

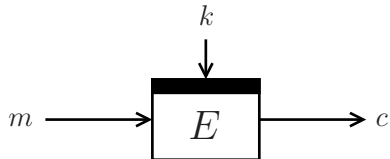
XPX: Generalized Tweakable Even-Mansour with Improved Security Guarantees

Bart Mennink
KU Leuven (Belgium)

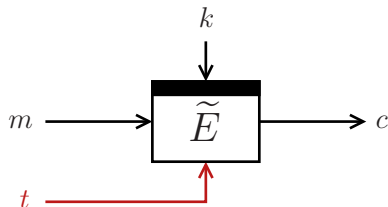
CRYPTO 2016
August 15, 2016



Tweakable Blockciphers

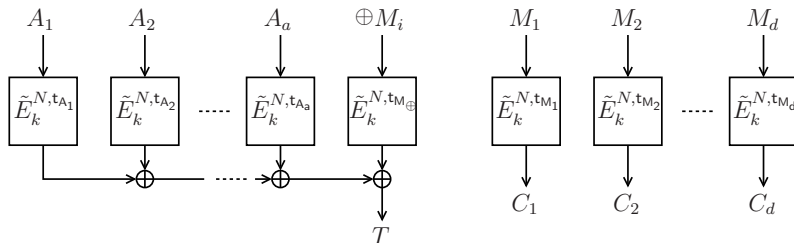


Tweakable Blockciphers



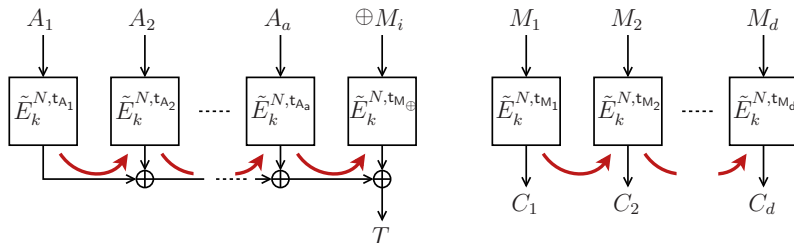
- Tweak: flexibility to the cipher
- Each tweak gives different permutation

Tweakable Blockciphers in OCBx



- OCBx by Rogaway et al. [RBBK01,Rog04,KR11]
- Internally based on tweakable blockcipher \tilde{E}
 - Tweak (N , position) is unique for **every** evaluation
 - Different blocks **always** transformed under different tweak

Tweakable Blockciphers in OCBx



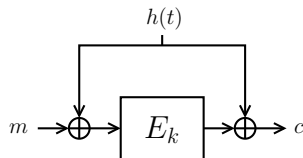
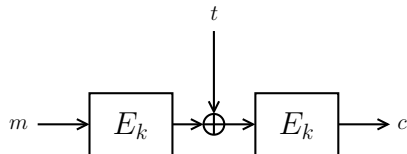
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- Internally based on tweakable blockcipher \tilde{E}
 - Tweak (N , position) is unique for **every** evaluation
 - Different blocks **always** transformed under different tweak
- Change of tweak should be **efficient**

Tweakable Blockciphers from Scratch

- Hasty Pudding Cipher [Sch98]
 - AES submission, “first tweakable cipher”
- Mercy [Cro01]
 - Disk encryption
- Threefish [FLS+07]
 - SHA-3 submission Skein
- TWEAKEY [JNP14]
 - CAESAR submissions Deoxys, Joltik, KIASU

Tweakable Blockciphers from Blockcipher

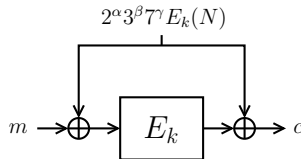
- LRW_1 and LRW_2 by Liskov et al. (2002):



- h is XOR-universal hash

Tweakable Blockciphers from Blockcipher

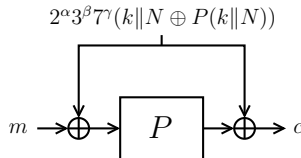
- XEX by Rogaway (2004):



- $(\alpha, \beta, \gamma, N)$ is tweak (simplified)
- Used in OCB2 and in about 14 CAESAR submissions

