Minnesota Department of Transportation Plow Vehicle Scheduling Problem

Bryan Melchert

Production and Operations Analysis

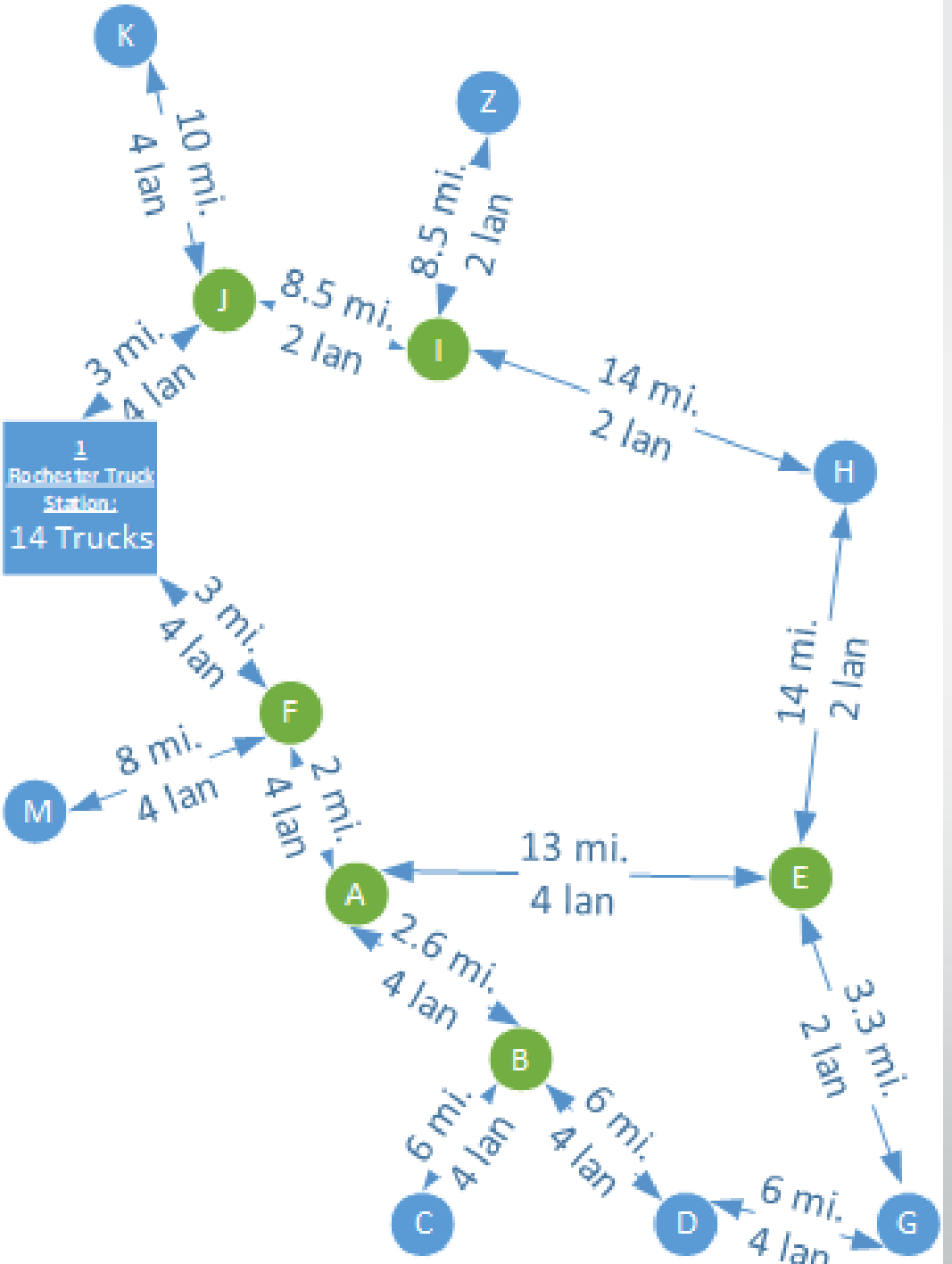
Spring 2018

Introduction

- Project Overview
  - Stress to drivers due to uneven scheduling
  - Mixed Integer programming model
- Objectives
  - Level driver hours
  - Arc based vehicle scheduling

