Position Sensing

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1 Introduction

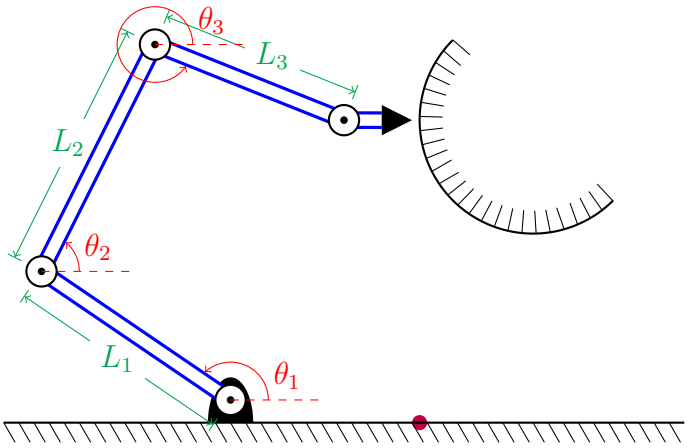


Figure 1

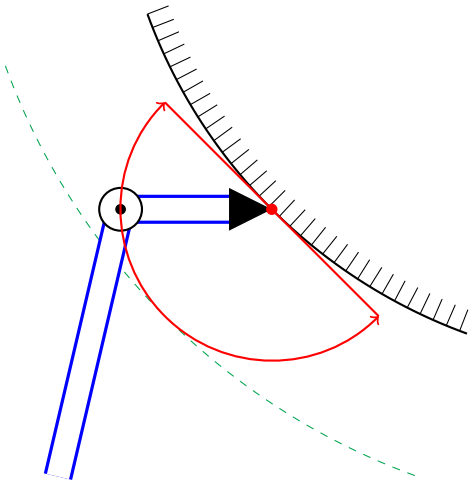


Figure 2

Error 1	Error 2	Error 3	Error 4
-0.07420	0.07420	-0.04497	-0.00179
-0.07420	0.02742	0.02853	-0.07487
-0.02412	-0.16026	0.27843	-0.27794
0.07272	1.20678	4.78410	1.30220
-0.71895	0.71895	-0.52670	-0.15747

Table 1: Sample data.

2 Model

2.1 Assumptions

-rods are rigid and 0 width

-first hinge is on ceiling at 0,0

nd

θ is current position / state

$\varphi = \theta - \delta$ is initial position / state

$$x = \sum_{i=1}^{N} L_i \cos\theta_i; \qquad y = \sum_{i=1}^{N} L_i \sin\theta_i \tag{1}$$

2.2 Calibration

Known position

$$1 = \sum_{i=1}^{N} L_i \cos\varphi_i; \qquad 0 = \sum_{i=1}^{N} L_i \sin\varphi_i \tag{2}$$

second position

$$1 = \sum_{i=1}^{N} L_i \cos(\varphi_i + \delta_i); \qquad 0 = \sum_{i=1}^{N} L_i \sin(\varphi_i + \delta_i) \tag{3}$$

gives system of 4 eqns and 4 unknowns -¿ unique solution (up to modulus)

3 Results

$$x_{\text{err}} = \frac{1}{2}\left|\sum_{i=1}^{N} L_i \left(\cos\left(\theta_i + \sum_{j=1}^{i} \phi_j^{\text{tol}}\right) - \cos\left(\theta_i - \sum_{j=1}^{i} \phi_j^{\text{tol}}\right)\right)\right|$$

trig identities

simplify

note $\phi << 1$

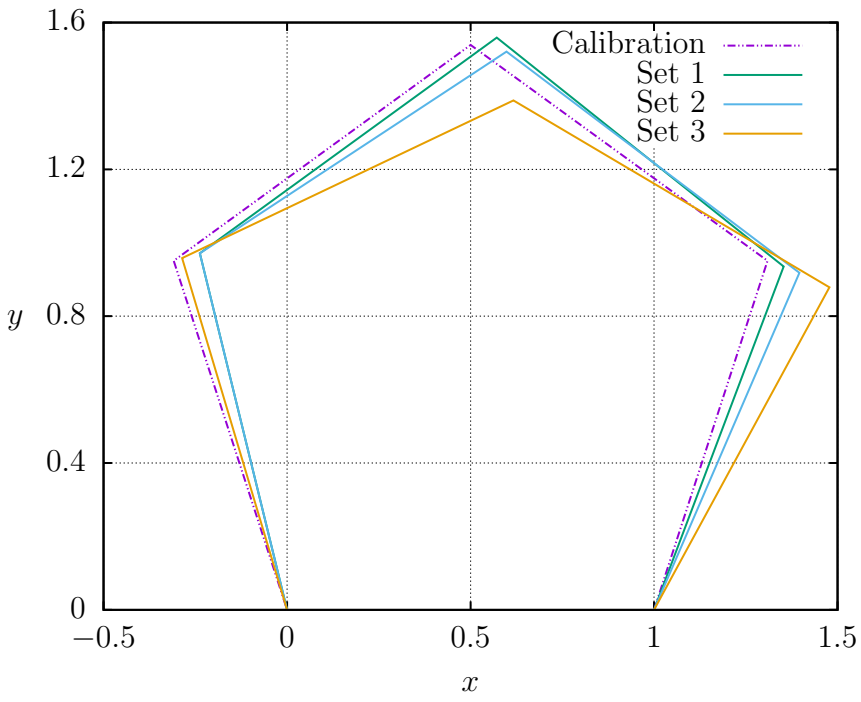


Figure 3

$$x_{\text{err}} = \left| \sum_{i=1}^{N} L_i \sin\theta_i \sum_{j=1}^{i} \phi_j^{\text{tol}} \right|$$

$$y_{\text{err}} = \left| \sum_{i=1}^{N} L_i \cos\theta_i \sum_{j=1}^{i} \phi_j^{\text{tol}} \right|$$

$$\frac{\partial x_{\text{err}}}{\partial L_k} = \left| \sin\theta_k \sum_{j=1}^{k} \phi_j^{\text{tol}} \right|$$

