Introduction to Provable Security

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IACR School on Design and Security of Cryptographic Algorithms and Devices

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What is Provable Security?

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absolute security

reductionist security

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(e.g., lower bound on the number of active S-boxes)

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crypto primitive

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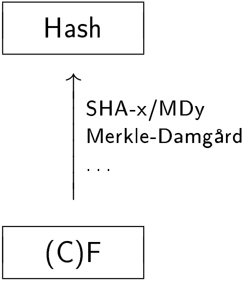
What is Provable Security?

- absolute security
  (e.g., lower bound on the number of active S-boxes)
- reductionist security
  - based on crypto primitive
    - keyed
    - keyless
  - based on math problem

Introduction

$\boxed{\text{Hash}}$

Introduction



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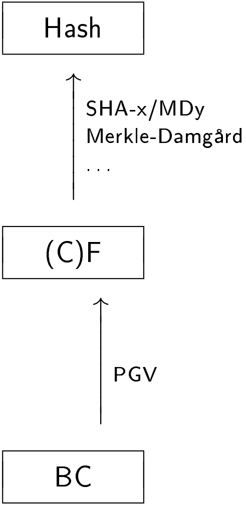

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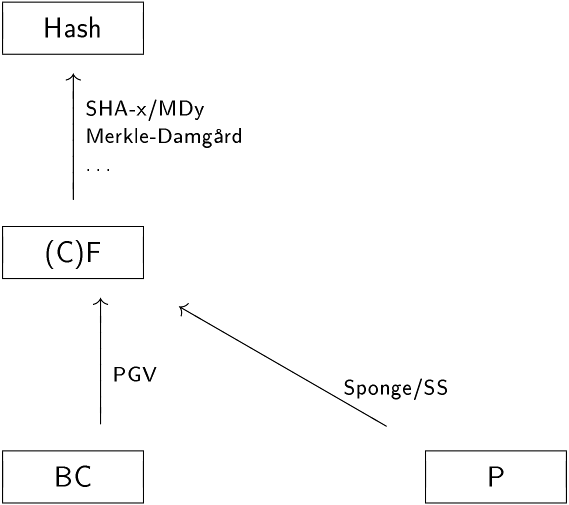

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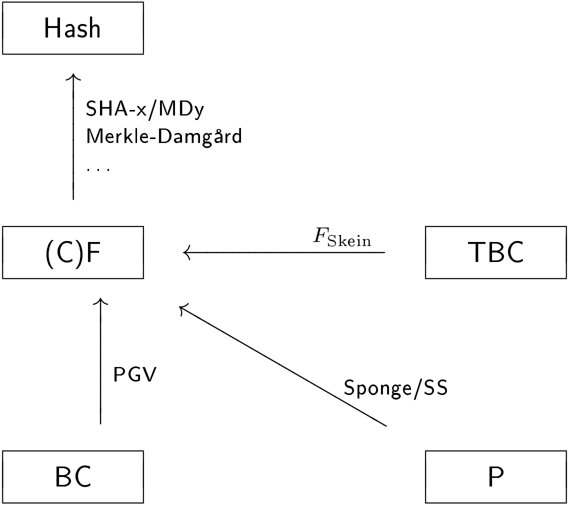

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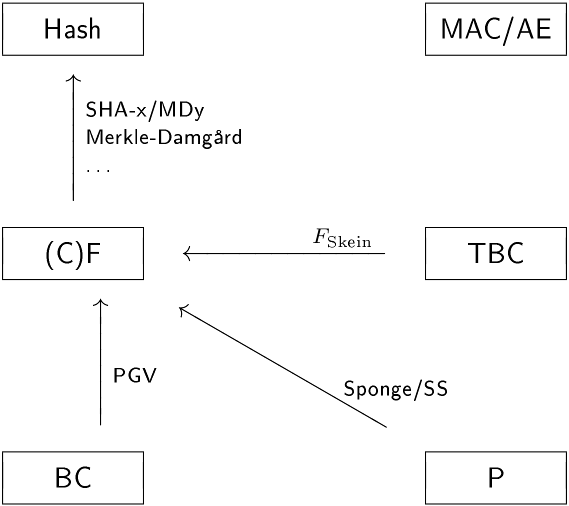

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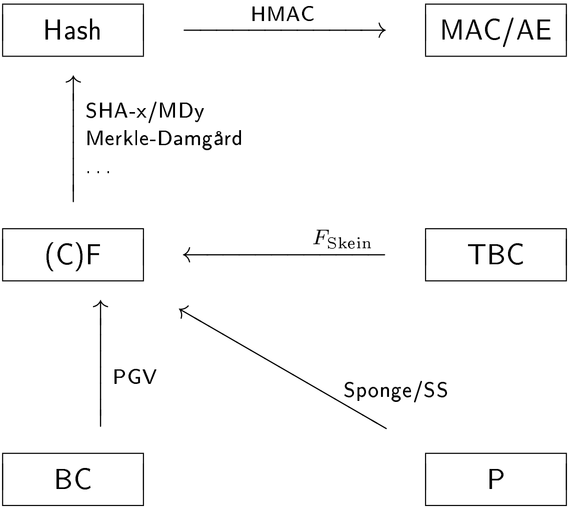

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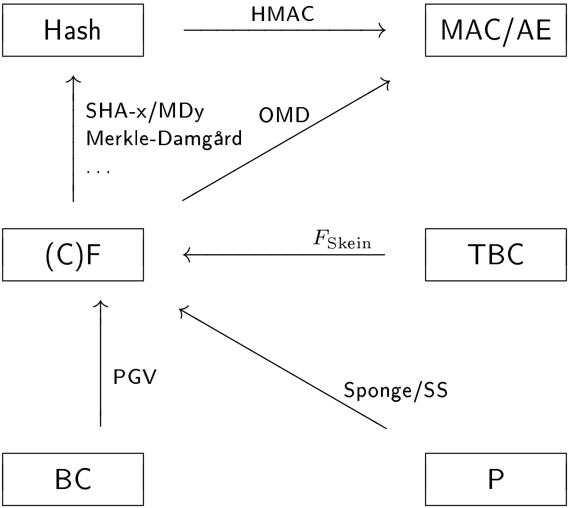

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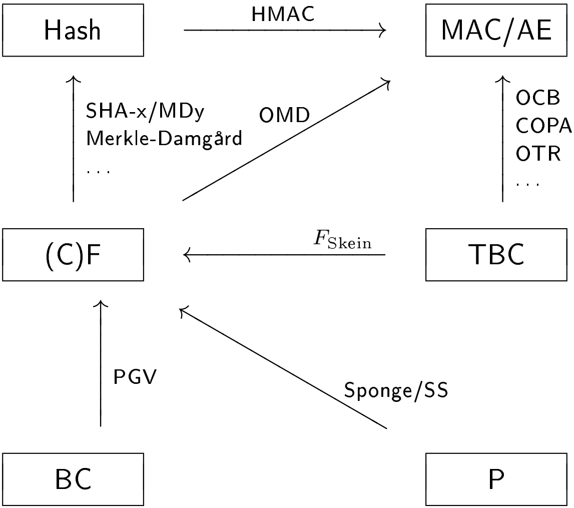

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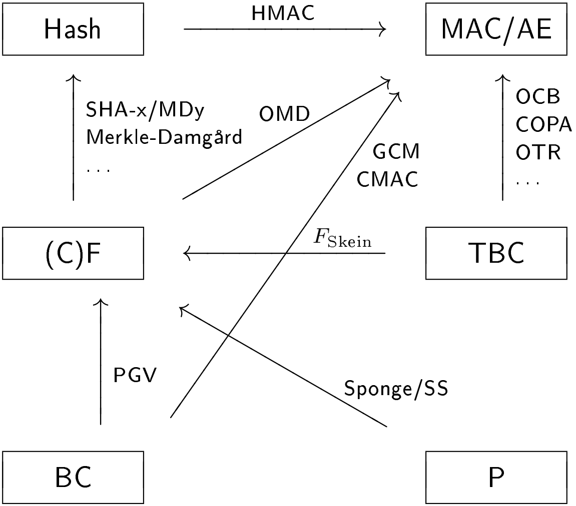

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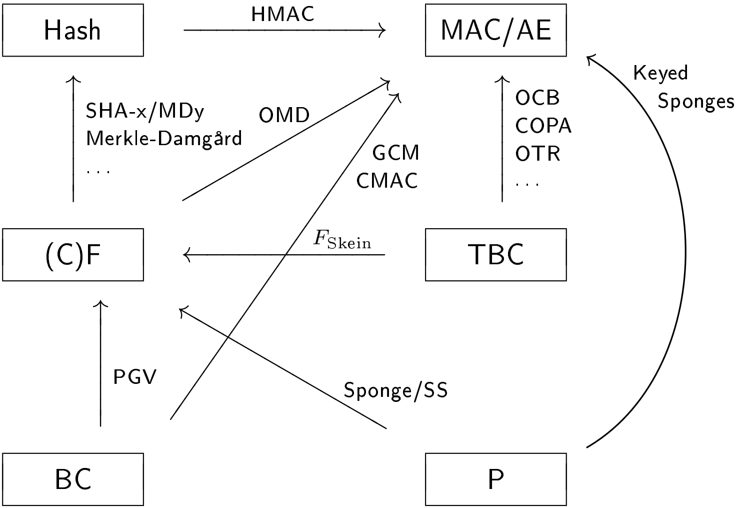

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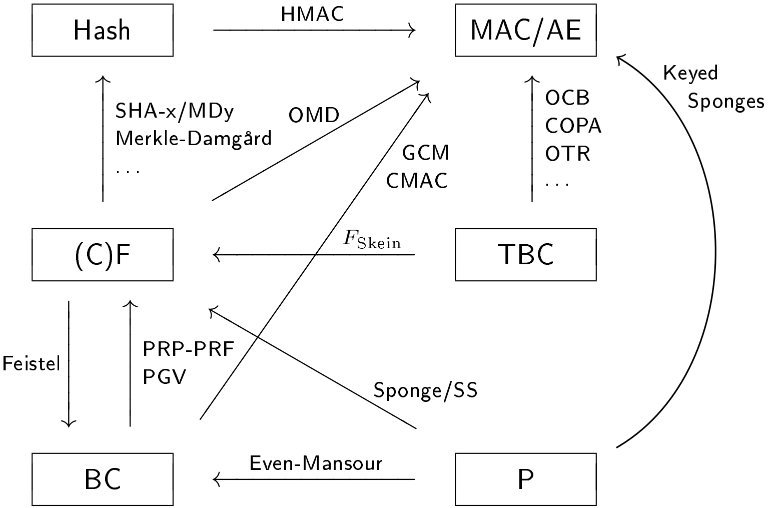

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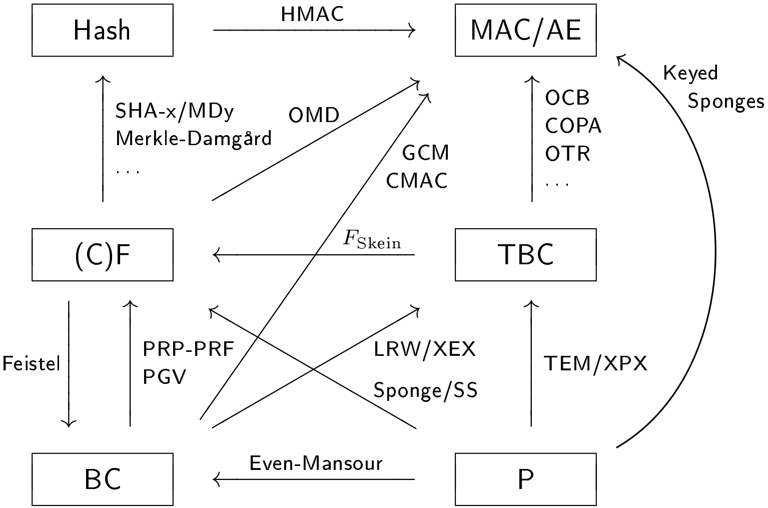
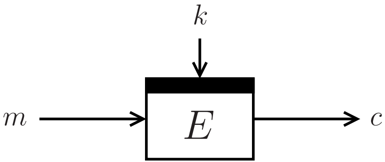