Assumptions

- Two lanes on a given road is considered to simplify the model
- Focusing on the Rochester, MN station only
- Trucks will travel an average of 55 miles per hour



Model Building (M.I.P.)

- 15 route sections between 16 nodes
  - Each truck gets at least 1 section
- 14 trucks assigned to entire route network
- Objective function: Minimize sum of truck distance traveled
- Square added ensures multiple trucks chosen
  - Encourages "Leveling" of Decision Variables

$$\sqrt{\sum_{i=1}^{14} T_i^{\,2}}$$

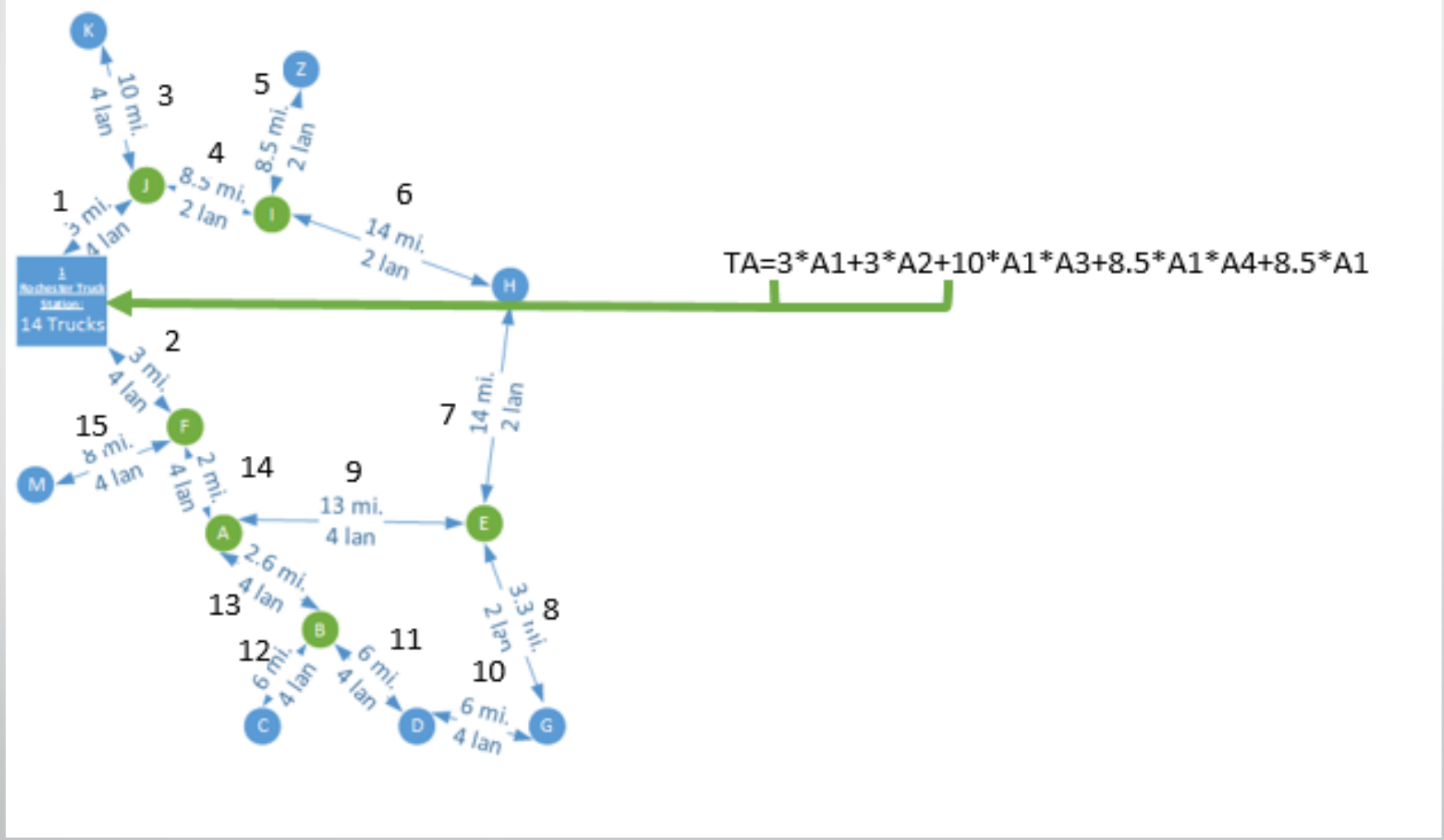
Where T_i = the total distance traveled by a truck "i"

