Module 10A – Introduction to Circuitry

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Outline

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1. Basic Circuit Elements

2. Kirchoff's Laws

Resistors

Definition

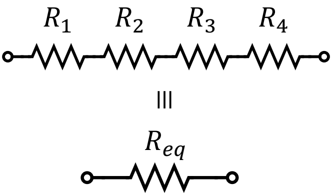
A **resistor** is a linear, two-terminal circuit element whose function is to reduce current flow along a path.

- The symbol for a resistor is as follows:

- The characteristic equation for a resistor is defined by **Ohm's Law**

$$v \equiv iR \tag{1}$$

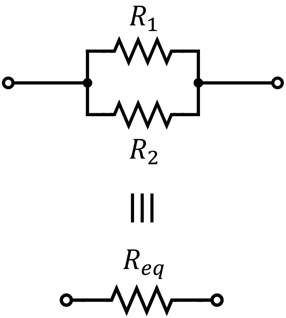
Resistors in Series



- We can define an equivalent resistor R_{eq} for a set of resistors in series by the following relationship:

$$R_{eq} = R_1 + R_2 + ... + R_N = \sum_{i=1}^{N} R_i \tag{2}$$

Resistors in Parallel



- We can define an equivalent resistor R_{eq} for a set of resistors in parallel by the following relationship:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N} = \sum_{i=1}^{N} \frac{1}{R_i} \tag{3}$$

Power Dissipated by a Resistor

- The power dissipated by a resistor is given by:

$$P = iv \tag{4}$$

- Combining equations (1) and (4) gives us two other ways to define power:

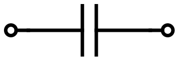
$$P = i^2 R = \frac{v^2}{R} \tag{5}$$

Capacitors

Definition

A **capacitor** is a linear, two-terminal circuit element whose function is to store energy in an *electric* field $\vec{E}$.

- The symbol for a capacitor is as follows:



- The characteristic equation for a capacitor is defined as follows:

$$i \equiv C\frac{dv}{dt} \tag{6}$$

Capacitors in Series and Parallel

- We can define an equivalent capacitor C_{eq} for a set of capacitors in series by the following relationship:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_N} = \sum_{i=1}^{N} \frac{1}{C_i} \tag{7}$$

- We can define an equivalent capacitor C_{eq} for a set of capacitors in parallel by the following relationship:

$$C_{eq} = C_1 + C_2 + ... + C_N = \sum_{i=1}^{N} C_i \tag{8}$$

Inductors

Definition

An **inductor** is a linear, two-terminal circuit element whose function is to store energy in a *magnetic* field $\vec{B}$.

- The symbol for a capacitor is as follows:

- The characteristic equation for an inductor is defined as follows:

$$v \equiv L\frac{di}{dt} \tag{9}$$

Inductors in Series and Parallel

- We can define an equivalent inductor L_{eq} for a set of inductors in series by the following relationship:

$$L_{eq} = L_1 + L_2 + ... + L_N = \sum_{i=1}^{N} L_i \tag{10}$$

- We can define an equivalent inductor L_{eq} for a set of inductors in parallel by the following relationship:

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + ... + \frac{1}{L_N} = \sum_{i=1}^{N} \frac{1}{L_i} \tag{11}$$

Principle of Duality

- In Physics, the *principle of duality* holds for a lot of concepts and quantities
- The constitutive equations between duals are opposites of one another
- For instance:
  - A parallel circuit is the dual of a series circuit
  - An inductor L is the dual of a capacitor C

$$v_L = L\frac{di}{dt} \iff i_C = C\frac{dv_c}{dt}$$

$$L_{eq,series} = \sum_{i=1}^{N} L_i \iff C_{eq,series} = \sum_{i=1}^{N} \frac{1}{C_i}$$

$$L_{eq,parallel} = \sum_{i=1}^{N} \frac{1}{L_i} \iff C_{eq,parallel} = \sum_{i=1}^{N} C_i$$

Steady State Analysis of DC Circuits

- Capacitors act as an open circuit at $t \to \infty$
  - Will later show this in the derivation of v_c in an RC circuit
- Inductors act as a short circuit at $t \to \infty$

Outline

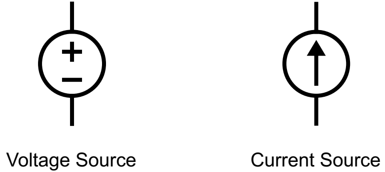
1. Basic Circuit Elements
2. Kirchoff's Laws

Background

- Kirchoff's Laws are used as the starting point for derivations of complex circuits
- After combining Kirchoff's Laws with the characteristic equations described in (1), (6), and (9), we end up with an ordinary differential equation
- This differential equation can be used to solve for voltage and current at certain points in the circuit

Voltage and Current Sources

- Batteries are the source for voltage and current in a circuit
- Typically, only one quantity is given and not the other



- For a voltage source, the current travels through the circuit from the positive end to the negative end
- For a current source, the current travels in the direction of the arrow

Kirchoff's Voltage Law

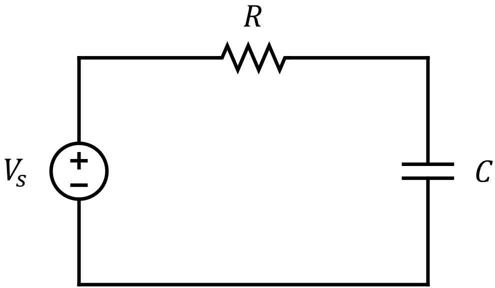
Definition

Kirchoff's Voltage Law (KVL) states that the sum of all voltage drops around a loop is equal to zero.

$$\sum_{\circlearrowright} v_j = 0$$

KVL Example

- For the circuit below, write down the characteristic differential equation using Kirchoff's Voltage Law as a starting point.



Kirchoff's Current Law

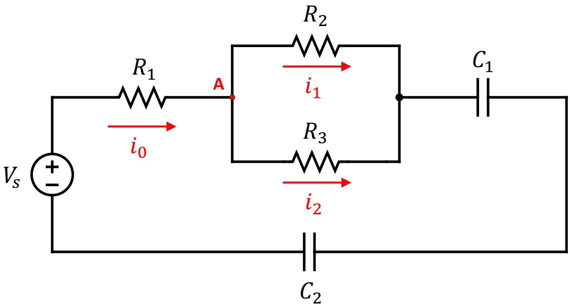
Definition

Kirchoff's Current Law (KCL) states that the sum of all currents entering or leaving a node is equal to zero.

$$\sum_{\perp} i_j = 0$$

KCL Example

- For the circuit below, write down the KCL equation for node A in terms of i_0, i_1, and i_2.



Applications of Kirchoff's Laws

We will derive the characteristic equations for the following circuits in the slides to come.

- 1st Order Circuits
  - RC Circuits
  - RL Circuits
- 2nd Order Circuits
  - RLC Circuits