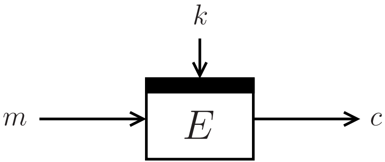
Keyed Constructions

Keyed Blockcipher



- Blockcipher: a family of permutations indexed by key

Keyed Blockcipher



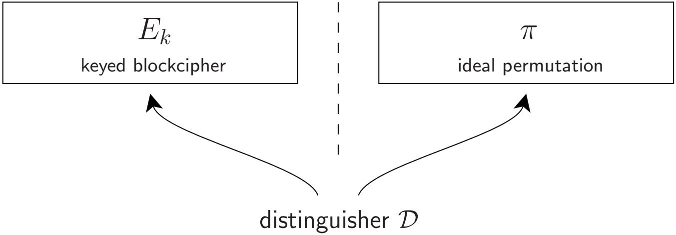
- Blockcipher: a family of permutations indexed by key
- E_k for secret k should behave like permutation π

Keyed Blockcipher: (S)PRP Security

E_k keyed blockcipher	π ideal permutation

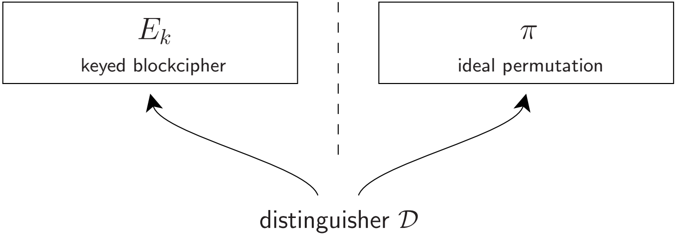
- Two oracles: E_k (for secret random key k) and π

Keyed Blockcipher: (S)PRP Security



- Two oracles: E_k (for secret random key k) and π
- Distinguisher $\mathcal{D}$ has query access to either E_k or π

Keyed Blockcipher: (S)PRP Security



- Two oracles: E_k (for secret random key k) and π
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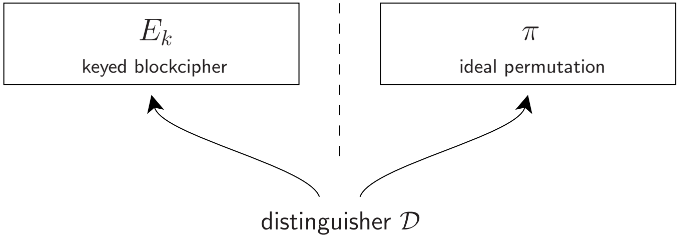
  - Forward query: $m \longrightarrow E_k(m)$ or $\pi(m)$
  - Inverse query: $c \longrightarrow E_k^{-1}(c)$ or $\pi^{-1}(c)$

Keyed Blockcipher: (S)PRP Security



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- $\mathcal{D}$ can also make "offline" evaluations of E

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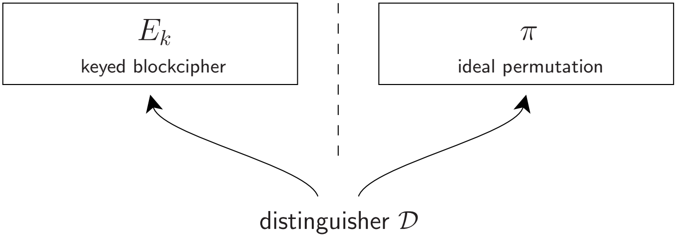

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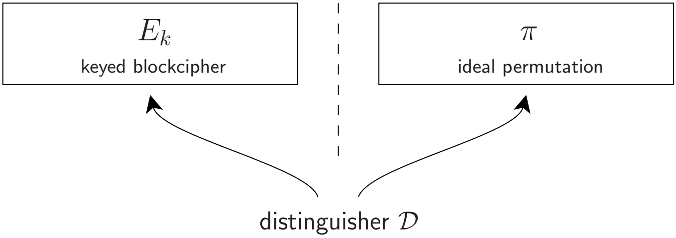
- Distinguisher $\mathcal{D}$ has query access to either E_k or π
  - Forward query: $m \quad \longrightarrow \quad E_k(m)$ or $\pi(m)$
  - Inverse query: $c \quad \longrightarrow \quad E_k^{-1}(c)$ or $\pi^{-1}(c)$
- $\mathcal{D}$ can also make "offline" evaluations of E
- $\mathcal{D}$ tries to determine which oracle it communicates with

Keyed Blockcipher: (S)PRP Security



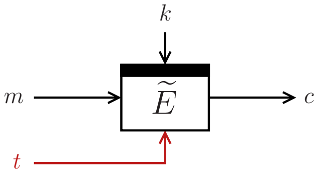
Definition

E is a (strong) pseudorandom permutation if

$$\mathbf{Adv}_E^{\mathrm{(s)prp}}(\mathcal{D}) = \left| \mathbf{Pr}\left[\mathcal{D}^{E_k^{(\pm)}} = 1\right] - \mathbf{Pr}\left[\mathcal{D}^{\pi^{(\pm)}} = 1\right] \right|$$

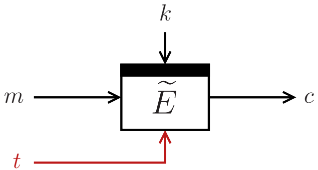
is small. $\mathcal{D}$ is parametrized by:
- Q online queries
- T offline evaluations

Keyed Tweakable Blockcipher



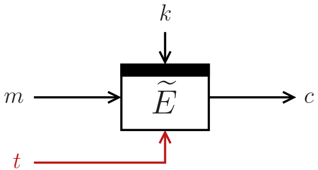
- Tweak: flexibility to the cipher
- Each key and tweak gives a different permutation

Keyed Tweakable Blockcipher



- Tweak: flexibility to the cipher
- Each key and tweak gives a different permutation
- $\widetilde{E}_k$ for secret k should behave like tweakable permutation $\widetilde{\pi}$

Keyed Tweakable Blockcipher



- Tweak: flexibility to the cipher
- Each key and tweak gives a different permutation
- $\widetilde{E}_k$ for secret k should behave like tweakable permutation $\widetilde{\pi}$

Definition

$\widetilde{E}$ is a (strong) tweakable pseudorandom permutation if

$$\mathbf{Adv}_{\widetilde{E}}^{\mathrm{(s)tprp}}(\mathcal{D}) = \left|\mathbf{Pr}\left[\mathcal{D}^{\widetilde{E}_k^{(\pm)}} = 1\right] - \mathbf{Pr}\left[\mathcal{D}^{\widetilde{\pi}^{(\pm)}} = 1\right]\right|$$

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- Q online queries
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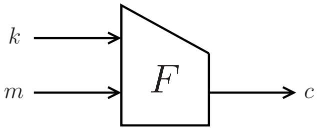
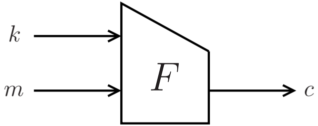
Keyed One-Way Function



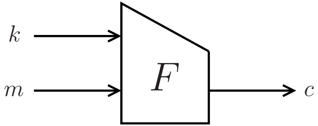
- Keyed one-way function (could be compressing)

Keyed One-Way Function



- Keyed one-way function (could be compressing)
- F_k for secret k should behave like random function \$

Keyed One-Way Function



- Keyed one-way function (could be compressing)
- F_k for secret k should behave like random function $\$$

Definition

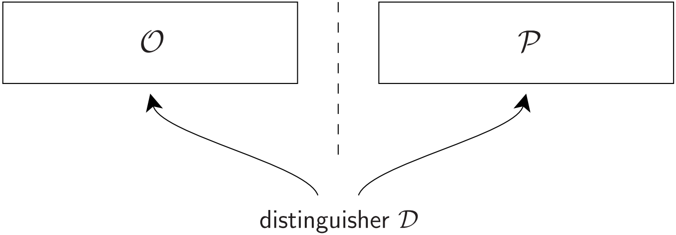
F is a pseudorandom function if

$$\mathbf{Adv}_F^{\mathrm{prf}}(\mathcal{D}) = \left|\mathbf{Pr}\left[\mathcal{D}^{F_k} = 1\right] - \mathbf{Pr}\left[\mathcal{D}^{\$} = 1\right]\right|$$

is small. $\mathcal{D}$ is parametrized by:
- Q online queries of length ℓ
- T offline evaluations

Indistinguishability

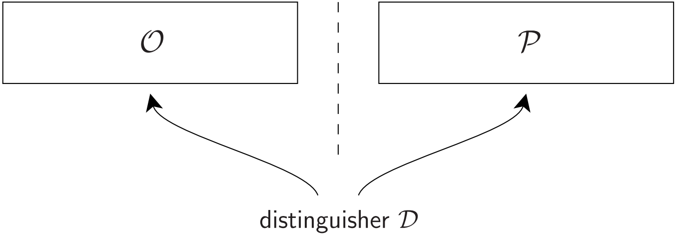
Generalization: Indistinguishability of Random Systems



$$\mathbf{Adv}^{\mathrm{ind}}(\mathcal{D}) = \left|\mathbf{Pr}\left[\mathcal{D}^{\mathcal{O}} = 1\right] - \mathbf{Pr}\left[\mathcal{D}^{\mathcal{P}} = 1\right]\right| = \Delta_{\mathcal{D}}(\mathcal{O}\ ;\ \mathcal{P})$$

Indistinguishability

Generalization: Indistinguishability of Random Systems



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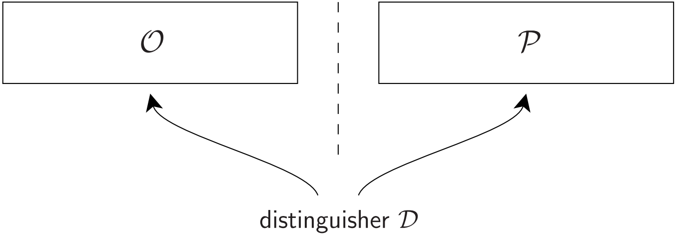
How to Prove that $\mathbf{Adv}^{\mathrm{ind}}(\mathcal{D})$ is Small?

Indistinguishability

Generalization: Indistinguishability of Random Systems



$$\mathbf{Adv}^{\mathrm{ind}}(\mathcal{D}) = \left|\mathbf{Pr}\left[\mathcal{D}^{\mathcal{O}} = 1\right] - \mathbf{Pr}\left[\mathcal{D}^{\mathcal{P}} = 1\right]\right| = \Delta_{\mathcal{D}}(\mathcal{O}\ ;\ \mathcal{P})$$

How to Prove that $\mathbf{Adv}^{\mathrm{ind}}(\mathcal{D})$ is Small?