Constraints Section: Path Cost Logic

```
TA=3*A1+3*A2+10*A1*A3+8.5*A1*A4+8.5*A1*A4*A5+14*A1*A4*A6+14*A1*A4*A6*A7+3.3*A1*A4*A
6*A7*A8+13*A1*A4*A6*A7*A9+6*A1*A4*A6*A7*A8*A10+6*A1*A4*A6*A7*A8*A10*A11+6*A1*A4*A
6*A7*A8*A10*A11*A12+2.6*A1*A4*A6*A7*A8*A10*A11*A13+13*A1*A4*A6*A7*A8*A10*A11*A13*
A9+2*A1*A4*A6*A7*A8*A10*A11*A13*A14+8*A1*A4*A6*A7*A8*A10*A11*A13*A14*A15+3*A1*A4*
A6*A7*A8*A10*A11*A13*A14*A2+15*A1*A4*A6*A7*A8*A10*A11*A13*A9*A14+8*A1*A4*A6*A7*A8
*A10*A11*A13*A9*A14*A15+3*A1*A4*A6*A7*A8*A10*A11*A13*A9*A14*A2+2*A1*A4*A6*A7*A9*A
14+8*A1*A4*A6*A7*A9*A14*A15+3*A1*A4*A6*A7*A9*A14*A2+2.6*A1*A4*A6*A7*A9*A13+6*A1*A
4*A6*A7*A9*A13*A12+6*A1*A4*A6*A7*A9*A13*A11+6*A1*A4*A6*A7*A9*A13*A11*A10+3.3*A1*A
4*A6*A7*A9*A13*A11*A8+8*A2*A15+2*A2*A14+13*A2*A14*A9+2.6*A2*A14*A13+6*A2*A14*A13*A
12+6*A2*A14*A13*A11+6*A2*A14*A13*A11*A10+3.3*A2*A14*A13*A11*A10*A8+14*A2*A14*A13*A
11*A10*A8*A7+14*A2*A14*A13*A11*A10*A8*A7*A6+8.5*A2*A14*A13*A11*A10*A8*A7*A6*A5+8.5
*A2*A14*A13*A11*A10*A8*A7*A6*A4+10*A2*A14*A13*A11*A10*A8*A7*A6*A4*A3+3*A2*A14*A13
*A11*A8*A7*A6*A4*A1+15.6*A2*A14*A9*A13+6*A2*A14*A9*A13*A12+6*A2*A14*A9*A13*A11+6*
A2*A14*A9*A13*A11*A10+3.3*A2*A14*A9*A13*A11*A10*A8+14*A2*A14*A9*A13*A11*A10*A8*A7
+14*A2*A14*A9*A13*A11*A10*A8*A7*A6+8.5*A2*A14*A9*A13*A11*A10*A8*A7*A6*A5+8.5*A2*A1
4*A9*A13*A11*A10*A8*A7*A6*A4+10*A2*A14*A9*A13*A11*A10*A8*A7*A6*A4*A3+3*A2*A14*A9*
A13*A11*A10*A8*A7*A6*A4*A1+3.3*A2*A14*A9*A8*6*A2*A14*A9*A8*A10+6*A2*A14*A9*A8*A10
*A11+6*A2*A14*A9*A8*A10*A11*A12+2.6*A2*A14*A9*A8*A10*A11*A13+14*A2*A14*A9*A7+14*A2
*A14*A9*A7*A6+8.5*A2*A14*A9*A7*A6*A5+8.5*A2*A14*A9*A7*A6*A4+10*A2*A14*A9*A7*A6*A4*
A3+3*A2*A14*A9*A7*A6*A4*A1;
```