Tweakable Blockciphers from Permutation

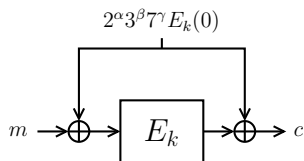
- Tweakable Even-Mansour (TEM):



- $(\alpha, \beta, \gamma, N)$ is tweak (simplified)
- Introduced in CAESAR candidate Minalpher (2014)
- Generalized by Cogliati et al. (2015)

Tweakable Blockciphers from Permutation

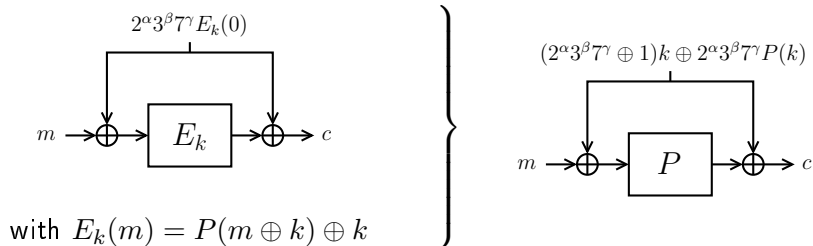
- Related to XEX with Even-Mansour:



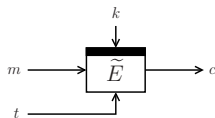
with $E_k(m) = P(m \oplus k) \oplus k$

Tweakable Blockciphers from Permutation

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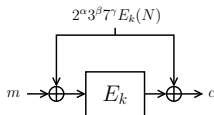


Tweakable Blockciphers in CAESAR



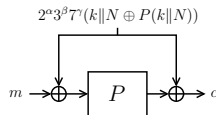
Dedicated

Deoxys,
Joltik,
KIASU,
SCREAM



XEX-inspired

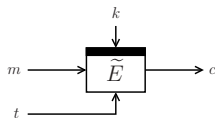
AEZ, CBA, COBRA,
COPA, **ELmD**, iFeed,
Marble, **OCB**, **OMD**,
OTR, **POET**, **SHELL**



TEM-inspired

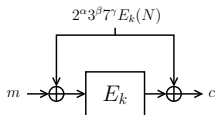
Minalpher,
Prøst

Tweakable Blockciphers in CAESAR



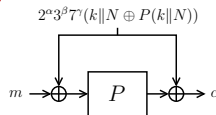
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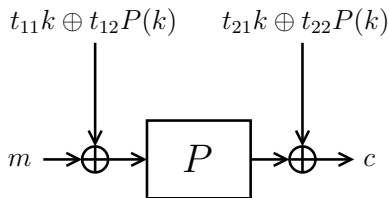


TEM-inspired

Minalpher,
Prøst

We generalize this

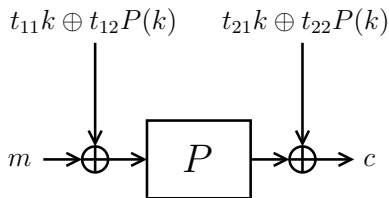
XPX



Tweak Set

- $(t_{11}, t_{12}, t_{21}, t_{22})$ from some tweak set $\mathcal{T} \subseteq (\{0, 1\}^n)^4$
- \mathcal{T} can (still) be any set

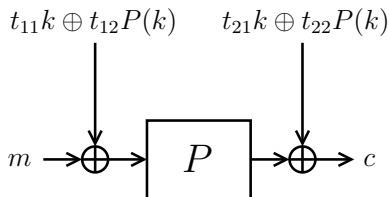
XPX



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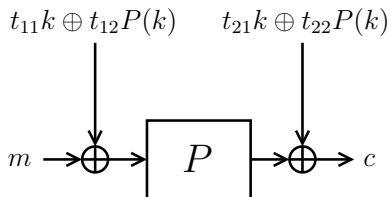
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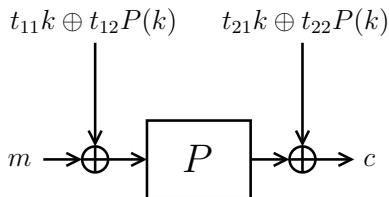
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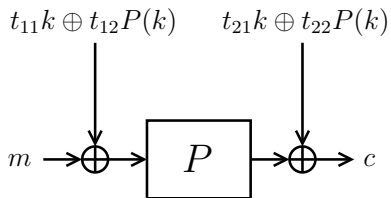
XPX