- Game-playing technique
- H-coefficient technique

Game-Playing Technique

- Bellare and Rogaway [BR06]
- Similar to Maurer's methodology [Mau02]

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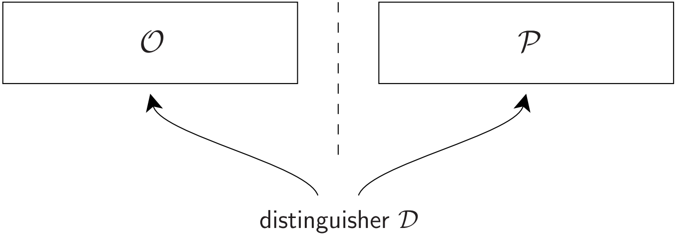

Game-Playing Technique

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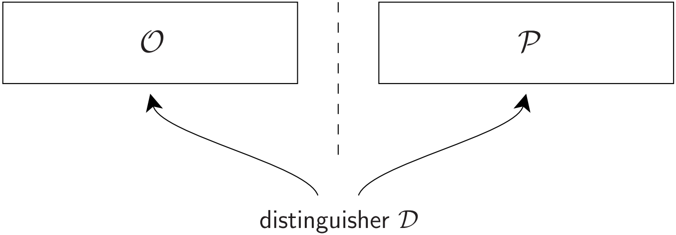

- Basic idea:
  - From $\mathcal{O}$ to $\mathcal{P}$ in small steps

Game-Playing Technique

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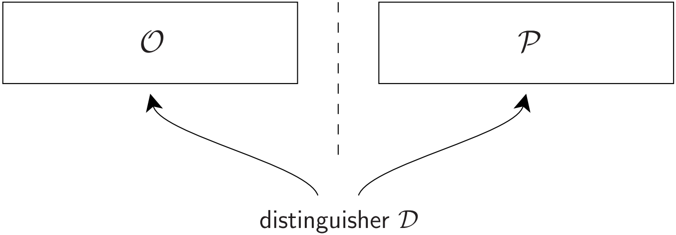


- Basic idea:
  - From $\mathcal{O}$ to $\mathcal{P}$ in small steps
  - Intermediate steps (presumably) easy to analyze

Game-Playing Technique

Triangle Inequality

Fundamental Lemma

Game-Playing Technique

Triangle Inequality

$$\Delta(\mathcal{O};\mathcal{P}) \leq \Delta(\mathcal{O};\mathcal{R}) + \Delta(\mathcal{R};\mathcal{P})$$

Fundamental Lemma

Game-Playing Technique

Triangle Inequality

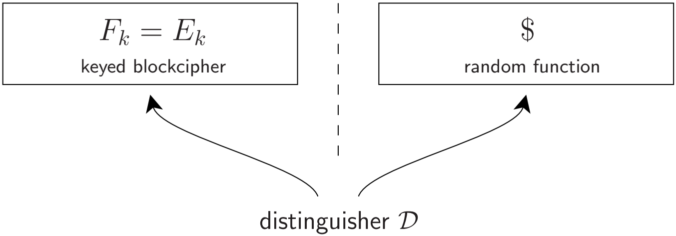
$$\Delta(\mathcal{O};\mathcal{P}) \leq \Delta(\mathcal{O};\mathcal{R}) + \Delta(\mathcal{R};\mathcal{P})$$

Fundamental Lemma

If $\mathcal{O}$ and $\mathcal{P}$ are identical until bad, then:

$$\Delta(\mathcal{O};\mathcal{P}) \leq \mathbf{Pr}\left[\mathcal{P} \text{ sets bad}\right]$$

Example: PRP-PRF Switch (1/4)



Theorem

For any distinguisher $\mathcal{D}$ making Q queries to $E_k/\$$ and T offline evaluations

$$\Delta_{\mathcal{D}}(E_k; \$) \leq \mathbf{Adv}_E^{\mathrm{prp}}(\mathcal{D}) + \frac{\binom{Q}{2}}{2^n}$$

Example: PRP-PRF Switch (2/4)

$$\Delta_{\mathcal{D}}(E_k; \$)$$

Example: PRP-PRF Switch (2/4)

Step 1. "Replace" E_k by Random Permutation π

$$\Delta_{\mathcal{D}}(E_k; \$)$$

Example: PRP-PRF Switch (2/4)

Step 1. "Replace" E_k by Random Permutation π

- Triangle inequality:

$$\Delta_{\mathcal{D}}(E_k; \$) \leq \Delta_{\mathcal{D}}(E_k; \pi) + \Delta_{\mathcal{D}}(\pi; \$)$$

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- $\Delta_{\mathcal{D}}(E_k; \pi) = \mathbf{Adv}_E^{\mathrm{prp}}(\mathcal{D})$ by definition

Example: PRP-PRF Switch (2/4)

Step 1. "Replace" E_k by Random Permutation π

- Triangle inequality:

$$\Delta_{\mathcal{D}}(E_k;\$) \leq \Delta_{\mathcal{D}}(E_k;\pi) + \Delta_{\mathcal{D}}(\pi;\$)$$

- $\Delta_{\mathcal{D}}(E_k;\pi) = \mathbf{Adv}_E^{\mathrm{prp}}(\mathcal{D})$ by definition
- $\Delta_{\mathcal{D}}(\pi;\$)$
  - $\mathcal{D}$ is parametrized by Q queries to $\pi/\$$

Example: PRP-PRF Switch (3/4)

Step 2. Random Permutation to Random Function

- Consider lazily sampled π and $\$$
  - Initially empty list of responses $\mathcal{L}$
  - Randomly generated response for every new query

Example: PRP-PRF Switch (3/4)

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Oracle π
$y \xleftarrow{\$} \{0,1\}^n \backslash \mathcal{L}$ $\mathcal{L} \xleftarrow{\cup} y$ return y

Example: PRP-PRF Switch (3/4)

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Oracle π	Oracle $\$$
$y \xleftarrow{\$} \{0,1\}^n \backslash \mathcal{L}$	$y \xleftarrow{\$} \{0,1\}^n$
$\mathcal{L} \xleftarrow{\cup} y$	
return y	return y

Example: PRP-PRF Switch (3/4)

Step 2. Random Permutation to Random Function

- Consider lazily sampled π and $\$$
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Oracle π	Oracle π'	Oracle $\$$

$y \xleftarrow{\$} \{0,1\}^n \setminus \mathcal{L}$	$y \xleftarrow{\$} \{0,1\}^n$
	if $y \in \mathcal{L}$
	    $y \xleftarrow{\$} \{0,1\}^n \setminus \mathcal{L}$
	    bad
		$y \xleftarrow{\$} \{0,1\}^n$

$\mathcal{L} \xleftarrow{\cup} y$ return y	$\mathcal{L} \xleftarrow{\cup} y$ return y	return y

Example: PRP-PRF Switch (4/4)

Oracle π	Oracle π'	Oracle $\$$

$y \xleftarrow{\$} \{0,1\}^n \setminus \mathcal{L}$	$y \xleftarrow{\$} \{0,1\}^n$
	if $y \in \mathcal{L}$
	    $y \xleftarrow{\$} \{0,1\}^n \setminus \mathcal{L}$
	    bad
		$y \xleftarrow{\$} \{0,1\}^n$

$\mathcal{L} \xleftarrow{\cup} y$ return y	$\mathcal{L} \xleftarrow{\cup} y$ return y	return y

$$\Delta_{\mathcal{D}}(\pi; \$)$$

Example: PRP-PRF Switch (4/4)

Oracle π	Oracle π'	Oracle $\$$

$y \xleftarrow{\$} \{0,1\}^n \backslash \mathcal{L}$	$y \xleftarrow{\$} \{0,1\}^n$
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$\mathcal{L} \xleftarrow{\cup} y$ return y	$\mathcal{L} \xleftarrow{\cup} y$ return y	return y

- Triangle inequality:

$$\Delta_{\mathcal{D}}(\pi; \$) \leq \Delta_{\mathcal{D}}(\pi; \pi') + \Delta_{\mathcal{D}}(\pi'; \$)$$

Example: PRP-PRF Switch (4/4)

Oracle π

$y \xleftarrow{\$} \{0,1\}^n \backslash \mathcal{L}$

$\mathcal{L} \xleftarrow{\cup} y$
return y

Oracle π'

$y \xleftarrow{\$} \{0,1\}^n$
if $y \in \mathcal{L}$
  $y \xleftarrow{\$} \{0,1\}^n \backslash \mathcal{L}$
  bad
$\mathcal{L} \xleftarrow{\cup} y$
return y

Oracle $\$$

$y \xleftarrow{\$} \{0,1\}^n$

return y

identical

- Triangle inequality:

$$\Delta_{\mathcal{D}}(\pi;\$) \leq \Delta_{\mathcal{D}}(\pi;\pi') + \Delta_{\mathcal{D}}(\pi';\$)$$

$$\leq \quad 0 \quad +$$

Example: PRP-PRF Switch (4/4)

Oracle π	Oracle π'	Oracle \$

$y \xleftarrow{\$} \{0,1\}^n \backslash \mathcal{L}$	$y \xleftarrow{\$} \{0,1\}^n$
	if $y \in \mathcal{L}$
	    $y \xleftarrow{\$} \{0,1\}^n \backslash \mathcal{L}$
	    bad
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$\mathcal{L} \xleftarrow{\cup} y$ return y	$\mathcal{L} \xleftarrow{\cup} y$ return y	return y

identical   identical until bad

- Triangle inequality:

$$\begin{aligned}\Delta_{\mathcal{D}}(\pi;\$) &\leq \Delta_{\mathcal{D}}(\pi;\pi') + \Delta_{\mathcal{D}}(\pi';\$)\\ &\leq \quad 0 \quad + \mathbf{Pr}\left[\pi' \text{ sets bad}\right]\end{aligned}$$

Example: PRP-PRF Switch (4/4)

Oracle π	Oracle π'	Oracle \$
$y \xleftarrow{\$} \{0,1\}^n \backslash \mathcal{L}$	$y \xleftarrow{\$} \{0,1\}^n$	$y \xleftarrow{\$} \{0,1\}^n$
	if $y \in \mathcal{L}$	
	$\quad y \xleftarrow{\$} \{0,1\}^n \backslash \mathcal{L}$	
	$\quad$ bad	
$\mathcal{L} \xleftarrow{\cup} y$	$\mathcal{L} \xleftarrow{\cup} y$	
return y	return y	return y

identical

identical until bad

- Triangle inequality:

$$\begin{aligned} \Delta_{\mathcal{D}}(\pi;\$) &\leq \Delta_{\mathcal{D}}(\pi;\pi') + \Delta_{\mathcal{D}}(\pi';\$) \\ &\leq \quad 0 \quad + \mathbf{Pr}\left[\pi' \text{ sets bad}\right] \leq \frac{\binom{Q}{2}}{2^n} \end{aligned}$$

H-Coefficient Technique

- Patarin [Pat91,Pat08]
- Popularized by Chen and Steinberger [CS14]
- Similar to “Strong Interpolation Technique” [Ber05]

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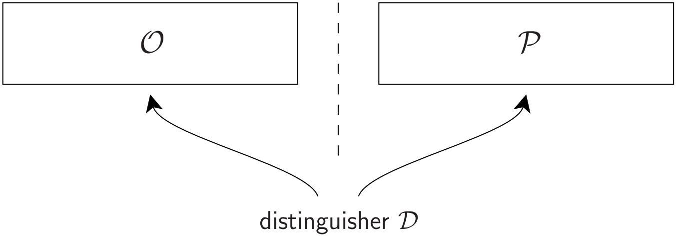

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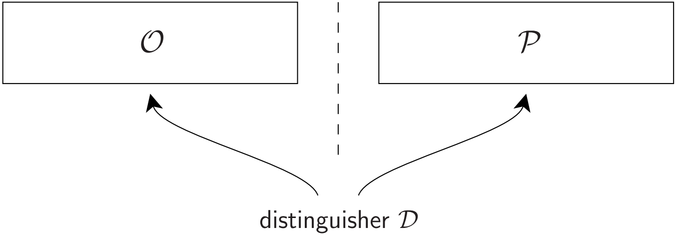
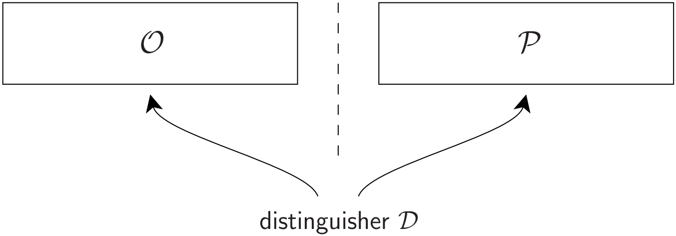