Note: Constraint shown above only represents the constraints for Truck A.

Constraints Section: Path Cost Logic (Continued)

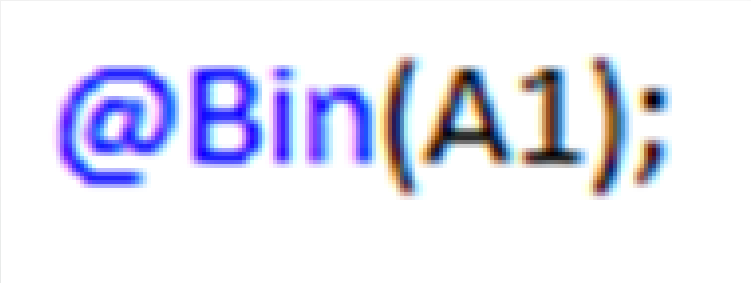


Constraints Section: Guaranty of Path Transverse

$$A1+B1+C1+D1+E1+F1+G1+H1+I1+J1+K1+L1+M1+N1>=1;$$

Note: Constraint shown above only represents the constraints for path 1.

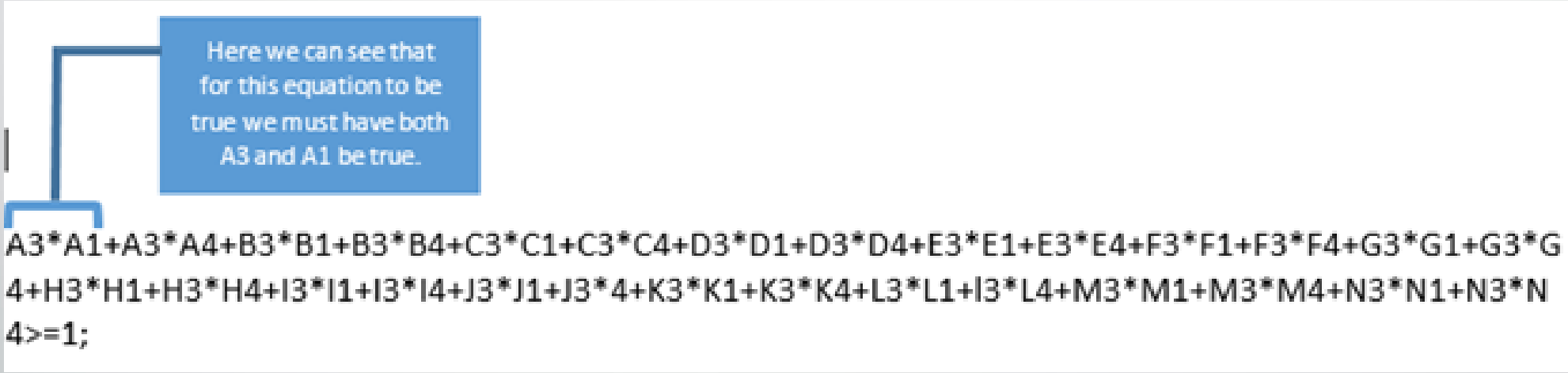
Constraint Section: Binary Logic Constraint



There are 14 Trucks and 15 possible paths which means a total of 240 Bin constraints were required to be made.

