Brady Metherall

**Theorem 1** (Eckart–Young–Mirsky)**.** *The $r$-truncated SVD of a matrix, $A$ is the best rank $r$ approximation in the spectral norm.*

*Proof.* this is the proof

$$\begin{aligned}
\|A - B\|_2^2 &\geq \|(A - B)x\|_2^2 \\
&= \|Ax\|_2^2 \\
&= \|U\Sigma V^T x\|_2^2 \\
&= \|\Sigma V^T x\|_2^2 \\
&= \text{expand} \\
&\geq \sigma_{r+1}^2 \\
&= \|A - A_r\|_2^2
\end{aligned}$$

□