From Mathematics to Generic Programming

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Solution.

When $n = 2$, we see that any element of S_n is a transposition or the identity permutation (do nothing). So commutativity trivially follows. When $n > 2$, we might intuitively suspect that permutations are not commutative because functions are, in general, not commutative: $f(g(x)) \neq g(f(x))$. Intuitively, pure functions depend on their inputs, so swapping functions around in terms of composition seems like it would, in general, not preserve inputs for the respective functions.

Consider s_1, s_2, and s_3 as elements in S_n where $n > 3$. Let α be the permutation which transposes s_1 and s_2 and let $s\beta$ be the permutation which transposes s_2 and s_3.

$$\alpha \circ \beta = (3 \;\; 1 \;\; 2)$$

$$\beta \circ \alpha = (2 \;\; 3 \;\; 1) \neq \alpha \circ \beta$$

We see that permutations which *intersect* will tend to not allow for commutativity, as this counterexample shows. S_n is not abelian for $n > 3$.