- Basic idea:
  - Each conversation defines a transcript τ

H-Coefficient Technique

- Patarin [Pat91,Pat08]
- Popularized by Chen and Steinberger [CS14]
- Similar to "Strong Interpolation Technique" [Ber05]



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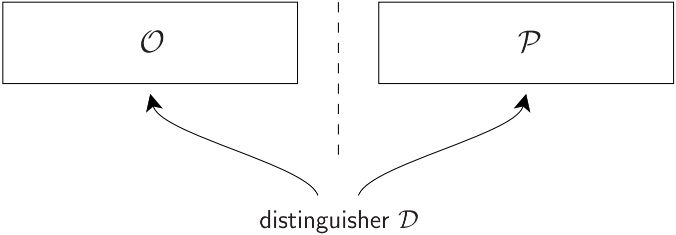

- Basic idea:
  - Each conversation defines a transcript τ
  - $\mathcal{O} \approx \mathcal{P}$ for most of the transcripts
  - Remaining transcripts occur with small probability

H-Coefficient Technique

- $\mathcal{D}$ is computationally unbounded and deterministic
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Lemma

Let $\varepsilon > 0$ be such that for all good transcripts τ:

$$\frac{\mathbf{Pr}\left[\mathcal{O} \text{ gives } \tau\right]}{\mathbf{Pr}\left[\mathcal{P} \text{ gives } \tau\right]} \geq 1 - \varepsilon$$

Then, $\Delta_{\mathcal{D}}(\mathcal{O}; P) \leq \varepsilon + \mathbf{Pr}\left[\text{bad transcript for } \mathcal{P}\right]$

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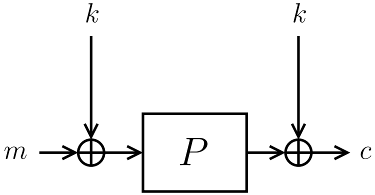
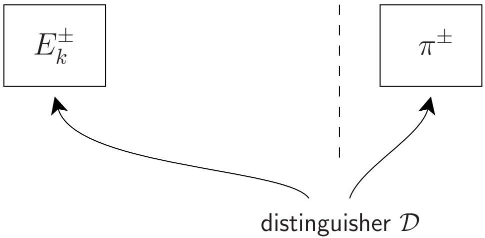
Trade-off: define bad transcripts smartly!

Example: Even-Mansour (1/10)



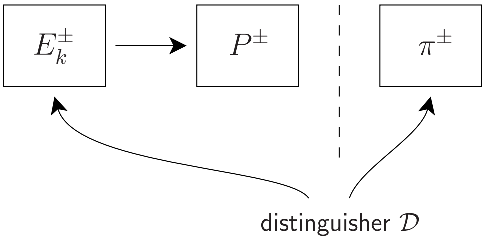
$$E_k(m) = P(m \oplus k) \oplus k$$

Example: Even-Mansour (2/10)



Slightly Different Security Model

Example: Even-Mansour (2/10)



Slightly Different Security Model

- Underlying permutation

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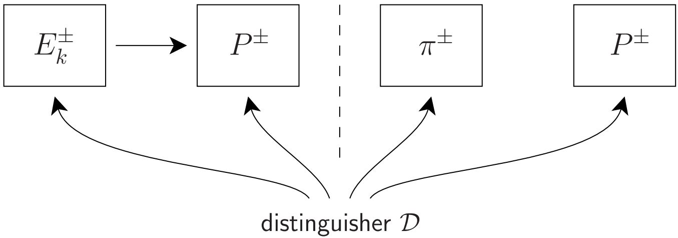

Slightly Different Security Model

- Underlying permutation randomized
- Information-theoretic distinguisher $\mathcal{D}$
  - Q construction queries
  - T offline evaluations $\approx T$ primitive queries

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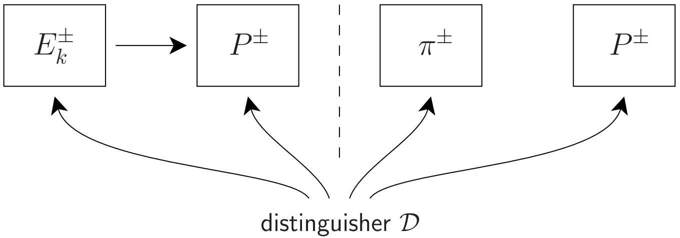

Slightly Different Security Model

- Underlying permutation randomized
- Information-theoretic distinguisher $\mathcal{D}$
  - Q construction queries
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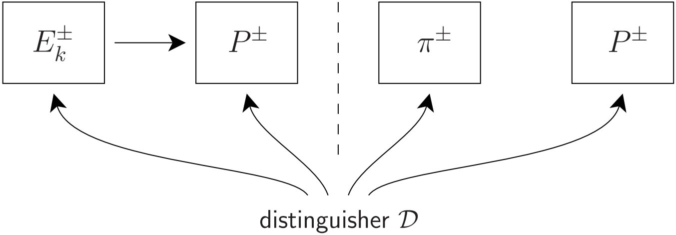
  - Unbounded computational power

Example: Even-Mansour (3/10)



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- Without loss of generality, $\mathcal{D}$ is deterministic
  - No random choices

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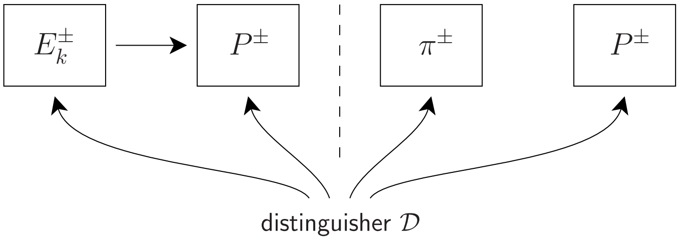
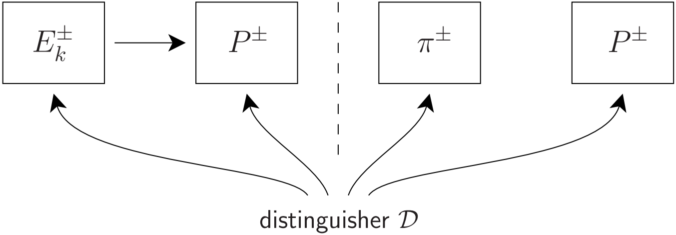


Slightly Different Security Model

- Without loss of generality, $\mathcal{D}$ is deterministic
  - No random choices
- Reason: at the end we maximize over all distinguishers

Example: Even-Mansour (4/10)



Theorem

For any deterministic distinguisher $\mathcal{D}$ making Q queries to $E_k/\$$ and T primitive queries

$$\mathbf{Adv}_E^{\mathrm{sprp}}(\mathcal{D}) = \Delta_{\mathcal{D}}(E_k^{\pm}, P^{\pm}; \pi^{\pm}, P^{\pm}) \leq \frac{2QT}{2^n}$$

Example: Even-Mansour (5/10)

Step 1. Define how transcripts look like

Step 2. Define good and bad transcripts

Step 3. Upper bound $\mathbf{Pr}\left[\text{bad transcript for } (\pi^{\pm}, P^{\pm})\right]$

Step 4. Lower bound $\frac{\mathbf{Pr}\left[(E_k^{\pm}, P^{\pm}) \text{ gives } \tau\right]}{\mathbf{Pr}\left[(\pi^{\pm}, P^{\pm}) \text{ gives } \tau\right]} \geq 1 - \varepsilon$ ($\forall$ good τ)

Example: Even-Mansour (6/10)

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   - Construction queries:
     $$\tau_E = \{(m_1, c_1), \ldots, (m_Q, c_Q)\}$$
   - Primitive queries:
     $$\tau_P = \{(x_1, y_1), \ldots, (x_T, y_T)\}$$

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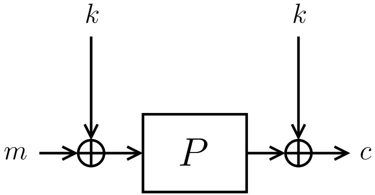
  - Ideal world $(\pi^{\pm}, P^{\pm})$: dummy key $k \xleftarrow{\$} \{0,1\}^n$

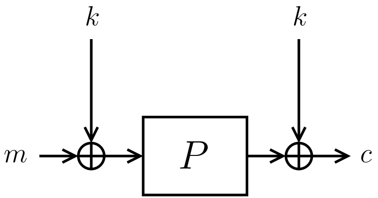
Example: Even-Mansour (7/10)



2. Define good and bad transcripts

- Intuition:

Example: Even-Mansour (7/10)



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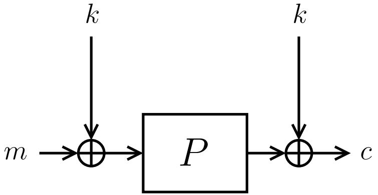

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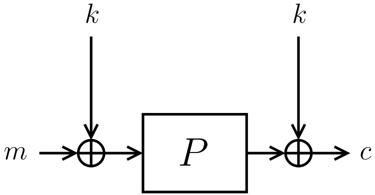

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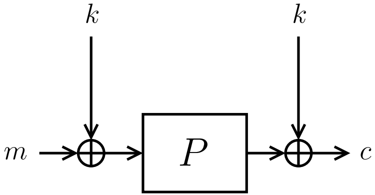

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- Note: no internal collisions in τ_E and τ_P

Example: Even-Mansour (8/10)

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$\uparrow$

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Example: Even-Mansour (9/10)

4. Lower bound $\dfrac{\mathbf{Pr}\left[(E_k^{\pm}, P^{\pm}) \text{ gives } \tau\right]}{\mathbf{Pr}\left[(\pi^{\pm}, P^{\pm}) \text{ gives } \tau\right]} \geq 1 - \varepsilon \ (\forall \text{ good } \tau)$

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- Counting "compatible" oracles (modulo details):

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- For real world $(E_k^{\pm}, P^{\pm})$:

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- For ideal world $(\pi^\pm, P^\pm)$:

$$\mathbf{Pr}\left[(\pi^\pm, P^\pm) \text{ gives } \tau\right] = \frac{(2^n - Q)!(2^n - T)!}{2^n \cdot (2^n!)^2}$$

Example: Even-Mansour (10/10)

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- Putting things together:

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- We put $\varepsilon = 0$

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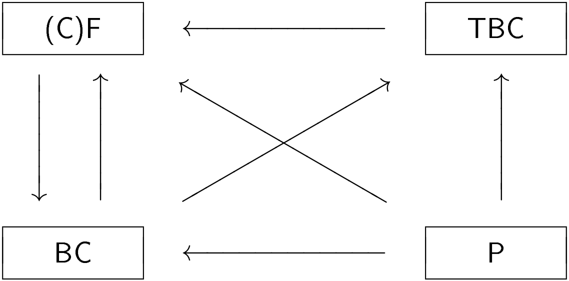
$$\geq 1$$

- We put $\varepsilon = 0$

- Conclusion:

$$\mathbf{Adv}_E^{\text{sprp}}(\mathcal{D}) = \Delta_{\mathcal{D}}(E_k^{\pm}, P^{\pm}; \pi^{\pm}, P^{\pm}) \leq \frac{2QT}{2^n} + 0$$

