

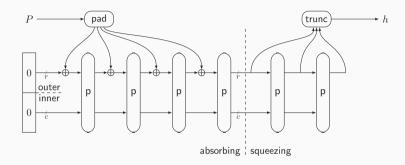
Understanding the Duplex and Its Security

Bart Mennink Radboud University (The Netherlands) Permutation-Based Crypto 2023, Lyon April 23, 2023



History on Sponges and Duplexes

Sponges [BDPV07]



- p is a b-bit permutation, with b = r + c
 - r is the rate
 - c is the capacity (security parameter)
- SHA-3, XOFs, lightweight hashing, ...
- ullet Behaves as RO up to query complexity $pprox 2^{c/2}$ [BDPV08]

Keying Sponges

Keyed Sponge

- PRF(K, P) = sponge(K||P)
- Message authentication with tag size t: MAC(K, P, t) = sponge(K||P, t)
- Keystream generation of length ℓ : $SC(K, D, \ell) = sponge(K||D, \ell)$
- ullet (All assuming K is fixed-length)

Keying Sponges

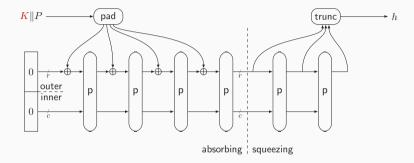
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- (All assuming *K* is fixed-length)

Keyed Duplex

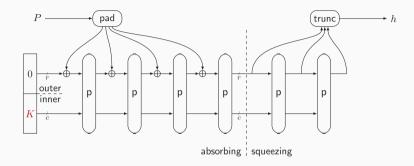
- Authenticated encryption
- Multiple CAESAR and NIST LWC submissions

Evolution of Keyed Sponges



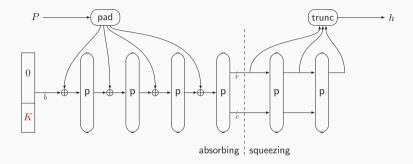
• Outer-Keyed Sponge [BDPV11b, ADMV15, NY16, Men18]

Evolution of Keyed Sponges

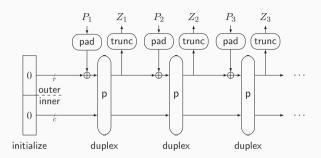


- Outer-Keyed Sponge [BDPV11b, ADMV15, NY16, Men18]
- Inner-Keyed Sponge [CDH+12, ADMV15, NY16]

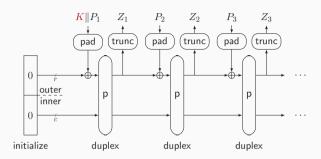
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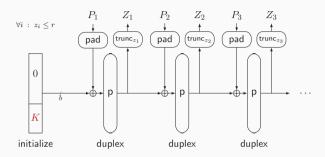
- Outer-Keyed Sponge [BDPV11b, ADMV15, NY16, Men18]
- Inner-Keyed Sponge [CDH+12, ADMV15, NY16]
- Full-Keyed Sponge [BDPV12, GPT15, MRV15]



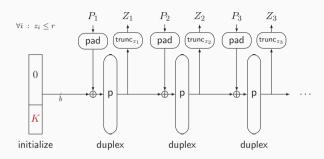
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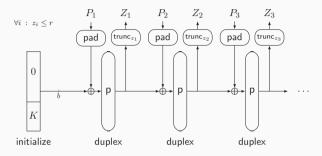


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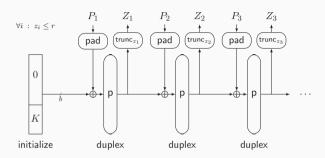


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Full-Keyed Duplex of [MRV15] (1)



Full-Keyed Duplex of [MRV15] (1)

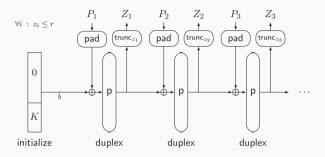


- *M*: data complexity (calls to construction)
- N: time complexity (calls to primitive)
- $\mu \leq 2M$: multiplicity ("maximum outer collision of p")

Simplified Security Bound

$$\frac{\mu N}{2^k} + \frac{M^2}{2^c}$$

Full-Keyed Duplex of [MRV15] (1)



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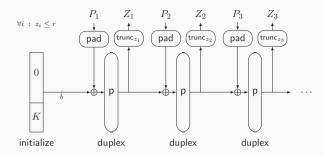
Simplified Security Bound



scheme behaves "randomly" as long as this term is \ll

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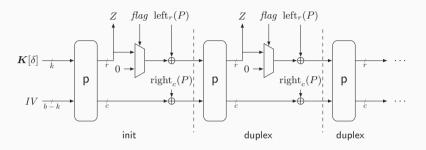
Full-Keyed Duplex of [MRV15] (2)



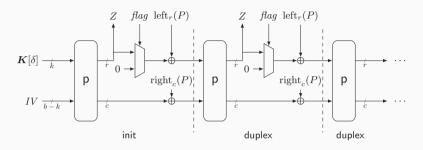
Limitations

- ullet Multiplicity μ only known a posteriori
- \bullet Dominating term $\mu N/2^k$ rather than $\mu N/2^c$
- Limited flexibility in modeling adversarial power (multi-user security, blockwise adaptive behavior, nonces, ...)

Full-Keyed Duplex of [DMV17] (1)

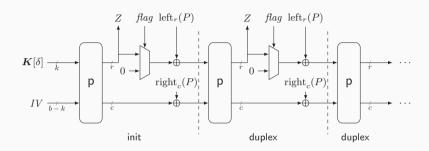


Full-Keyed Duplex of [DMV17] (1)



- Multi-user by design: index δ specifies key in array
- ullet Initial state: concatenation of $oldsymbol{K}[\delta]$ and IV
- Full-state absorption, no padding
- Rephasing: p, Z, P instead of P, p, Z
- Refined adversarial strength

Full-Keyed Duplex of [DMV17] (2)

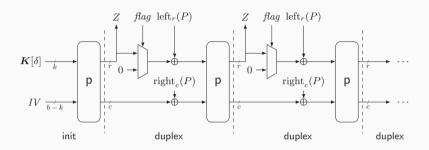


- *M*: data complexity (calls to construction)
- N: time complexity (calls to primitive)
- Q: number of init calls
- Q_{IV} : max # init calls for single IV
- L: # queries with repeated path (e.g., nonce-violation)
- Ω : # queries with overwriting outer part (e.g., RUP)
- $\nu_{r,c}^M$: some multicollision coefficient (often small)

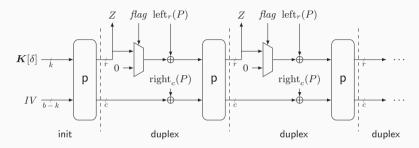
Simplified Security Bound

$$\frac{Q_{IV}N}{2^k} + \frac{(L + \Omega + \nu_{r,c}^M)N}{2^c}$$

Full-Keyed Duplex of [DM19] (1)

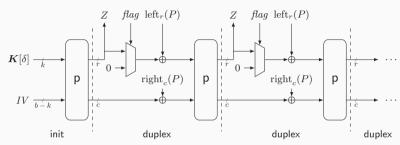


Full-Keyed Duplex of [DM19] (1)



- Initialization can be rotated (not depicted)
- Another rephasing: Z, P, p instead of p, Z, P instead of P, p, Z
- Security analysis in leaky setting
- Even further refined adversarial strength
- Comparable bound

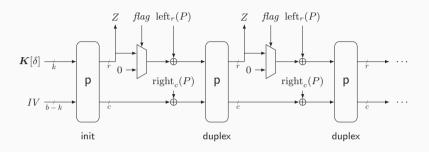
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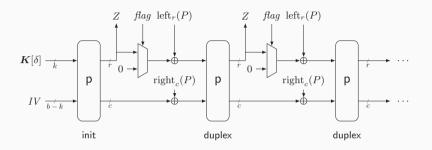


- M: data complexity (calls to construction)
- N: time complexity (calls to primitive)
- Q: number of init calls
- Q_{IV} : max # init calls for single IV
- Q_{δ} : maximum # init calls for single δ
- ullet L: # queries with repeated path (e.g., nonce-violation)
- Ω : # queries with overwriting outer part (e.g., RUP)
- R: max # duplexing calls for single non-empty path
- $\nu_{r.c}^{M}$: some multicollision coefficient (often small)

Simplified Security Bound

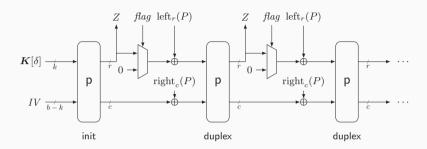
$$\frac{Q_{IV}N}{2^{k-Q_{\delta}\lambda}} + \frac{(L+\Omega+\nu_{r,c}^{M})N}{2^{c-(R+1)\lambda}}$$





Scheme: versatile but complex

- What about these rephasings?
- What about the flag?

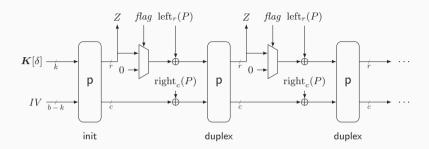


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Security bounds: strong but complex

- Security parameters hard to understand
- Bound quickly misunderstood
- Unclear how use case affects bound



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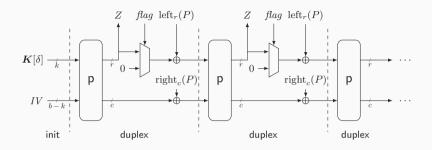
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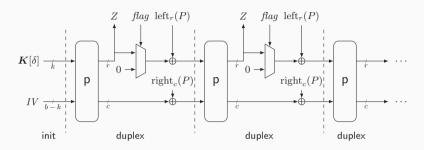
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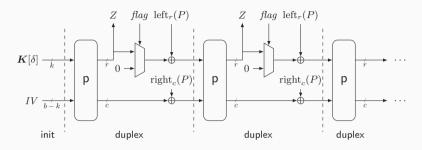
This work: explanation of the duplex, its security, and some applications

Understanding the Duplex

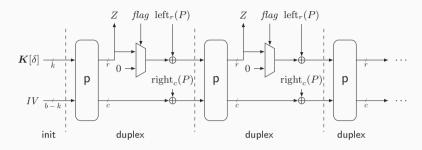




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 - including possible initial state rotation (not depicted)
 - yet another rephasing



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- Basically the scheme of [DMV17] and [DM19], but:
 - including possible initial state rotation (not depicted)
 - yet another rephasing
- Security results of [DMV17] and [DM19] carry over
- First: understanding phasing and flagging

Α	Р	S	Α	Р	S	Α	Р	S	Α	

	Α	Р	S	Α	Р	S	Α	Р	S	А	
[BDPV11a]	init		duplex				duplex				

• [BDPV11a]: duplex security reduced to sponge indifferentiability

	А	Р	S	А	Р	S	Α	Р	S	А	
[BDPV11a]	init			duplex			duplex				
[MRV15]	init				duplex			duplex			

- [BDPV11a]: duplex security reduced to sponge indifferentiability
- \bullet [MRV15]: same structure but tighter bound

