2,382.55	\$ 1,154.53	\$ 2,648.63	\$ 681.80	\$ 1,673.35	\$ 738.73	\$ 1,276.89	\$ 1,535.20
Shanghai, China	\$ 1,118.52	\$ 1,810.38	\$ 219.94	\$ 225.75	\$ 811.30	\$ 225.75	\$ 1,701.76	\$ 695.81
Hamburg, Germany	\$ 2,009.05	\$ 1,421.87	\$ 1,423.73	\$ 799.87	\$ 313.55	\$ 856.80	\$ 2,138.93	\$ 617.17
Geraldton, Australia	\$ 2,004.46	<u>\$ 4,348.24</u>	\$ 688.77	\$ 1,060.92	\$ 3,187.10	\$ 1,117.85	\$ 4,046.93	\$ 2,780.36

Table 1: From/To chart with freight shipping charges assessed per route, courtesy of WorldFreightRate.com.

# Appendix

Manufacturing	Anticipated Truckloads	Distributers	Percentage Production	Distributers	Shipment Quantity
Dubuque, IA	110	Los Angeles	0.12	Los Angeles	70
Peosta, IA	30	New York	0.17	New York	99
Horn Lake, MS	85	Singapore	0.2	Singapore	117
Albuquerque, NM	45	Seoul	0.07	Seoul	41
Rio de Janeiro, Brazil	95	Randstad	0.1	Randstad	59
Shanghai, China	115	Osaka	0.12	Osaka	70
Hamburg, Germany	60	Houston	0.09	Houston	53
Geraldton, Australia	45	Barcelona	0.13	Barcelona	76
<b>Total:</b>	585				

Table 2, 3, and 4: Anticipated manufacturing production, distributor demand percentages, and distributor quantities needed.

# Appendix

From/ To	Los Angeles	New York	Singapore	Seoul	Randstad	Osaka	Houston	Barcelona
Dubuque, IA	0	69	0	34	0	0	0	7
Peosta, IA	0	30	0	0	0	0	0	0
Horn Lake, MS	32	0	0	0	0	0	53	0
Albuquerque, NM	38	0	0	7	0	0	0	0
Rio de Janeiro, Brazil	0	0	0	0	0	70	0	25
Shanghai, China	0	0	72	0	0	0	0	43
Hamburg, Germany	0	0	0	0	59	0	0	1
Geraldton, Australia	0	0	45	0	0	0	0	0

Table 5: Revised From/ To Chart showing the LINDO output for optimal truckloads between locations. Most options are not used.

## Appendix – Sensitivity Analysis

Route	Value	Reduced Cost	
Dubuque - Japan	0.00	55.47	
Shanghai - Singapore	72.00 (truckloads)	0.00	
Route	Coefficient	Allowable Increase	Allowable Decrease
Dubuque - Japan	2060.00 (\$)	INFINITY	55.47 (\$)
Shanghai - Singapore	219.94 (\$)	59.87 (\$)	749.68 (\$)
Constraint	Dual Price		
Dubuque Manufacturing	38.19 (\$)		
Japan Demand	-2042.72 (\$)		



# Appendix – LINDO Inputs

## LINDO Transportation Input

Min

1499.871663x11+775x12+2385x13+1887x14+2547x15+2060x16+903.9014374x17+2801x18+1492.7104  
72x21+813.1930185x22+2423.193018x23+1925.193018x24+2585.193018x25+2098.193018x26+896.74  
02464x27+2839.193018x28+1161.617193x31+710.4476424x32+2320.447642x33+1822.447642x34+248  
2.447642x35+1995.447642x36+374.1562561x37+2736.447642x38+591.8872435x41+1494.702596x42+  
1826.887244x43+1170.887244x44+3275.887244x45+1343.887244x46+660.0666602x47+3343.887244x  
48+2382.55x51+1154.53x52+2648.63x53+681.8x54+1673.35x55+738.73x56+1276.89x57+1535.2x58+11  
18.52x61+1810.38x62+219.94x63+225.75x64+811.3x65+225.75x66+1701.76x67+695.81x68+2009.05x7  
1+1421.87x72+1423.73x73+799.87x74+313.55x75+856.8x76+2138.93x77+617.17x78+2004.46x81+4348  
.24x82+688.77x83+1060.92x84+3187.1x85+1117.85x86+4046.93x87+2780.36x88

SUBJECT TO

x11+x21+x31+x41+x51+x61+x71+x81>=70

x11+x12+x13+x14+x15+x16+x17+x18>=110

x12+x22+x32+x42+x52+x62+x72+x82>=99

x21+x22+x23+x24+x25+x26+x27+x28>=30

x13+x23+x33+x43+x53+x63+x73+x83>=117

x31+x32+x33+x34+x35+x36+x37+x38>=85

x14+x24+x34+x44+x54+x64+x74+x84>=41

x41+x42+x43+x44+x45+x46+x47+x48>=45

x15+x25+x35+x45+x55+x65+x75+x85>=59

x51+x52+x53+x54+x55+x56+x57+x58>=95

x16+x26+x36+x46+x56+x66+x76+x86>=70

x61+x62+x63+x64+x65+x66+x67+x68>=115

x17+x27+x37+x47+x57+x67+x77+x87>=53

x71+x72+x73+x74+x75+x76+x77+x78>=60

x18+x28+x38+x48+x58+x68+x78+x88>=76

x81+x82+x83+x84+x85+x86+x87+x88>=45

end