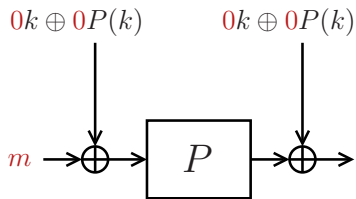
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 - ③ “Strong” $\mathcal{T} \longrightarrow$ related-key secure

XPX: Valid Tweaks

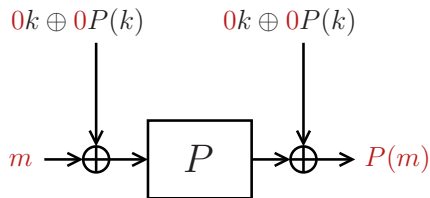


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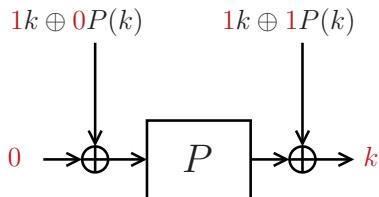
$$(0, 0, 0, 0) \in \mathcal{T}$$

XPX: Valid Tweaks



$$(0, 0, 0, 0) \in \mathcal{T} \implies \text{XPX}_k((0, 0, 0, 0), m) = P(m)$$

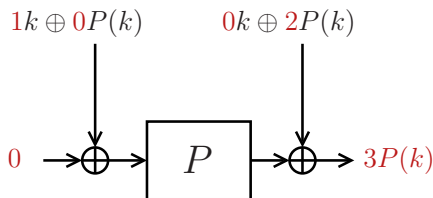
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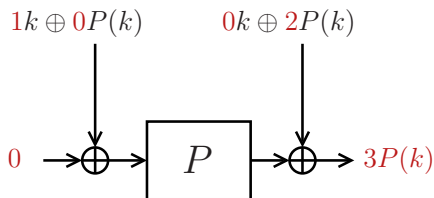


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XPX: Valid Tweaks



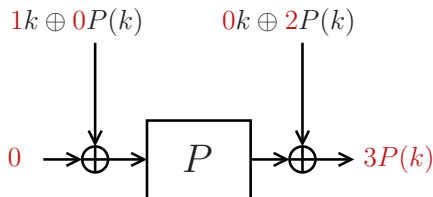
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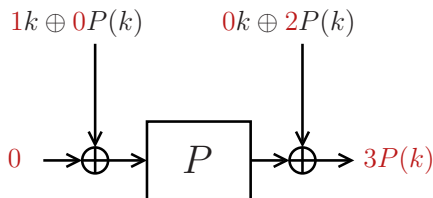
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“Valid” Tweak Sets

- Technical definition to eliminate weak cases

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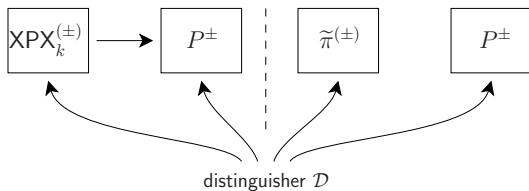
...

“Valid” Tweak Sets

- Technical definition to eliminate weak cases
- Proven to be minimal: \mathcal{T} invalid \Rightarrow XPX insecure

XPX: Single-Key Security

(Strong) Tweakable PRP

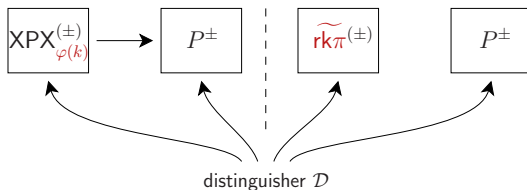


- Information-theoretic indistinguishability
 - $\tilde{\pi}$ ideal tweakable permutation
 - P ideal permutation
 - k secret key