	Α	Р	S	Α	Р	S	Α	Р	S	А		
[BDPV11a]	init				duplex		duplex					
[MRV15]	init			duplex			duplex					
[DMV17]	init			duplex			duplex					

- [BDPV11a]: duplex security reduced to sponge indifferentiability
- [MRV15]: same structure but tighter bound
- [DMV17]: improved bound by re-structuring, but *flag* needed

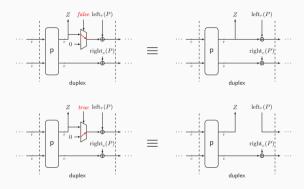
	A	Р	S	Α	Р	S	Α	Р	S	Α	
[BDPV11a]	init				duplex			duplex			
[MRV15]	init			duplex			duplex				
[DMV17]	init			duplex			duplex				
[DM19]	init	init duplex				duplex					

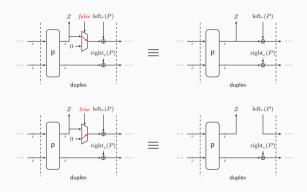
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- [DM19]: security analysis in leaky setting, include upcoming p

Generalized Keyed Duplex: Phasing

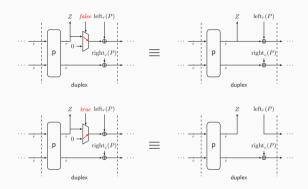
	Α	Р	S	А	Р	S	Α	Р	S	Α	
[BDPV11a]	init		duplex		duplex						
[MRV15]		init			duplex		duplex				
[DMV17]		init		duplex			duplex				
[DM19]	in	it	duplex			duplex					
now	init	duplex			duplex		duplex				

- [BDPV11a]: duplex security reduced to sponge indifferentiability
- [MRV15]: same structure but tighter bound
- [DMV17]: improved bound by re-structuring, but *flag* needed
- [DM19]: security analysis in leaky setting, include upcoming p
- now: seemingly most useful phasing





• Typical use case: authenticated encryption using duplex

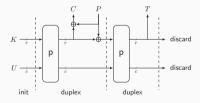


- Typical use case: authenticated encryption using duplex
- Security decreases for increasing number of calls with flag = true
- Earlier P, p, Z phasing allowed outer part overwriting by default

- Consider extreme simplification of SpongeWrap authenticated encryption
- Key K, plaintext P, ciphertext C, and tag T all r bits; nonce U c bits
- General case will be discussed later in this presentation

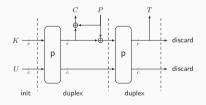
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Encryption

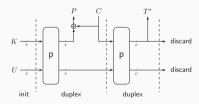


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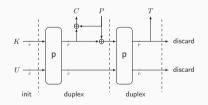


Decryption

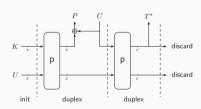


- Consider extreme simplification of SpongeWrap authenticated encryption
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Encryption



Decryption



- Duplex call with flag = true upon decryption
- \bullet Adversary can choose C and thus fix outer part to value of its choice

Understanding Duplex Security

Algorithm Keyed duplex construction $\mathsf{KD}[\mathsf{p}]_K$

```
Interface: KD.init Input: (\delta, IV) \in \{1, \dots, \mu\} \times \mathcal{IV}
Output: \varnothing
S \leftarrow \operatorname{rot}_{\alpha}(K[\delta] \parallel IV)
return \varnothing
Interface: KD.duplex
Input: (flag, P) \in \{true, false\} \times \{0, 1\}^b
Output: Z \in \{0, 1\}^r
S \leftarrow \operatorname{p}(S)
Z \leftarrow \operatorname{left}_r(S)
S \leftarrow S \oplus [flag] \cdot (Z \parallel 0^{b-r}) \oplus P
return Z
```

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```
Algorithm Ideal extendable input function IXIF[ro]
```

Interface: |X|F.init Input: $(\delta, IV) \in \{1, \dots, \mu\} \times \mathcal{IV}$ Output: \varnothing $path \leftarrow \operatorname{encode}[\delta] \parallel IV$ $\mathsf{return} \ \varnothing$

Interface: IXIF.duplex Input: $(flag, P) \in \{true, false\} \times \{0, 1\}^b$

Output: $Z \in \{0,1\}^r$

$$Z \leftarrow \text{ro}(path, r) \\ path \leftarrow path \parallel ([flag] \cdot (Z \parallel 0^{b-r}) \oplus P) \\ \textbf{return} \ Z$$

Algorithm Keyed duplex construction $KD[p]_K$

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Z \leftarrow \operatorname{left}_r(S)
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return Z
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```
\label{eq:algorithm} \begin{tabular}{l} \textbf{Algorithm} & \begin{tabular}{l} \textbf{Interface:} & \begin{tabular}{l} \textbf{INIF.} & \begin{tabular}{l} \textbf{Interface:} & \begin{tabular}{l} \textbf{INIF.} & \begin{tabular}{l} \textbf{IV} \\ \textbf{Output:} & \varnothing \\ & path \leftarrow \operatorname{encode}[\delta] \parallel IV \\ & \textbf{return} & \varnothing \\ \end{tabular} \\ \begin{tabular}{l} \textbf{Interface:} & \begin{tabular}{l} \textbf{INIF.} & \begin{tabular}{l} \textbf{Algorithm} & \begin{tabular}{l} \textbf{IV} \\ & \textbf{return} & \varnothing \\ \end{tabular} \\ \begin{tabular}{l} \textbf{Interface:} & \begin{tabular}{l} \textbf{INIF.} & \begin{tabular}{l} \textbf{Algorithm} & \begin{tabular}{l} \textbf{Algorithm}
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$$\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{KD}[\mathsf{p}]_{K},\mathsf{p}^{\pm}\;;\;\mathsf{IXIF}[\mathsf{ro}],\mathsf{p}^{\pm}\right)$$

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• IXIF[ro] is basically random oracle in disguise

$\label{eq:local_problem} \textbf{Algorithm} \ \, \texttt{Keyed} \ \, \texttt{duplex} \ \, \texttt{construction} \ \, \texttt{KD}[\texttt{p}]_{\pmb{K}}$ $\label{eq:local_problem} \textbf{Interface:} \ \, \texttt{KD.init}$ $\label{eq:local_problem} \textbf{Interface:} \ \, \texttt{KD.duplex}$ $\label{eq:local_problem} \textbf{Interface:} \ \,$

```
\label{eq:local_state} \begin{tabular}{l} \textbf{Algorithm} & \textbf{Ideal} & \textbf{extendable} & \textbf{input} & \textbf{function} & \textbf{IXIF}[\textbf{ro}] \\ \hline \textbf{Interface:} & \textbf{IXIF.init} \\ \textbf{Input:} & (\delta, IV) \in \{1, \dots, \mu\} \times \mathcal{IV} \\ \textbf{Output:} & \varnothing \\ & path \leftarrow \textbf{encode}[\delta] \parallel IV \\ & \textbf{return} & \varnothing \\ \hline \textbf{Interface:} & \textbf{IXIF.duplex} \\ \textbf{Input:} & (flag, P) \in \{true, false\} \times \{0, 1\}^b \\ \textbf{Output:} & Z \in \{0, 1\}^r \\ \hline & Z \leftarrow \textbf{ro}(path, r) \\ & path \leftarrow path \parallel ([flag] \cdot (Z \parallel 0^{b-r}) \oplus P) \\ & \textbf{return} & Z \\ \hline \end{tabular}
```

$$\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{KD}[\mathsf{p}]_{K},\mathsf{p}^{\pm}\;;\;\mathsf{IXIF}[\mathsf{ro}],\mathsf{p}^{\pm}\right)$$

- IXIF[ro] is basically random oracle in disguise
- If $KD[p]_K$ is hard to distinguish from IXIF[ro] for certain bound on adversarial resources, $KD[p]_K$ roughly "behaves like" random oracle

$\label{eq:Algorithm} \begin{tabular}{l} \textbf{Algorithm} & \texttt{Keyed duplex construction KD[p]}_{\textbf{\textit{K}}} \\ \hline \textbf{Interface:} & \texttt{KD.init} \\ \textbf{Input:} & (\delta, IV) \in \{1, \dots, \mu\} \times \mathcal{IV} \\ \textbf{Output:} & \varnothing \\ S \leftarrow \mathsf{rot}_{\alpha}(\textbf{\textit{K}}[\delta] \parallel IV) \\ \texttt{return} & \varnothing \\ \hline \\ \textbf{Interface:} & \texttt{KD.duplex} \\ \textbf{Input:} & (flag, P) \in \{true, false\} \times \{0, 1\}^b \\ \textbf{Output:} & Z \in \{0, 1\}^r \\ S \leftarrow \mathsf{p}(S) \\ Z \leftarrow \mathsf{left}_r(S) \\ S \leftarrow S \oplus |flag| \cdot (Z || 0^{b-r}) \oplus P \\ \hline \end{tabular}$

return Z

