Optimization Assignment

Brady Metherall

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Problem 1. The new system consist of

$$\sum_{j}^{n} a_{ij}x_j \leq b_i, \qquad i \in M_0, \tag{1}$$

$$\sum_{j}^{n} (a_{ik}a_{lj} - a_{lk}a_{ij})x_j, \leq a_{ik}b_l - a_{lk}b_i \qquad (i,l) \in M_+ \times M_-. \tag{2}$$

The coefficient of x_k in (1) and (2) is 0 since $i \in M_0$ and $a_{ik}a_{lk} - a_{lk}a_{ik} = 0$, respectively. Therefore, x_k is not in this new system.

Problem 2.

i) If both systems had a solution, that would imply

$$\begin{aligned} \mathbf{0}^T &= yA, \\ 0 &= (yA)x, \\ &= y(Ax), \\ &\leq yb, \\ &< 0, \end{aligned}$$

which is a contradiction. Thus, both systems cannot have solutions.

ii) There must exist a k_* such that $d_{k_*} < 0$, otherwise, our new system is in fact consistent.

Problem 3.

i) Both y_t and z_t are binary variables. If production occurs within period t, then $y_t = 1$. Furthermore, if production is switched on within period t, then $z_t = 1$.

ii)

$$\begin{pmatrix} \mathbf{0}^T & \mathbf{1}^T \\ I - L & -I \\ -I & I \\ I & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix} \leq \begin{pmatrix} k \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{1} \\ \mathbf{1} \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} \mathbf{0} & I - L^T & -I \\ \mathbf{1} & -I & I \end{pmatrix} \tag{4}$$

Problem 4.

i) The constraint matrix is the vertex-edge incidence matrix, A. Therefore, each column contains exactly two 1s. We can then choose the partitions to be $M_1 = V_1$ and $M_2 = V_2$, and the constraint matrix is totally unimodular.

ii) The relaxation of the LP is

$$\begin{aligned} \max_x \sum_{e\in E} & x_e, \\ \text{s.t.} \quad Ax &\leq 1, \\ x_e &\geq 0. \end{aligned} \tag{5}$$

The dual is then given by

$$\begin{aligned} \min_y \sum_{v\in V} & y_v, \\ \text{s.t} \quad A^T y &\geq 1, \\ y_v &\geq 0. \end{aligned} \tag{6}$$

Since A^T is still totally unimodular this solves the IP problem as well. Additionally, for the minimum cardinality node covering, we do not require y_v to be greater than one, and so $y_v \in \{0,1\}$.

iii) We can find trivial feasible solutions to (5) and (6) is the empty matching and the full graph. Then by the Strong Duality Theorem and since A is totally unimodular, König's Theorem holds.

Problem 5.
Problem 6.
Problem 7.
Problem 8.
Problem 9.