Keyed Authenticated Encryption



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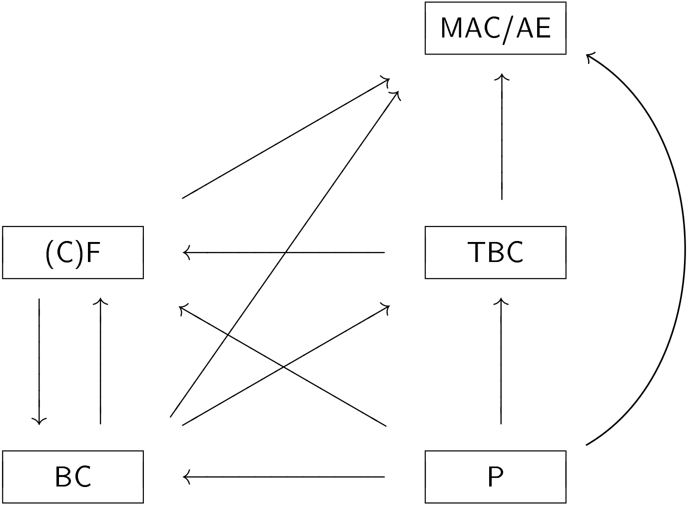

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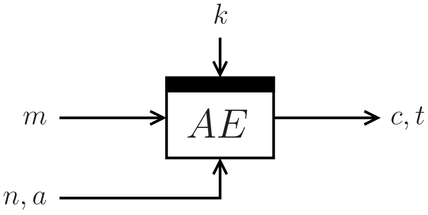
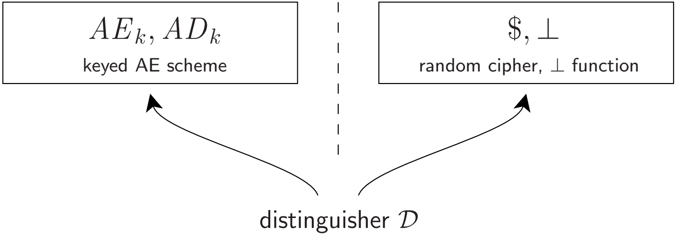


- Confidentiality: c should always look random
- Authenticity: t is "hard to forge"

- (AE_k, AD_k) for secret k should behave like $(\$, \perp)$

Keyed Authenticated Encryption: AE Security



Definition

(AE, AD) is a secure authenticated encryption scheme if[1]

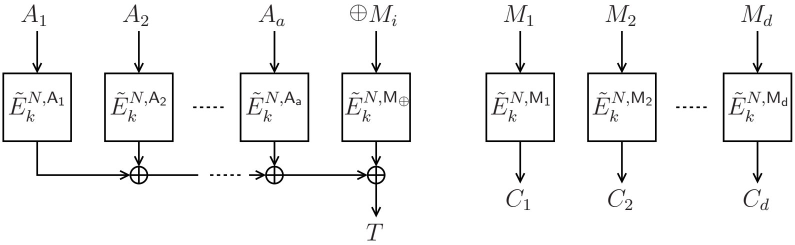
$$\mathbf{Adv}_{AE}^{\mathrm{ae}}(\mathcal{D}) = \left| \mathbf{Pr}\left[\mathcal{D}^{AE_k, AD_k} = 1\right] - \mathbf{Pr}\left[\mathcal{D}^{\$,\perp} = 1\right] \right|$$

is small. $\mathcal{D}$ is parametrized by:

- Q online queries of length ℓ (nonce-respecting/misusing)
- T offline evaluations

[1] Also known as CCA3 security

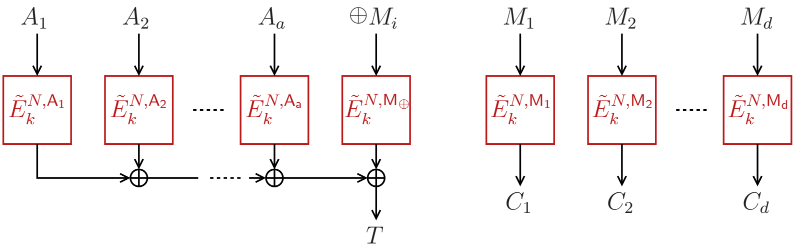
Example: OCBx (1/2)



- Generalized OCB by Rogaway et al. [RBBK01,Rog04,KR11]

$$\Delta_{\mathcal{D}}(AE_k \;, AD_k \;; \$, \perp)$$

Example: OCBx (1/2)



- Generalized OCB by Rogaway et al. [RBBK01,Rog04,KR11]
- Internally based on tweakable blockcipher $\widetilde{E}$
  - Tweak (N, tweak) is unique for every evaluation

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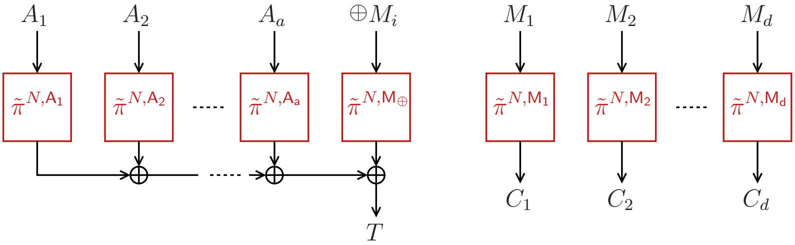

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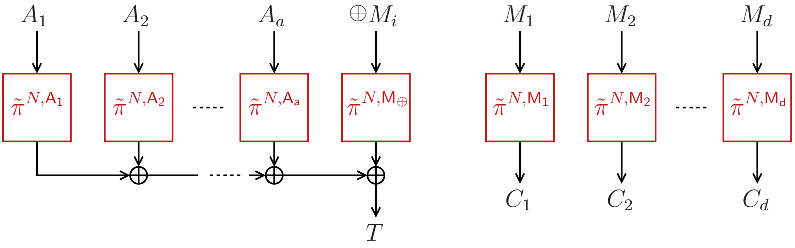

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STPRP security of $\widetilde{E}$

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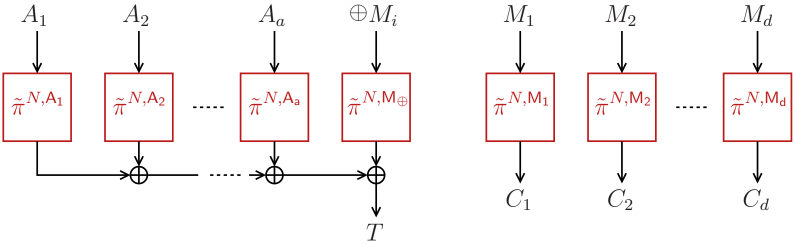

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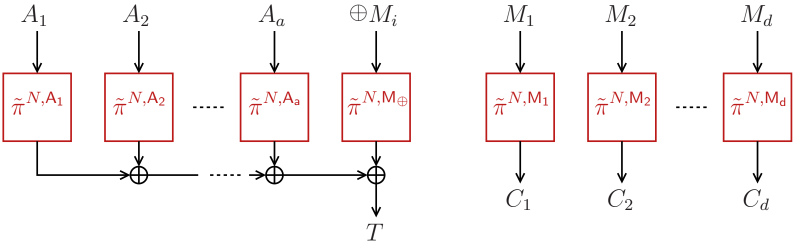

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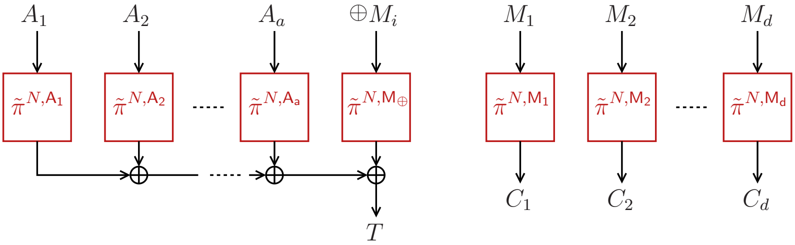

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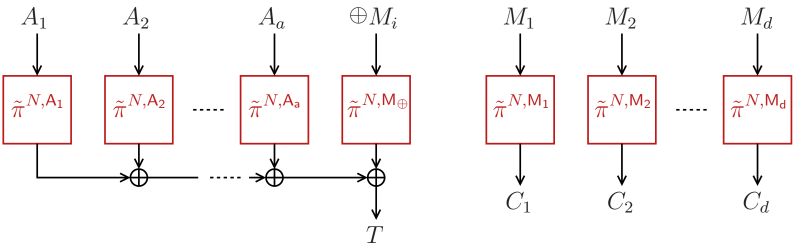

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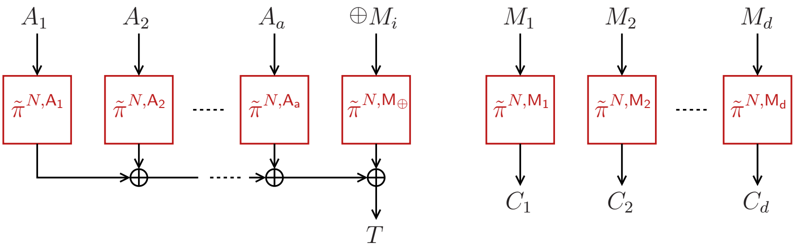

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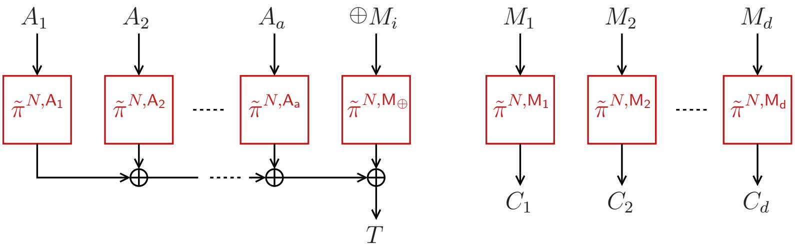

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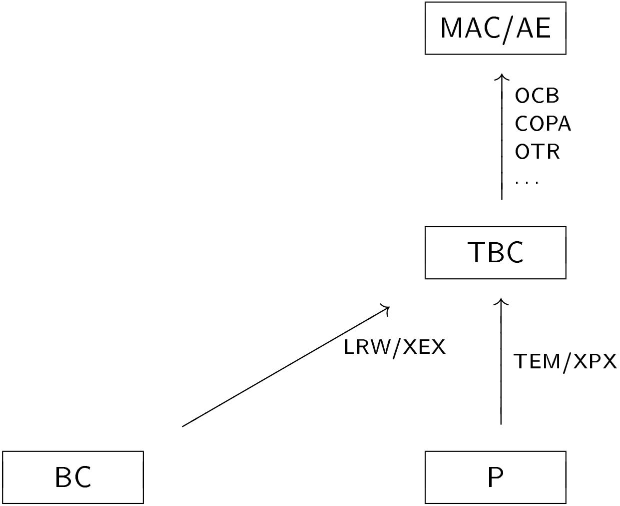
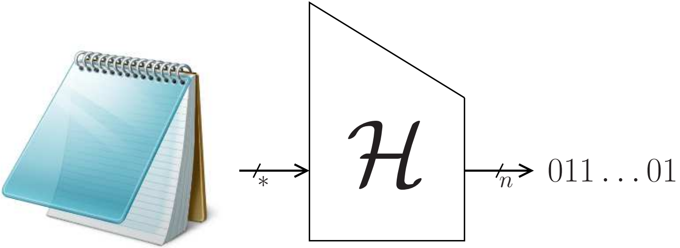
Tomorrow's Talk