```
\label{eq:local_local_local_local} \begin{tabular}{l} \textbf{Algorithm} & \textbf{Ideal} & \textbf{extendable} & \textbf{input} & \textbf{function} & \textbf{IXIF}[\textbf{ro}] \\ \hline \textbf{Interface:} & \textbf{IXIF.init} \\ \textbf{Input:} & (\delta, IV) \in \{1, \dots, \mu\} \times \mathcal{IV} \\ \textbf{Output:} & \varnothing \\ & path \leftarrow \textbf{encode}[\delta] \parallel IV \\ & \textbf{return} & \varnothing \\ \hline \\ \textbf{Interface:} & \textbf{IXIF.duplex} \\ \textbf{Input:} & (flag, P) \in \{true, false\} \times \{0, 1\}^b \\ \textbf{Output:} & Z \in \{0, 1\}^r \\ \hline & \textbf{Z} \leftarrow \textbf{ro}(path, r) \\ & path \leftarrow path \parallel ([flag] \cdot (Z \parallel 0^{b-r}) \oplus P) \\ & \textbf{return} & Z \\ \hline \end{tabular}
```

$$\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{KD}[\mathsf{p}]_{K},\mathsf{p}^{\pm}\;;\;\mathsf{IXIF}[\mathsf{ro}],\mathsf{p}^{\pm}\right)$$

- IXIF[ro] is basically random oracle in disguise
- If $KD[p]_K$ is hard to distinguish from IXIF[ro] for certain bound on adversarial resources, $KD[p]_K$ roughly "behaves like" random oracle
- Bound on adversarial resources is in turn determined by use case!

Security Bounds From [DMV17] and [DM19]

- *M*: data complexity (calls to construction)
- *N*: time complexity (calls to primitive)
- Q: number of init calls
- Q_{IV} : max # init calls for single IV
- L: # queries with repeated path (e.g., nonce-violation)
- Ω : # queries with overwriting outer part (e.g., RUP)
- $\nu_{r,c}^M$: some multicollision coefficient (often small)

Simplified Security Bound

$$\frac{Q_{IV}N}{2^k} + \frac{(L + \Omega + \nu_{r,c}^M)N}{2^c}$$

Security Bounds From [DMV17] and [DM19]

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- $\nu_{r,c}^M$: some multicollision coefficient (often small)

Simplified Security Bound

$$\frac{Q_{IV}N}{2^k} + \frac{(L+\Omega+\nu_{r,c}^M)N}{2^c}$$

Actual Security Bounds (Retained)

• [DMV17]:

$$\mathbf{Adv_{KD}}(\mathsf{D}) \leq \frac{(L+\Omega)N}{2^c} + \frac{2\nu_{r,c}^{2(M-L)}(N+1)}{2^c} + \frac{\binom{L+\Omega+1}{2}}{2^c} + \frac{(M-L-Q)Q}{2^b - Q} + \frac{M(M-L-1)}{2^b} + \frac{Q(M-L-Q)}{2^{\min\{c+k,\max\{b-\alpha,c\}\}}} + \frac{Q_{IV}N}{2^k} + \frac{\binom{\mu}{2}}{2^k} + \frac{\binom{\mu}{2}}{2^k} + \binom{\mu}{2} + \binom$$

• [DM19] (with one simplification):

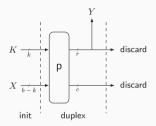
$$\mathbf{Adv_{KD}}(\mathsf{D}) \leq \frac{(L+\Omega)N}{2^c} + \frac{2\nu_{r,c}^M(N+1)}{2^c} + \frac{\nu_{r,c}^M(L+\Omega) + \binom{L+\Omega}{2}}{2^c} + \frac{\binom{M-L-Q}{2} + (M-L-Q)(L+\Omega)}{2^b} + \frac{\binom{M+N}{2} + \binom{N}{2}}{2^b} + \frac{Q(M-Q)}{2^{\min\{c+k, \max\{b-\alpha,c\}\}}} + \frac{Q_{IV}N}{2^k} + \frac{\binom{M}{2}N}{2^k} + \frac{\binom{M}{2}N}{2^k} + \frac{N}{2^k} +$$

Coefficient (Skipped)

Intermezzo: Multicollision

Use Case 1: Truncated Permutation

Truncated Permutation

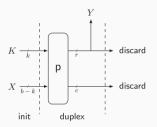


Algorithm Truncated permutation TP[p]

```
\begin{split} & \textbf{Input:} \ (K,X) \in \{0,1\}^k \times \{0,1\}^{b-k} \\ & \textbf{Output:} \ Y \in \{0,1\}^r \\ & \textbf{Underlying keyed duplex:} \ \text{KD[p]}_{(K)} \\ & \text{KD.init}(1,X) \\ & Y \leftarrow \text{KD.duplex}(false,0^b) \\ & \textbf{return} \ Y \end{split}
```

• PRP-to-PRF conversion: SoP/EDM/EDMD/truncation/STH/...

Truncated Permutation

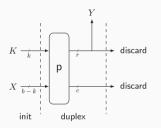


Algorithm Truncated permutation TP[p]

```
\begin{split} & \textbf{Input:} \ (K,X) \in \{0,1\}^k \times \{0,1\}^{b-k} \\ & \textbf{Output:} \ Y \in \{0,1\}^r \\ & \textbf{Underlying keyed duplex:} \ \text{KD[p]}_{(K)} \\ & \text{KD.init}(1,X) \\ & Y \leftarrow \text{KD.duplex}(false,0^b) \\ & \textbf{return } Y \end{split}
```

- PRP-to-PRF conversion: SoP/EDM/EDMD/truncation/STH/...
- Trend towards RP-to-PRF conversion:
 - Sum of externally keyed permutations [CLM19]
 - Permutation-based EDM [DNT21]

Truncated Permutation



Algorithm Truncated permutation TP[p]

```
\begin{aligned} & \textbf{Input:} \  \, (K,X) \in \{0,1\}^k \times \{0,1\}^{b-k} \\ & \textbf{Output:} \  \, Y \in \{0,1\}^r \\ & \textbf{Underlying keyed duplex:} \  \, \mathsf{KD[p]}_{(K)} \\ & \mathsf{KD.init}(1,X) \\ & Y \leftarrow \mathsf{KD.duplex}(false,0^b) \\ & \mathsf{return} \  \, Y \end{aligned}
```

- PRP-to-PRF conversion: SoP/EDM/EDMD/truncation/STH/...
- Trend towards RP-to-PRF conversion:
 - Sum of externally keyed permutations [CLM19]
 - Permutation-based EDM [DNT21]
- Truncation of externally keyed permutation can be described using duplex

Consider distinguisher D against PRF security of TP[p]

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{TP}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{TP}[\mathsf{p}]_K, \mathsf{p}^{\pm}\;;\;\mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right)$$

ullet D can make q construction queries $+\ N$ primitive queries

 \bullet Consider distinguisher D against PRF security of $\mathsf{TP}[p]$

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{TP}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{TP}[\mathsf{p}]_K, \mathsf{p}^{\pm}\;;\;\mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right)$$

- ullet D can make q construction queries + N primitive queries
- $\mathsf{TP}[\mathsf{p}]_K$ is basically just $\mathsf{TP}[\mathsf{KD}[\mathsf{p}]_K]$

Consider distinguisher D against PRF security of TP[p]

$$\mathbf{Adv}_{\mathsf{TP}}^{\mathsf{prf}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{TP}[\mathsf{p}]_K, \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathsf{prf}}, \mathsf{p}^{\pm}\right)$$

- ullet D can make q construction queries +N primitive queries
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- Triangle inequality:

$$\begin{split} \mathbf{Adv}_{\mathsf{TP}}^{\mathsf{prf}}(\mathsf{D}) &= \Delta_{\mathsf{D}} \left(\mathsf{TP}[\mathsf{p}]_K, \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathsf{prf}}, \mathsf{p}^{\pm} \right) \\ &= \Delta_{\mathsf{D}} \left(\mathsf{TP}[\mathsf{KD}[\mathsf{p}]_K], \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathsf{prf}}, \mathsf{p}^{\pm} \right) \\ &\leq \Delta_{\mathsf{D}} \left(\mathsf{TP}[\mathsf{KD}[\mathsf{p}]_K], \mathsf{p}^{\pm} \; ; \; \mathsf{TP}[\mathsf{IXIF}[\mathsf{ro}]], \mathsf{p}^{\pm} \right) + \Delta_{\mathsf{D}} \left(\mathsf{TP}[\mathsf{IXIF}[\mathsf{ro}]], \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathsf{prf}}, \mathsf{p}^{\pm} \right) \end{split}$$

Consider distinguisher D against PRF security of TP[p]

$$\mathbf{Adv}_{\mathsf{TP}}^{\mathsf{prf}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{TP}[\mathsf{p}]_K, \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathsf{prf}}, \mathsf{p}^{\pm}\right)$$

- D can make q construction queries + N primitive queries
- ullet TP[p] $_K$ is basically just TP[KD[p] $_K$]
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$$\begin{split} \mathbf{Adv}_{\mathsf{TP}}^{\mathsf{prf}}(\mathsf{D}) &= \Delta_{\mathsf{D}} \left(\mathsf{TP}[\mathsf{p}]_{\mathit{K}}, \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathsf{prf}}, \mathsf{p}^{\pm} \right) \\ &= \Delta_{\mathsf{D}} \left(\mathsf{TP}[\mathsf{KD}[\mathsf{p}]_{\mathit{K}}], \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathsf{prf}}, \mathsf{p}^{\pm} \right) \\ &\leq \Delta_{\mathsf{D}} \left(\mathsf{TP}[\mathsf{KD}[\mathsf{p}]_{\mathit{K}}], \mathsf{p}^{\pm} \; ; \; \mathsf{TP}[\mathsf{IXIF}[\mathsf{ro}]], \mathsf{p}^{\pm} \right) + \Delta_{\mathsf{D}} \left(\mathsf{TP}[\mathsf{IXIF}[\mathsf{ro}]], \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathsf{prf}}, \mathsf{p}^{\pm} \right) \end{split}$$

Consider distinguisher D against PRF security of TP[p]

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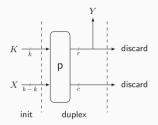
Consider distinguisher D against PRF security of TP[p]

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{TP}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{TP}[\mathsf{p}]_K, \mathsf{p}^{\pm}\;;\;\mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right)$$

- ullet D can make q construction queries + N primitive queries
- $\mathsf{TP}[\mathsf{p}]_K$ is basically just $\mathsf{TP}[\mathsf{KD}[\mathsf{p}]_K]$
- Triangle inequality:

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What are the resources of D'?



Algorithm Truncated permutation TP[p]

 $\begin{array}{ll} \textbf{Input:} & (K,X) \in \{0,1\}^k \times \{0,1\}^{b-k} \\ \textbf{Output:} & Y \in \{0,1\}^r \\ \textbf{Underlying keyed duplex:} & \mathsf{KD[p]}_{(K)} \\ & \mathsf{KD.init}(1,X) \\ & Y \leftarrow \mathsf{KD.duplex}(false,0^b) \\ & \mathbf{return} & Y \end{array}$

resources of D'

in terms of resources of D

M: data complexity (calls to construction)

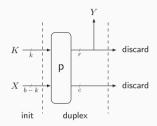
N: time complexity (calls to primitive)

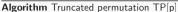
 ${\it Q}$: number of init calls

 $Q_{IV}\colon \max \,\#\, \operatorname{init}\, \operatorname{calls}\, \operatorname{for}\, \operatorname{single}\, IV$

L: # queries with repeated path

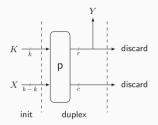
 $\Omega \colon \#$ queries with overwriting outer part





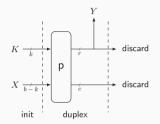
 $\begin{aligned} & \textbf{Input:} \ (K,X) \in \{0,1\}^k \times \{0,1\}^{b-k} \\ & \textbf{Output:} \ Y \in \{0,1\}^r \\ & \textbf{Underlying keyed duplex:} \ \mathsf{KD}[\mathsf{p}]_{(K)} \\ & \mathsf{KD.init}(1,X) \\ & Y \leftarrow \mathsf{KD.duplex}(false,0^b) \\ & \text{return } Y \end{aligned}$

resources of D'	in terms of	resources of D
M : data complexity (calls to construction) N : time complexity (calls to primitive) Q : number of init calls Q_{IV} : max $\#$ init calls for single IV L : $\#$ queries with repeated path Ω : $\#$ queries with overwriting outer part		N



 $\begin{aligned} & \mathsf{KD.init}(1,X) \\ & Y \leftarrow \mathsf{KD.duplex}(false,0^b) \\ & \mathbf{return} \ Y \end{aligned}$

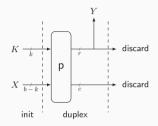
resources of D'	in terms of	resources of D
M: data complexity (calls to construction)		q
N: time complexity (calls to primitive)	\longrightarrow	N
Q: number of init calls	$\xrightarrow{\hspace*{1cm}}$	q
$Q_{IV}\colon$ max $\#$ init calls for single IV		
L: # queries with repeated path		
Ω : # queries with overwriting outer part		



$\label{eq:Algorithm} \begin{array}{l} \textbf{Algorithm Truncated permutation TP[p]} \\ \\ \textbf{Input:} \ \ (K,X) \in \{0,1\}^k \times \{0,1\}^{b-k} \\ \textbf{Output:} \ \ Y \in \{0,1\}^r \\ \textbf{Underlying keyed duplex:} \ \ \mathsf{KD[p]}_{(K)} \\ \mathsf{KD.init}(1,X) \\ Y \leftarrow \mathsf{KD.duplex}(false,0^b) \end{array}$

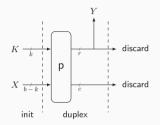
resources of D'	in terms of	resources of D
M : data complexity (calls to construction) N : time complexity (calls to primitive) Q : number of init calls Q_{IV} : max $\#$ init calls for single IV L : $\#$ queries with repeated path Ω : $\#$ queries with overwriting outer part		q N q 1

 $\mathbf{return}\ Y$



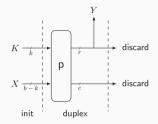
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resources of D'	in terms of	resources of D
M : data complexity (calls to construction) N : time complexity (calls to primitive) Q : number of init calls Q_{IV} : max $\#$ init calls for single IV L : $\#$ queries with repeated path		q N q 1
Ω : $\#$ queries with overwriting outer part		



$\label{eq:Algorithm} \begin{array}{l} \textbf{Algorithm} \ \, \text{Truncated permutation} \ \, \text{TP}[\textbf{p}] \\ \textbf{Input:} \ \, (K,X) \in \{0,1\}^k \times \{0,1\}^{b-k} \\ \textbf{Output:} \ \, Y \in \{0,1\}^r \\ \textbf{Underlying keyed duplex:} \ \, \text{KD}[\textbf{p}]_{(K)} \\ \text{KD.init}(1,X) \\ Y \leftarrow \text{KD.duplex}(false,0^b) \\ \textbf{return} \ \, Y \end{array}$

resources of D'	in terms of	resources of D
M: data complexity (calls to construction)	\longrightarrow	q
N: time complexity (calls to primitive)	$\!$	N
Q: number of init calls	$\longrightarrow\hspace{-0.8cm}\longrightarrow$	q
Q_{IV} : max $\#$ init calls for single IV	$\!$	1
L: # queries with repeated path	$\longrightarrow\hspace{-0.8cm}\longrightarrow$	0
Ω : # queries with overwriting outer part	$\!$	0



Algorithm Truncated permutation $\mathsf{TP}[\mathsf{p}]$ Input: $(K,X) \in \{0,1\}^k \times \{0,1\}^{b-k}$ Output: $Y \in \{0,1\}^r$ Underlying keyed duplex: $\mathsf{KD}[\mathsf{p}]_{(K)}$ $\mathsf{KD.init}(1,X)$ $Y \leftarrow \mathsf{KD.duplex}(false,0^b)$

resources of D'	in terms of	resources of D
M: data complexity (calls to construction)		q
N: time complexity (calls to primitive)	$\!$	N
Q: number of init calls	$-\!$	q
Q_{IV} : max $\#$ init calls for single IV	\longrightarrow	1
L: $#$ queries with repeated path	$\!$	0
Ω : # queries with overwriting outer part	\longrightarrow	0

return Y

From [DMV17] (in single-user setting): $\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}') \leq \frac{2\nu_{r,c}^{2q}(N+1)}{2^c} + \frac{2\binom{q}{2}}{2^b} + \frac{N}{2^k}$

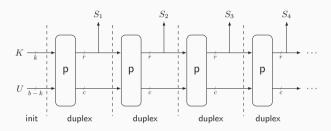
Use Case 2: Parallel Keystream

Generation (Skipped)

Use Case 3: Sequential Keystream

Generation

Sequential Keystream Generation

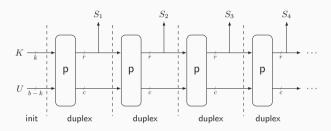


- Input: key K, nonce U
- ullet Output: keystream S of requested length

Algorithm Sequential keystream generation S-SC[p]