\mathcal{T} is valid \implies XPX is (S)TPRP up to $\mathcal{O}\left(\frac{q^2 + qr}{2^n}\right)$

XPX: Related-Key Security

Related-Key (Strong) Tweakable PRP



- Information-theoretic indistinguishability
 - $\widetilde{\text{rk}\pi}$ ideal tweakable **related-key** permutation
 - P ideal permutation
 - k secret key
- \mathcal{D} restricted to some set of **key-deriving functions** Φ

XPX: Related-Key Security

Key-Deriving Functions

- Φ_{\oplus} : all functions $k \mapsto k \oplus \delta$

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Results

if \mathcal{T} is valid, and for all tweaks:	security	Φ
$t_{12} \neq 0$	TPRP	Φ_{\oplus}
$t_{12}, t_{22} \neq 0$ and $(t_{21}, t_{22}) \neq (0, 1)$	STPRP	Φ_{\oplus}

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$t_{11}, t_{12}, t_{21}, t_{22} \neq 0$	STPRP	$\Phi_{P\oplus}$

XPX Covers Even-Mansour



for $\mathcal{T} = \{(1, 0, 1, 0)\}$

XPX Covers Even-Mansour



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- Single-key STPRP secure (surprise?)

XPX Covers Even-Mansour



for $\mathcal{T} = \{(1, 0, 1, 0)\}$

- Single-key STPRP secure (surprise?)
- Generally, if $|\mathcal{T}| = 1$, XPX is a normal blockcipher

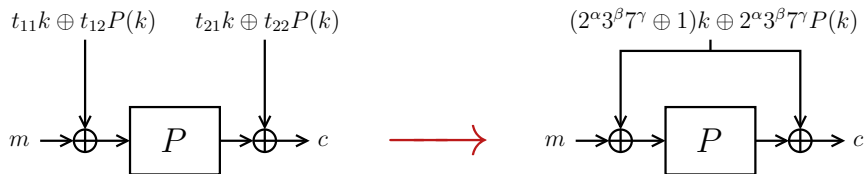
XPX Covers XEX With Even-Mansour



$$\text{for } \mathcal{T} = \left\{ \begin{pmatrix} 2^\alpha 3^\beta 7^\gamma \oplus 1, 2^\alpha 3^\beta 7^\gamma, \\ 2^\alpha 3^\beta 7^\gamma \oplus 1, 2^\alpha 3^\beta 7^\gamma \end{pmatrix} \mid (\alpha, \beta, \gamma) \in \{\text{XEX-tweaks}\} \right\}$$

- (α, β, γ) is in fact the “real” tweak

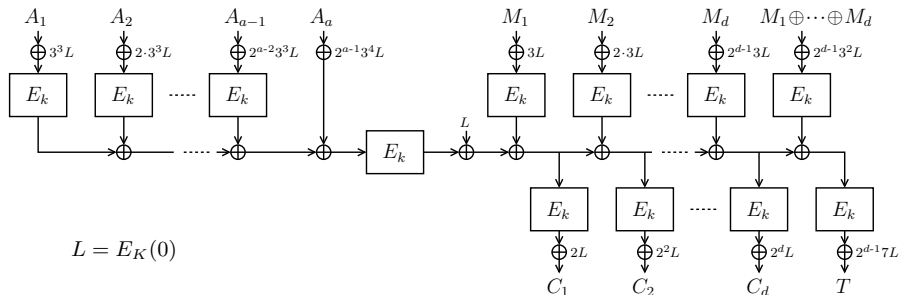
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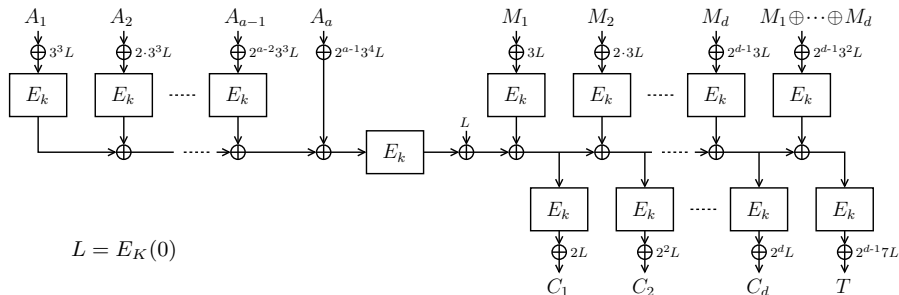
- (α, β, γ) is in fact the “real” tweak
- $\Phi_{P \oplus}$ -related-key STPRP secure (if $2^\alpha 3^\beta 7^\gamma \neq 1$)