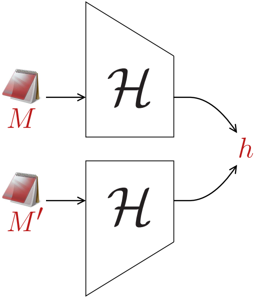
Keyless Constructions

Hash Functions



Hash Functions: Classical Security Requirements

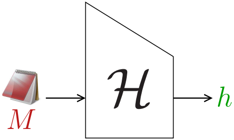
Collision



Find $M \neq M'$

Application:
2012 Flame virus

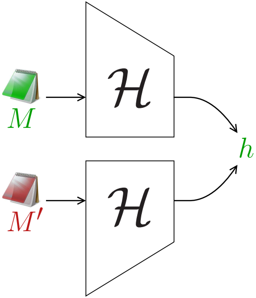
Preimage



Given h, find M

Application:
passphrase protection

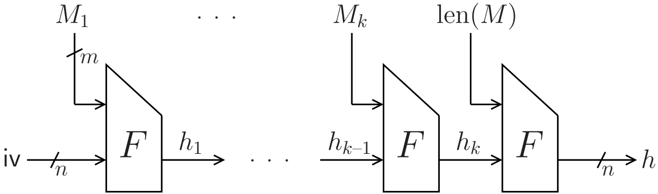
Second Preimage



Given M, find $M' \neq M$

Application:
data integrity

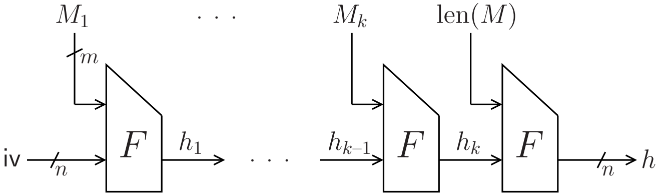
Hash Functions from Compression Functions



Merkle-Damgård with Strengthening

- Damgård [Dam89] and Merkle [Mer89]
- Consecutive evaluation of compression function F
- Length encoding at the end

Hash Functions from Compression Functions



Security of Merkle-Damgård

- $\mathcal{H}$ and F have same security models

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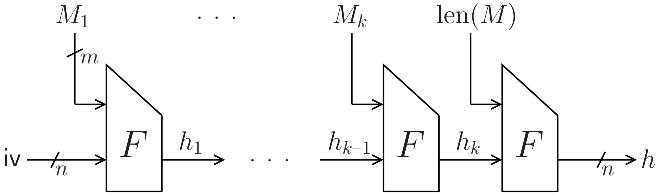


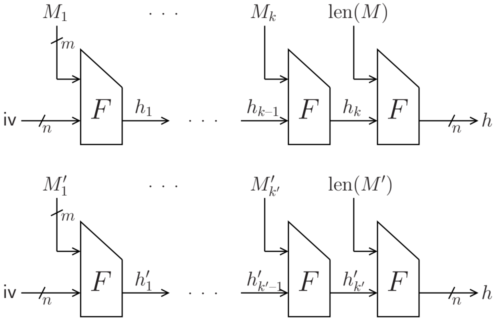
Security of Merkle-Damgård

- $\mathcal{H}$ and F have same security models
- Ideally, we want:

$$F \text{ is col/sec/pre secure} \implies \mathcal{H} \text{ is col/sec/pre secure}$$

Collision Security of Merkle-Damgård

- Suppose we are given a collision $\mathcal{H}(M) = \mathcal{H}(M')$



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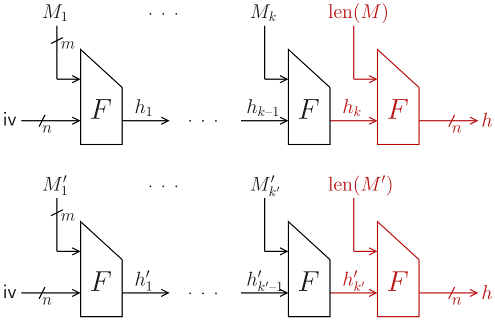

- Then, $F(h_k, \mathrm{len}(M)) = F(h'_{k'}, \mathrm{len}(M'))$

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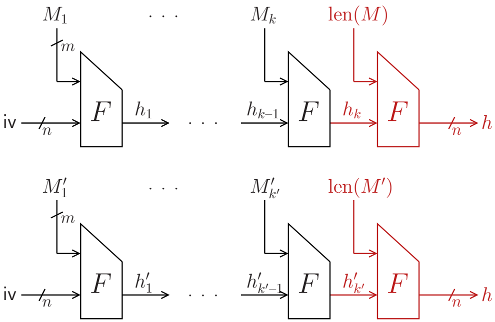

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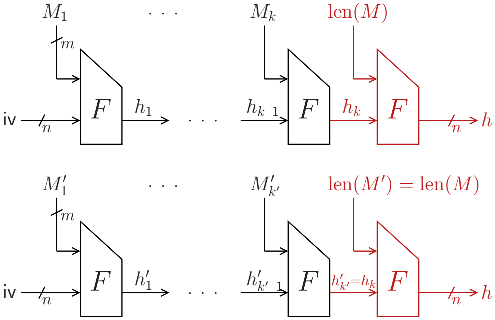

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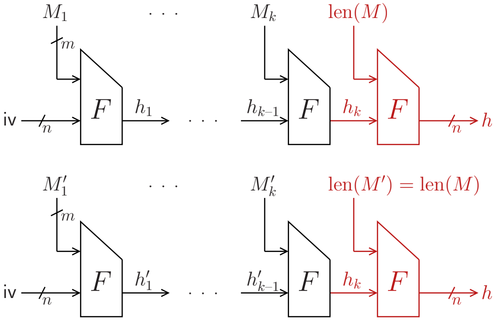

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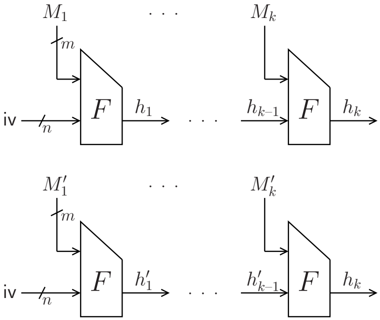

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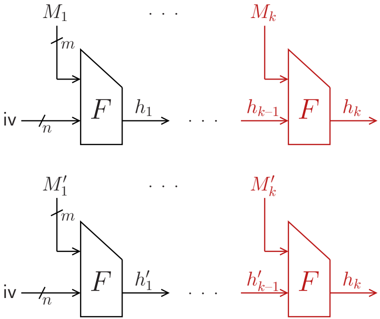

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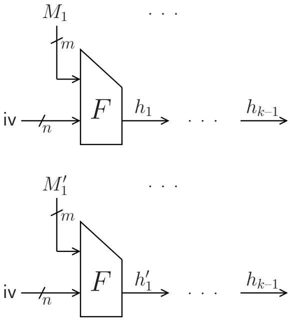

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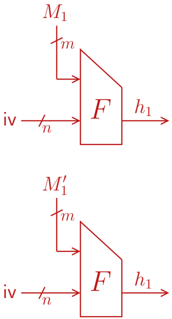

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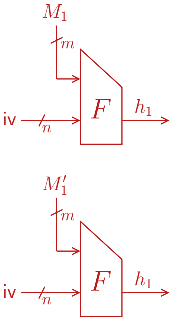

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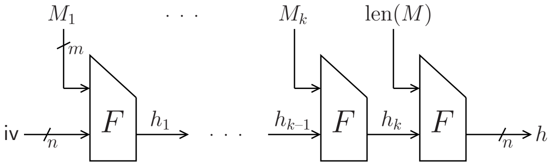
    - $\dots$
- We can find F-collision

Preimage Security of Merkle-Damgård

- Let h be any range value
- Suppose we are given a preimage $\mathcal{H}(M) = h$



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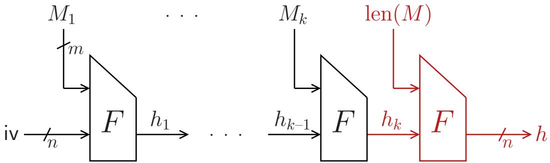

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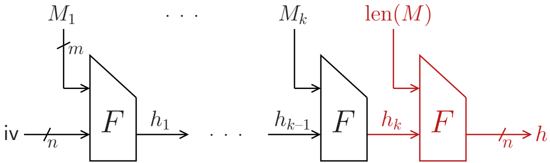

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Second Preimage Security of Merkle-Damgård

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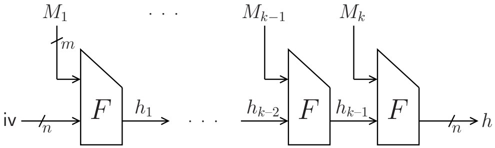

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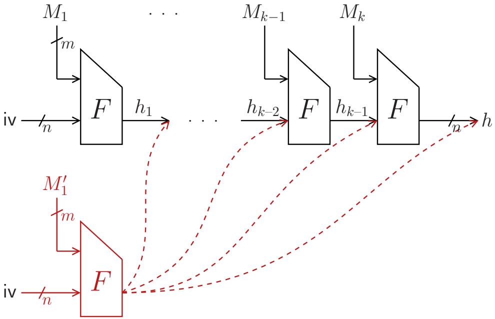

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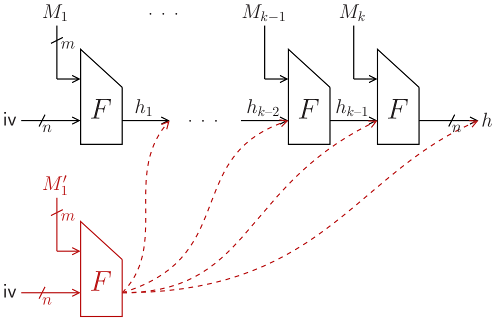
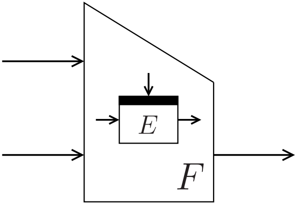
  - Work-around trick if $\mathrm{len}(M)$ is included



Compression Functions from Blockciphers/Permutations

Blockcipher Based



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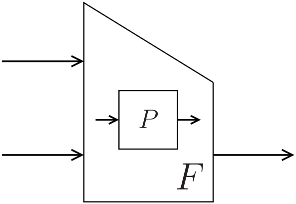

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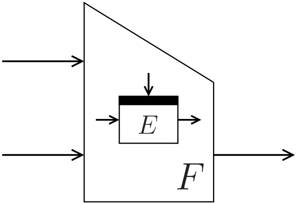

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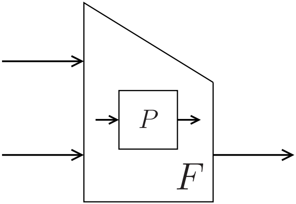

- Proofs in ideal model
  - E or P assumed to be uniformly random primitives
  - Proof via combinatorics and probability theory

Compression Functions from Blockciphers

- Classical approach
- PGV compression functions [PGV93]
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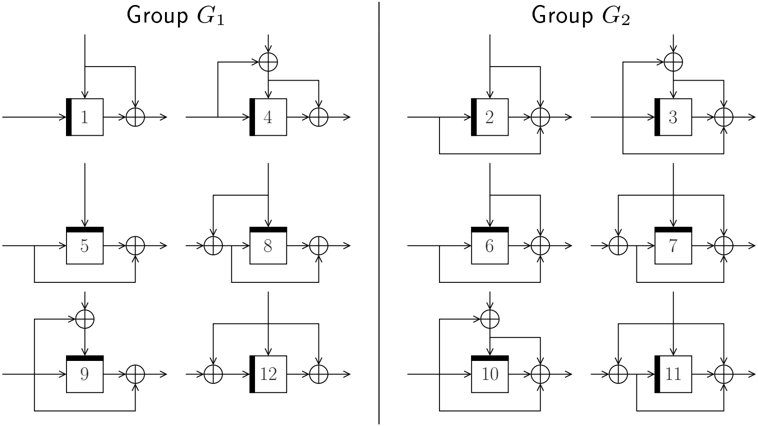