Note: Constraint shown above only represents path 1, truck 1.

Constraints Section: Adjusted Path Logic



Note: Constraint shown above only represents the constraints for path 3.

Truck Assignments (Linear Program)

- Paths for each truck have been determined… But what about specific section assigned to each plow?
- 1 Truck assigned 2 route sections; Other trucks assigned to just 1
- Given truck's path from first model, assigns section to each truck
- Objective Function: Minimize sum of assigned sections per truck

$$\sum_{i=A}^{1N} T_i$$

Where T_i = the total number of paths assigned to truck "i"

Constraints Section Truck Assignments:

$$A1+B1+C1+D1+E1+F1+G1+H1+I1+J1+K1+L1+M1+N1=1;$$

Note: Constraint shown above only represents the constraints for path 1.

Constraints Section Truck Assignments:

$$TA = A1 + A2 + A3 + \ldots + A15$$

Note: The summation shown above only represents truck A.

Modeling in Lingo

```
Min = (1/14)*(TA^2+TB^2+ TC^2+TD^2+ TE^2+TF^2+ TG^2+TH^2+ TI^2+TJ^2+ TK^2+TL^2+ TM^2+TN^2) ;
```

- Used Excel to model path constraints for each truck
- Min Function in Lingo

```
@Bin(A1);
```

```
TA=3*A1+3*A2+10*A1*A3+8.5*A1*A4+8.5*A1*A4*A5+14*A1*A4*A6+14*A1*A4*A6*A7+3.3*A1*A4*
TB=3*B1+3*B2+10*B1*B3+8.5*B1*B4+8.5*B1*B4*B5+14*B1*B4*B6+14*B1*B4*B6*B7+3.3*B1*B4*B6
TC=3*C1+3*C2+10*C1*C3+8.5*C1*C4+8.5*C1*C4*C5+14*C1*C4*C6+14*C1*C4*C6*C7+3.3*C1*C4*C6
TD=3*D1+3*D2+10*D1*D3+8.5*D1*D4+8.5*D1*D4*D5+14*D1*D4*D6+14*D1*D4*D6*D7+3.3*D1*D4*
TE=3*E1+3*E2+10*E1*E3+8.5*E1*E4+8.5*E1*E4*E5+14*E1*E4*E6+14*E1*E4*E6*E7+3.3*E1*E4*E6*E7
TF=3*F1+3*F2+10*F1*F3+8.5*F1*F4+8.5*F1*F4*F5+14*F1*F4*F6+14*F1*F4*F6*F7+3.3*F1*F4*F6*F7
TG=3*G1+3*G2+10*G1*G3+8.5*G1*G4+8.5*G1*G4*G5+14*G1*G4*G6+14*G1*G4*G6*G7+3.3*G1*G4*
TH=3*H1+3*H2+10*H1*H3+8.5*H1*H4+8.5*H1*H4*H5+14*H1*H4*H6+14*H1*H4*H6*H7+3.3*H1*H4*
TI=3*I1+3*I2+10*I1*I3+8.5*I1*I4+8.5*I1*I4*I5+14*I1*I4*I6+14*I1*I4*I6*I7+3.3*I1*I4*I6*I7*I8+13*I1*
TJ=3*J1+3*J2+10*J1*J3+8.5*J1*J4+8.5*J1*J4*J5+14*J1*J4*J6+14*J1*J4*J6*J7+3.3*J1*J4*J6*J7*J8+13
TK=3*K1+3*K2+10*K1*K3+8.5*K1*K4+8.5*K1*K4*K5+14*K1*K4*K6+14*K1*K4*K6*K7+3.3*K1*K4*K6
TL=3*L1+3*L2+10*L1*L3+8.5*L1*L4+8.5*L1*L4*L5+14*L1*L4*L6+14*L1*L4*L6*L7+3.3*L1*L4*L6*L7*L8
TM=3*M1+3*M2+10*M1*M3+8.5*M1*M4+8.5*M1*M4*M5+14*M1*M4*M6+14*M1*M4*M6*M7+3.3
TN=3*N1+3*N2+10*N1*N3+8.5*N1*N4+8.5*N1*N4*N5+14*N1*N4*N6+14*N1*N4*N6*N7+3.3*N1*
TO=3*O1+3*O2+10*O1*O3+8.5*O1*O4+8.5*O1*O4*O5+14*O1*O4*O6+14*O1*O4*O6*O7+3.3*O1*
TP=3*P1+3*P2+10*P1*P3+8.5*P1*P4+8.5*P1*P4*P5+14*P1*P4*P6+14*P1*P4*P6*P7+3.3*P1*P4*P6
```

```
A12+B12+C12+D12+E12+F12+G12+H12+I12+J12+K12+L12+M12+N12>=1;
```