```
\begin{split} & \textbf{Input:} \  \, (K,U,\ell) \in \{0,1\}^k \times \{0,1\}^{b-k} \times \mathbb{N} \\ & \textbf{Output:} \  \, S \in \{0,1\}^\ell \\ & \textbf{Underlying keyed duplex:} \  \, \mathsf{KD}[\mathsf{p}]_{(K)} \\ & S \leftarrow \varnothing \\ & \mathsf{KD.init}(1,U) \\ & \textbf{for} \  \, i = 1,\dots,\lceil\ell/r\rceil \  \, \textbf{do} \\ & S \leftarrow S \parallel \mathsf{KD.duplex}(false,0^b) \\ & \textbf{return} \  \, \mathsf{left}_\ell(S) \end{split}
```

Sequential Keystream Generation

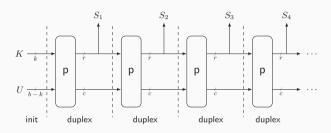


- Input: key K, nonce U
- ullet Output: keystream S of requested length
- PRF security of S-SC[p]:
 - Comparable analysis as for TP[p]

Algorithm Sequential keystream generation S-SC[p]

```
\begin{split} & \textbf{Input:} \  \, (K,U,\ell) \in \{0,1\}^k \times \{0,1\}^{b-k} \times \mathbb{N} \\ & \textbf{Output:} \  \, S \in \{0,1\}^\ell \\ & \textbf{Underlying keyed duplex:} \  \, \mathbf{KD[p]}_{(K)} \\ & S \leftarrow \varnothing \\ & \textbf{KD.init}(1,U) \\ & \textbf{for} \  \, i = 1,\dots,\lceil \ell/r \rceil \  \, \textbf{do} \\ & S \leftarrow S \parallel \mathbf{KD.duplex}(false,0^b) \\ & \textbf{return } \mathbf{left}_\ell(S) \end{split}
```

Sequential Keystream Generation



- Input: key K, nonce U
- ullet Output: keystream S of requested length
- PRF security of S-SC[p]:
 - Comparable analysis as for TP[p]
 - Resources of D' slightly differ

Algorithm Sequential keystream generation S-SC[p]

```
\begin{split} & \textbf{Input:} \  \, (K,U,\ell) \in \{0,1\}^k \times \{0,1\}^{b-k} \times \mathbb{N} \\ & \textbf{Output:} \  \, S \in \{0,1\}^\ell \\ & \textbf{Underlying keyed duplex:} \  \, \mathbf{KD[p]}_{(K)} \\ & S \leftarrow \varnothing \\ & \mathbf{KD.init}(1,U) \\ & \textbf{for} \  \, i = 1,\dots,\lceil \ell/r \rceil \  \, \textbf{do} \\ & S \leftarrow S \parallel \mathbf{KD.duplex}(false,0^b) \\ & \textbf{return} \  \, \text{left}_{\ell}(S) \end{split}
```

Consider distinguisher D against PRF security of S-SC[p]

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{S-SC}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{S-SC}[\mathsf{p}]_K, \mathsf{p}^{\pm}\;;\;\mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right)$$

• D can make q construction queries (total σ blocks) + N primitive queries

 \bullet Consider distinguisher D against PRF security of S-SC[p]

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{S-SC}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{S-SC}[\mathsf{p}]_K,\mathsf{p}^{\pm}\;;\;\mathsf{R}^{\mathrm{prf}},\mathsf{p}^{\pm}\right)$$

- D can make q construction queries (total σ blocks) + N primitive queries
- $\bullet \ \, \text{Triangle inequality:} \ \, \mathbf{Adv}^{\mathrm{prf}}_{\mathsf{S-SC}}(\mathsf{D}) \leq \Delta_{\mathsf{D}'}\left(\mathsf{KD}[\mathsf{p}]_K,\mathsf{p}^{\pm} \; ; \; \mathsf{IXIF}[\mathsf{ro}],\mathsf{p}^{\pm}\right)$

 \bullet Consider distinguisher D against PRF security of S-SC[p]

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{S-SC}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{S-SC}[\mathsf{p}]_K, \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right)$$

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- What are the resources of D'?

Consider distinguisher D against PRF security of S-SC[p]

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{S-SC}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{S-SC}[\mathsf{p}]_K, \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right)$$

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- What are the resources of D'?

M : data complexity (calls to construction) N : time complexity (calls to primitive) Q : number of init calls Q_{IV} : max $\#$ init calls for single IV L : $\#$ queries with repeated path Ω : $\#$ queries with overwriting outer part	resources of D'	in terms of	resources of D
	N : time complexity (calls to primitive) Q : number of init calls Q_{IV} : max $\#$ init calls for single IV L : $\#$ queries with repeated path		

Consider distinguisher D against PRF security of S-SC[p]

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{S-SC}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{S-SC}[\mathsf{p}]_K, \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right)$$

- ullet D can make q construction queries (total σ blocks) + N primitive queries
- Triangle inequality: $\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{S-SC}}(\mathsf{D}) \leq \Delta_{\mathsf{D}'}\left(\mathsf{KD}[\mathsf{p}]_K,\mathsf{p}^{\pm}\;;\;\mathsf{IXIF}[\mathsf{ro}],\mathsf{p}^{\pm}\right)$
- What are the resources of D'?

resources of D
N
10

 \bullet Consider distinguisher D against PRF security of S-SC[p]

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- What are the resources of D'?

resources of D'	in terms of	resources of D
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N: time complexity (calls to primitive)	\longrightarrow	N
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L: $#$ queries with repeated path		
Ω : # queries with overwriting outer part		

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resources of D'	in terms of	resources of D
M: data complexity (calls to construction)	\longrightarrow	σ
N: time complexity (calls to primitive)	$\!$	N
Q: number of init calls	$\!$	q
Q_{IV} : max $\#$ init calls for single IV	${\longrightarrow}$	1
L: $#$ queries with repeated path	$\longrightarrow\hspace{-0.8cm}\longrightarrow$	0
Ω : # queries with overwriting outer part		0

Consider distinguisher D against PRF security of S-SC[p]

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{S-SC}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{S-SC}[\mathsf{p}]_K, \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right)$$

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- What are the resources of D'?

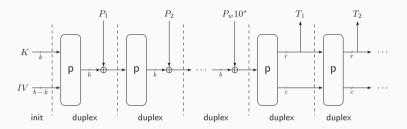
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M: data complexity (calls to construction)		σ
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Q: number of init calls	$\!$	q
Q_{IV} : max $\#$ init calls for single IV	\longrightarrow	1
L: $#$ queries with repeated path	\longrightarrow	0
$\Omega{:}~\#$ queries with overwriting outer part	\longrightarrow	0

From [DMV17] (in single-user setting):

$$\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}') \le \frac{2\nu_{r,c}^{2\sigma}(N+1)}{2^c} + \frac{(\sigma-q)q}{2^b-q} + \frac{2\binom{\sigma}{2}}{2^b} + \frac{q(\sigma-q)}{2^{\min\{c+k,b\}}} + \frac{N}{2^k}$$

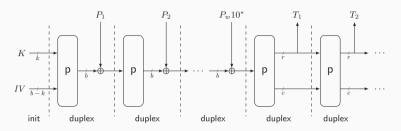
Use Case 4: Message

Authentication



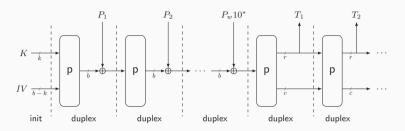
- Input: key K, initial value IV, message P
- ullet Output: tag T

