Continuous Optimization
InFoMM Assignment

Brady Metherall

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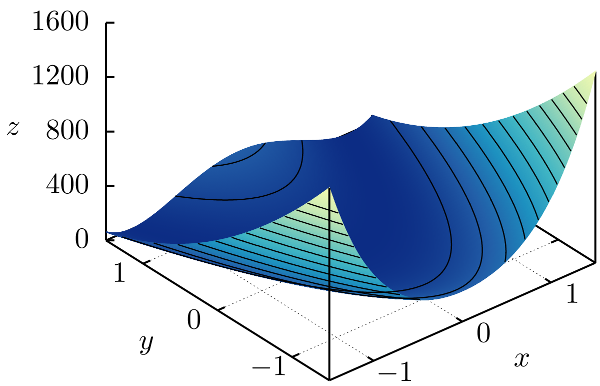


Figure 1

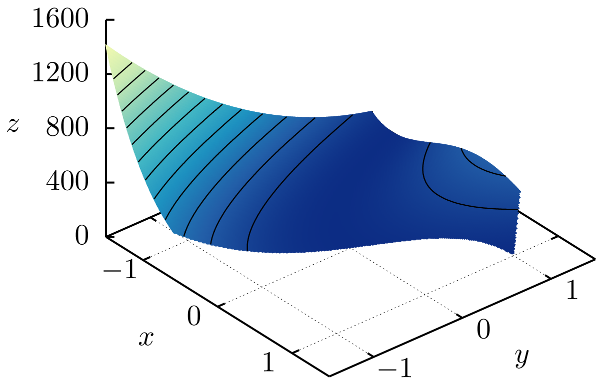


Figure 2

1 Rosenbrock Function

unconstrained rosenbrock

quadratic constrained rosenbrock wikipedia

$$\min_{x,y\in\mathbb{R}} 100(y-x^2)^2 + (1-x)^2 \tag{1}$$

$$(x-1)^2 + y \geq 1 \tag{2}$$
$$x + y \leq 2 \tag{3}$$

2 Catenary Problem

We now turn our attention to the catenary problem. The catenary problem is concerned with the shape of a freely hanging rope with fixed endpoints neglecting bending stiffness. The solution is the shape that minimizes the gravitational potential energy, and thus is a staple in calculus of variations and mechanics courses. In this section we focus our attention on the discrete analogue of this problem. We instead consider a series of n rigid beams attached together. By assuming the gravitational potential energy is focused entirely at the centre of each beam, we obtain the expression

$$U = mg\left(\frac{1}{2}y_0 + \frac{1}{2}y_n + \sum_{i=1}^{n-1} y_i\right) \tag{4}$$

for the total potential energy, as the midpoint is the average of the endpoints of each beam. Additionally, there are multiple constraints of the system. We assume both endpoints are fixed at the same height, and that each beam is the same length, L. Combining these constraints with the expression for the potential energy yields the optimization problem

$$\min_{\mathbf{x},\mathbf{y}\in\mathbb{R}^{n+1}} mg\left(\frac{1}{2}y_0 + \frac{1}{2}y_n + \sum_{i=1}^{n-1} y_i\right) \tag{5}$$

$$\begin{aligned} x_0 &= 0, & x_n &= \gamma n L, \\ y_0 &= 0, & y_n &= 0, \\ (x_i - x_{i+1})^2 &+ (y_i - y_{i+1})^2 = L^2, \end{aligned} \tag{6}$$

where γ is the ratio of the width of the span to the total length of the beams. While solving the problem we use the the following numerical values:

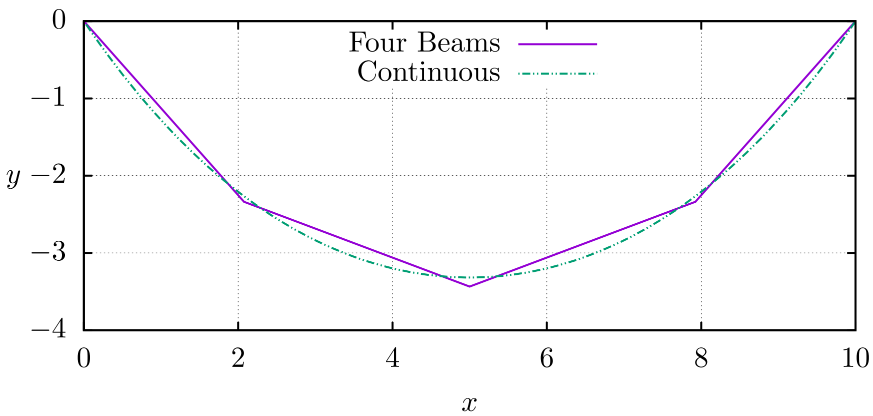
$$x_n = 10, \qquad g = 1, \qquad m = 1, \tag{7}$$
$$\gamma = \frac{4}{5}, \qquad L = \frac{x_n}{\gamma n}. \tag{8}$$