Application to AE: COPA and Prøst-COPA



- By Andreeva et al. (2014)
- Implicitly based on XEX based on AES

Application to AE: COPA and Prøst-COPA



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- Prøst-COPA by Kavun et al. (2014):
COPA based on XEX based on Even-Mansour

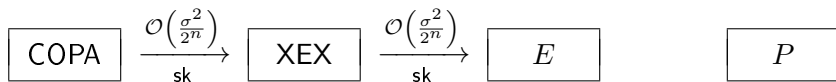
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Single-Key Security of COPA



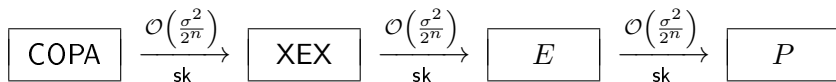
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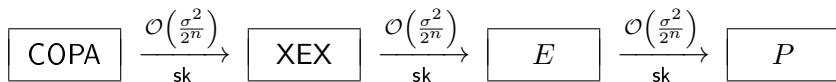
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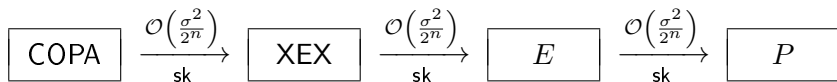
Related-Key Security of COPA

- Existing proof generalizes for any Φ



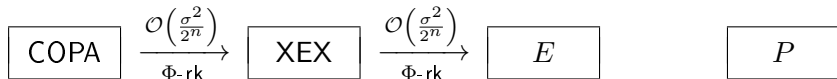
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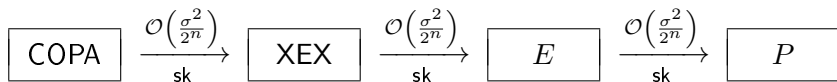
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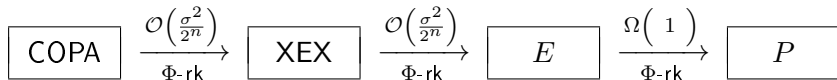
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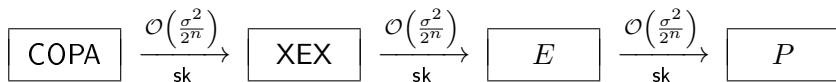
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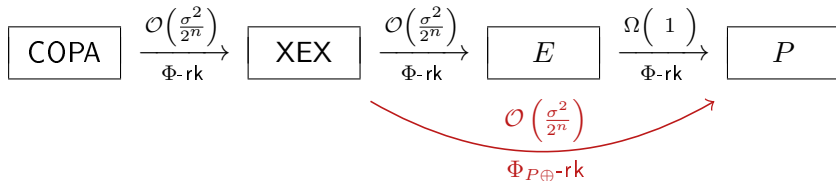
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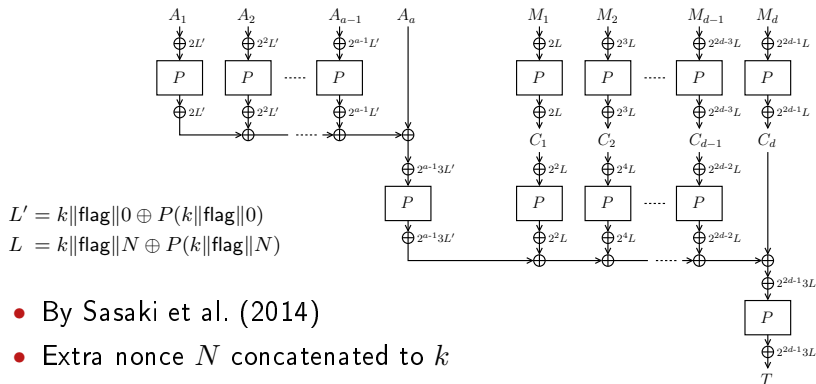