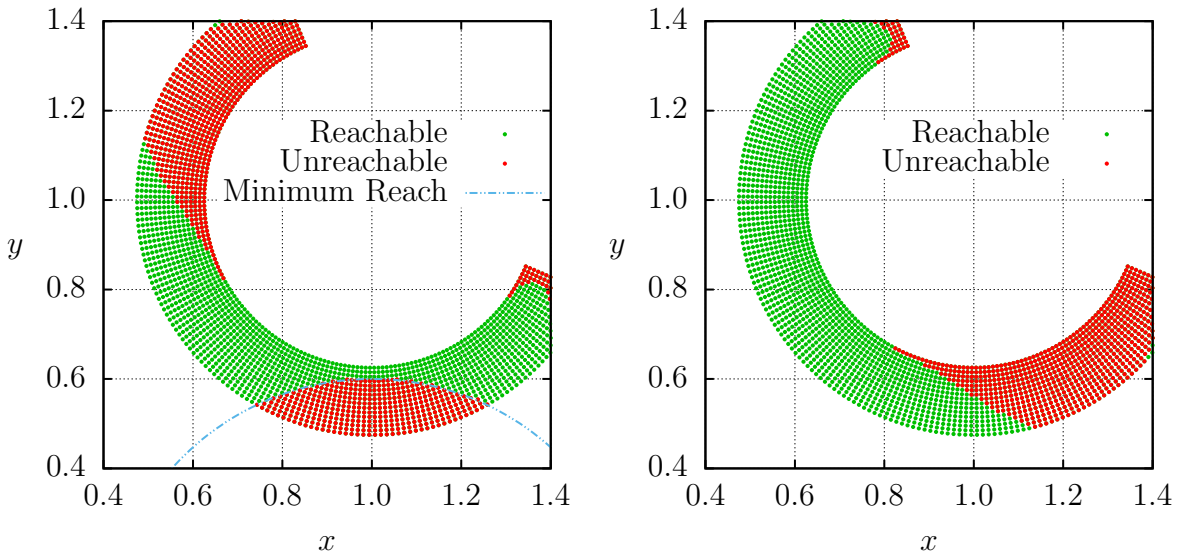
$$\frac{\partial x_{\text{err}}}{\partial \phi_k^{\text{tol}}} = \left| \sum_{i=k}^{N} L_i \sin\theta_i \right|$$

4 Conclusion

A Geometric Constraint

$$t = \frac{\sin(\theta_2) + \cos(\theta_2) - L_1 \cos(\theta_1 - \theta_2) \pm \sqrt{\Delta}}{L_2}$$

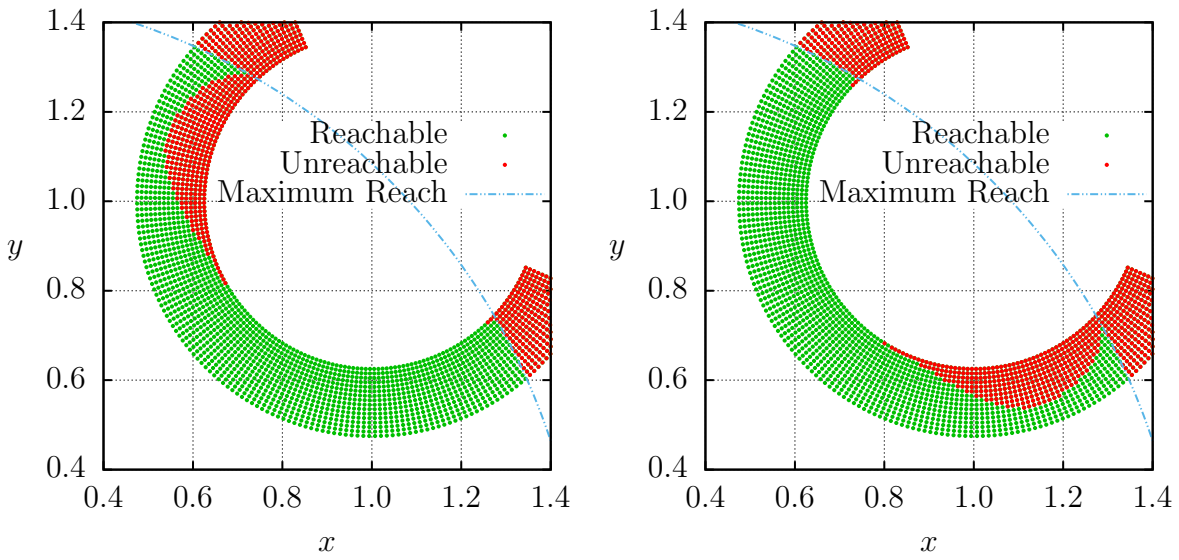
$$\Delta = -L_1^2 \sin^2(\theta_1 - \theta_2) + 2L_1 \sin(\theta_1 - \theta_2)(\cos(\theta_2) - \sin(\theta_2)) + \sin(2\theta_2) + R^2 - 1$$



(a)

(b)

Figure 4: $L_1 = 1$, $L_2 = 0.6$



(a)

(b)

Figure 5: $L_1 = 1$, $L_2 = 0.475$