```
Input: (K, IV, P) \in \{0, 1\}^k \times \mathcal{IV} \times \{0, 1\}^*
Output: T \in \{0, 1\}^k
Underlying keyed duplex: \mathsf{KD}[\mathsf{p}]_{(K)}
(P_1, P_2, \dots, P_w) \leftarrow \mathsf{pad}_b^{10^*}(P)
T \leftarrow \varnothing
\mathsf{KD.init}(1, IV)
for i = 1, \dots, w do
\mathsf{KD.duplex}(false, P_i)
\mathsf{for} \ i = 1, \dots, [t/r] \ \mathsf{do}
T \leftarrow T \parallel \mathsf{KD.duplex}(false, 0^b)
return \mathsf{left}_t(T)
```



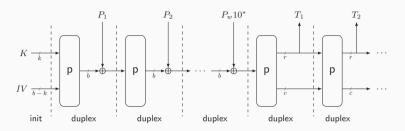
- Input: key K, initial value IV, message P
- ullet Output: tag T
- Analysis of [MRV15] applies

```
Input: (K, IV, P) \in \{0, 1\}^k \times \mathcal{IV} \times \{0, 1\}^*
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```



- Input: key K, initial value IV, message P
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- Analysis of [MRV15] applies
- PRF security of FSKS[p]:
 - Comparable analysis as for S-SC[p]

```
Input: (K, IV, P) \in \{0, 1\}^k \times \mathcal{IV} \times \{0, 1\}^*
Output: T \in \{0, 1\}^t
Underlying keyed duplex: \mathrm{KD}[\mathrm{p}]_{(K)}
(P_1, P_2, \ldots, P_w) \leftarrow \mathrm{pad}_b^{10^*}(P)
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```



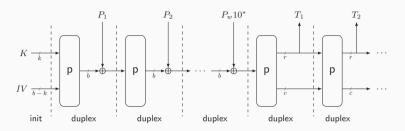
- Input: key K, initial value IV, message P
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- Analysis of [MRV15] applies
- PRF security of FSKS[p]:
 - Comparable analysis as for S-SC[p]
 - ... but distinguisher can repeat paths

Algorithm Full-state keyed sponge FSKS[p]

 $T \leftarrow T \parallel \mathsf{KD.duplex}(false, 0^b)$

return $left_t(T)$

```
\begin{aligned} & \textbf{Input:} \ (K, IV, P) \in \{0, 1\}^k \times \mathcal{IV} \times \{0, 1\}^* \\ & \textbf{Output:} \ T \in \{0, 1\}^t \\ & \textbf{Underlying keyed duplex:} \ \text{KD}[p]_{(K)} \\ & (P_1, P_2, \dots, P_w) \leftarrow \text{pad}_b^{1,0}(P) \\ & T \leftarrow \varnothing \\ & \text{KD.init}(1, IV) \\ & \textbf{for } i = 1, \dots, w \ \textbf{do} \\ & \text{KD.duplex}(false, P_i) \\ & \textbf{for } i = 1, \dots, \lceil t/r \rceil \ \textbf{do} \end{aligned} \quad \triangleright \ \text{discard output}
```



- Input: key K, initial value IV, message P
- ullet Output: tag T
- Analysis of [MRV15] applies
- PRF security of FSKS[p]:
 - Comparable analysis as for S-SC[p]
 - ... but distinguisher can repeat paths
 - Impacts resources of D'

```
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```

 $\bullet \ \ Consider \ distinguisher \ D \ against \ PRF \ security \ of \ FSKS[p] \\$

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{FSKS}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{FSKS}[\mathsf{p}]_K, \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right)$$

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From [DMV17] (in single-user setting):

$$\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}') \leq \frac{2\nu_{r,c}^{2\sigma}(N+1)}{2^c} + \frac{(q-1)N + \binom{q}{2}}{2^c} + \frac{(\sigma-q)q}{2^b-q} + \frac{2\binom{\sigma}{2}}{2^b} + \frac{q(\sigma-q)}{2^{\min\{c+k,b\}}} + \frac{N}{2^k}$$

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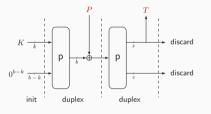
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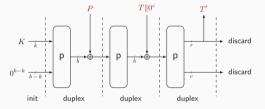
influence of $\it L$

• Repeated paths (i.e., large L) can seriously affect security

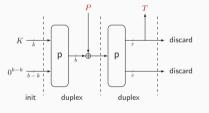
- ullet Repeated paths (i.e., large L) can seriously affect security
- Consider simplified FSKS[p]: no IV, no padding, r-bit tag

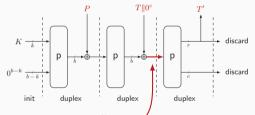
- Repeated paths (i.e., large L) can seriously affect security
- Consider simplified FSKS[p]: no IV, no padding, r-bit tag
- Distinguisher makes two queries: $P \mapsto T$ and $P||T||0^c \mapsto T'$





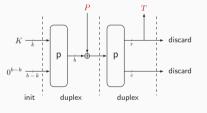
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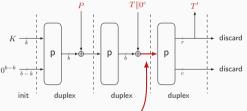




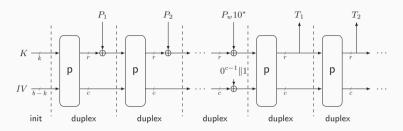
ullet State of second query before squeezing equals $0^r \| *^c$

- Repeated paths (i.e., large L) can seriously affect security
- ullet Consider simplified FSKS[p]: no IV, no padding, r-bit tag
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- \bullet State of second query before squeezing equals $0^r \| *^c$
- Key recovery attack:
 - Make q twin queries as above and N primitive queries of form $0^r \| *^c$
 - Construction-primitive collision (likely if $\frac{q \cdot N}{2^c} \approx 1$) \longrightarrow derive K

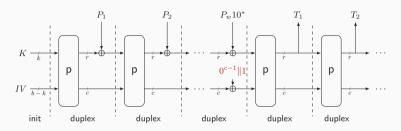


ullet Input: key K, initial value IV, message P

ullet Output: tag T

Algorithm Ascon-PRF[p]

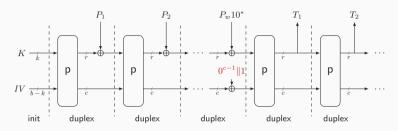
```
\begin{aligned} & \textbf{Input:} \ (K,IV,P) \in \{0,1\}^k \times \mathcal{IV} \times \{0,1\}^* \\ & \textbf{Output:} \ T \in \{0,1\}^t \\ & \textbf{Underlying keyed duplex:} \ KD[p]_{(K)} \\ & (P_1,P_2,\ldots,P_w) \leftarrow \operatorname{pad}_r^{10^*}(P) \\ & T \leftarrow \varnothing \\ & KD.\operatorname{init}(1,IV) \\ & \textbf{for} \ i = 1,\ldots,w-1 \ \textbf{do} \\ & KD.\operatorname{duplex}(false,P_i) \\ & KD.\operatorname{duplex}(false,P_i) \\ & KD.\operatorname{duplex}(false,P_w||0^{c-1}1) \\ & \textbf{for} \ i = 1,\ldots,\lceil t/r\rceil \ \textbf{do} \\ & T \leftarrow T \parallel KD.\operatorname{duplex}(false,0^b) \\ & \textbf{return } \operatorname{left}_t(T) \end{aligned}
```



- ullet Input: key K, initial value IV, message P
- ullet Output: tag T
- Domain separation solves problem of repeated paths

Algorithm Ascon-PRF[p]

```
\begin{split} & \textbf{Input:} \ (K,IV,P) \in \{0,1\}^k \times \mathcal{IV} \times \{0,1\}^* \\ & \textbf{Output:} \ T \in \{0,1\}^t \\ & \textbf{Underlying keyed duplex:} \ \mathsf{KD[p]}_{(K)} \\ & (P_1,P_2,\ldots,P_w) \leftarrow \mathsf{pad}_r^{10^*}(P) \\ & T \leftarrow \varnothing \\ & \mathsf{KD.init}(1,IV) \\ & \textbf{for } i=1,\ldots,w-1 \ \textbf{do} \\ & \mathsf{KD.duplex}(false,P_i) \\ & \mathsf{KD.duplex}(false,P_w||0^{c-1}1) \\ & \textbf{for } i=1,\ldots,[t/r] \ \textbf{do} \\ & T \leftarrow T \parallel \mathsf{KD.duplex}(false,0^b) \\ & \textbf{return } \mathsf{left}_t(T) \end{split}
```

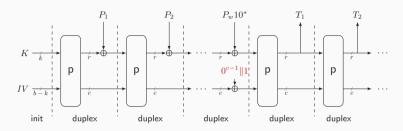


- ullet Input: key K, initial value IV, message P
- ullet Output: tag T
- Domain separation solves problem of repeated paths
 - Repeated paths may still occur...

Algorithm Ascon-PRF[p]

```
\begin{split} & \textbf{Input:} \ (K, IV, P) \in \{0, 1\}^k \times \mathcal{TV} \times \{0, 1\}^* \\ & \textbf{Output:} \ T \in \{0, 1\}^t \\ & \textbf{Underlying keyed duplex:} \ \mathsf{KD[p]}_{(K)} \\ & (P_1, P_2, \dots, P_w) \leftarrow \mathsf{pad}_r^{10^*}(P) \\ & T \leftarrow \varnothing \\ & \mathsf{KD.init}(1, IV) \\ & \textbf{for} \ i = 1, \dots, w - 1 \ \textbf{do} \\ & \mathsf{KD.duplex}(false, P_t) \\ & \mathsf{KD.duplex}(false, P_w||0^{c-1}1) \\ & \textbf{for} \ i = 1, \dots, \lceil t/r \rceil \ \textbf{do} \\ & T \leftarrow T \parallel \mathsf{KD.duplex}(false, 0^b) \\ & \textbf{return } \mathsf{left}_t(T) \end{split}
```

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- ullet Input: key K, initial value IV, message P
- ullet Output: tag T
- Domain separation solves problem of repeated paths
 - Repeated paths may still occur...
 - ... but adversary cannot exploit them