The solutions from KNITRO for two values of n can be found in Figure 3. We shall compare

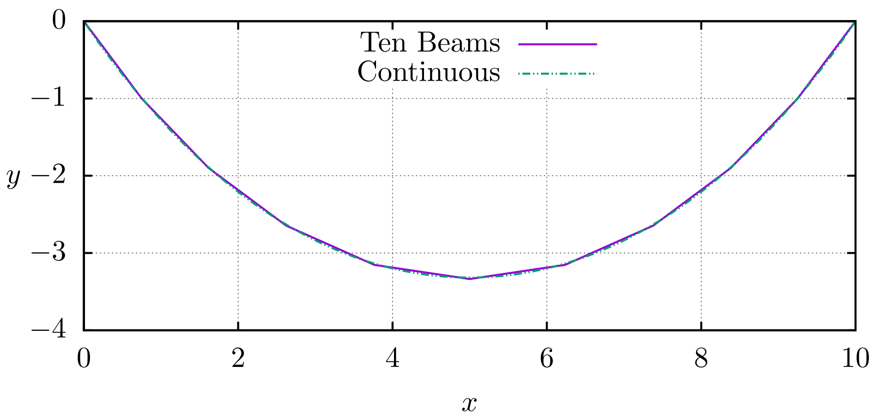
Starting value	$(-2,-3)$	$(2,1)$	$(-3,4)$	$(\pi,-e)$	$(0,0)$	$(2,4)$
Unconstrained						
No derivatives	15	43	28	35	22	35
Gradient	15	41	28	35	DNC[1]	DNC
Hessian	25	13	28	20	DNC	DNC
Constrained						
No derivatives	36	16	40	22	17	18
Gradient	36	16	41	22	DNC	DNC
Hessian	20	12	23	12	DNC	DNC

[1]Did not converge.

Table 1: Iterations needed to converge to the minimum of the constrained and unconstrained Rosenbrock function.



(a) $n = 4$.



(b) $n = 10$.

Figure 3: Solution of the discrete catenary problem.

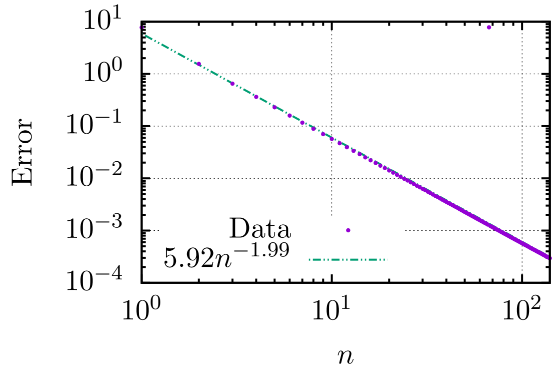


Figure 4: The L^2 norm of the difference between the discrete solution and the continuous solution.

our results to the solution of the continuous case, given by

$$f(x) = a\left(\cosh\left(\frac{2x - x_n}{2a}\right) - \cosh\left(\frac{x_n}{2a}\right)\right), \quad (9)$$

where a is the positive solution to

$$\frac{x_n}{2a} = \gamma \sinh\left(\frac{x_n}{2a}\right). \quad (10)$$

We see that for $n = 4$ (Figure 3a) the discrete solution closely approximates the continuous solution. Moreover, it is evident from $n = 10$ (Figure 3b) that as $n \to \infty$ the continuous solution will be obtained. This observation is justified by examining the L^2 norm of the difference of the two solutions for a range of number of beams—the result of this can be seen in Figure 4. We find that the discrete solution converges to the continuous solution approximately quadratically. However, when $n \gtrsim 140$ KNITRO is no longer able to solve the optimization problem since the number of variables is approaching 300.