a triangulation again, but not necessarily a Delaunay triangulation. This is because the addition of p_r can make some of the existing edges illegal. To

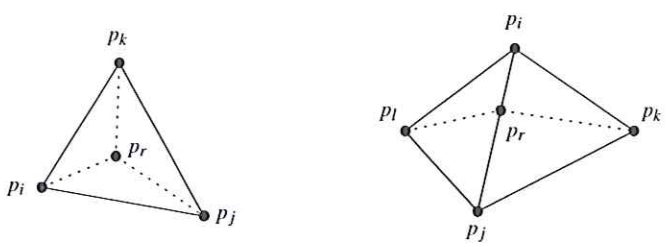

Figure 9.7
The two cases when adding a point p_r

remedy this, we call a procedure LEGALIZEEDGE with each potentially illegal edge. This procedure replaces illegal edges by legal ones through edge flips. Before we come to the details of this, we give a precise description of the main algorithm. It will be convenient for the analysis to let P be a set of $n+1$ points.

Algorithm DELAUNAYTRIANGULATION(P)
Input. A set P of $n+1$ points in the plane.
Output. A Delaunay triangulation of P.

1. Let p_0 be the lexicographically highest point of P, that is, the rightmost among the points with largest y-coordinate.
2. Let p_{-1} and p_{-2} be two points in $\mathbb{R}^2$ sufficiently far away and such that P is contained in the triangle $p_0p_{-1}p_{-2}$.
3. Initialize $\mathcal{T}$ as the triangulation consisting of the single triangle $p_0p_{-1}p_{-2}$.
4. Compute a random permutation $p_1, p_2, \ldots, p_n$ of $P \setminus \{p_0\}$.
5. **for** $r \leftarrow 1$ **to** n
6. **do** ($*$ Insert p_r into $\mathcal{T}$: $*$)
7. Find a triangle $p_ip_jp_k \in \mathcal{T}$ containing p_r.
8. **if** p_r lies in the interior of the triangle $p_ip_jp_k$
9. **then** Add edges from p_r to the three vertices of $p_ip_jp_k$, thereby splitting $p_ip_jp_k$ into three triangles.
10. LEGALIZEEDGE($p_r, \overline{p_ip_j}, \mathcal{T}$)
11. LEGALIZEEDGE($p_r, \overline{p_jp_k}, \mathcal{T}$)
12. LEGALIZEEDGE($p_r, \overline{p_kp_i}, \mathcal{T}$)
13. **else** ($*$ p_r lies on an edge of $p_ip_jp_k$, say the edge $\overline{p_ip_j}$ $*$)
14. Add edges from p_r to p_k and to the third vertex p_l of the other triangle that is incident to $\overline{p_ip_j}$, thereby splitting the two triangles incident to $\overline{p_ip_j}$ into four triangles.
15. LEGALIZEEDGE($p_r, \overline{p_ip_l}, \mathcal{T}$)
16. LEGALIZEEDGE($p_r, \overline{p_lp_j}, \mathcal{T}$)
17. LEGALIZEEDGE($p_r, \overline{p_jp_k}, \mathcal{T}$)
18. LEGALIZEEDGE($p_r, \overline{p_kp_i}, \mathcal{T}$)
19. Discard p_{-1} and p_{-2} with all their incident edges from $\mathcal{T}$.
20. **return** $\mathcal{T}$