Algorithm Ascon-PRF[p]

```
\begin{split} & \textbf{Input:} \ (K, IV, P) \in \{0, 1\}^k \times \mathcal{IV} \times \{0, 1\}^* \\ & \textbf{Output:} \ T \in \{0, 1\}^t \\ & \textbf{Underlying keyed duplex:} \ \mathsf{KD}[\mathsf{p}]_{(K)} \\ & (P_1, P_2, \dots, P_w) \leftarrow \mathsf{pad}_r^{10^*}(P) \\ & T \leftarrow \varnothing \\ & \mathsf{KD.init}(1, IV) \\ & \textbf{for } i = 1, \dots, w - 1 \ \textbf{do} \\ & \mathsf{KD.duplex}(false, P_i) \\ & \mathsf{KD.duplex}(false, P_w || 0^{c-1}1) \\ & \textbf{for } i = 1, \dots, \lceil t/r \rceil \ \textbf{do} \\ & T \leftarrow T \mid\!\mid \mathsf{KD.duplex}(false, 0^b) \\ & \textbf{return } \mathsf{left}_t(T) \end{split}
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- Improved bound from [DMV17]:
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- Improved bound from [DM19]:
 - Defines additional parameter $\nu_{\text{fix}} \leq L + \Omega$
 - In most cases $\nu_{\text{fix}} = L + \Omega$; for current case $\nu_{\text{fix}} = 0$
 - Dominant term $\frac{(q-1)N+\binom{q}{2}}{2^c}$ never appears in the first place

Multi-user bound from [DMV17]

$$\mathbf{Adv}^{\mu\text{-}\mathrm{prf}}_{\mathsf{Ascon-PRF}}(\mathsf{D}) \leq \tfrac{2\nu_{r,c}^{2\sigma}(N+1)}{2^c} + \tfrac{(\sigma-q)q}{2^b-q} + \tfrac{2\binom{\sigma}{2}}{2^b} + \tfrac{q(\sigma-q)}{2^{\min\{c+k,b\}}} + \tfrac{\mu N}{2^k} + \tfrac{\binom{\mu}{2}}{2^k}$$

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Application to Ascon-PRF Parameters

- (k, b, c, r) = (128, 320, 192, 128)
- Assume online complexity of $q,\sigma\ll 2^{64}$ (could be taken higher)
- The multicollision term $\nu_{128.192}^{265}$ is at most 5

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$$\downarrow \leq \qquad \qquad \downarrow \geq \qquad \qquad \downarrow \leq \qquad \qquad \downarrow \leq$$

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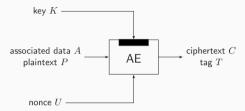
$$\downarrow \leq \qquad \qquad \downarrow \geq \qquad \qquad \downarrow$$

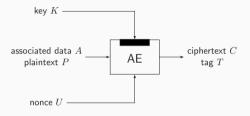
Application to Ascon-PRF Parameters

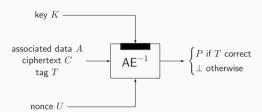
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- The multicollision term $u_{128.192}^{265}$ is at most 5
- \bullet Generic security as long as $N \ll 2^{128}/\mu$

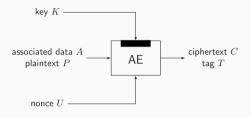
Encryption (Shortened)

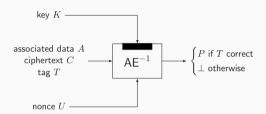
Use Case 5: Authenticated





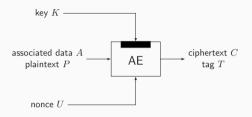


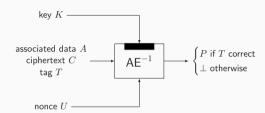




Role of Duplex

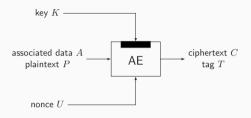
Blockwise construction allows for processing different types of in-/output

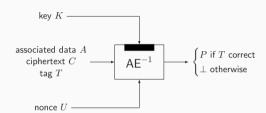




Role of Duplex

- Blockwise construction allows for processing different types of in-/output
- Usage of flag makes duplex-style encryption decryptable

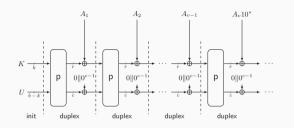




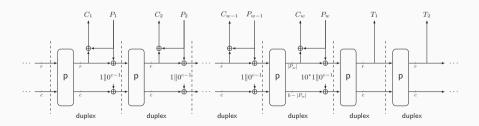
Role of Duplex

- Blockwise construction allows for processing different types of in-/output
- Usage of flag makes duplex-style encryption decryptable (Although the flag is not a necessity for this)

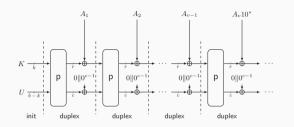
MonkeySpongeWrap: Encryption



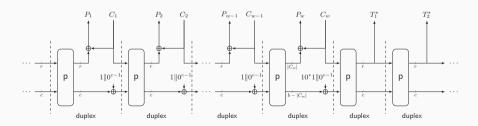
- State initialized using key and nonce
- Cleaned-up and synchronized domain separation
- Spill-over into inner part
- Used in Xoodyak and Gimli (a.o.)



MonkeySpongeWrap: Decryption



- Decryption similar to encryption
- Notable difference:
 - ullet Processing of C
 - Duplexing with flag = true



• Consider distinguisher D against AE security of MonkeySpongeWrap[p] $\mathbf{Adv}^{\mathrm{ae}}_{\mathsf{MonkeySpongeWrap}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{ENC}[\mathsf{p}]_K, \mathsf{DEC}[\mathsf{p}]_K, \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathrm{ae}}, \bot, \mathsf{p}^{\pm}\right)$

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- Triangle inequality derivation slightly more involved than before:

$$\mathbf{Adv}^{\mathrm{ae}}_{\mathsf{MonkeySpongeWrap}}(\mathsf{D}) \leq \Delta_{\mathsf{D}'}\left(\mathsf{KD}[\mathsf{p}]_K,\mathsf{p}^{\pm}\;;\;\mathsf{IXIF}[\mathsf{ro}],\mathsf{p}^{\pm}\right) + \frac{q_d}{2^t}$$

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What are the resources of D'?

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resources of D'	in terms of	resources of D
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attack of Gilbert et al. [GBKR23] "operates" here

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$$\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}') \leq \frac{2\nu_{r,c}^{2\sigma}(N+1)}{2^{c}} \underbrace{\left(\frac{\sigma_{d}N + \left(\frac{\sigma_{d}}{2}\right)}{2^{c}}\right) \frac{(\sigma - q)q}{2^{b} - q}}_{2^{b} - q} + \frac{2\left(\frac{\sigma}{2}\right)}{2^{b}} + \frac{q(\sigma - q)}{2^{\min\{c + k, b\}}} + \frac{N}{2^{k}}$$

attack of Gilbert et al. [GBKR23] "operates" here, with $\sigma_d, N \approx 2^{3c/4}$ 34

Generalized Keyed Duplex

- Versatile construction but application not always clear
- Five representative use cases
- Further use cases: PRNG, PBKDF, ...
- Generic security of ISAP v2 follows from duplex and SuKS [DEM⁺20]
- Caution: all presented results only hold in random permutation model

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Much More in Paper

- More detailed explanation on duplex, multicollisions, applications, ...
- Application of bounds of both [DMV17] and [DM19] to use cases
- Multi-user security

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Thank you for your attention!

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Supporting Slides

Intermezzo: Multicollision Coefficient

Definition

- M balls, 2^r bins
- $\nu^{M}_{r,c}$ is smallest ${m x}$ such that $\Pr\left(|{
 m fullest~bin}|>{m x}
 ight)\leq {{m x}\over 2^c}$

Definition

- M balls, 2^r bins
- $\nu^M_{r,c}$ is smallest x such that $\Pr\left(|\mathsf{fullest\ bin}|>x\right) \leq \frac{x}{2^c}$

- We often need upper bound on the maximum multicollision in outer part
- Denote this maximum by ν

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- ullet The value ${m
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- We could be unlucky: there could be a $> \nu$ -multicollision

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- We could be unlucky: there could be a $> \nu$ -multicollision
- ullet However, if we take $m{
 u}=
 u_{r,c}^M$, this happens with probability at most $rac{m{
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Definition

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- ullet The value ${\color{blue}
 u}$ appears in the security bound in a term of the form ${\color{blue}
 u\cdot N\over 2^c}$
- ullet We could be unlucky: there could be a >
 u-multicollision
- This term is negligible compared to the main probability bound

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Intuition of Behavior

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Intuition of Behavior

- If $M \ll 2^r$, all bins will likely be "reasonably" empty
- If $M\gg 2^r$, there will likely be a bin with around $linear(b)\cdot \frac{M}{2^r}$ balls

Definition

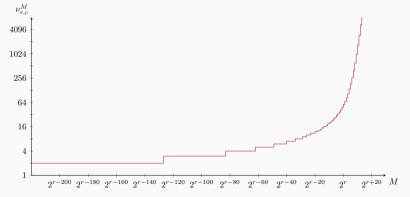
- M balls, 2^r bins
- $\nu^{M}_{r,c}$ is smallest ${m x}$ such that $\Pr\left(|{
 m fullest~bin}|>{m x}
 ight)\leq {{m x}\over 2^c}$

Intuition of Behavior

- If $M \ll 2^r$, all bins will likely be "reasonably" empty
- If $M \gg 2^r$, there will likely be a bin with around $linear(b) \cdot \frac{M}{2^r}$ balls
- Formula for $\nu_{r,c}^{M}$ and upper bounds in above 2 cases, derived in [DMV17]
- $\nu_{r,c}^{M}$ is (at most) smallest x that satisfies

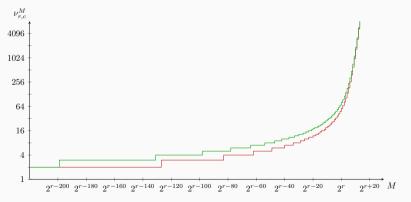
$$\frac{2^b e^{-M/2^r} (M/2^r)^x}{(x - M/2^r)x!} \le 1$$

Stairway to Heaven for b = 256



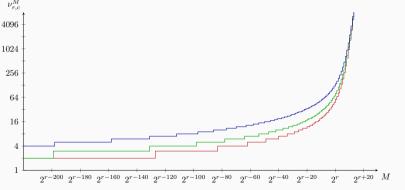
$M/2^r$	$ u_{r,c}^M$	
2^{-256}	_	
2^{-128}	2	
2^{-64}	4	
2^{-32}	8	
2^{-16}	14	
2^{-8}	23	
2^{0}	57	
2^{8}	601	
2^{16}	70205	
2^{19}	537313	

Stairway to Heaven for b = 256, b = 400



$M/2^r$	$ u_{r,c}^M$	$ u_{r,c}^M$	
2^{-256}	_	2	
2^{-128}	2	4	
2^{-64}	4	7	
2^{-32}	8	12	
2^{-16}	14	21	
2^{-8}	23	34	
2^{0}	57	80	
2^{8}	601	707	
2^{16}	70205	71484	
2^{19}	537313	540887	

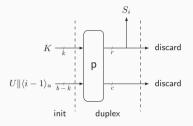
Stairway to Heaven for b = 256, b = 400, b = 800



$M/2^r$	$ u_{r,c}^{M}$	$ u_{r,c}^M$	$ u_{r,c}^M$
2^{-256}	_	2	4
2^{-128}	2	4	7
2^{-64}	4	7	12
2^{-32}	8	12	23
2^{-16}	14	21	40
2^{-8}	23	34	64
2^{0}	57	80	139
2^{8}	601	707	944
2^{16}	70205	71484	74119
2^{19}	537313	540887	548194

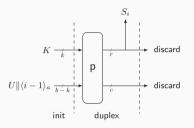
Use Case 2: Parallel Keystream

Generation



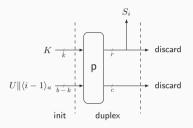
```
\begin{split} & \textbf{Input:} \  \, (K,U,\ell) \in \{0,1\}^k \times \{0,1\}^{b-k-a} \times \{0,\dots,r2^a\} \\ & \textbf{Output:} \  \, S \in \{0,1\}^\ell \\ & \textbf{Underlying keyed duplex:} \  \, \mathsf{KD}[\mathsf{p}]_{(K)} \\ & S \leftarrow \varnothing \\ & \textbf{for} \  \, i = 1,\dots,\lceil \ell/r\rceil \  \, \textbf{do} \\ & \mathsf{KD.init}(1,U\|\langle i-1\rangle_a)) \\ & S \leftarrow S \parallel \mathsf{KD.duplex}(false,0^b) \\ & \textbf{return } \mathsf{left}_\ell(S) \end{split}
```

- Input: key K, nonce U
- ullet Output: keystream S of requested length



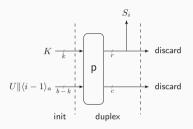
```
\begin{aligned} & \textbf{Input:} \  \, (K,U,\ell) \in \{0,1\}^k \times \{0,1\}^{b-k-a} \times \{0,\dots,r2^a\} \\ & \textbf{Output:} \  \, S \in \{0,1\}^\ell \\ & \textbf{Underlying keyed duplex:} \  \, \text{KD}[\mathbf{p}]_{(K)} \\ & S \leftarrow \varnothing \\ & \textbf{for} \  \, i = 1,\dots,\lceil \ell/r \rceil \  \, \textbf{do} \\ & \text{KD.init}(1,U\|\langle i-1\rangle_a)) \\ & S \leftarrow S \parallel \textbf{KD.duplex}(false,0^b) \\ & \textbf{return } \text{left}_\ell(S) \end{aligned}
```

- ullet Input: key K, nonce U
- ullet Output: keystream S of requested length
- \bullet P-SC[p] can be seen as TP[p] in counter mode



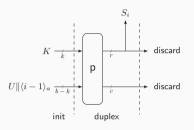
```
\begin{split} & \textbf{Input:} \  \, (K,U,\ell) \in \{0,1\}^k \times \{0,1\}^{b-k-a} \times \{0,\dots,r2^a\} \\ & \textbf{Output:} \  \, S \in \{0,1\}^\ell \\ & \textbf{Underlying keyed duplex:} \  \, \mathsf{KD}[\mathsf{p}]_{(K)} \\ & S \leftarrow \varnothing \\ & \textbf{for} \  \, i = 1,\dots,\lceil\ell/r\rceil \  \, \textbf{do} \\ & \quad \mathsf{KD.init}(1,U\|\langle i-1\rangle_a)) \\ & \quad S \leftarrow S \parallel \mathsf{KD.duplex}(false,0^b) \\ & \textbf{return } \mathsf{left}_\ell(S) \end{split}
```

- ullet Input: key K, nonce U
- ullet Output: keystream S of requested length
- ullet P-SC[p] can be seen as TP[p] in counter mode
- PRF security of P-SC[p] easily follows:



```
\begin{aligned} & \textbf{Input:} \  \, (K,U,\ell) \in \{0,1\}^k \times \{0,1\}^{b-k-a} \times \{0,\dots,r2^a\} \\ & \textbf{Output:} \  \, S \in \{0,1\}^\ell \\ & \textbf{Underlying keyed duplex:} \  \, \text{KD}[\mathbf{p}]_{(K)} \\ & S \leftarrow \varnothing \\ & \textbf{for} \  \, i = 1,\dots,\lceil \ell/r\rceil \  \, \textbf{do} \\ & \textbf{KD.init}(1,U\|\langle i-1\rangle_a)) \\ & S \leftarrow S \parallel \textbf{KD.duplex}(false,0^b) \\ & \textbf{return } \  \, \text{left}_\ell(S) \end{aligned}
```

- Input: key K, nonce U
- ullet Output: keystream S of requested length
- P-SC[p] can be seen as TP[p] in counter mode
- PRF security of P-SC[p] easily follows:
 - TP[p] behaves like a PRF (up to good bound)

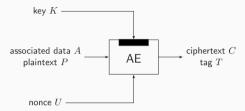


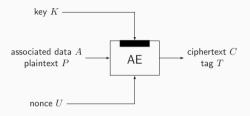
```
\begin{split} & \textbf{Input:} \  \, (K,U,\ell) \in \{0,1\}^k \times \{0,1\}^{b-k-a} \times \{0,\dots,r2^a\} \\ & \textbf{Output:} \  \, S \in \{0,1\}^k \\ & \textbf{Underlying keyed duplex:} \  \, \mathsf{KD}[\mathsf{p}]_{(K)} \\ & S \leftarrow \varnothing \\ & \textbf{for} \  \, i = 1,\dots,\lceil \ell/r\rceil \  \, \textbf{do} \\ & \quad \quad \, \mathsf{KD.init}(1,U\|\langle i-1\rangle_a)) \\ & \quad \quad \, S \leftarrow S \parallel \mathsf{KD.duplex}(false,0^b) \\ & \textbf{return } \mathsf{left}_\ell(S) \end{split}
```

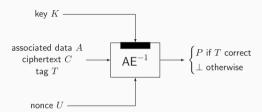
- ullet Input: key K, nonce U
- ullet Output: keystream S of requested length
- P-SC[p] can be seen as TP[p] in counter mode
- PRF security of P-SC[p] easily follows:
 - TP[p] behaves like a PRF (up to good bound)
 - Counter mode with a PRF generates uniform random keystream (provided nonce/counter never repeats)

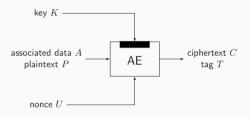
Use Case 5: Authenticated

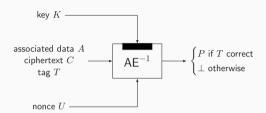
Encryption





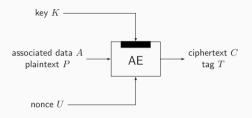


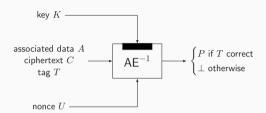




Role of Duplex

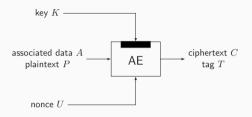
Blockwise construction allows for processing different types of in-/output

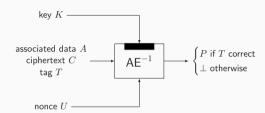




Role of Duplex

- Blockwise construction allows for processing different types of in-/output
- Usage of flag makes duplex-style encryption decryptable

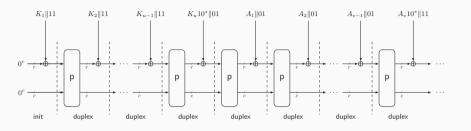


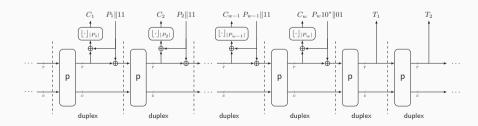


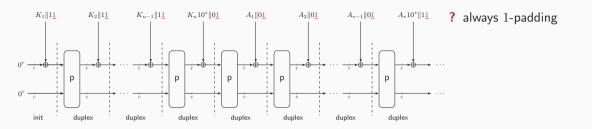
Role of Duplex

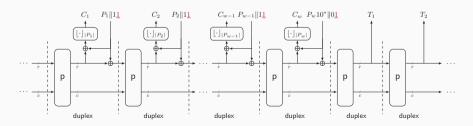
- Blockwise construction allows for processing different types of in-/output
- Usage of flag makes duplex-style encryption decryptable (Although the flag is not a necessity for this)

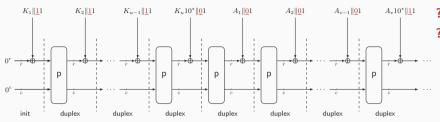
SpongeWrap [BDPV11a]



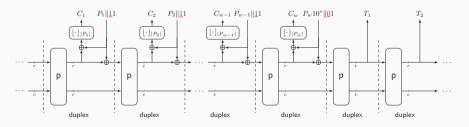


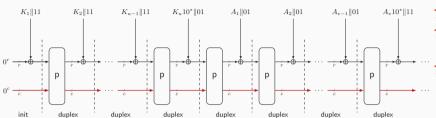




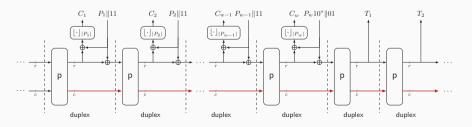


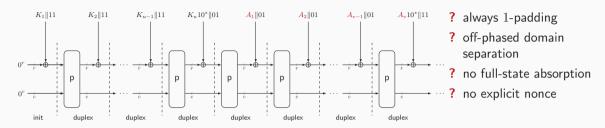
- ? always 1-padding
- ? off-phased domain separation

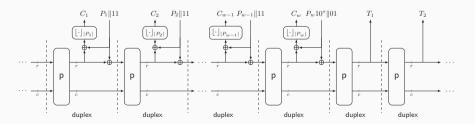


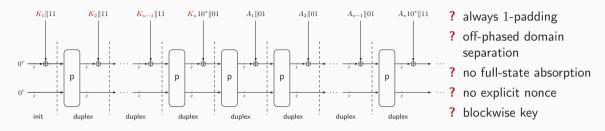


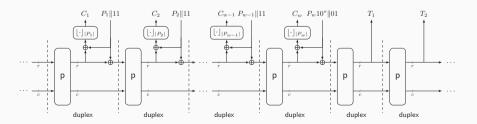
- ? always 1-padding
- ? off-phased domain separation
 - no full-state absorption



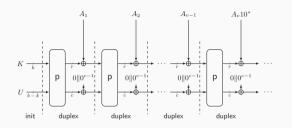




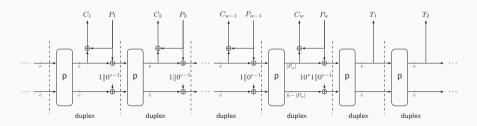




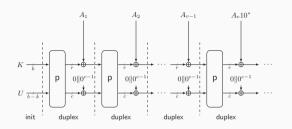
MonkeySpongeWrap: Encryption



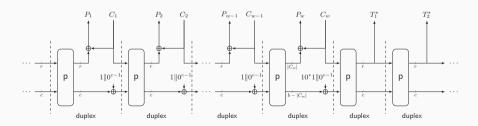
- State initialized using key and nonce
- Cleaned-up and synchronized domain separation
- Spill-over into inner part
- Used in Xoodyak and Gimli (a.o.)



MonkeySpongeWrap: Decryption



- Decryption similar to encryption
- Notable difference:
 - ullet Processing of C
 - Duplexing with flag = true



MonkeySpongeWrap: Algorithm

Algorithm MonkeySpongeWrap[p]: ENC	Algorithm MonkeySpongeWrap[p]: DEC	
Input: $(K,U,A,P) \in \{0,1\}^k \times \{0,1\}^{b-k} \times \{0,1\}^* \times \{0,1\}^*$ Output: $(C,T) \in \{0,1\}^{ P } \times \{0,1\}^t$	$ \begin{array}{c} \hline & \textbf{Input:} \ (K,U,A,C,T) \in \{0,1\}^k \times \{0,1\}^{b-k} \times \{0,1\}^* \times \{0,1\}^* \times \{0,1\}^t \\ \textbf{Output:} \ P \in \{0,1\}^{ C } \ \text{or} \ \bot \\ \end{array} $	
Underlying keyed duplex: $KD[p]_{(K)}$	Underlying keyed duplex: $KD[p]_{(K)}$	
$(A_1,A_2,\ldots,A_v) \leftarrow \operatorname{pad}_r^{10^*}(A)$ $(P_1,P_2,\ldots,P_w) \leftarrow \operatorname{pad}_r^{10^*}(P)$ $C \leftarrow \varnothing$ $T \leftarrow \varnothing$ $KD.init(1,U)$ $for \ i = 1,\ldots,v \ do$ $KD.duplex(\mathit{false},A_i \ 0 \ 0^{c-1})$ $\triangleright \ discard output(A_i)$	$(A_1,A_2,\ldots,A_v) \leftarrow \operatorname{pad}_r^{10^*}(A)$ $(C_1,C_2,\ldots,C_w) \leftarrow \operatorname{pad}_r^{10^*}(C)$ $P \leftarrow \varnothing$ $T^* \leftarrow \varnothing$ $\operatorname{KD.init}(1,U)$ for $i=1,\ldots,v$ do $\operatorname{KD.duplex}(false,A_i\ 0\ 0^{c-1})$ \Rightarrow discard output	
$\begin{split} &\text{for } i=1,\ldots,w \text{ do} \\ &C \leftarrow C \parallel \text{KD.duplex}(false,P_i \lVert 1 \rVert 0^{c-1}) \oplus P_i \\ &\text{for } i=1,\ldots,\lceil t/r \rceil \text{ do} \\ &T \leftarrow T \parallel \text{KD.duplex}(false,0^b) \\ &\text{return } (\text{left}_{ P }(C),\text{left}_t(T)) \end{split}$	$\begin{split} &\text{for } i=1,\dots,w \text{ do} \\ &P \leftarrow P \parallel KD.duplex(true,C_i \lVert 1 \rVert 0^{c-1}) \oplus C_i \\ &\text{for } i=1,\dots,\lceil t/r \rceil \text{ do} \\ &T^* \leftarrow T^* \parallel KD.duplex(false,0^b) \\ &\text{return } \mathrm{left}_t(T) = \mathrm{left}_t(T^*) ? \mathrm{left}_{ C }(P) : \bot \end{split}$	

• Consider distinguisher D against AE security of MonkeySpongeWrap[p] $\mathbf{Adv}_{\mathsf{MonkeySpongeWrap}}^{\mathrm{ae}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{ENC}[\mathsf{p}]_K, \mathsf{DEC}[\mathsf{p}]_K, \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathrm{ae}}, \bot, \mathsf{p}^{\pm}\right)$

• D can make: q_e encryption queries (total σ_e blocks), q_d decryption queries (total σ_d blocks), N primitive queries

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- Decryption calls: nonce may repeat, flag may be true

• Consider distinguisher D against AE security of MonkeySpongeWrap[p]

$$\mathbf{Adv}^{\mathrm{ae}}_{\mathsf{MonkeySpongeWrap}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{ENC}[\mathsf{p}]_K, \mathsf{DEC}[\mathsf{p}]_K, \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathrm{ae}}, \bot, \mathsf{p}^{\pm}\right)$$

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- Triangle inequality derivation slightly more involved than before:

$$\mathbf{Adv}^{\mathrm{ae}}_{\mathsf{MonkeySpongeWrap}}(\mathsf{D}) \leq \Delta_{\mathsf{D}'}\left(\mathsf{KD}[\mathsf{p}]_K,\mathsf{p}^{\pm}\;;\;\mathsf{IXIF}[\mathsf{ro}],\mathsf{p}^{\pm}\right) + \frac{q_d}{2^t}$$

 $\bullet \ \ Consider \ distinguisher \ D \ against \ AE \ security \ of \ MonkeySpongeWrap[p]$

$$\mathbf{Adv}^{\mathrm{ae}}_{\mathsf{MonkeySpongeWrap}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{ENC}[\mathsf{p}]_K, \mathsf{DEC}[\mathsf{p}]_K, \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathrm{ae}}, \bot, \mathsf{p}^{\pm}\right)$$

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What are the resources of D'?

- D can make: q_e encryption queries (total σ_e blocks), q_d decryption queries (total σ_d blocks), N primitive queries
- Encryption calls: unique nonce, *flag* always *false*
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- D can make: q_e encryption queries (total σ_e blocks), q_d decryption queries (total σ_d blocks), N primitive queries
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resources of D'	in terms of	resources of D
M: data complexity (calls to construction)		
N: time complexity (calls to primitive)	\longrightarrow	N
Q: number of init calls		
Q_{IV} : max # init calls for single IV		
L: # queries with repeated path		
Ω : # queries with overwriting outer part		

- D can make: q_e encryption queries (total σ_e blocks), q_d decryption queries (total σ_d blocks), N primitive queries
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resources of D'	in terms of	resources of D
M: data complexity (calls to construction)		$\sigma_e + \sigma_d$
N: time complexity (calls to primitive)	$\!$	N
Q: number of init calls	${}$	$q_e + q_d$
Q_{IV} : max # init calls for single IV		
L: # queries with repeated path		
Ω : # queries with overwriting outer part		

- D can make: q_e encryption queries (total σ_e blocks), q_d decryption queries (total σ_d blocks), N primitive queries
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resources of D'	in terms of	resources of D
M: data complexity (calls to construction)	\longrightarrow	$\sigma_e + \sigma_d$
N: time complexity (calls to primitive)	\longrightarrow	N
Q: number of init calls	${\longrightarrow}$	$q_e + q_d$
$Q_{IV}\colon max\ \#$ init calls for single IV	$ \longrightarrow $	1
$L{:}$ $\#$ queries with repeated path		
Ω : # queries with overwriting outer part		

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resources of D'	in terms of	resources of D
M: data complexity (calls to construction)	\longrightarrow	$\sigma_e + \sigma_d$
N: time complexity (calls to primitive)	\longrightarrow	N
Q: number of init calls	$\!$	$q_e + q_d$
Q_{IV} : max $\#$ init calls for single IV	$\!$	1
L: $#$ queries with repeated path	${\longrightarrow}$	$\leq q_d$
Ω : # queries with overwriting outer part		

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resources of D'	in terms of	resources of D
M: data complexity (calls to construction)		$\sigma_e + \sigma_d$
N: time complexity (calls to primitive)	\longrightarrow	N
Q: number of init calls	\longrightarrow	$q_e + q_d$
Q_{IV} : max $\#$ init calls for single IV	\longrightarrow	1
L: $#$ queries with repeated path	\longrightarrow	$\leq q_d$
Ω : # queries with overwriting outer part		$\leq \sigma_d - 2q_d$

- D can make: q_e encryption queries (total σ_e blocks), q_d decryption queries (total σ_d blocks), N primitive queries
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resources of D'	in terms of	resources of D
M: data complexity (calls to construction)		$\sigma_e + \sigma_d$
N: time complexity (calls to primitive)	\longrightarrow	N
Q: number of init calls	\longrightarrow	$q_e + q_d$
Q_{IV} : max $\#$ init calls for single IV	$\longrightarrow\hspace{-0.8cm}\longrightarrow$	1
L: $#$ queries with repeated path	\longrightarrow	$\leq q_d$
$\Omega{:}~\#$ queries with overwriting outer part	\longrightarrow	$\leq \sigma_d - 2q_d$

$$\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}') \leq \frac{2\nu_{r,c}^{2\sigma}(N+1)}{2^c} + \frac{\sigma_d N + \binom{\sigma_d}{2}}{2^c} + \frac{(\sigma - q)q}{2^b - q} + \frac{2\binom{\sigma}{2}}{2^b} + \frac{q(\sigma - q)}{2^{\min\{c + k, b\}}} + \frac{N}{2^k}$$

- D can make: q_e encryption queries (total σ_e blocks), q_d decryption queries (total σ_d blocks), N primitive queries
- Encryption calls: unique nonce, *flag* always *false*
- Decryption calls: nonce may repeat, *flag* may be *true*

resources of D'	in terms of	resources of D
M: data complexity (calls to construction)		$\sigma_e + \sigma_d$
N: time complexity (calls to primitive)	\longrightarrow	N
Q: number of init calls	$\longrightarrow\hspace{-0.5cm}\longrightarrow$	$q_e + q_d$
Q_{IV} : max $\#$ init calls for single IV	$\longrightarrow\hspace{-0.5cm}\longrightarrow$	1
L: $#$ queries with repeated path	\longrightarrow	$\leq q_d$
Ω : # queries with overwriting outer part	\longrightarrow	$\leq \sigma_d - 2q_d$

$$\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}') \leq \frac{2\nu_{r,c}^{2\sigma}(N+1)}{2^c} \underbrace{\left(\frac{\sigma_d N + \left(\frac{\sigma_d}{2}\right)}{2^c}\right)}_{2^c} \underbrace{\frac{(\sigma - q)q}{2^b - q}}_{\frac{2^b - q}{2^b}} + \frac{\frac{q(\sigma - q)}{2^{\min\{c + k, b\}}}}{2^k} + \frac{N}{2^k}$$
 attack of Gilbert et al. [GBKR23] "operates" here

- D can make: q_e encryption queries (total σ_e blocks), q_d decryption queries (total σ_d blocks), N primitive queries
- Encryption calls: unique nonce, flag always false
- Decryption calls: nonce may repeat, flag may be true

resources of D'	in terms of	resources of D
M: data complexity (calls to construction)		$\sigma_e + \sigma_d$
N: time complexity (calls to primitive)	\longrightarrow	N
Q: number of init calls	\longrightarrow	$q_e + q_d$
Q_{IV} : max $\#$ init calls for single IV	\longrightarrow	1
$L\colon\#$ queries with repeated path	$\xrightarrow{\hspace*{1cm}}$	$\leq q_d$
Ω : $\#$ queries with overwriting outer part	$\!$	$\leq \sigma_d - 2q_d$

$$\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}') \leq \frac{2\nu_{r,c}^{2\sigma}(N+1)}{2^c} \underbrace{\left(\frac{\sigma_d N + \left(\frac{\sigma_d}{2}\right)}{2^c}\right)^2 \frac{(\sigma-q)q}{2^b - q} + \frac{2\left(\frac{\sigma}{2}\right)^s}{2^b} + \frac{q(\sigma-q)}{2^{\min\{c+k,b\}}} + \frac{N}{2^k}}_{2^k}$$
 attack of Gilbert et al. [GBKR23] "operates" here, with $\sigma_d, N \approx 2^{3c/4}$