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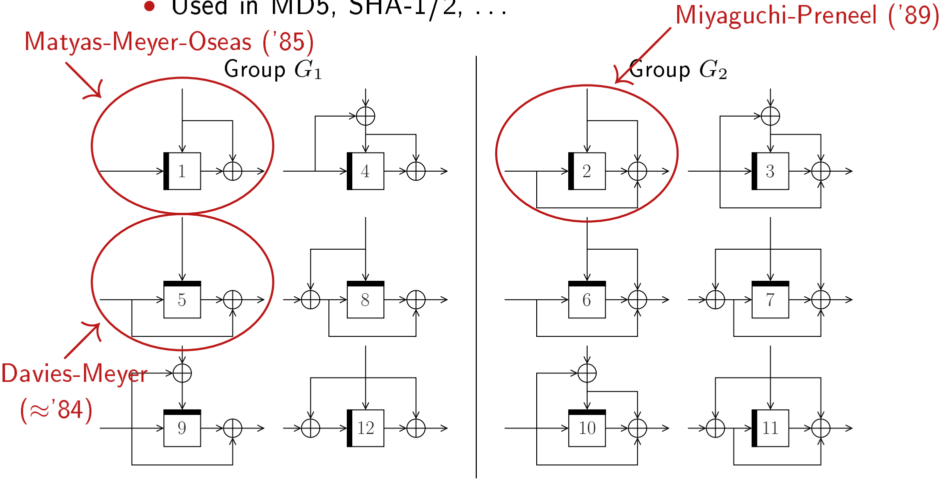
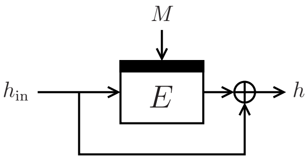


Security of Davies-Meyer Compression Function



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- Assume that E is an ideal cipher

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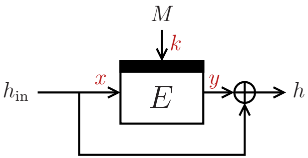

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  - Forward query $(k, x) \to y$
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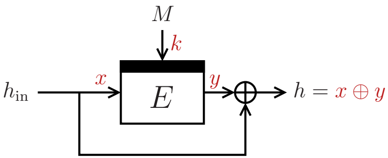

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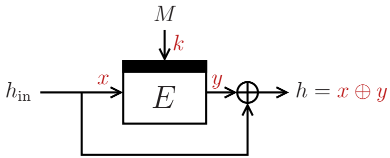

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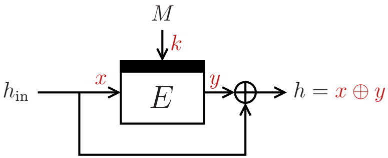
- $\mathcal{A}$ must make the required queries for the attack

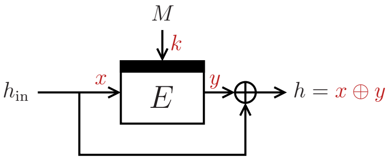
Security of Davies-Meyer Compression Function



Collision Resistance

- Consider ith query (k, x, y) (forward or inverse)

Security of Davies-Meyer Compression Function



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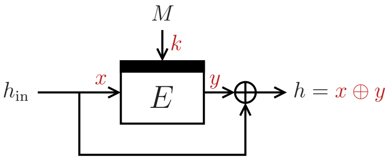

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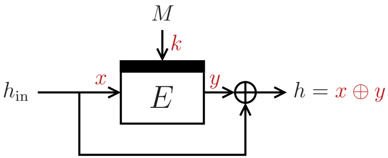

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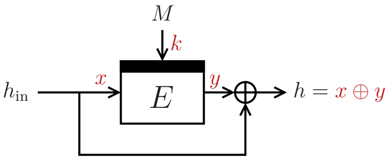

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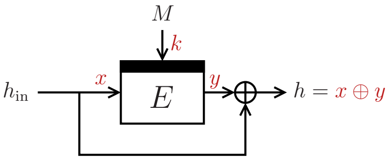

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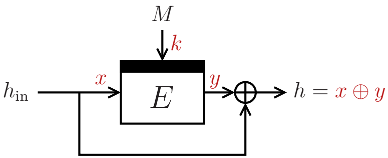

Preimage Resistance

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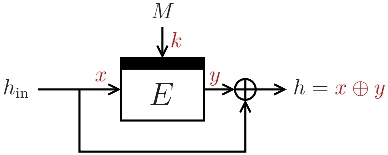

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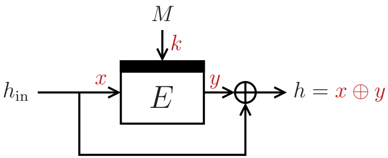

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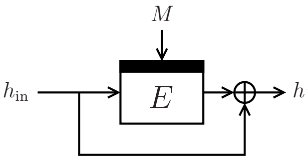
Security of Davies-Meyer Compression Function



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- What if E is SPRP?

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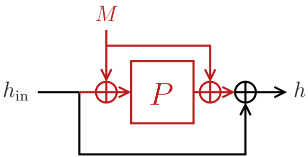

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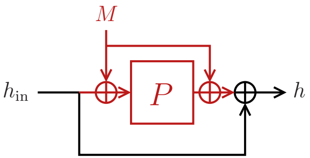

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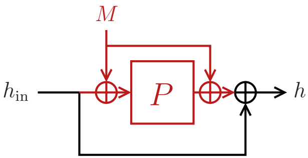

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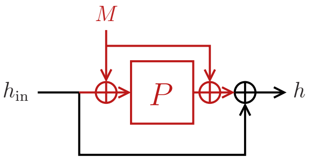

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- Bottom line:
  - Be careful with choice of security model
  - Be careful with instantiation of the construction

Compression Functions from Blockciphers

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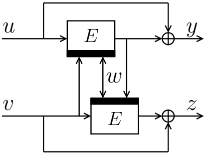
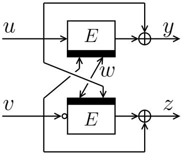
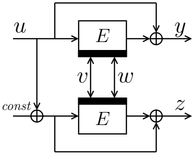
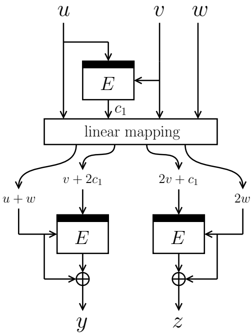


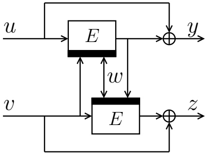


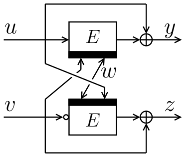


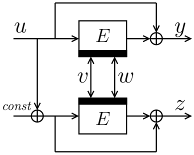


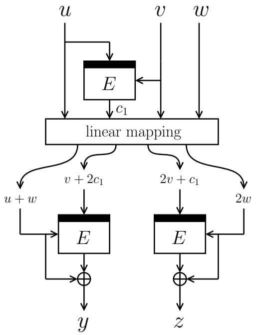
Compression Functions from Blockciphers

- Similar ideas for other PGVs
- Additional proof tricks for double length functions
  - "Wish lists" [LSS10]
  - "Free super queries" [LSS11]
  - Many case distinctions (up to ≈ 40 in worst case)









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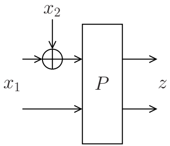

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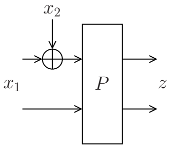

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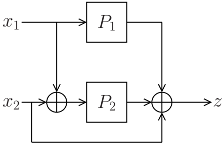

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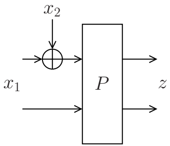

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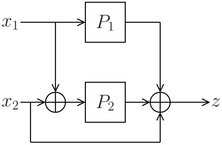

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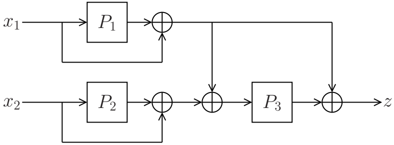

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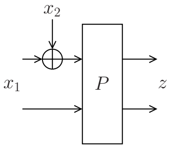

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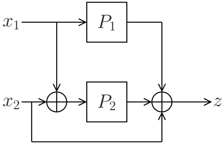

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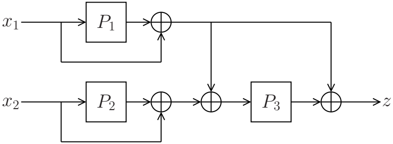

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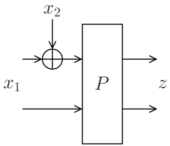

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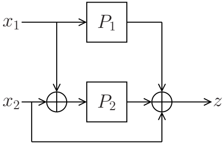

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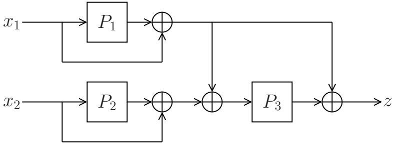

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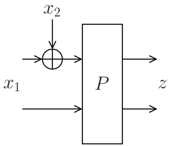

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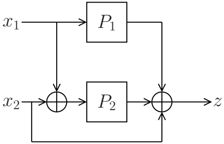

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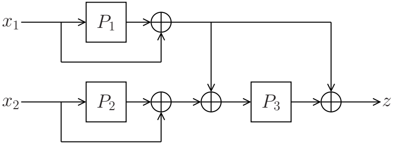

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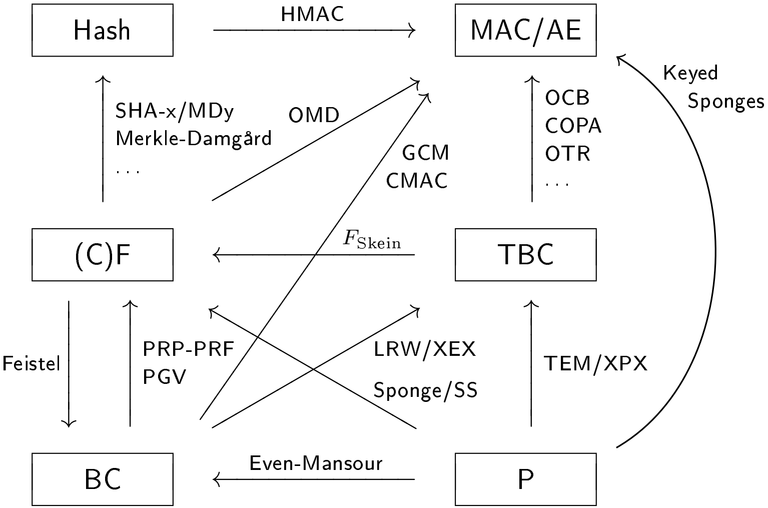

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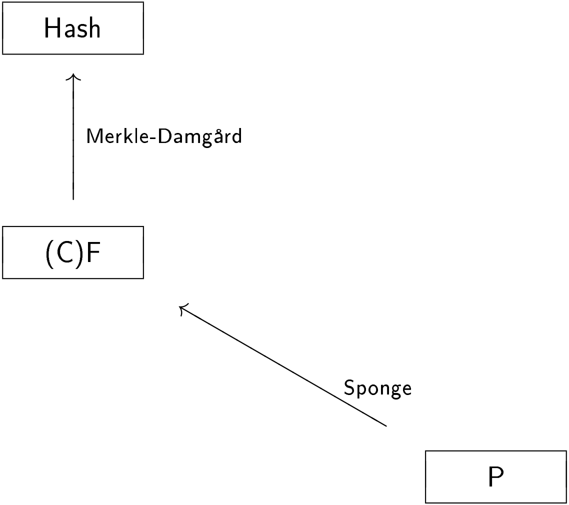