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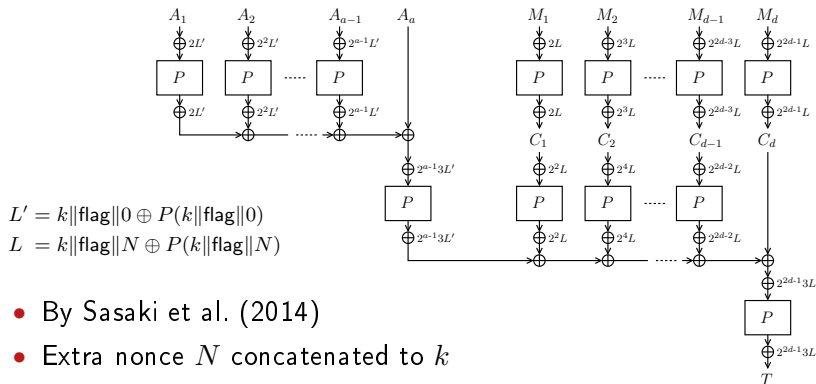


Application to AE: Minalpher



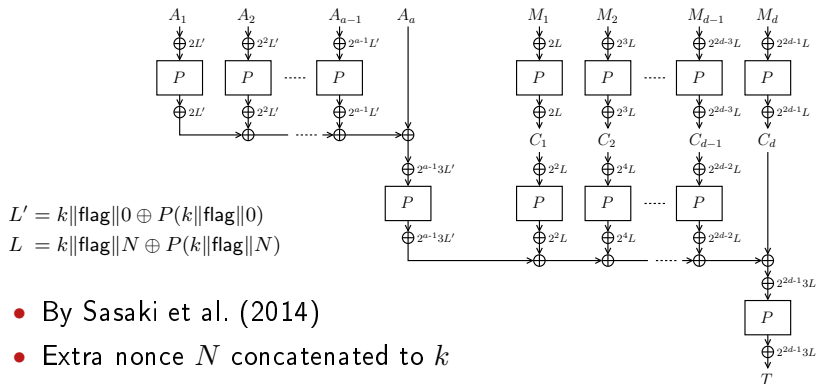
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- Extra nonce N concatenated to k

Application to AE: Minalpher

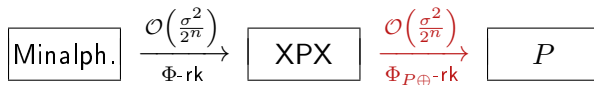


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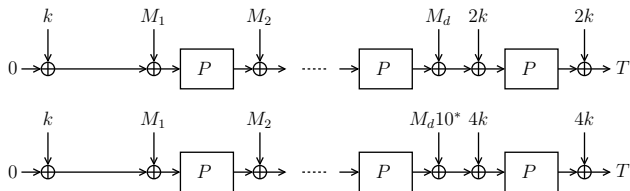
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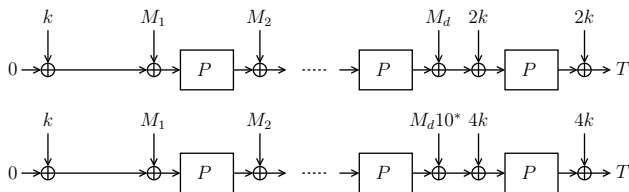


Application to MAC: Chaskey



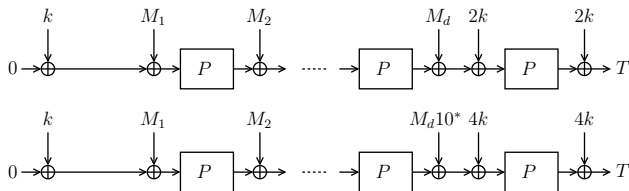
- By Mouha et al. (2014)
- Original proof based on 3 EM's:
$$\begin{cases} E_k(m) = P(m \oplus k) \oplus k \\ E_k(m) = P(m \oplus 3k) \oplus 2k \\ E_k(m) = P(m \oplus 5k) \oplus 4k \end{cases}$$