Results

- **30.47 miles** per truck on average x 2 for the return trip = 61 miles
- 61miles / 55mph = **1.1 hours**
- Decreases route time by **39%.**

- "Fun" Facts: While testing and tinkering with model variations of it ran over 10 million iterations (~4 hours compute time) when put together.
  - "Winning" model finished with 3.6 million iterations

Sensitivity Analysis

2	0.000000	-0.8571454
3	0.000000	-1.142857
4	0.000000	-1.114285
5	0.000000	-1.114285
6	0.000000	-1.114285
7	0.000000	-1.114285
8	0.000000	-1.114285
9	0.000000	-1.464286
10	0.000000	-1.114285
11	0.000000	-1.464286
12	0.000000	-1.114285
13	0.000000	-1.114285
14	0.000000	-1.114285
15	0.000000	-1.114285
16	0.000000	-0.8571429
17	0.000000	-0.8571429
18	0.000000	-2.571445
19	0.000000	0.000000
20	1.000000	0.000000
21	1.000000	0.000000
22	1.000000	0.000000
23	0.000000	0.000000
24	0.000000	0.000000
25	1.000000	0.000000
26	0.000000	0.000000
27	0.000000	0.000000
28	0.000000	0.000000
29	0.000000	0.000000
30	0.000000	0.000000
31	0.000000	0.000000
32	1.000000	0.000000
33	1.000000	0.000000
34	0.000000	0.000000
35	0.000000	-5.714286
36	0.000000	2.285714
37	1.000000	0.000000
38	1.000000	0.000000
39	1.000000	0.000000
40	0.000000	0.000000
41	0.000000	0.000000
42	1.000000	0.000000
43	1.000000	0.000000
44	1.000000	0.000000
45	1.000000	0.000000
46	1.000000	0.000000
47	1.000000	0.000000
48	1.000000	0.000000

Lingo Output

```
Local optimal solution found.
  Objective value:                              58.25179
  Objective bound:                              58.25179
  Infeasibilities:                              0.000000
  Extended solver steps:                               2
  Total solver iterations:                           245
  Elapsed runtime seconds:                          0.30

  Model Class:                                     MINLP

  Total variables:                   256
  Nonlinear variables:               254
  Integer variables:                  28

  Total constraints:                 270
  Nonlinear constraints:              18

  Total nonzeros:                    773
  Nonlinear nonzeros:                295


                        Variable           Value        Reduced Cost
                              TA        6.000000            0.000000
                              TB        6.800000            0.000000
                              TC        6.800000            0.000000
                              TD        11.25000            0.000000
                              TE        6.800000            0.000000
                              TF        6.800000            0.000000
                              TG        6.800000            0.000000
                              TH        6.800000            0.000000
                              TI        6.800000            0.000000
                              TJ        8.000000            0.000000
                              TK        11.25000            0.000000
```

Questions?