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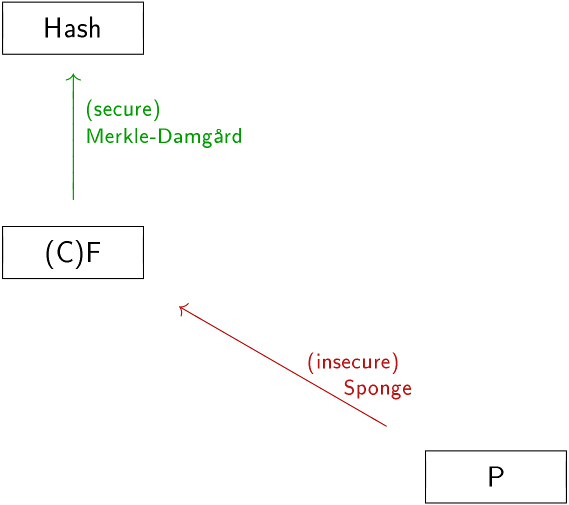

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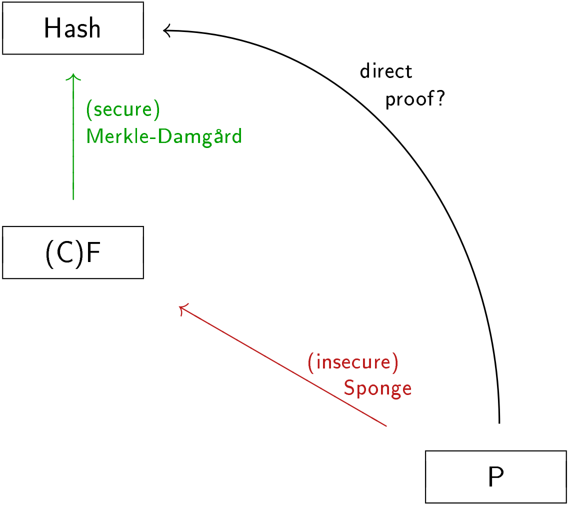

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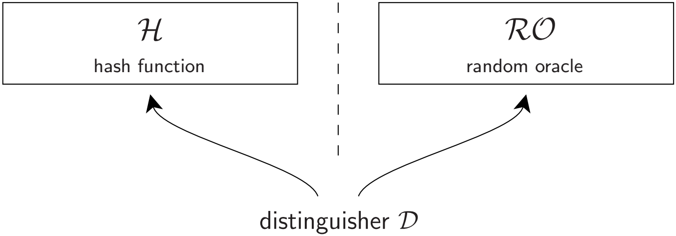

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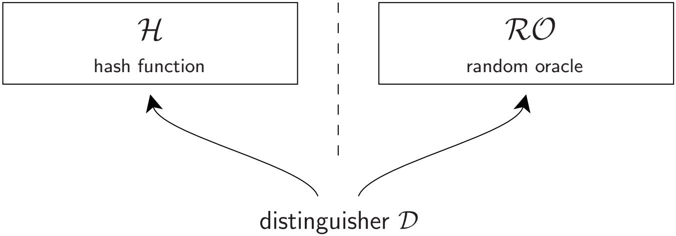

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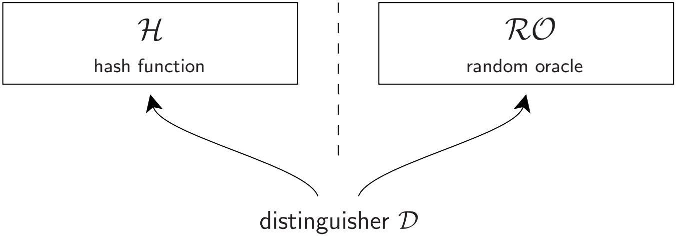

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- Solution: indifferentiability

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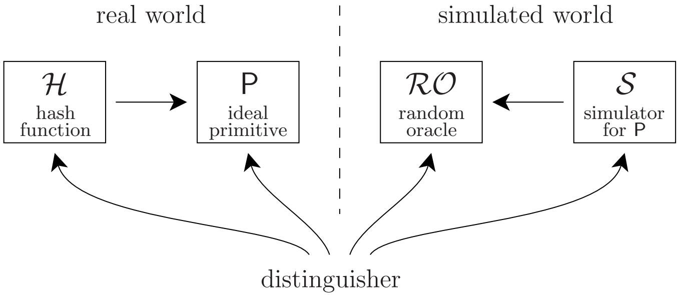

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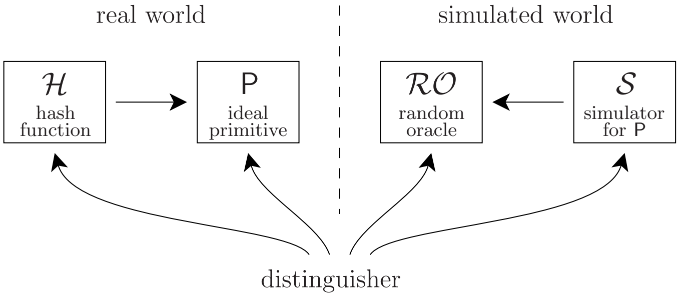

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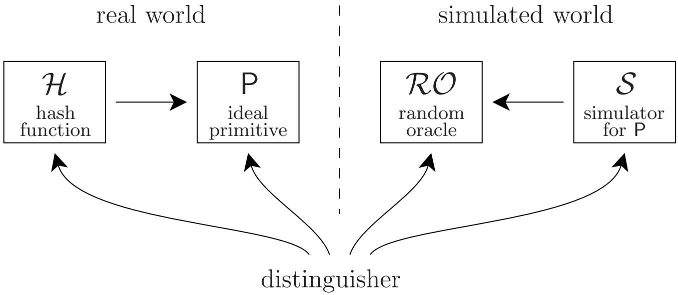

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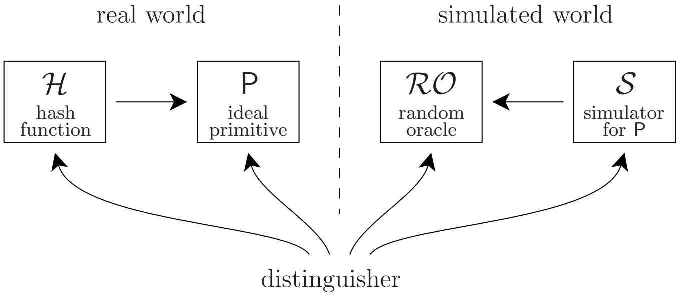
- Proof idea:

  Step 1. Construct a clever simulator $\mathcal{S}$

  Step 2. Use game-playing or H-coefficient technique

Indifferentiability of Hash Functions



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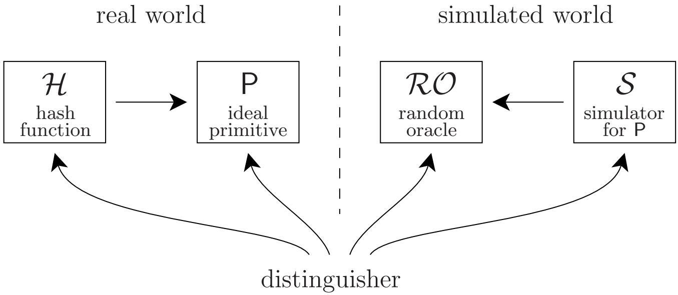

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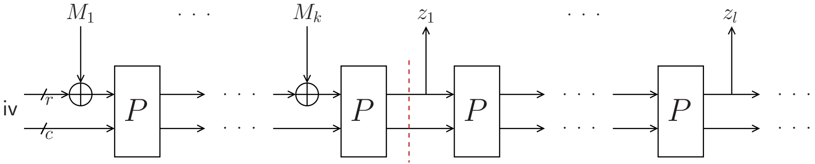
- Indifferentiability $\Longrightarrow$ coll/pre/sec security

Indifferentiability of Sponge



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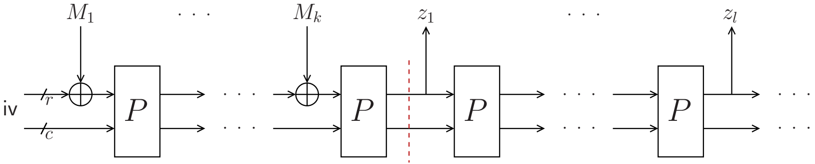
Indifferentiability of Sponge



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- Sponge indifferentiable from random oracle

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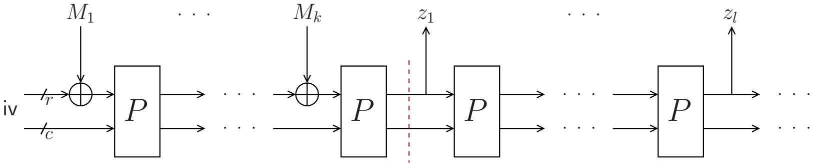
(with Q the total complexity)

Indifferentiability of Sponge



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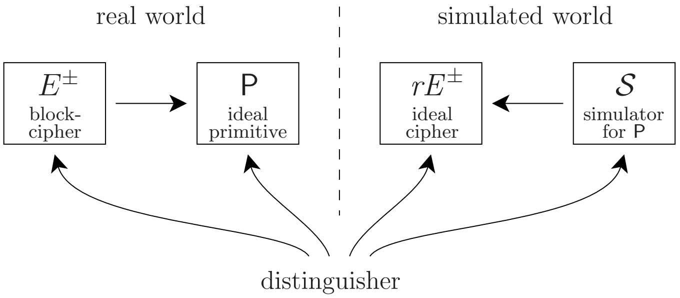
Indifferentiability of Blockciphers



- Similar model for keyless blockciphers
- Blockcipher behaves like ideal cipher
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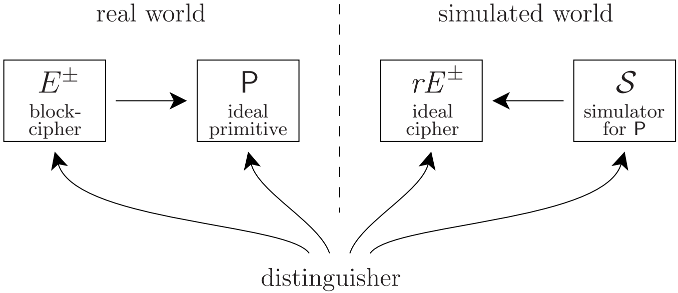

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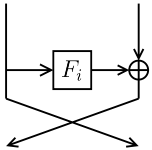
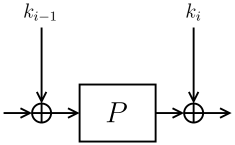
- Much (!!) harder to prove

Indifferentiability of Blockciphers





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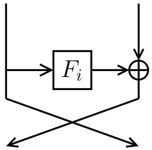

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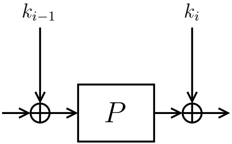

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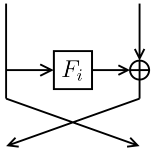

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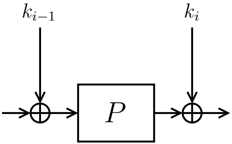

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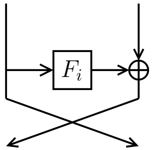

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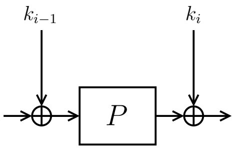

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Extremely hard research question!

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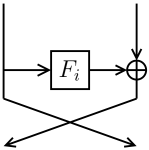

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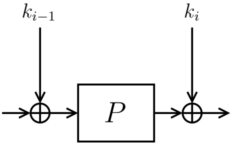

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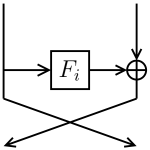

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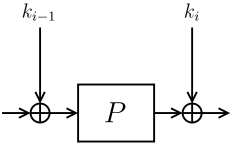

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Conclusion

Recipe

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  - Keyed: usually indistinguishability
  - Keyless: indifferentiability or dedicated security model
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Thank you for your attention!