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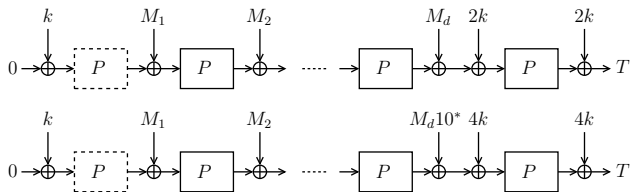
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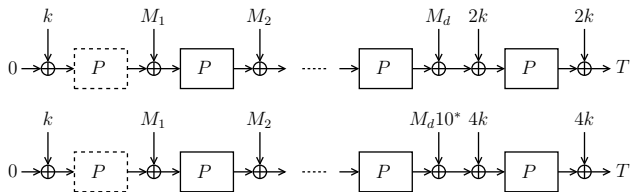


Application to MAC: Adjusted Chaskey



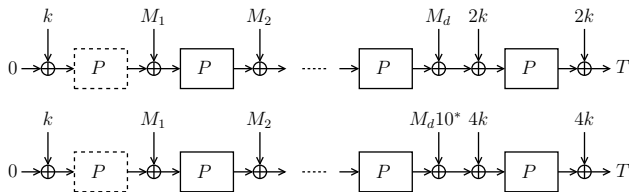
- Extra P -call

Application to MAC: Adjusted Chaskey



- Extra P -call
- Based on XPX with $\mathcal{T}' = \{(0, 1, 0, 1), (2, 1, 2, 0), (4, 1, 4, 0)\}$

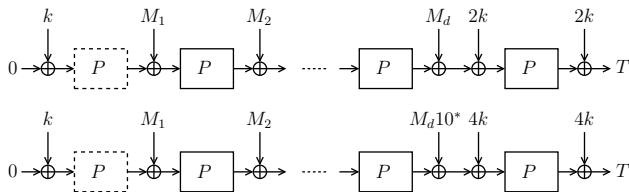
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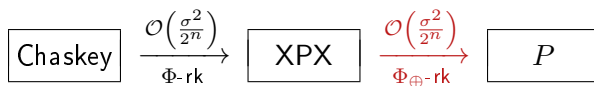
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$$\boxed{\text{Chaskey}} \xrightarrow[\Phi\text{-rk}]{\mathcal{O}\left(\frac{\sigma^2}{2^n}\right)} \boxed{\text{XPX}} \xrightarrow[\Phi_{\oplus}\text{-rk}]{\mathcal{O}\left(\frac{\sigma^2}{2^n}\right)} \boxed{P}$$

Application to MAC: Adjusted Chaskey



- Extra P -call
- Based on XPX with $\mathcal{T}' = \{(0, 1, 0, 1), (2, 1, 2, 0), (4, 1, 4, 0)\}$



- Approach can also be applied to:
 - Keyed Sponge and Duplex
 - 10 Sponge-inspired CAESAR candidates

Conclusions

XPX

- Generalized tweakable Even-Mansour
- Various levels of security
 - Single-key to related-key
- Applications to
 - AE schemes (including 12 CAESAR candidates)
 - MAC functions

Further Research

- Beyond birthday bound?
- Other related-key settings?

Thank you for your attention!

SUPPORTING SLIDES

XPX: Security Proof Techniques

Patarin's H-coefficient Technique

- Each conversation defines a transcript
- Define **good** and **bad** transcripts

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\uparrow prob. ratio for **good** transcripts

- Trade-off: define **bad** transcripts smartly!

XPX: Security Proof Techniques

Before the Interaction

- Reveal “dedicated” oracle queries

After the Interaction

- Reveal key information
 - Single-key: k and $P(k)$
 - Φ_{\oplus} -related-key: k and $P(k \oplus \delta)$
 - $\Phi_{P \oplus}$ -related-key: k and $P(k \oplus \delta)$ and $P^{-1}(P(k) \oplus \varepsilon)$

Bounding the Advantage

- Smart definition of **bad** transcripts